2025 第44届中国控制会议
The 44th Chinese Control Conference（CCC2025）
Paper ID: 1552
编 号

Direct and Indirect Methods: The Development and Benchmarking of a PMP&SQP Algorithm for Optimal Control

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Abstract

Optimal control problems (OCPs) are crucial in various scientific and engineering domains, necessitating efficient and robust numerical methods for their resolution. This paper introduces a numerical method, denoted as PMP\&SQP, which combines the Pontryagin Minimum Principle (PMP) and Sequential Quadratic Programming (SQP). The method innovatively employs PMP for dimensionality reduction by incorporating covalent states and optimizes their initial values according to the OCP's cost function, rather than directly solving the PMP conditions. This approach takes advantage of PMP's capacity for dimensionality reduction and SQP's optimization strengths, thereby substantially enhancing computational efficiency and reducing sensitivity to initial guess variability. Benchmarking against traditional methods demonstrates the superior performance of PMP\&SQP in solving large-scale OCPs and its robustness across different initial conditions.

Object / Approach	Initial Value	Control Sequence
Solve	PMP&S	PMP&BVP
Optimize	PMP&SQP	NLP
	Indirect	Direct

Fig. 1 Commonly used methods classification.

A PMP&SQP method

Consider an OCP based on a nonlinear system:

$$J=\int_{t_0}^{t_f} L(x,u,t)\mathrm{d}t+\phi(x(t_f)) \qquad \dot{x}=f(x,u,t)$$

The Hamiltonian function and optimal control are:

$$H(x,\lambda,u,t)=L(x,u,t)+\lambda^{\mathrm{T}}f(x,u,t) \quad u^*(t)=h(x,\lambda,t)$$

The optimal covalent states' dynamis and transversality conditions are:

$$\dot{\lambda}=-\frac{\partial H}{\partial x} \qquad \lambda(t_f)=\left.\frac{\partial \phi}{\partial x}\right|_{t_f}+\gamma\left.\left(\frac{\partial \Psi}{\partial x}\right)^{\mathrm{T}}\right|_{t_f}$$

Given the initial covalent states, there is an autonomous system:

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix}=\begin{bmatrix} f(x,h(x,\lambda,t),t) \\ -\frac{\partial H}{\partial x} \end{bmatrix}$$

The original OCP could be reconstructed as a new optimal problem with respect to the initial covalent states :

$$\min_{\lambda_0\in\Lambda} J'=\int_{t_0}^{t_f} L(x,u,t)\mathrm{d}t+\phi(x(t_f))+\boxed{Q(\Psi(x(t_f)))}$$

An additional punishment for not archiving desired terminal constraints

A PMP&SQP method

The method comprises five steps:

1. Apply PMP to the OCP, constructing an autonomous system with state and covalent state dynamics, including transversality conditions.
2. Choose an intuitive initial control trace guess, $u_{g(t)}$, as λ_0 lacks direct physical interpretation.
3. Determine $x_g(t_f)$ from $u_{g(t)}$ through iteration, and solve for $\lambda_g(t_f)$ using transversality conditions.
4. Backward iterate to find $\lambda_g(0)$, leveraging the system's dynamics and $\lambda_g(t_f)$.
5. Formulate the optimization problem as shown and solve it via SQP, using $\lambda_g(0)$ as the initial guess.

Example and Benchmarking

A point starts from the origin and moves toward a terminal circle, aiming to minimize the travel time under constant speed, with the velocity direction as the control input.

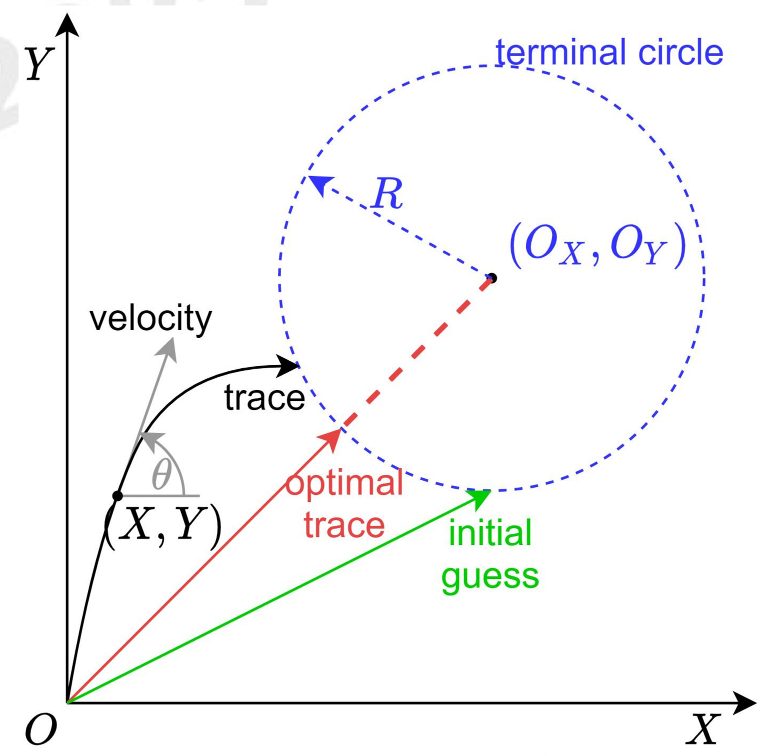

Fig. 2 An example.

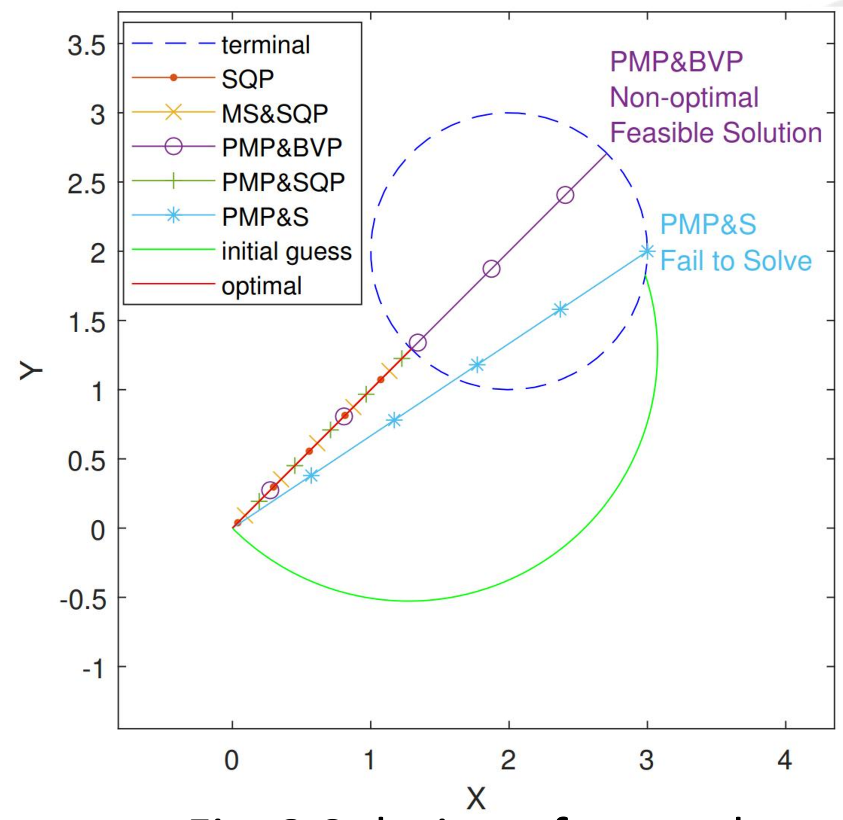

Fig. 3 Solution of example.

Robustness: acquire the optimal solution with bad initial guess.

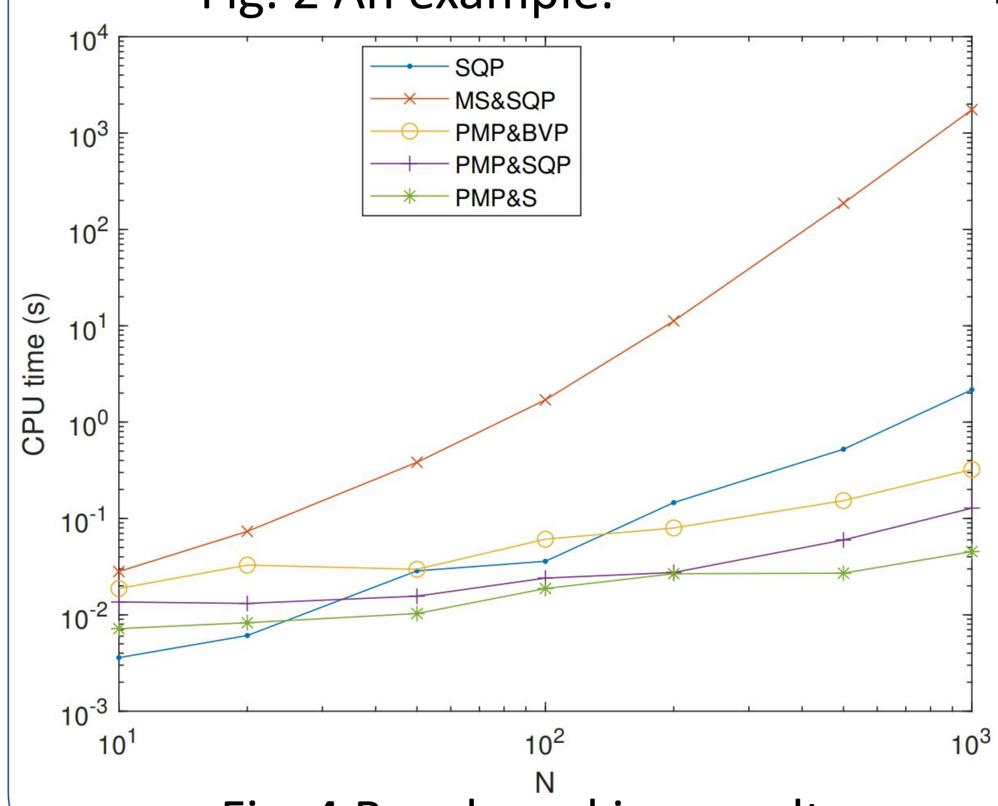

Fig. 4 Benchmarking results.

computational efficiency : lower computational time with larger OCP size.

Conclusion

Through a synthesis of direct and indirect methods, the proposed PMP&SQP approach capitalizes on the dimensionality reduction capabilities of PMP, enhanced computational efficiency, and improved robustness against initial guess variability.