Harnessing Complex Systems for Control:
from design challenges to new opportunities

by

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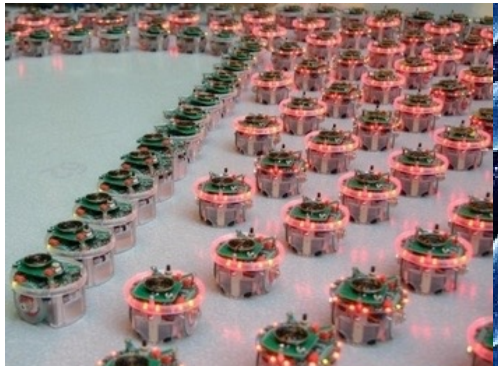
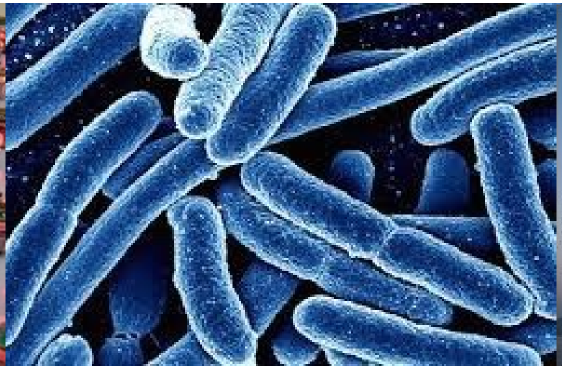
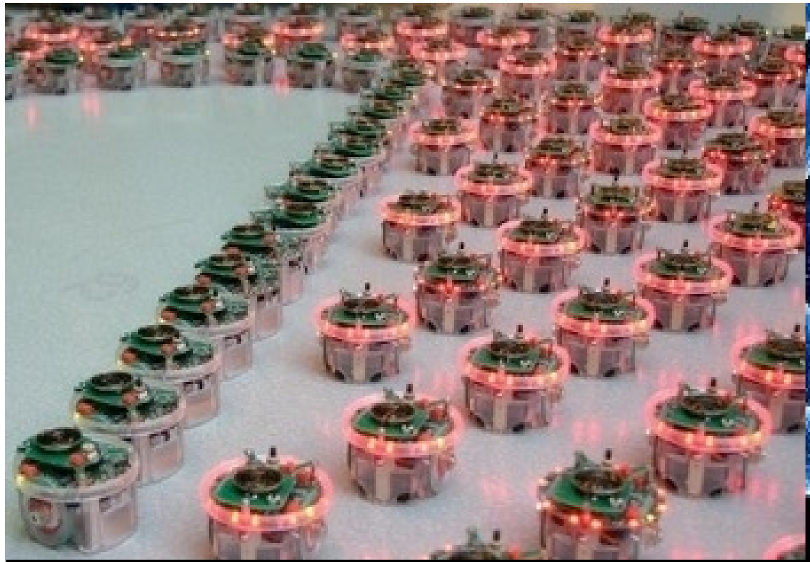
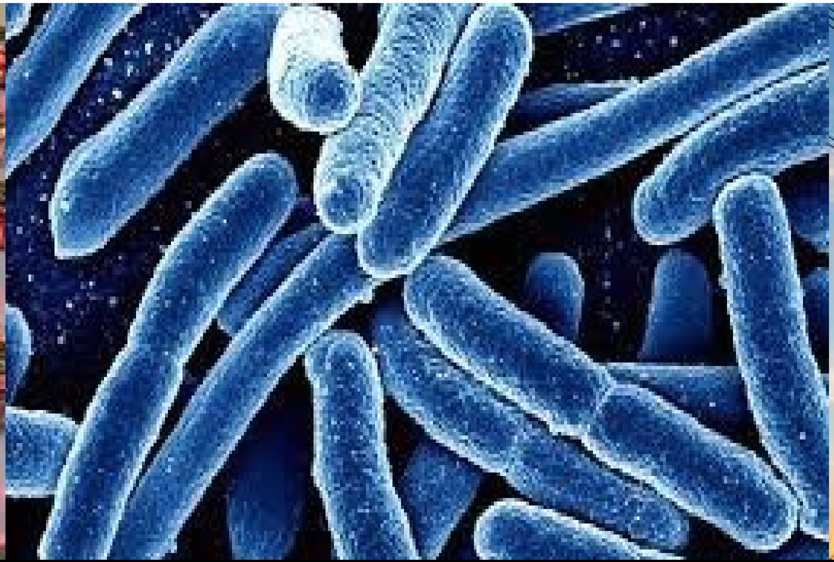
Outline

- Introduction and motivation
- Controlling complex systems
- A multi-scale problem
- A paradigmatic problem: shepherding
- Controlling across scales
- Conclusions, perspectives and open problems

Introduction and Motivation

Controlling complex systems


How to orchestrate in real-time the collective behaviour of a complex system?



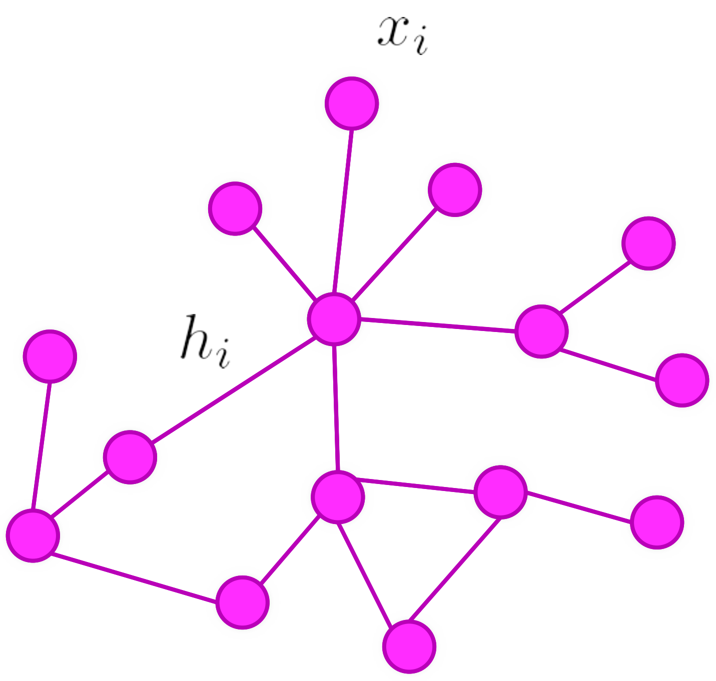
What do these systems have in common?

1. Large collection of **agents**
2. Nontrivial **interactions**
3. Complex network (graph)

$$\dot{x}_i = f_i(t, x_i) + h_i(t, \{x_j\})$$

$$x_i \in \mathbb{R}^n \quad f_i : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$$

$$h_i = \sigma \sum_{j=1}^{N} a_{ij} [g(x_i) - g(x_j)]$$



4. Emergent collective behaviour

L. Torres, A. Blevins, D. Bassett, T. Eliassi-Rad, *The Why, How and When of Representations for Complex Systems*, SIAM Review 2021

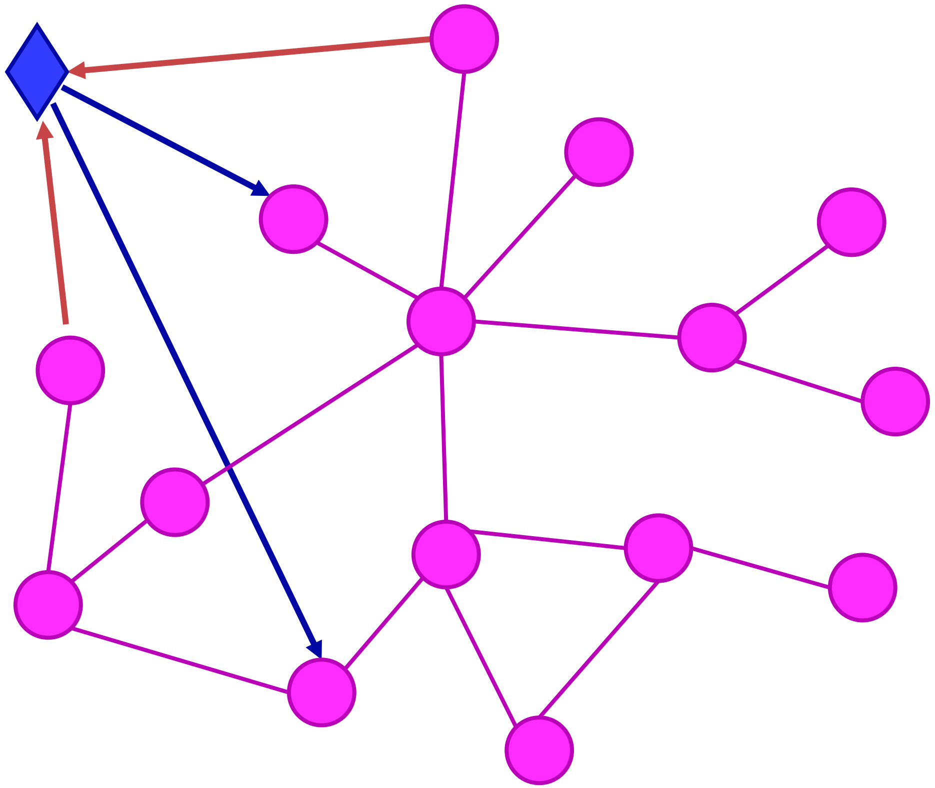
Complex systems vs Classical Control Systems

Feedback Control = Sense + Compute + Actuate

Whom do we sense?
observability

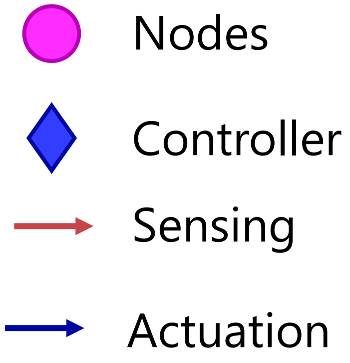
Whom do we control?
controllability

What do we compute?
control design



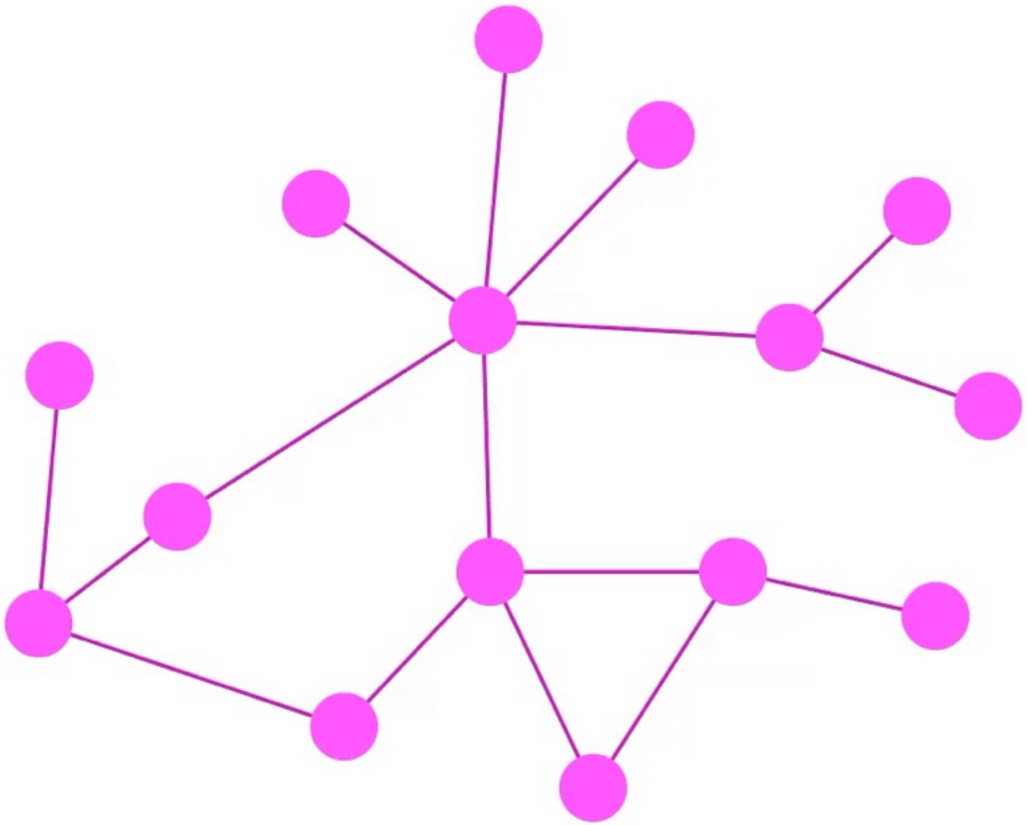
Wish List

- *Distributed*
- *Real-Time*
- *Robust*
- *Stable*

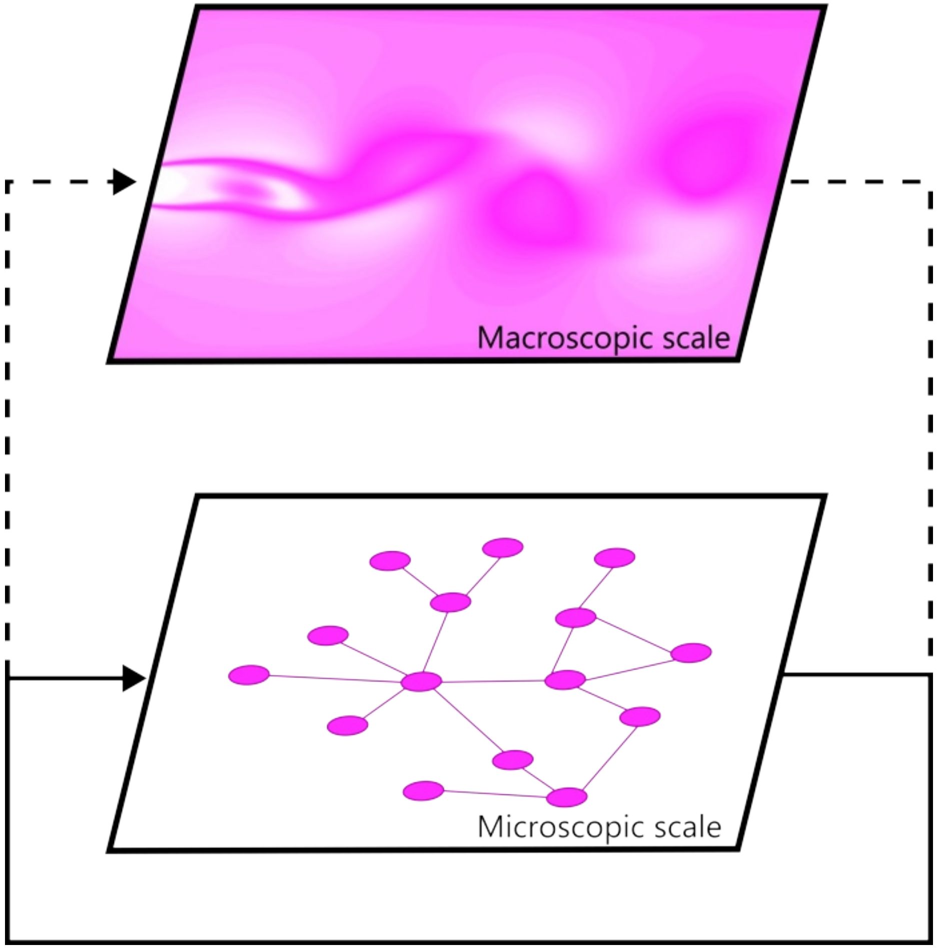


A multi-scale problem

Microscopic description

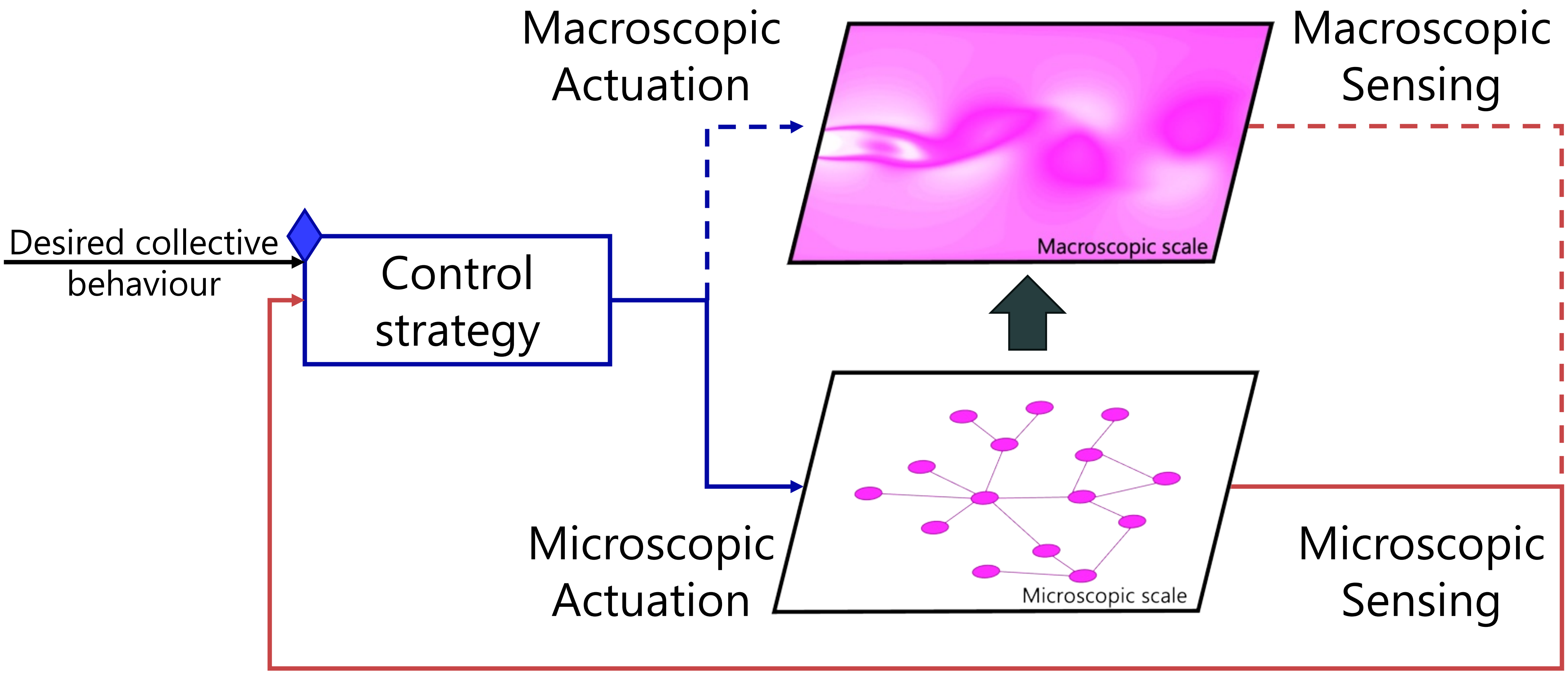


Macroscopic description



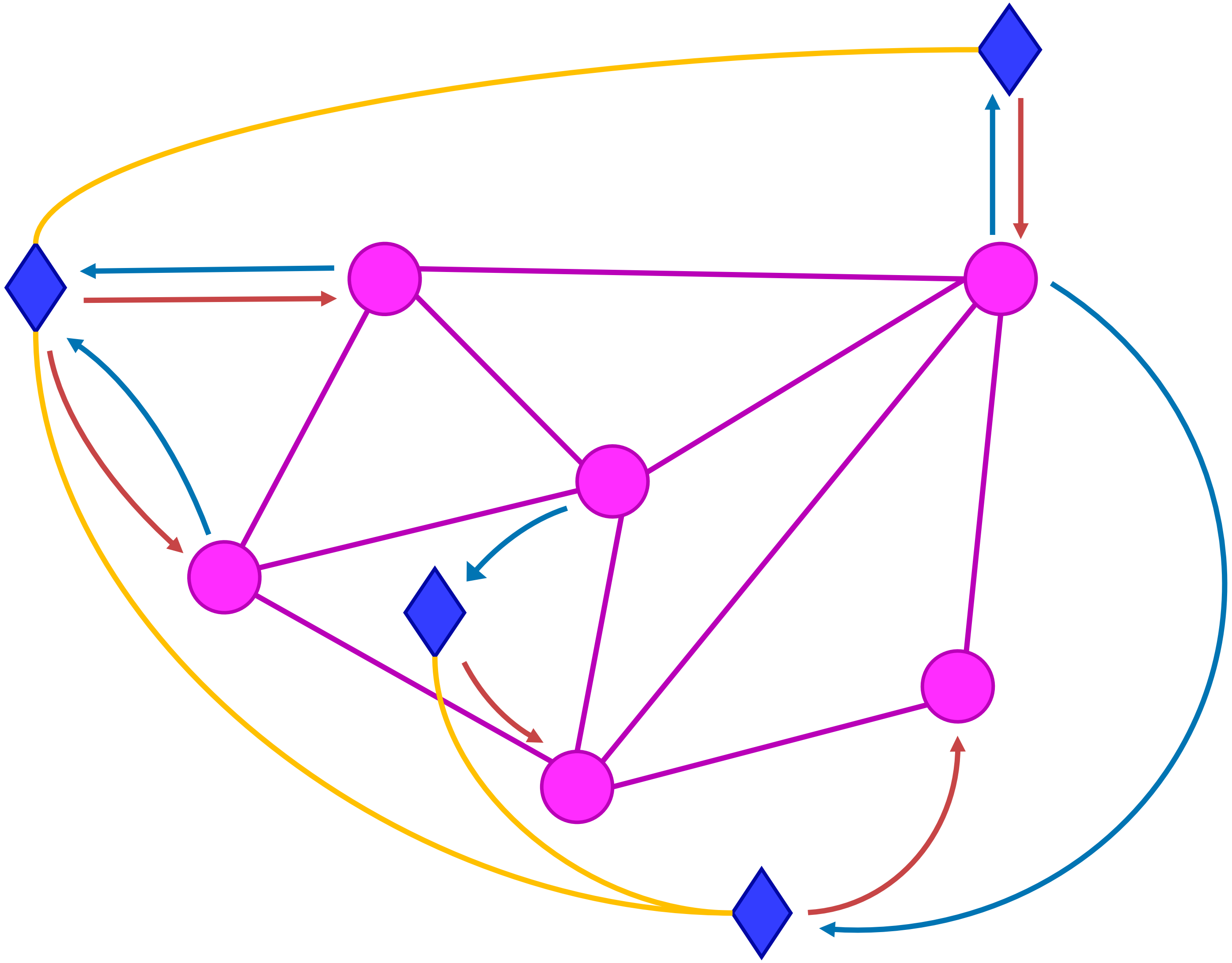
M. di Bernardo, *Controlling collective behaviour in complex systems*, Encyclopedia of Systems and Control, Springer, 2020

A multi-scale control problem



M. di Bernardo, *Controlling collective behaviour in complex systems*, Encyclopedia of Systems and Control, Springer, 2020
R. D'Souza, M. di Bernardo, Y.Y. Liu, *Controlling complex networks with complex nodes*, Nature Reviews Physics, 2023

A distributed control strategy



nature reviews physics https://doi.org/10.1038/s42254-023-00566-3

Perspective Check for updates

Controlling complex networks with complex nodes

Raissa M. D'Souza[1,2,3], Mario di Bernardo[4,5] & Yang-Yu Liu[6,7]

Network

Controller

Sensing

Actuation

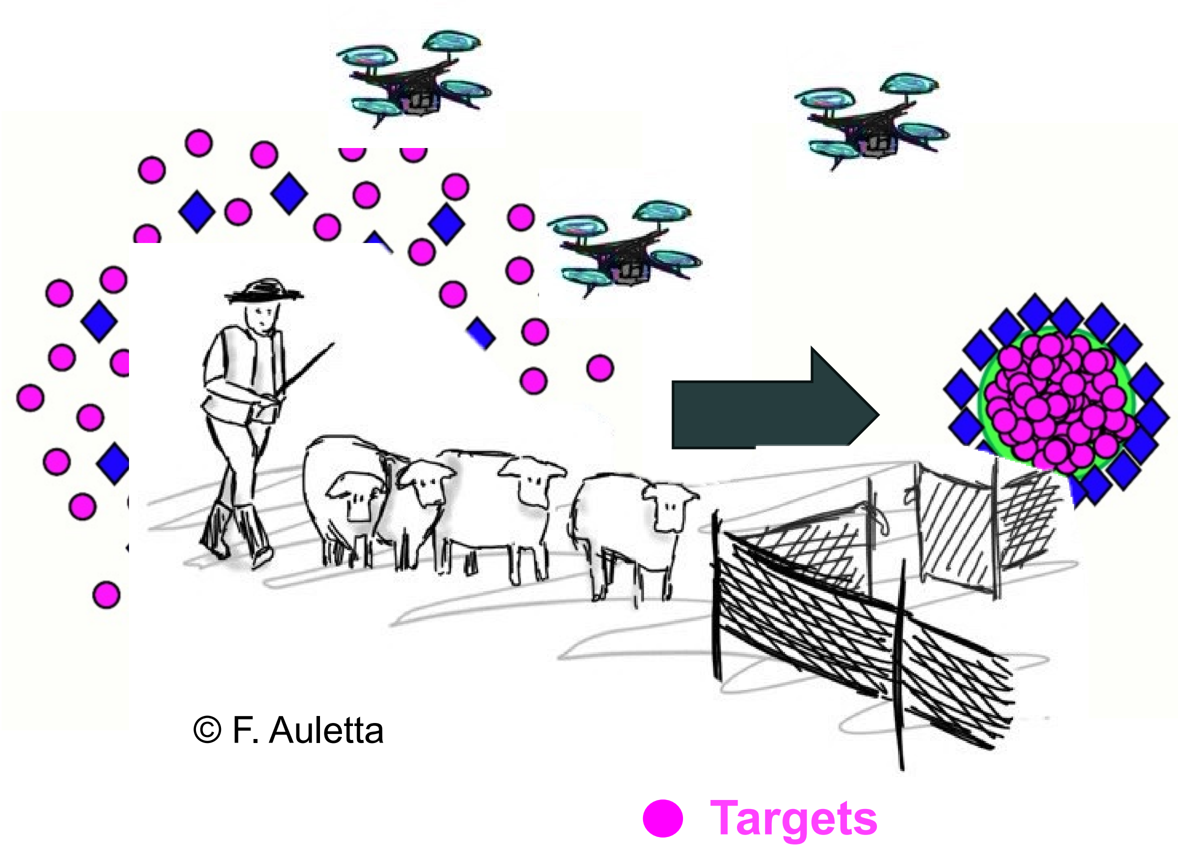
Collaboration

Relevance in applications

A paradigmatic problem

The shepherding control problem

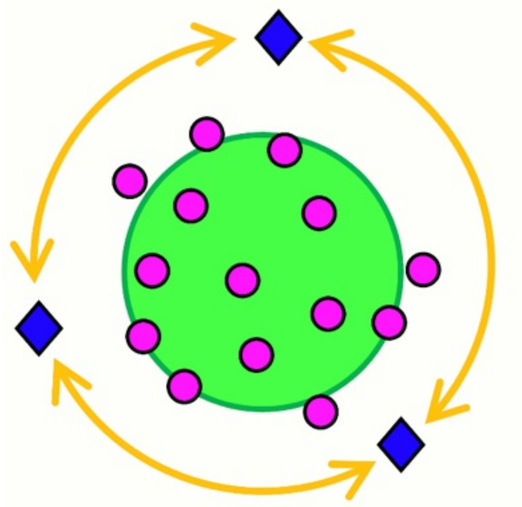
- Here a group of agents, *the* ***herders***, need to steer the collective dynamics of another group of agents, *the* ***targets***, in some desired way

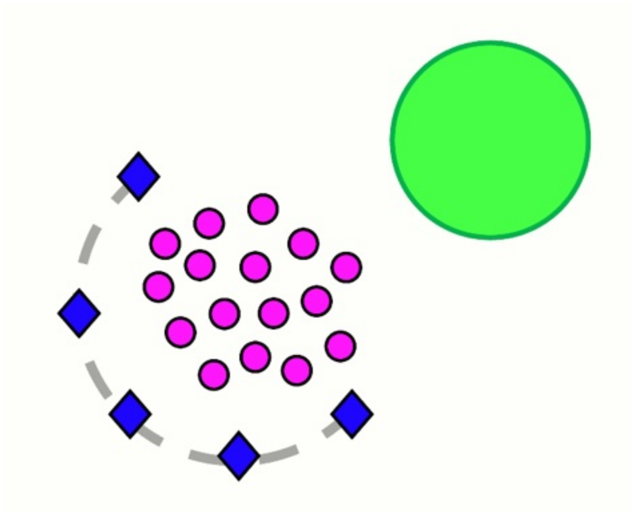


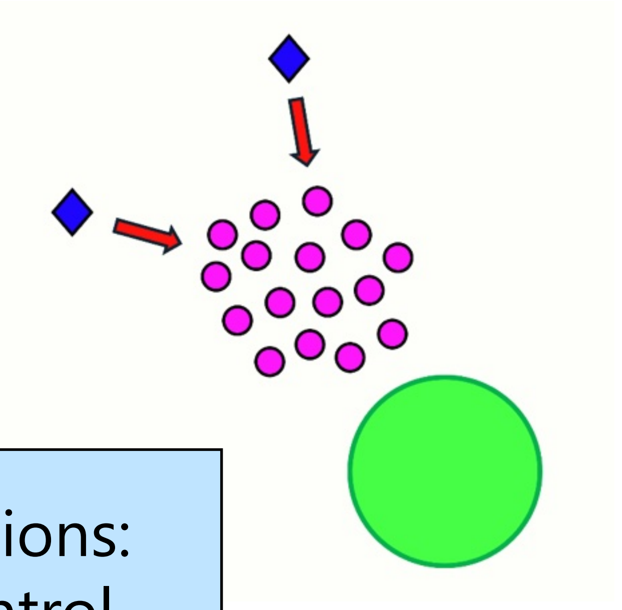
Deciding the herding behaviour

- <u>Crucial problem</u>: design the distributed control strategy driving the herders

Herders must cooperate: collective sensing and decision-making



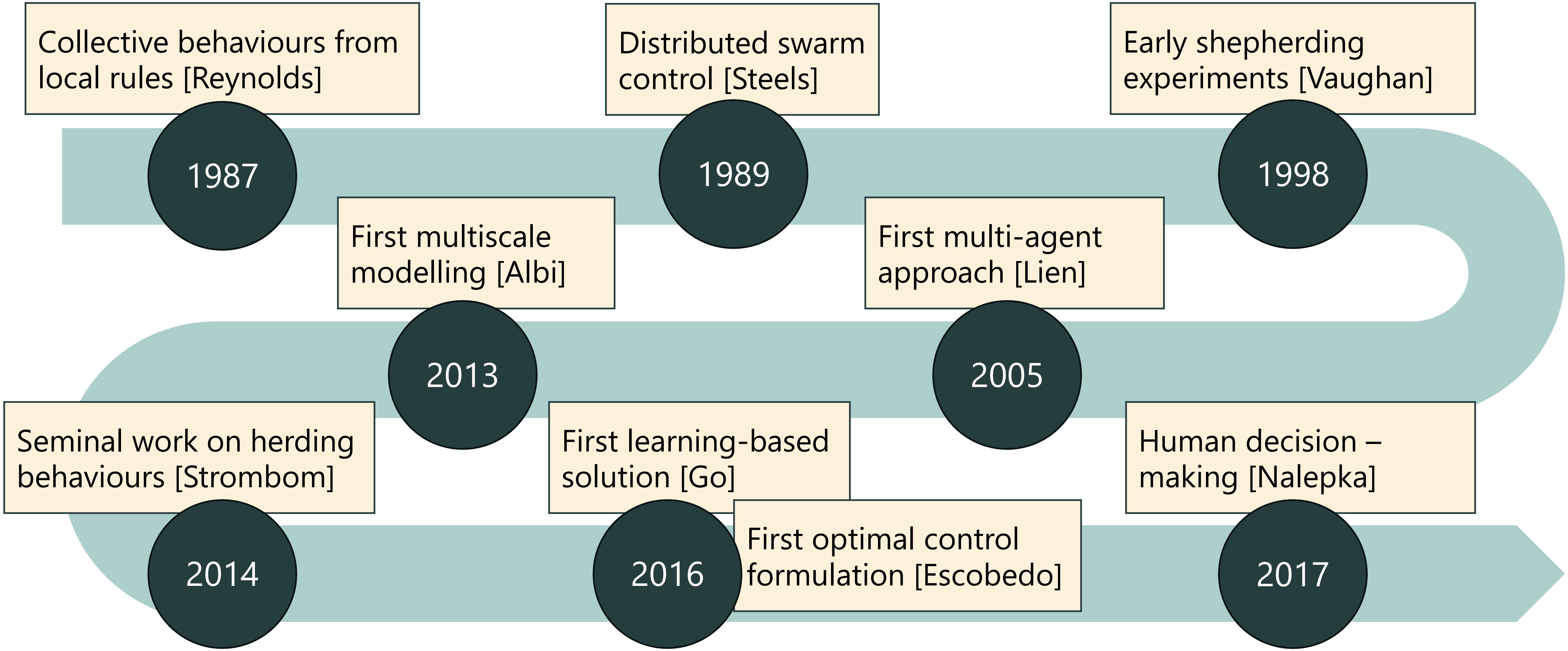




Possible solutions: formation control, virtual force fields

A Pierson, M Schwager, *Controlling noncooperative herds with robotic herders*, IEEE Trans Robotics 2018
R.A. Licitra, Z Bell, W Dixon, *Single-agent indirect herding of mutliple targets with uncertain dynamics*, IEEE Trans Robotics, 2019
D. Ko, E. Zuazua, *Asymptotic behaviour and control of "a guidance by repulsion model*, Math Models Methods Appl Sci, 2020

Timeline


Collective behaviours from local rules [Reynolds]
1987
Distributed swarm control [Steels]
1989
Early shepherding experiments [Vaughan]
1998
First multi-agent approach [Lien]
2005
First multiscale modelling [Albi]
2013
Seminal work on herding behaviours [Strombom]
2014
First learning-based solution [Go]
2016
First optimal control formulation [Escobedo]
Human decision – making [Nalepka]
2017

References (previous slide)

Reynolds, “Flocks, herds and schools: A distributed behavioral model,” SIGGRAPH Computer Graphics, 21(4), 1987

Steels, “Cooperation between distributed agents through self-organization,” Technical Report: AI-Memo 89-05, 1989

Vaughan et al., “Robot Sheepdog Project achieves automatic flock control,” International Conference on Simulation of Adaptive Behaviour, 1998

Lien et al., “Shepherding behaviours with Multiple Shepherds,” IEEE International Conference on Robotics and Automation, 2005

Albi and Pareschi, “Modeling of self-organized systems interacting with a few individuals: From microscopic to macroscopic dynamics.” Applied Mathematics Letters, 26(4), 2013

Strömbom et al., "Solving the shepherding problem: heuristics for herding autonomous, interacting agents." Journal of The Royal Society Interface, 11(100), 2014

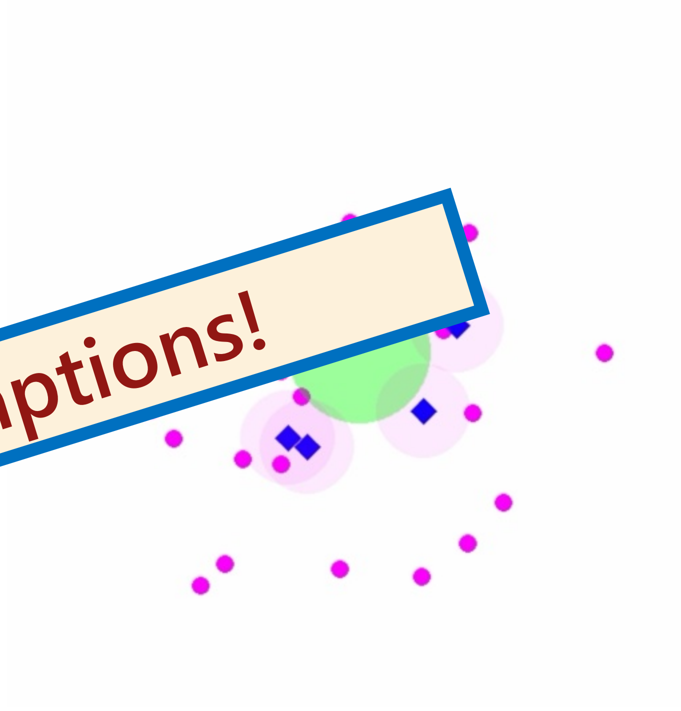
Escobedo et al., “Optimal strategies for driving a mobile agent in a “guidance by repulsion” model,” Communications in Nonlinear Science and Numerical Simulation, 39, pp. 58-72, 2016

Go et al., “A reinforcement learning approach to the shepherding task using SARSA,” International Joint Conference on Neural Networks (IJCNN), pp. 3833-3836, 2016

Nalepka et al., “Herd Those Sheep: Emergent Multiagent Coordination and Behavioral-Mode Switching,” Psychological Science, 28(5), pp. 630-650, 2017

Two key assumptions

Targets’ cohesiveness (e.g. flocking)
[e.g., Pierson et al, 2017]



We relax both of these assumptions!

Herders’ Unlimited sensing
[Auletta et al, Auton, Rob., 2022]

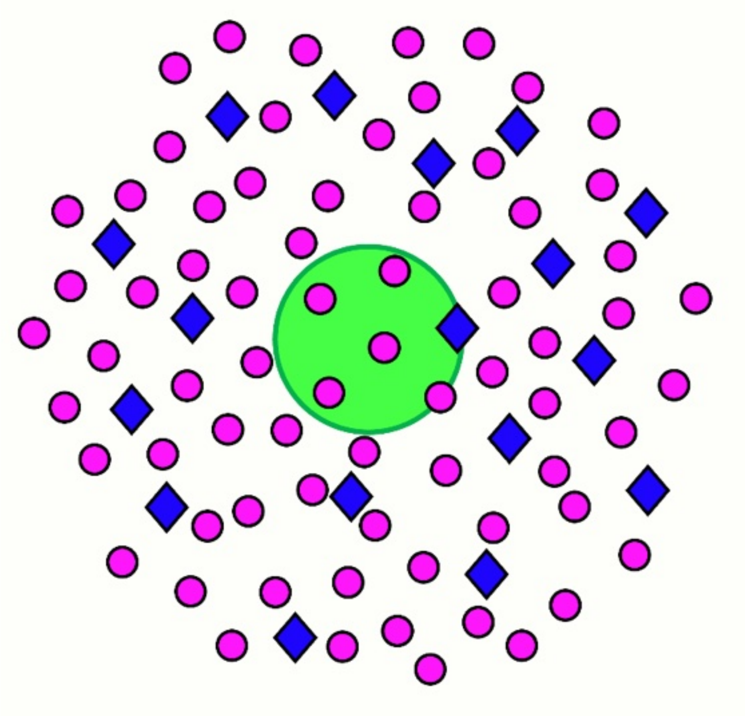
Key research questions

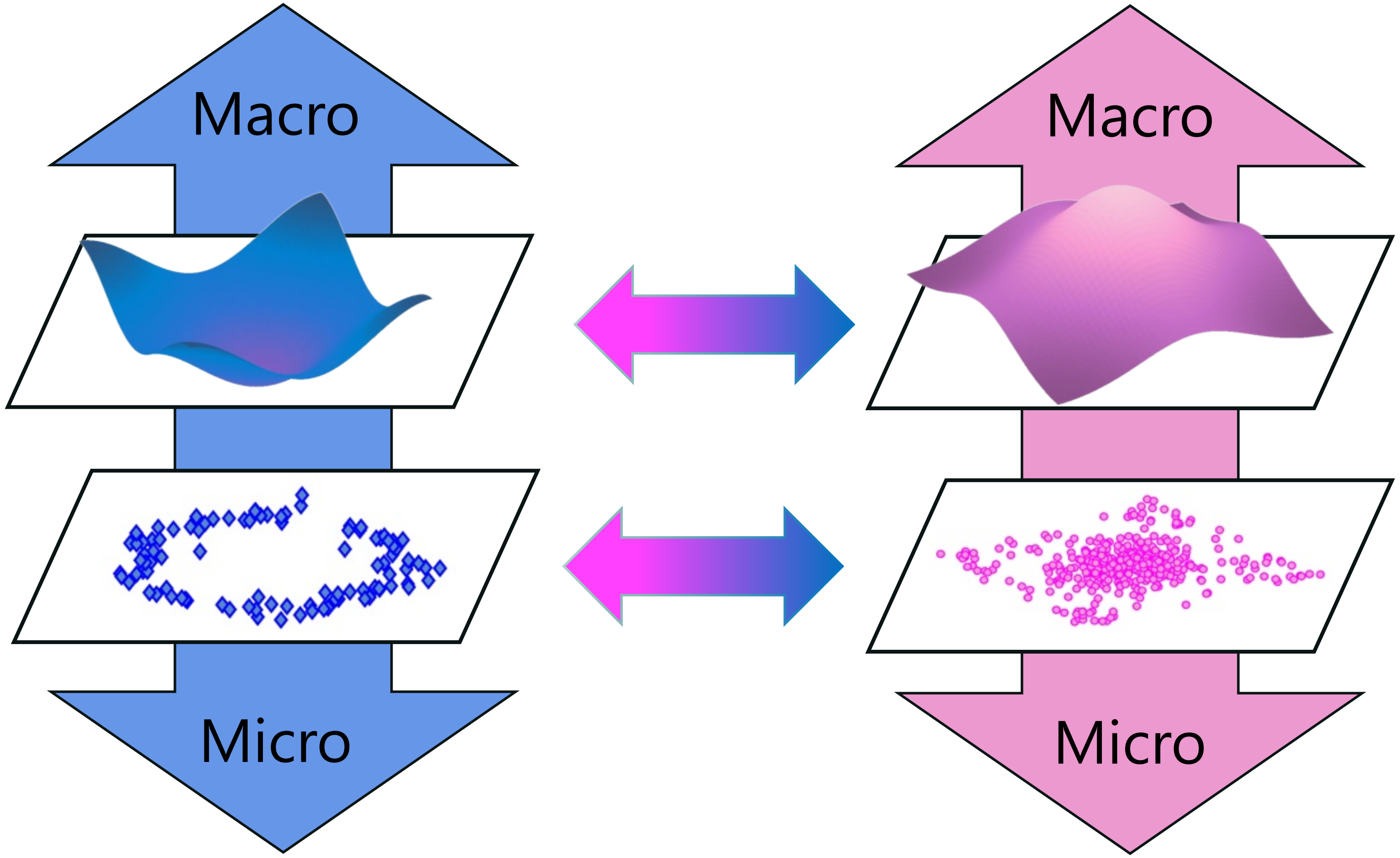
Can local feedback rules solve the global shepherding control problem in the presence of limited sensing and non-cohesive targets?

Under what “herdability” conditions multiple operating herders can effectively shepherd a group of targets towards a desired state?

Controlling across scales

- The shepherding problem can be solved at different description levels





A microscopic approach

Andrea
Lama

Stefano
Covone

Italo
Napolitano

Francesco
De Lellis

A minimal shepherding model

- We studied the problem of removing the assumptions of *targets' cohesiveness* (e.g. flocking targets) and *herders' global sensing*

Cooperation

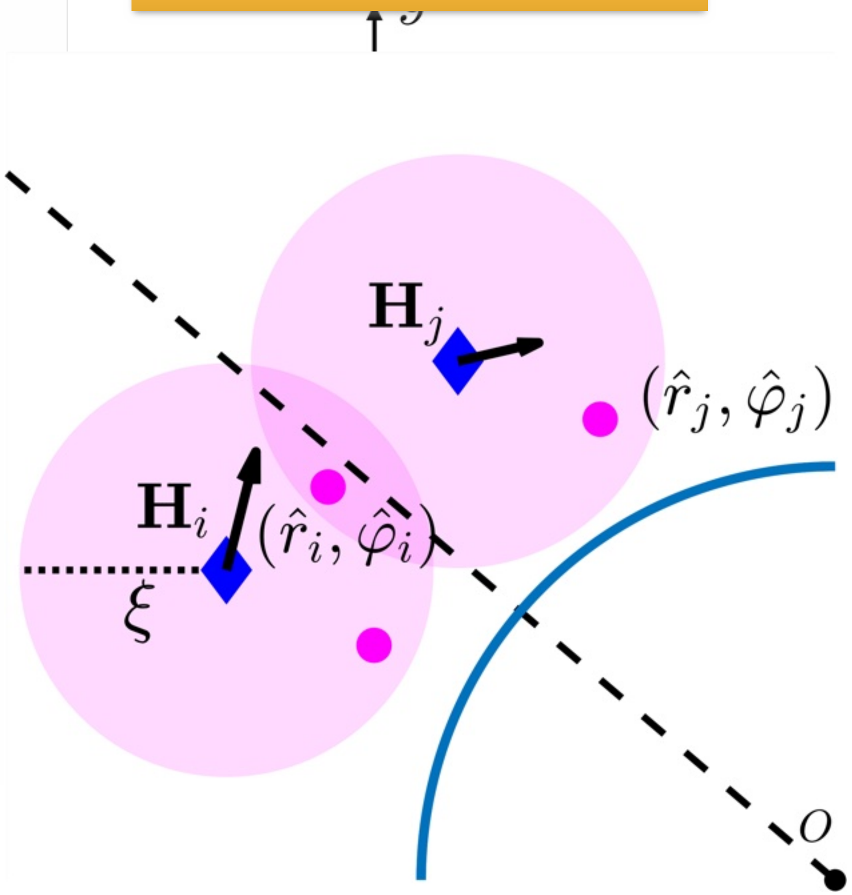
Driving

$$\dot{\mathbf{T}}_a = \sqrt{2D}\mathcal{N}$$

$$\dot{\mathbf{H}}_i = \mathbf{u}_i$$

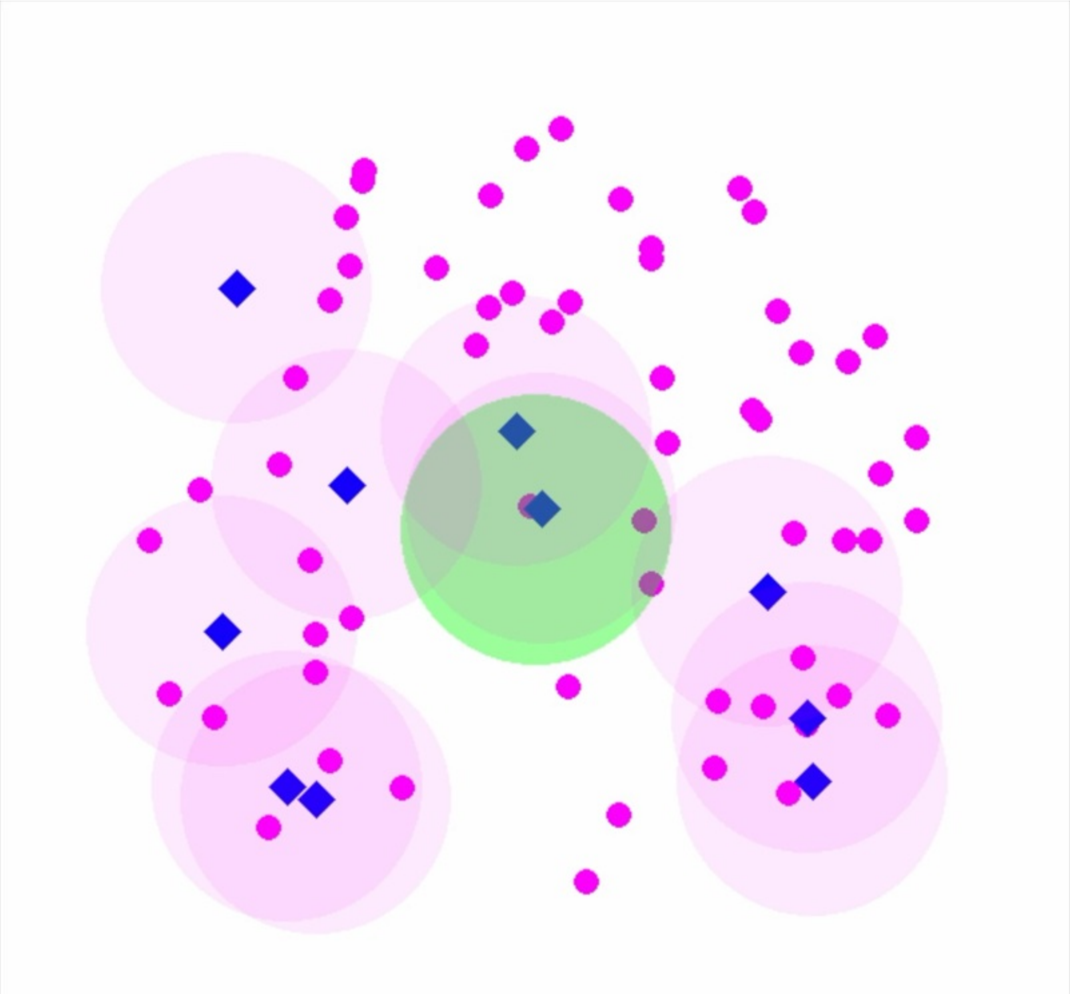
$$\mathbf{u}_i = -K^H \left[\mathbf{H}_i - \left(\mathbf{T}_i^*(\xi) + \delta\hat{\mathbf{T}}_i^*\right)\right]$$

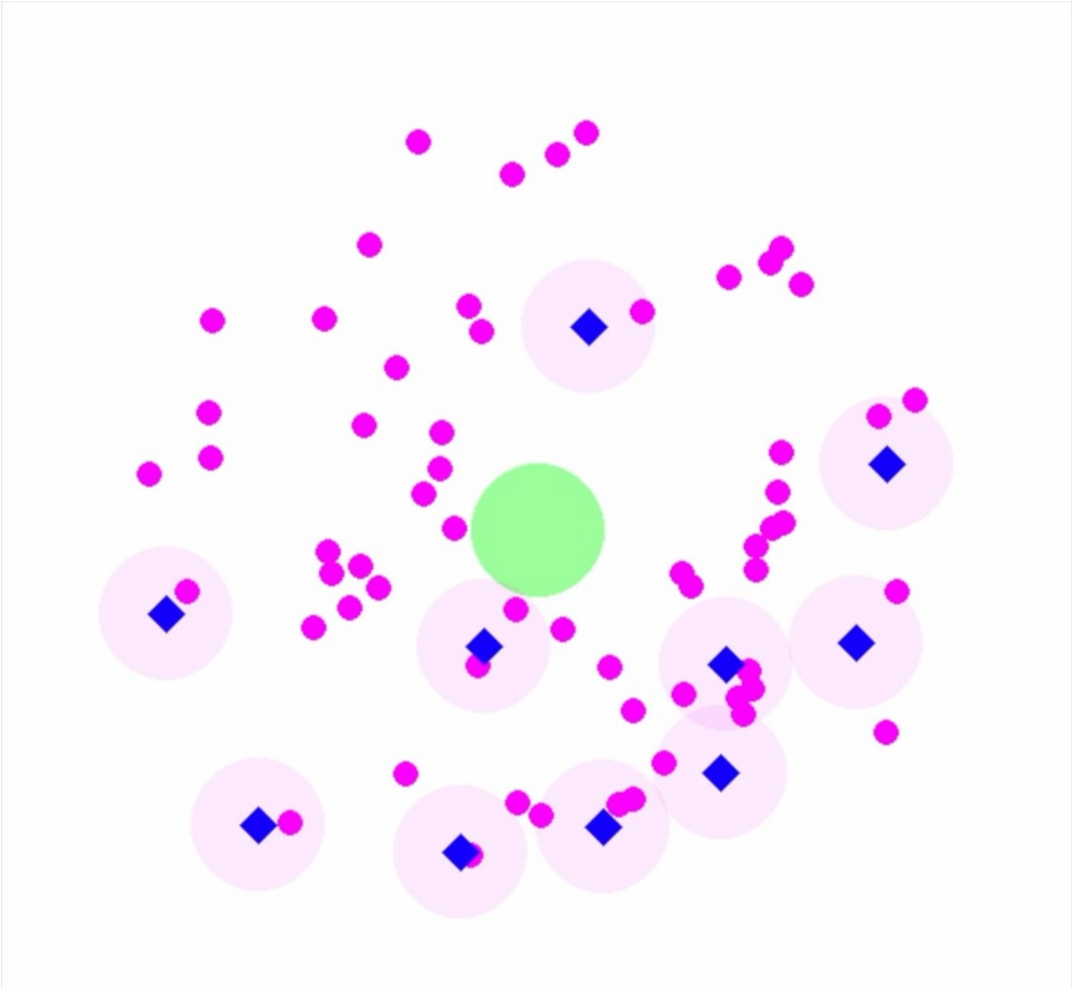
Target Selection



A. Lama, MdB, *Shepherding control and Herdability in Complex Multiagent Systems*, Physical Review Research, 2024

The herdability problem



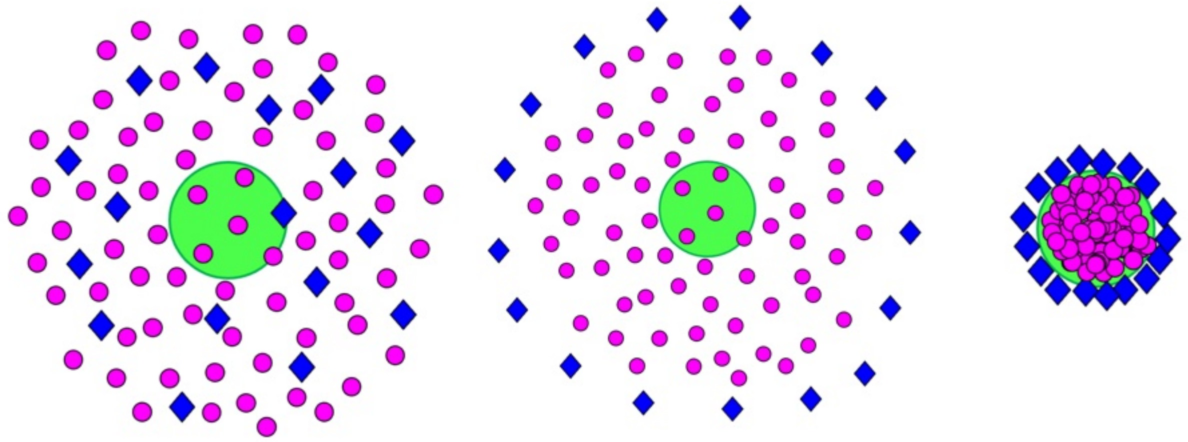


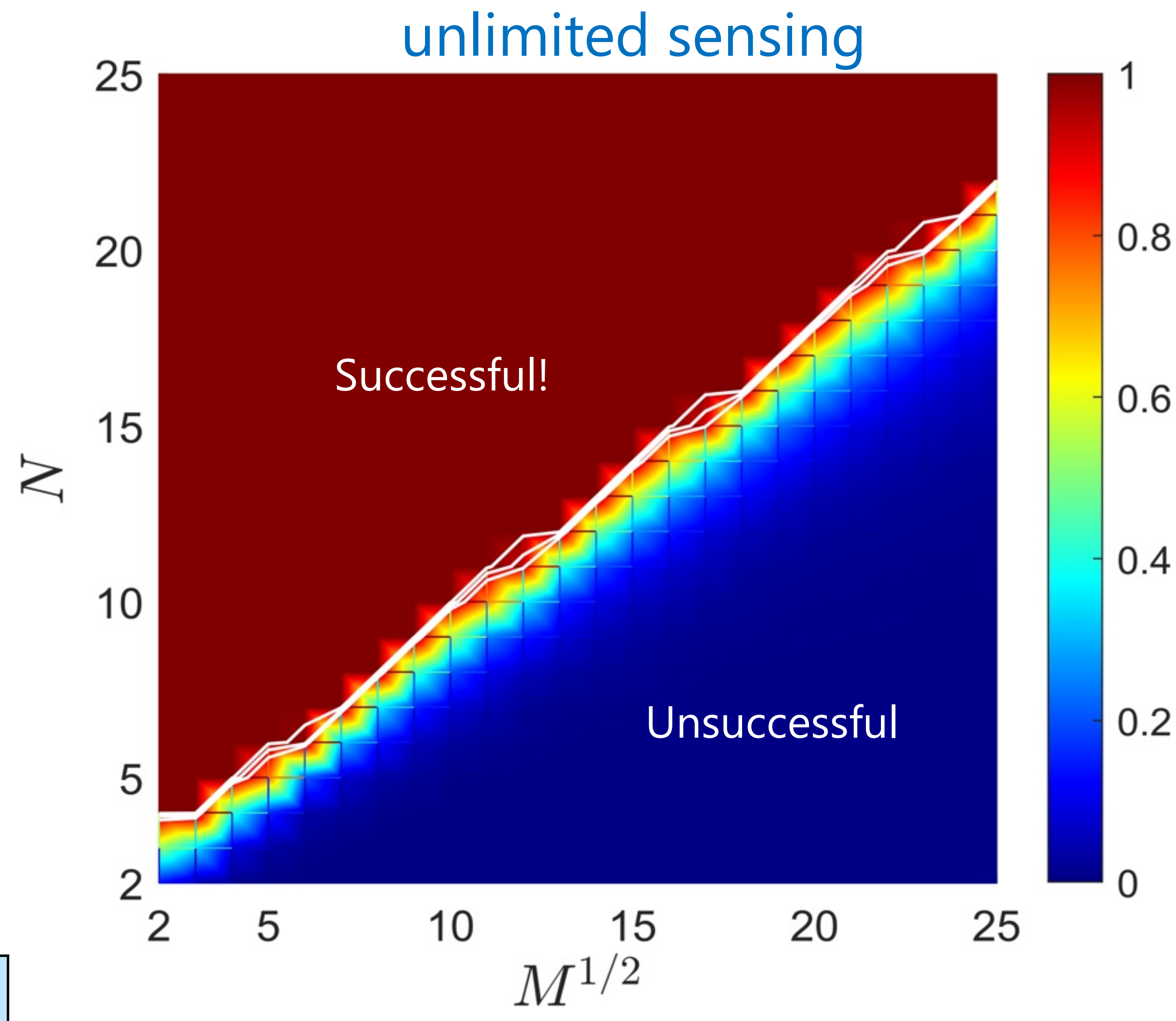
Under what conditions on the repulsion zone, the sensing area and the density of the targets can we achieve herdability of a given number of targets?

Herdability charts

What is the minimum number of herders $N^*(M)$ necessary to herd M targets?

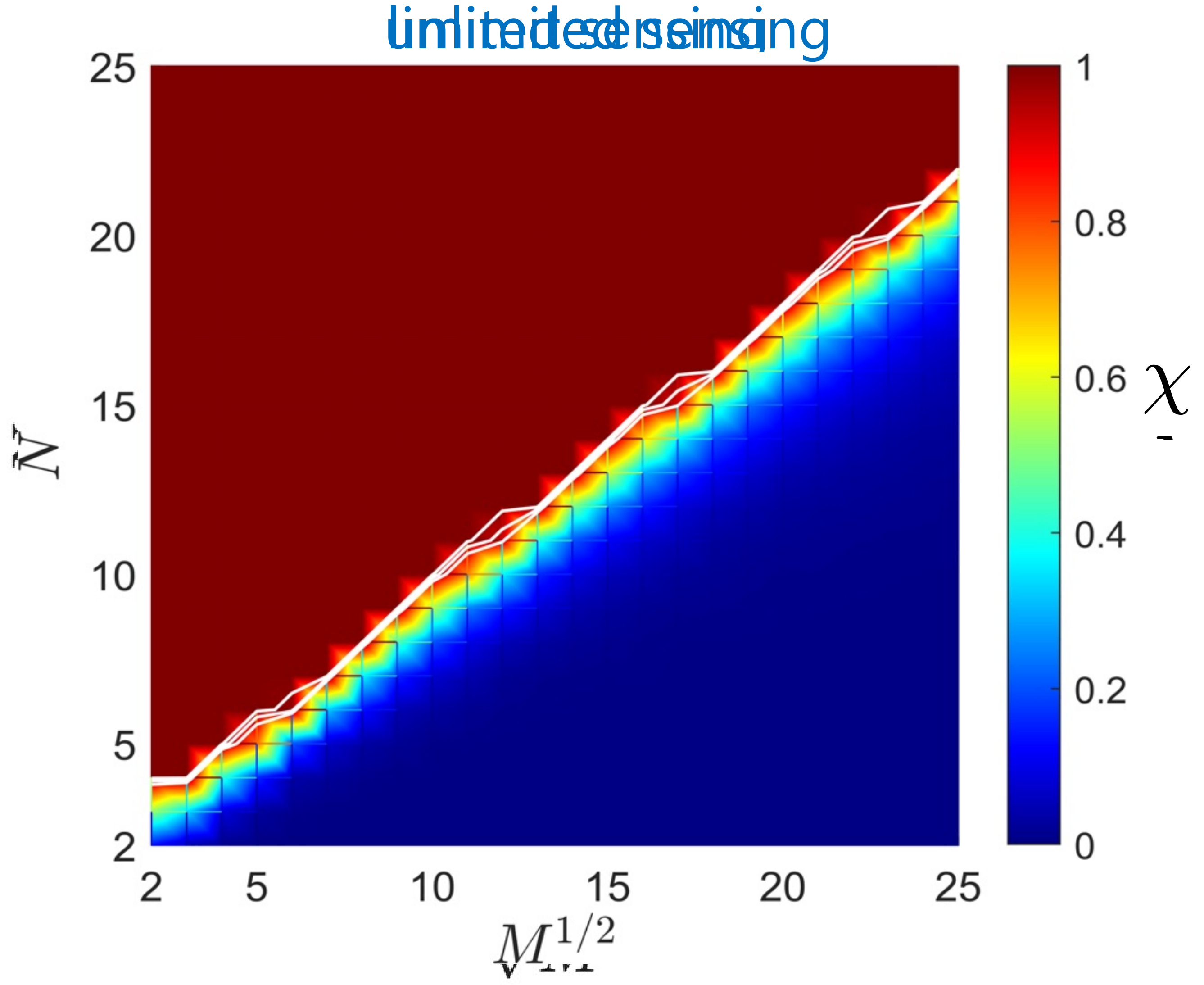
$$N^*(M) \propto \sqrt{M}$$

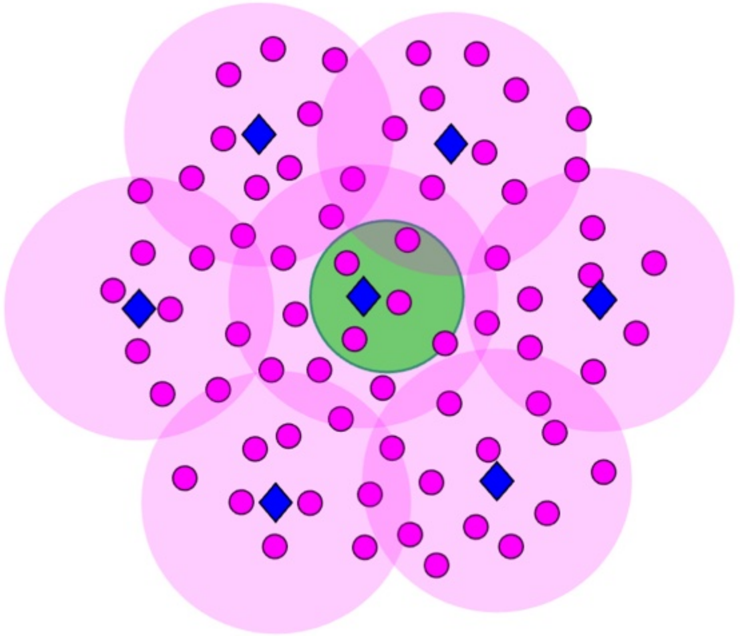




Herders need to counterbalance diffusion of the M targets with the transport flow they induce

Finite sensing effect





Herders need to collectively sense all the targets

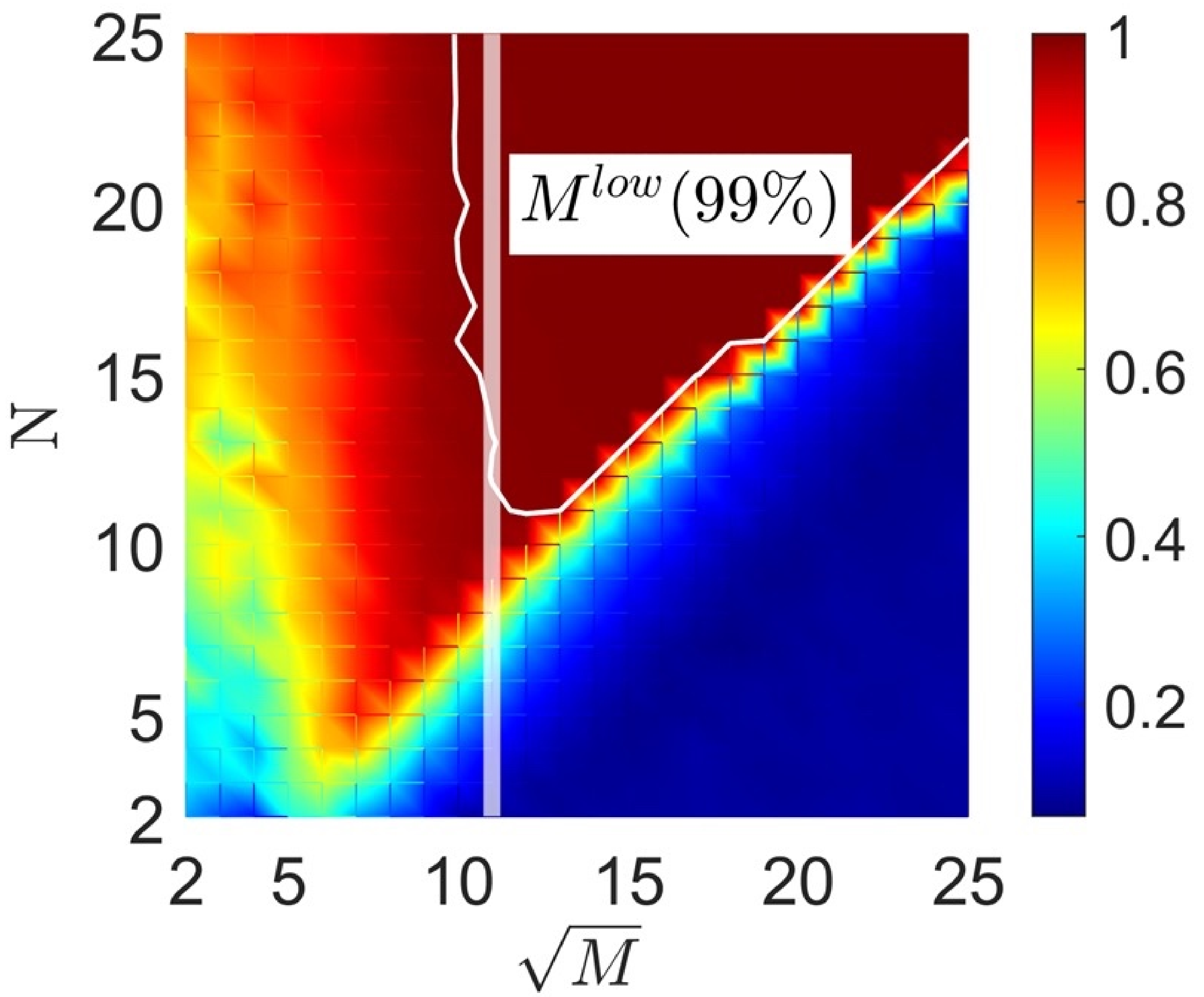
The herdability graph

- We define the *herdability graph*

$$G_{ab}(\boldsymbol{T}, \xi) = 1$$

if $|T_a - T_b| \leq \xi$

- If there is a path on G between a and e the herder is able to switch from a to e
- Then, herdability can be linked to the *percolation* of this graph

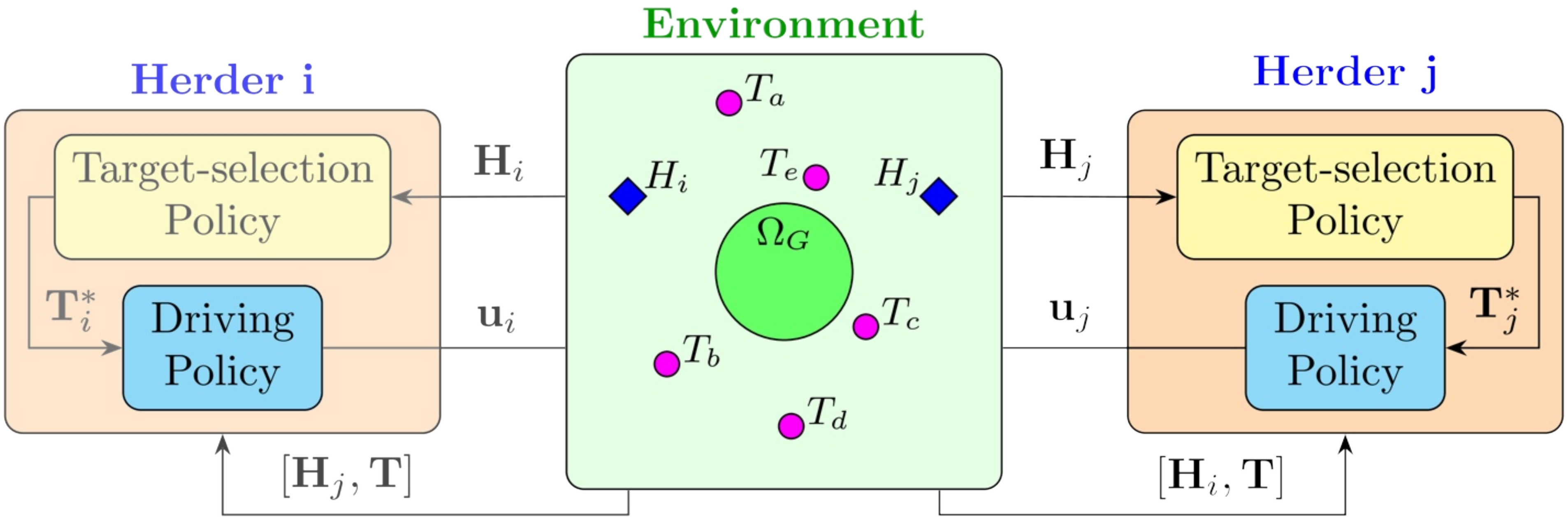


A. Lama, MdiB, *Shepherding and herdability in complex multiagent systems*, Physical Review Research, 2024

A hierarchical learning-based approach

Can herders *learn* to solve the problem without a model?

Does spontaneous cooperation emerge without any explicity rule?



I. Napolitano et al., *Emergent Cooperative Strategies for Multi-Agent Shepherding via Reinforcement Learning*, Eur. Control Conf, 2025
S. Covone et al., *Hierarchical Policy-Gradient Reinforcement Learning for Multi-Agent Shepherding Control*, IEEE CDC, 2025

Driving policy

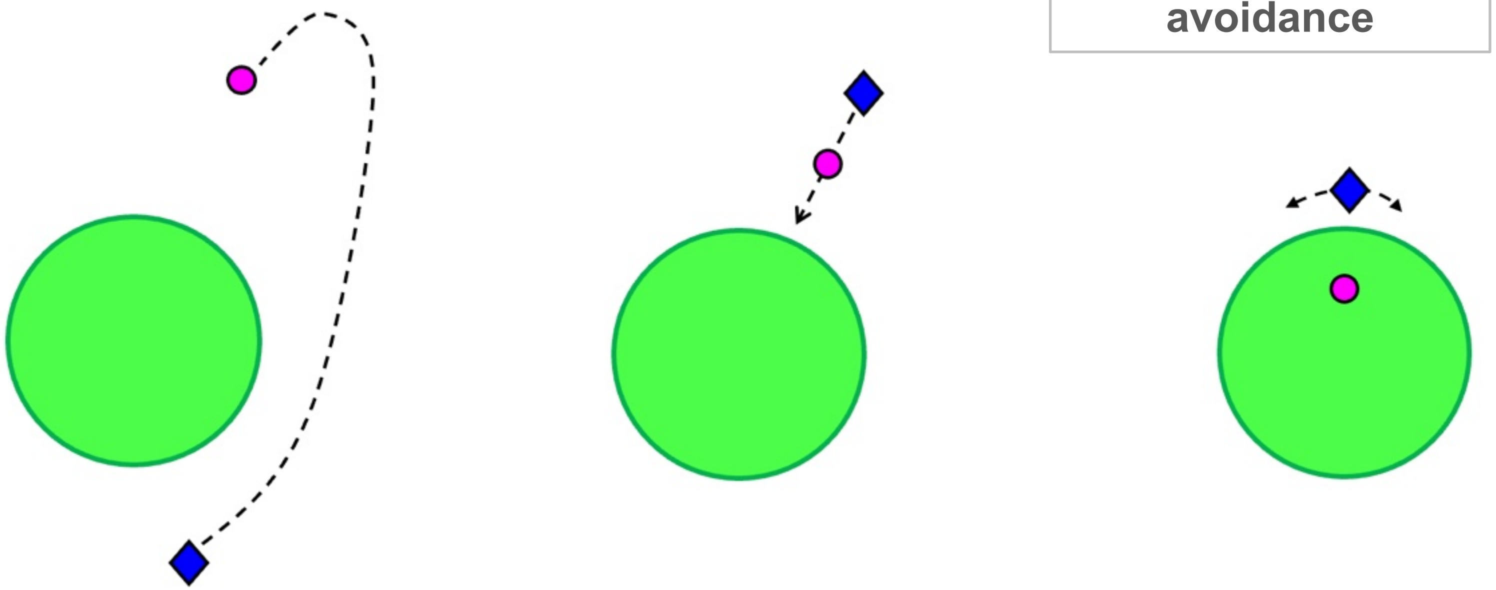
$$r_{\mathrm{D}} = -k_1 \|\mathbf{T} - \mathbf{H}\| - k_2 \|\mathbf{T}\| \mathbb{1}_{\{\mathbf{T} \notin \Omega_G\}} - k_3 \mathbb{1}_{\{\mathbf{H} \in \Omega_G\}} - k_4 \|\mathbf{u}\|$$

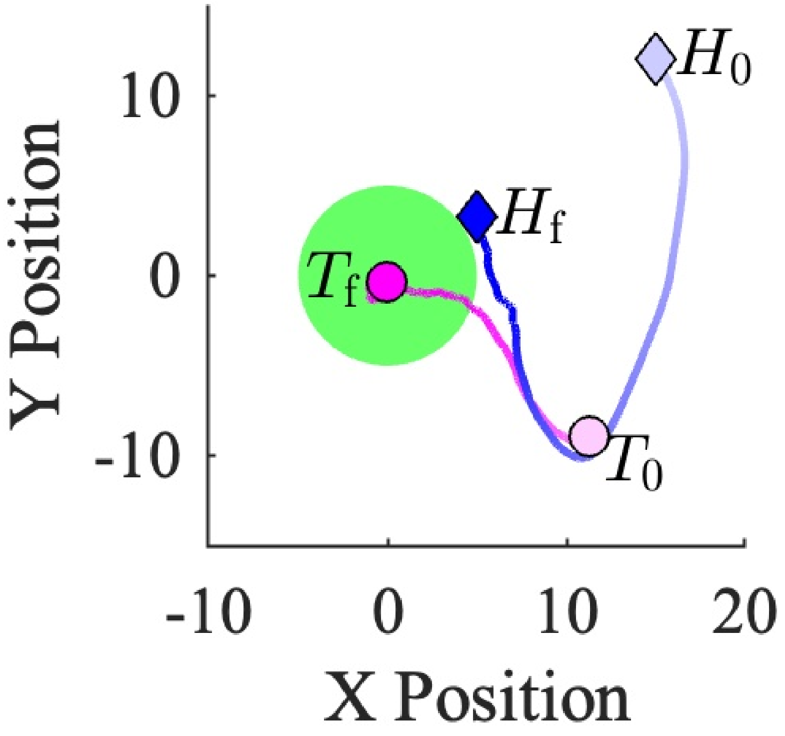
Approaching phase

Steering phase

Goal region avoidance

Control effort

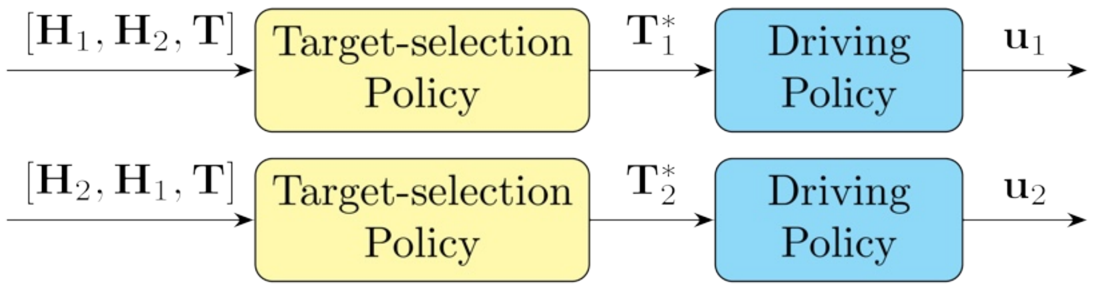


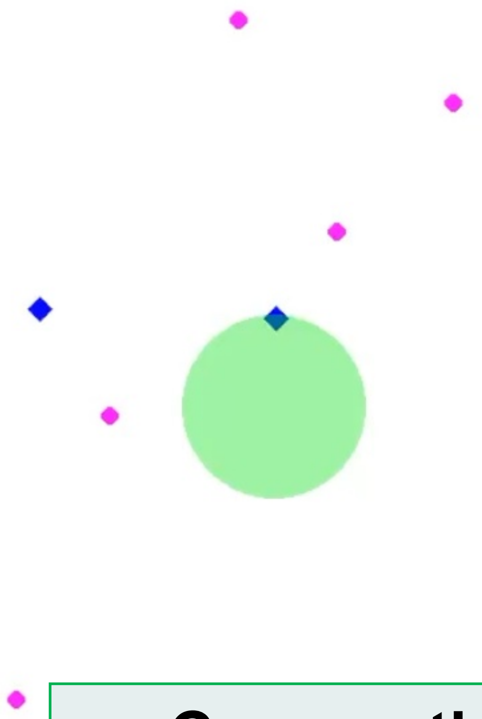


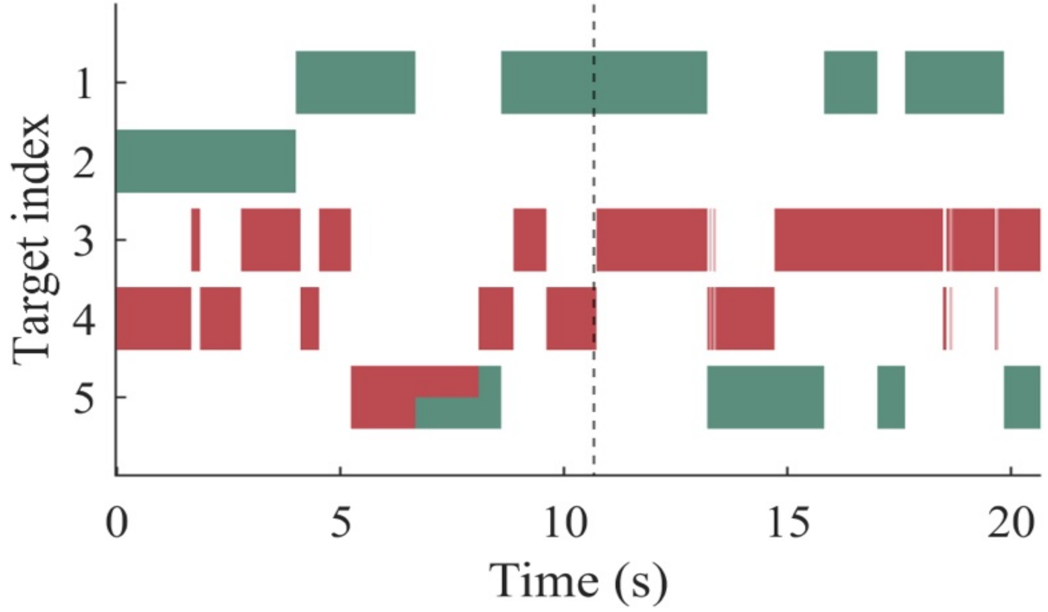
Decision-making policy (decentralized)

$$r_{\mathrm{T},k} = -k_5 \sum_{i=1}^{M} \left(||\mathbf{T}_i(k)|| - \rho_{\mathrm{G}}\right) \mathbb{1}_{\{\mathbf{T}_i(k) \notin \Omega_G\}}$$

Distance from goal



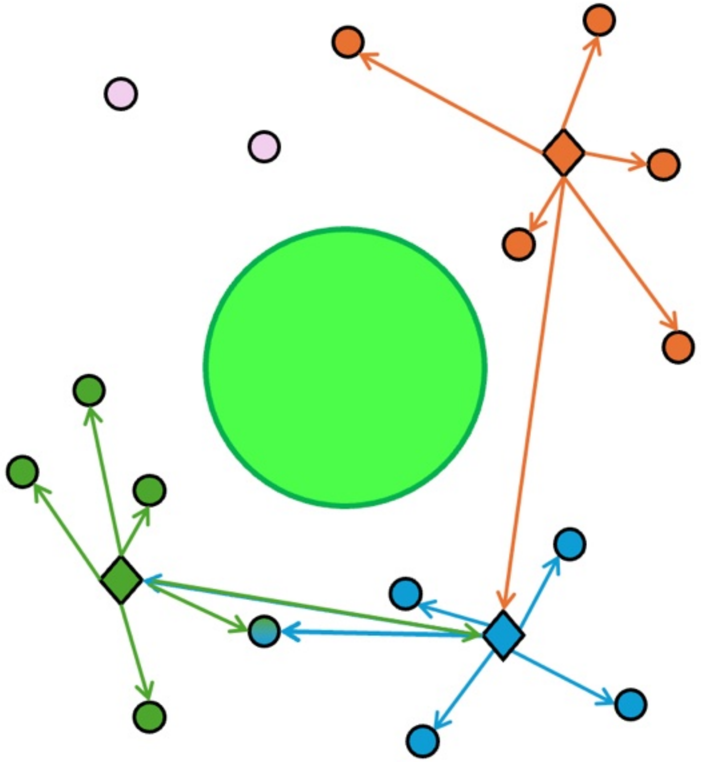


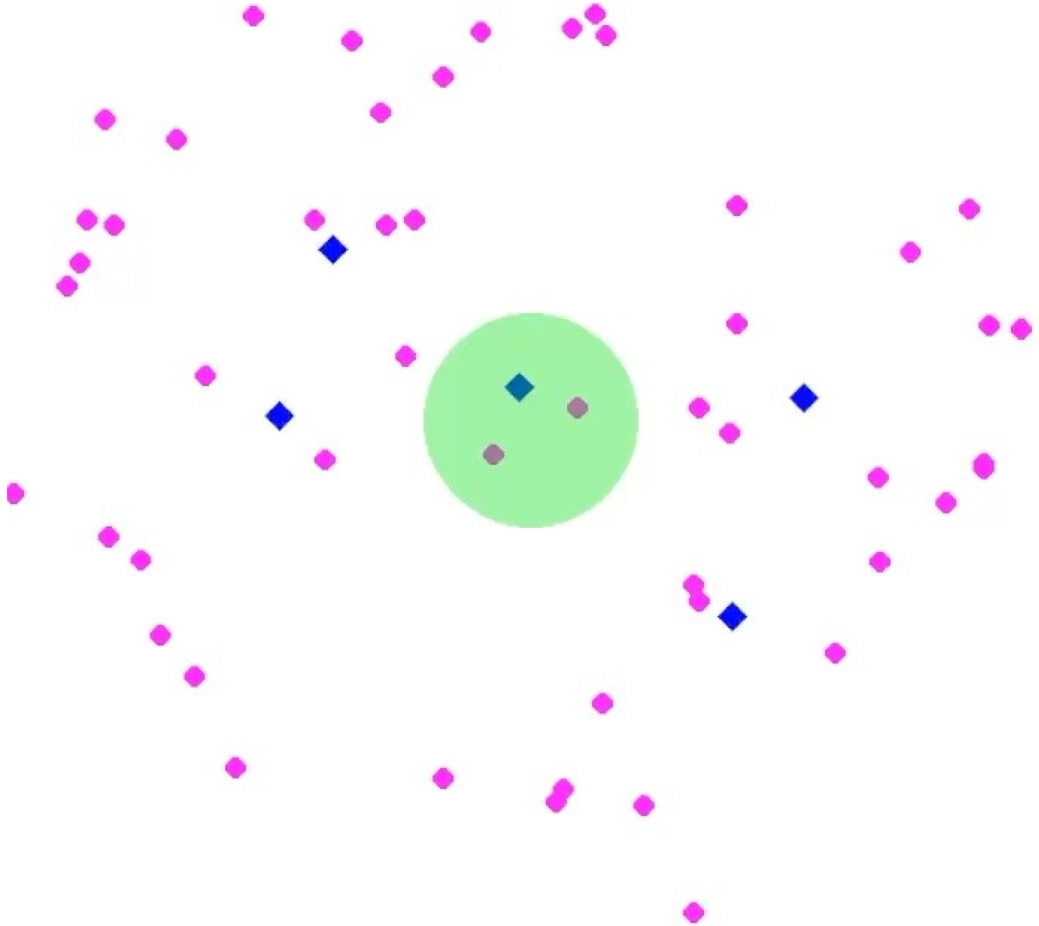


Cooperation among herders emerges spontaneously

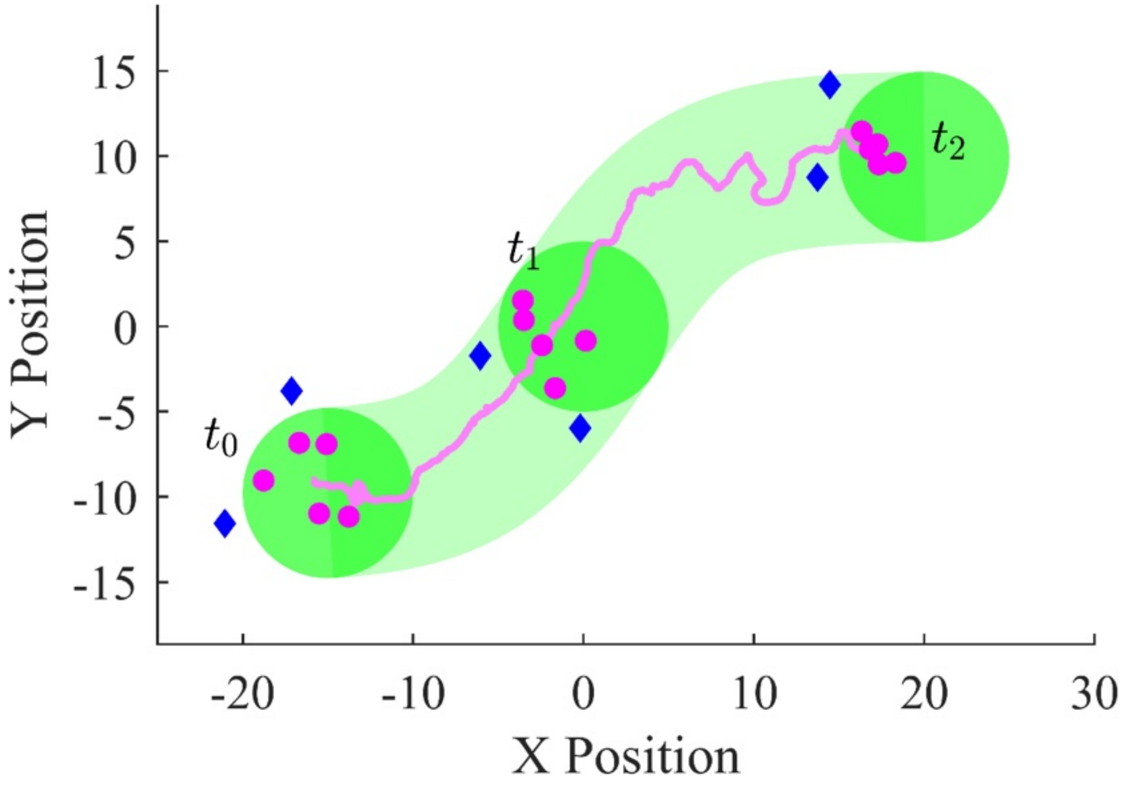
Towards a scalable solution

- The trained policy is extended to large-scale systems through **topological sensing**.

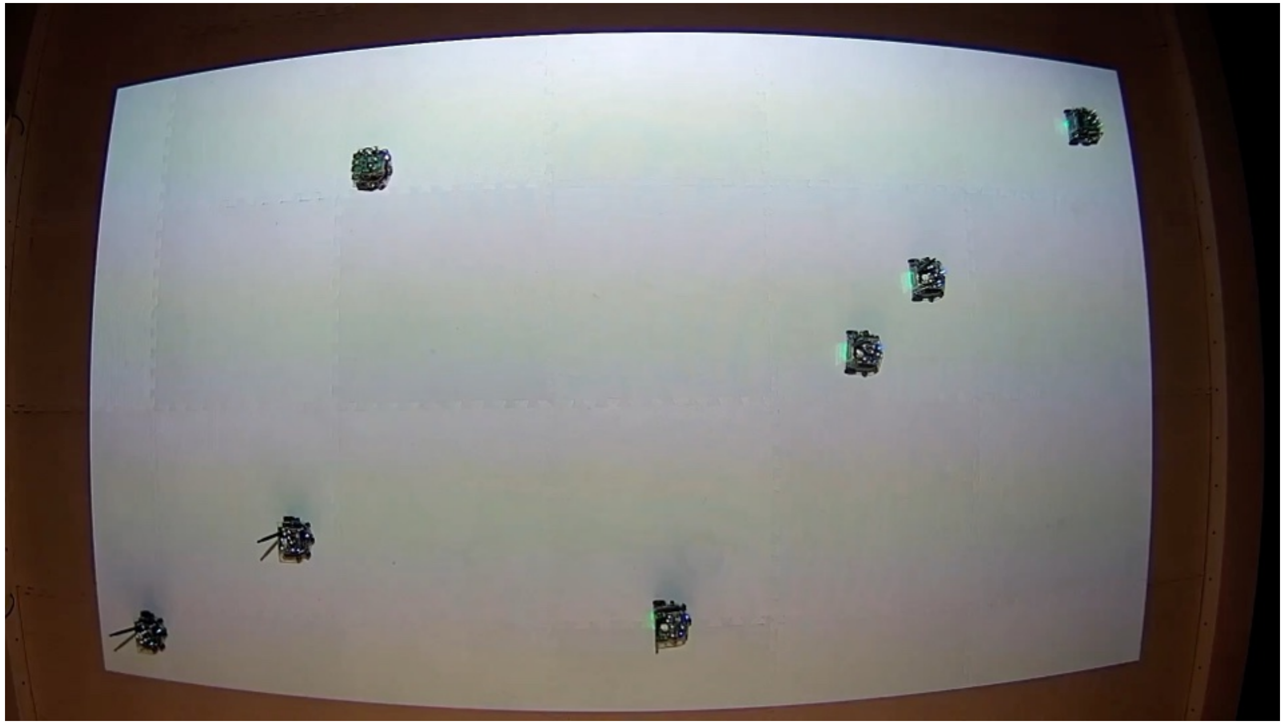




Experimental validation and further tasks



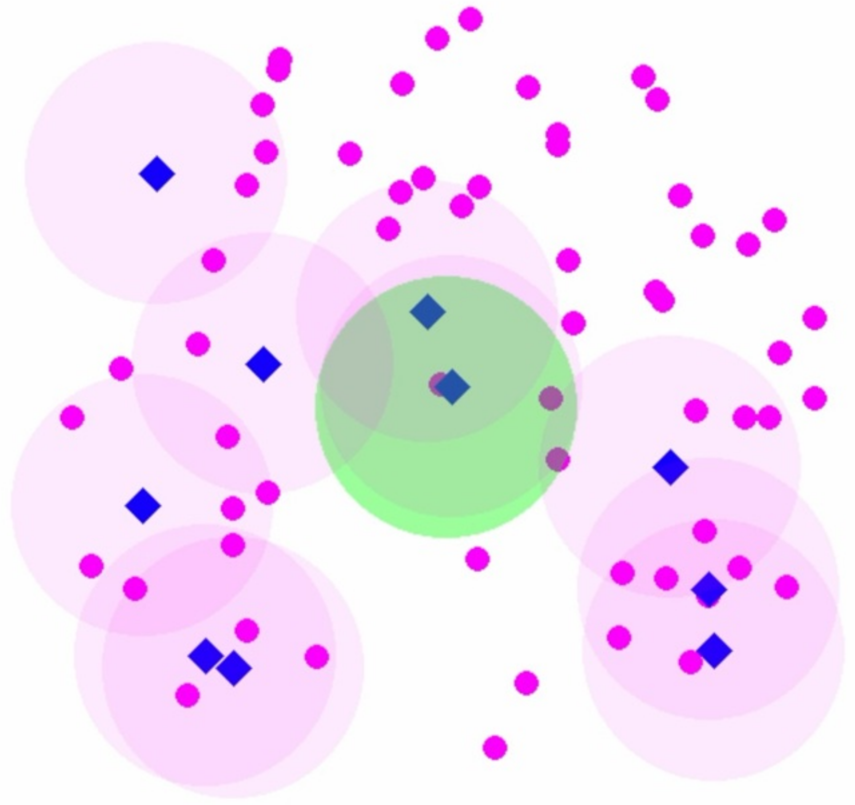
ROBOTARIUM
by GeorgiaTech



S. Covone, I. Napolitano et al. *A hierarchical machine learning approach to solve the shepherding problem*, IEEE Trans. on Robotics, 2025

To sum up

- Control laws can be designed to solve the shepherding problem
- Herdability requires a minimum target density if herders possess *limited sensing*
- Herders can *learn* to cooperate and *self organize* to be successful
- **How can we prove convergence?**
- **What about more sophisticated shepherding goals, e.g. achieving a desired density?**

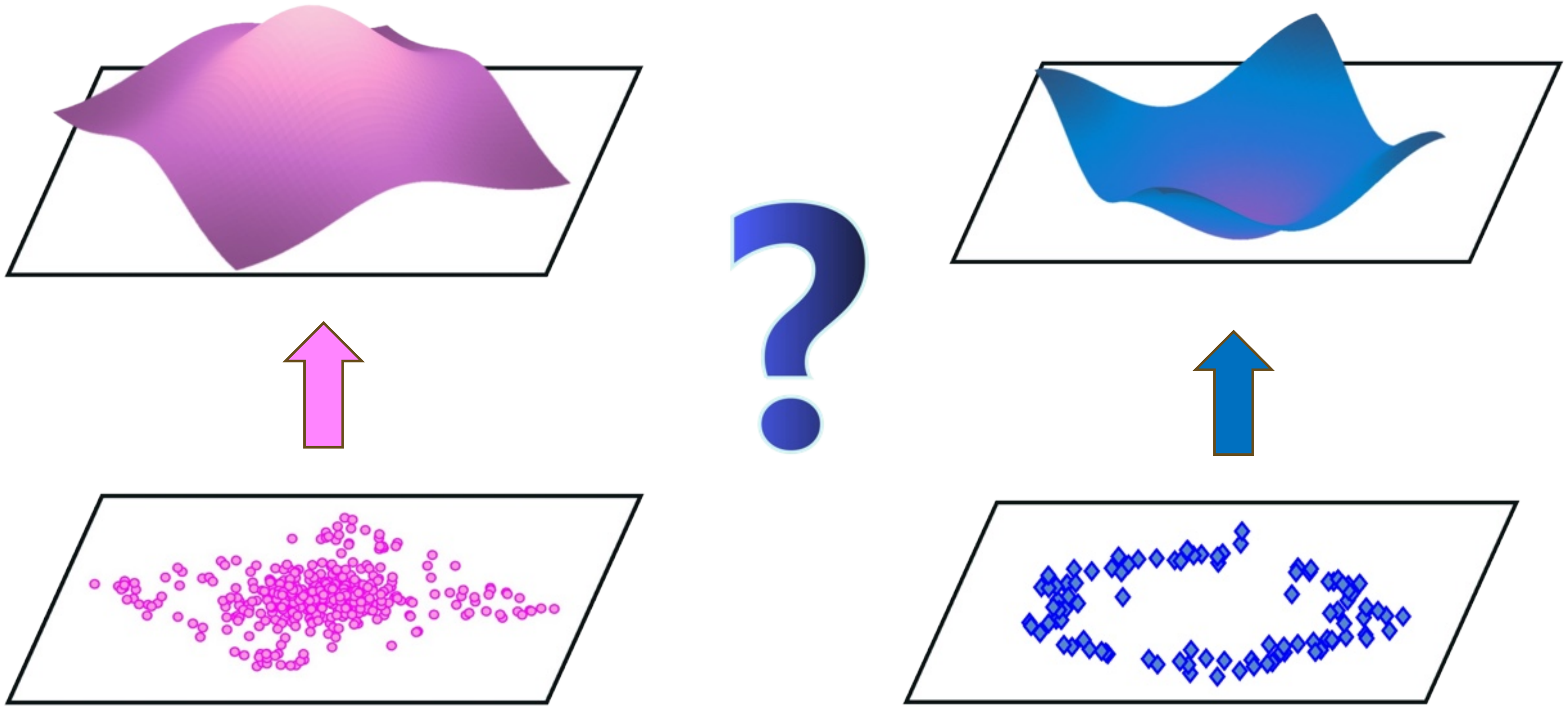


Limited sensing

No cohesiveness

Bridging microscopic and macroscopic scales

- We need to work at a different description level



Crucial problem: how to incorporate decision-making in fields equations?

A macroscopic approach

Beniamino
Di Lorenzo

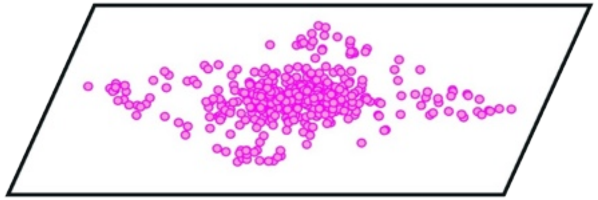
Andrea
Lama

Gian Carlo
Maffettone

From micro to macro

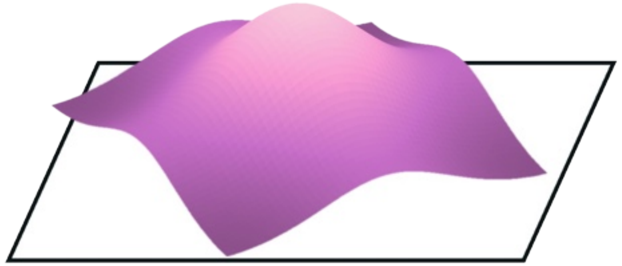
- As is typical in mean-field, given stochastic equations of the agents dynamics:

$$\dot{x}_i = \sqrt{2D}\eta + F_i = \sqrt{2D}\eta + \sum_{j \in \mathcal{N}_i} F_{ji}$$



- We consider average forces acting on each "particle":

$$F_i = \sum_{j \in \mathcal{N}_i} F_{ji} \rightarrow \langle F(x) \rangle = \int_{\mathbf{B}(x)} F(x,y)\rho(y)\, dy$$

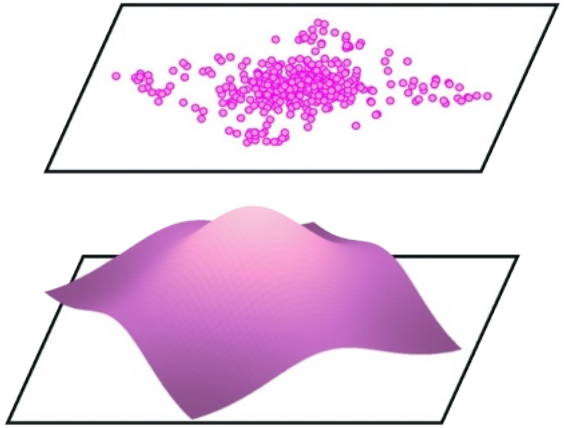


- We obtain a PDE describe the evolution of the agents' densities:

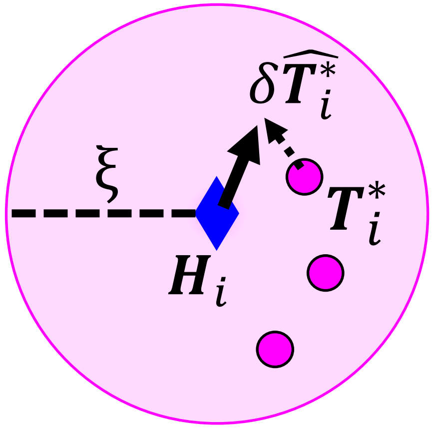
$$\partial_t \rho(x,t) + \nabla \cdot \left[\langle F \rangle \rho\right](x,t) = D\nabla^2 \rho(x,t)$$

You et al, *Nonreciprocity as a generic route to traveling states*, PNAS, 2020

Shepherding in the continuum



$$\dot{\mathbf{T}}_a = \sqrt{2D}\mathcal{N} + \beta \sum_{i \in \mathcal{N}_a} (\lambda - |\mathbf{d}_{ia}|)\, \hat{\mathbf{d}}_{ia}$$

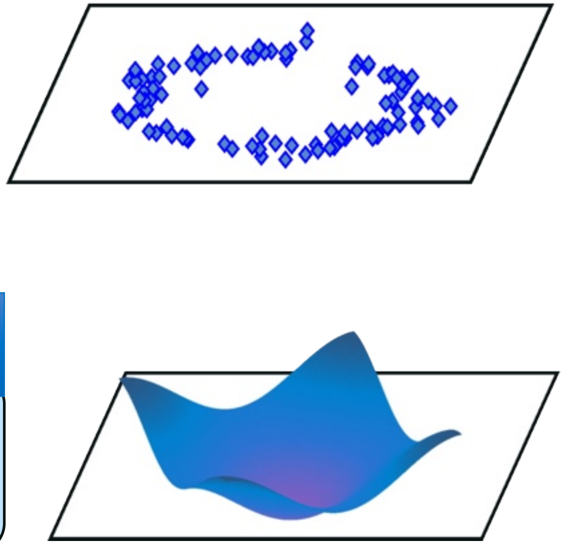


$$\partial_t \rho^T + \nabla \cdot \left[-\alpha \rho^T \nabla \rho^H\right] = D\nabla^2 \rho^T$$

Targets

$$\dot{\mathbf{H}}_i = -k^H \left[\mathbf{H}_i - \left(\mathbf{T}_i^*(\xi) + \delta\hat{\mathbf{T}}_i^*\right)\right] + \sqrt{2D}\mathcal{N}$$

$$\mathbf{T}_i^* = \lim_{\gamma \to \infty} \frac{\sum_{N_i(\xi)} e^{\gamma|\mathbf{T}_a|}\mathbf{T}_a}{\sum_{N_i(\xi)} e^{\gamma|\mathbf{T}_a|}}$$



Herders

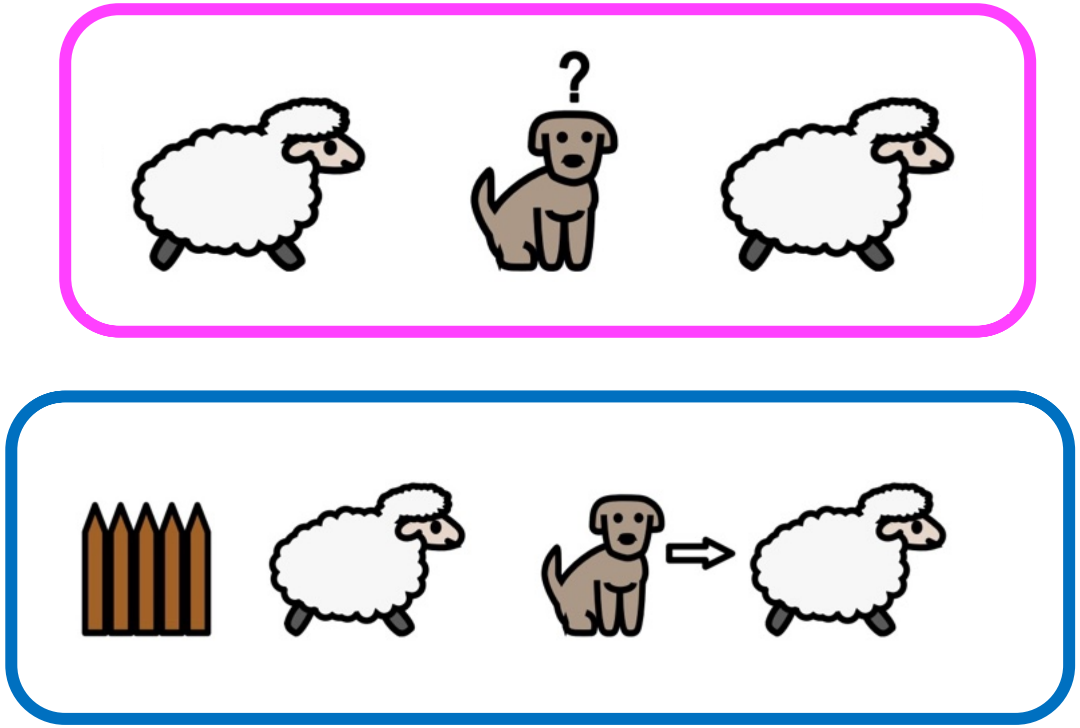
$$\partial_t \rho^H + \nabla \cdot \left[v_1(\gamma,\delta,x)\rho^H \rho^T + v_2(\gamma,\delta,x)\rho^H \nabla \rho^T\right] = D\nabla^2 \rho^H$$

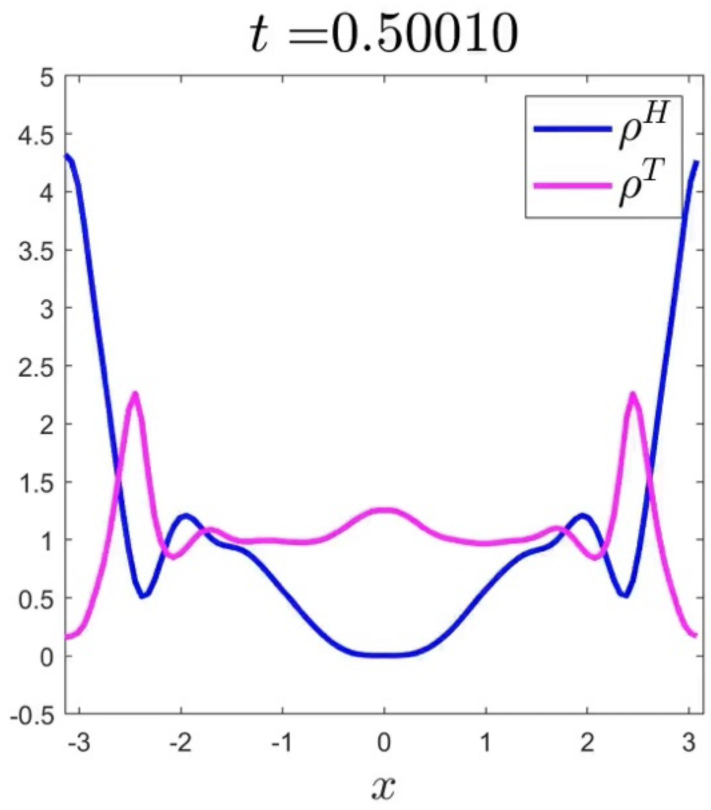
A. Lama, MdiB, S. Klapp, *Nonreciprocal field theory of decision-making in multi-agent control systems,* Nature Communications, 2025

Density evolution

$$\partial_t \rho^T + \nabla \cdot \left[-\alpha \rho^T \nabla \rho^H\right] = D\nabla^2 \rho^T$$

$$\partial_t \rho^H + \nabla \cdot \left[v_1(\gamma,\delta,x)\rho^H \rho^T + v_2(\gamma,\delta,x)\rho^H \nabla \rho^T\right] = D\nabla^2 \rho^H$$

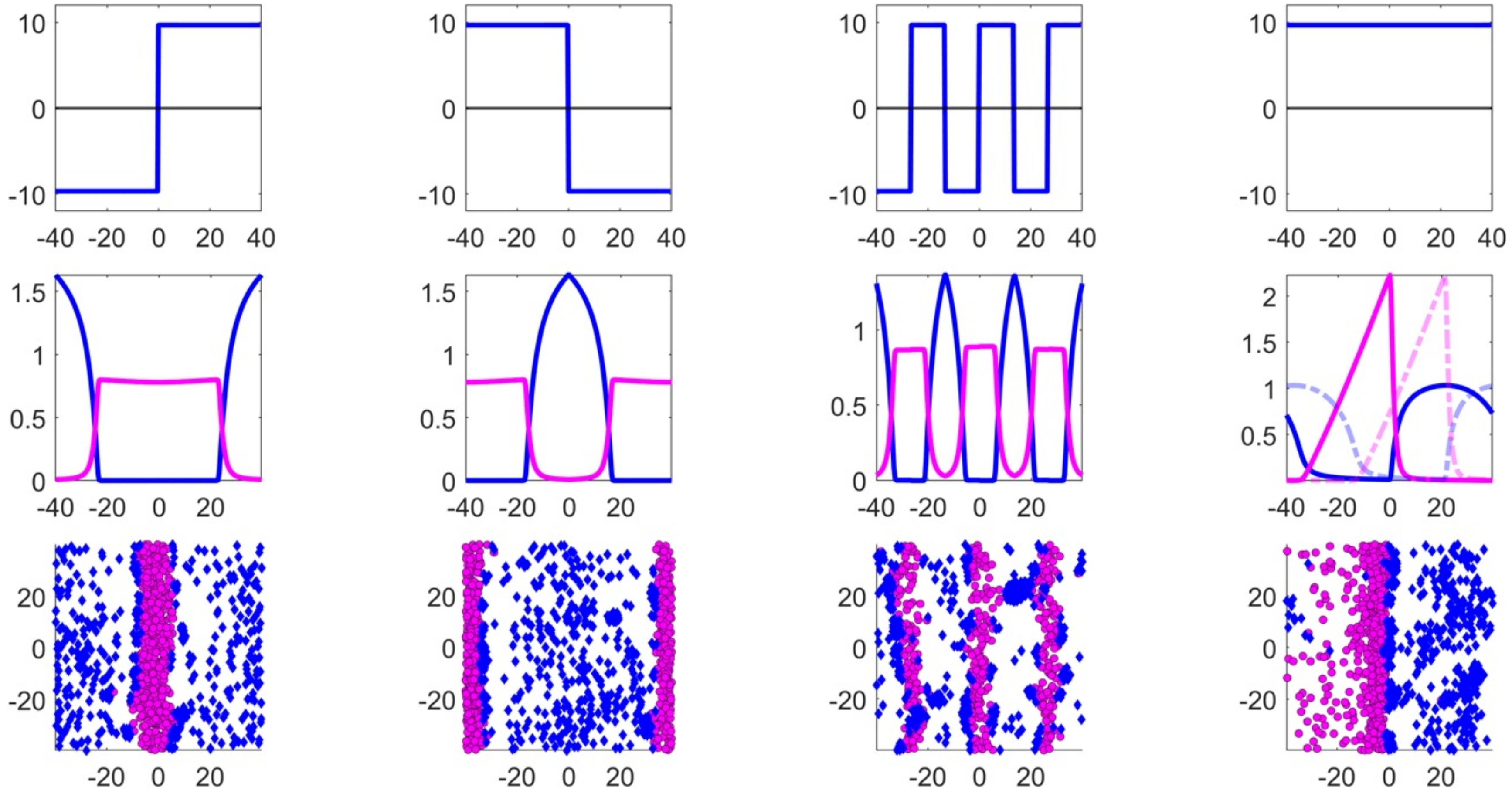




A. Lama, MdiB, S. Klapp, *Nonreciprocal field theory of decision-making in multi-agent control systems,* Nature Communications, 2025

Macroscopic control design

$$\partial_t \rho^H + \nabla \cdot \left[\rho^H \left(v_1(\gamma,\delta,x)\rho^T + v_2(\gamma,\delta,x)\nabla\rho^T\right)\right] = D\nabla^2\rho^H$$



Leader-follower density control

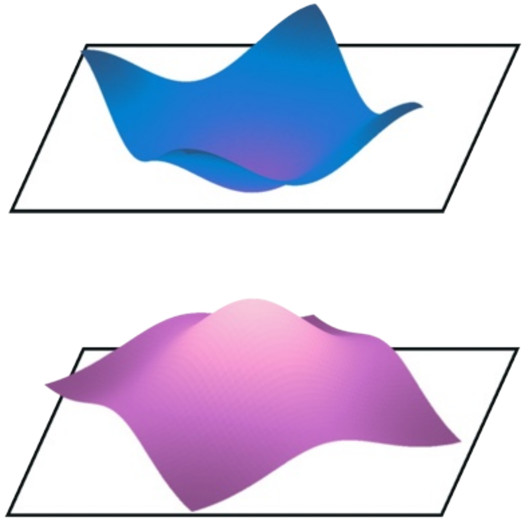
- We can formulate shepherding as a more general density control problem

$$\partial_t \rho^H(x,t) + \nabla \cdot \left[\rho^H \left(v_1(\gamma,\delta,x)\rho^T + v_2(\gamma,\delta,x)\nabla\rho^T\right)\right] = D\nabla^2\rho^H$$

$$\partial_t \rho^T(x,t) + \nabla \cdot \left[\rho^T(x,t)\mathbf{v}^{TH}(x,t)\right] = D\nabla^2\rho^T(x,t)$$

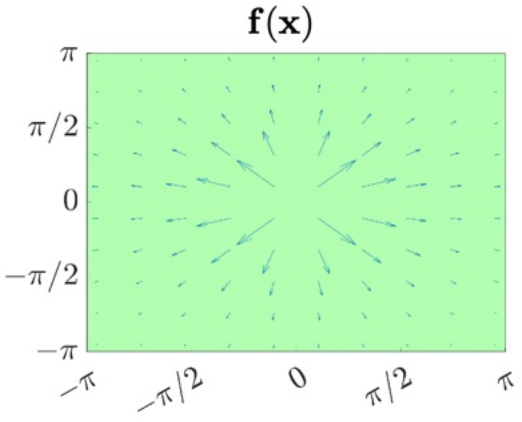
$$\mathbf{v}^{TH}(\mathbf{x},t) = (\mathbf{f} * \rho^H)(\mathbf{x},t)$$

$$\int_\Omega \left[\rho^T(\mathbf{x},t) + \rho^H(\mathbf{x},t)\right] \mathrm{d}\mathbf{x} = M^T + M^H = 1$$



- We wish to design the control field to achieve

$$\lim_{t\to\infty} \|\bar{\rho}^T(x) - \rho^T(x,t)\|_2 = 0$$



GC Maffettone et al, *Leader-Follower Density Control of Spatial Dynamics in Large-Scale Multi-Agent Systems*, IEEE TAC, 2025

Feasibility conditions (herdability)

- We derived a condition on the leaders' mass needed for a feasible solution

Theorem: the LF density control problem admits a feasible solution if and only if

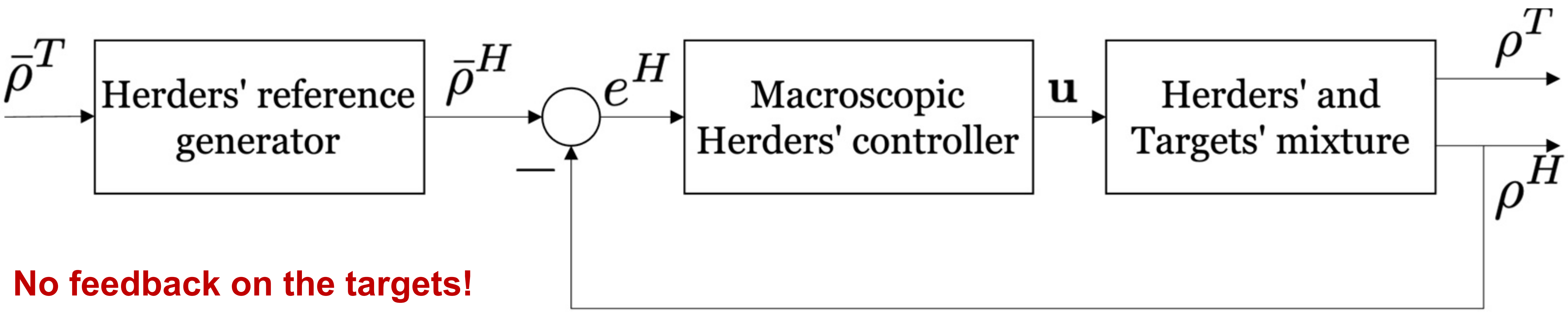
$$\widehat{M}^H \leq M^H \leq 1$$

where

$$\widehat{M}^H = \max_x \left\{ -\pi D g_1(x) + \frac{\pi D}{L^2} g_2(x) - \frac{DC}{2L^2} \right\}$$

$$g_1(x) = \left[\log(\hat{\rho}^T)\right]_{xx} \qquad g_2(x) = \log(\hat{\rho}^T) \qquad C = \int_{\mathcal{S}} \log(\hat{\rho}^T)\,\mathrm{d}x$$

Macroscopic Control Strategy



No feedback on the targets!

Theorem: *in a feasible scenario, if* $\nabla \cdot [\rho^H(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)] = -K_H e^H(\mathbf{x},t)$ *and*

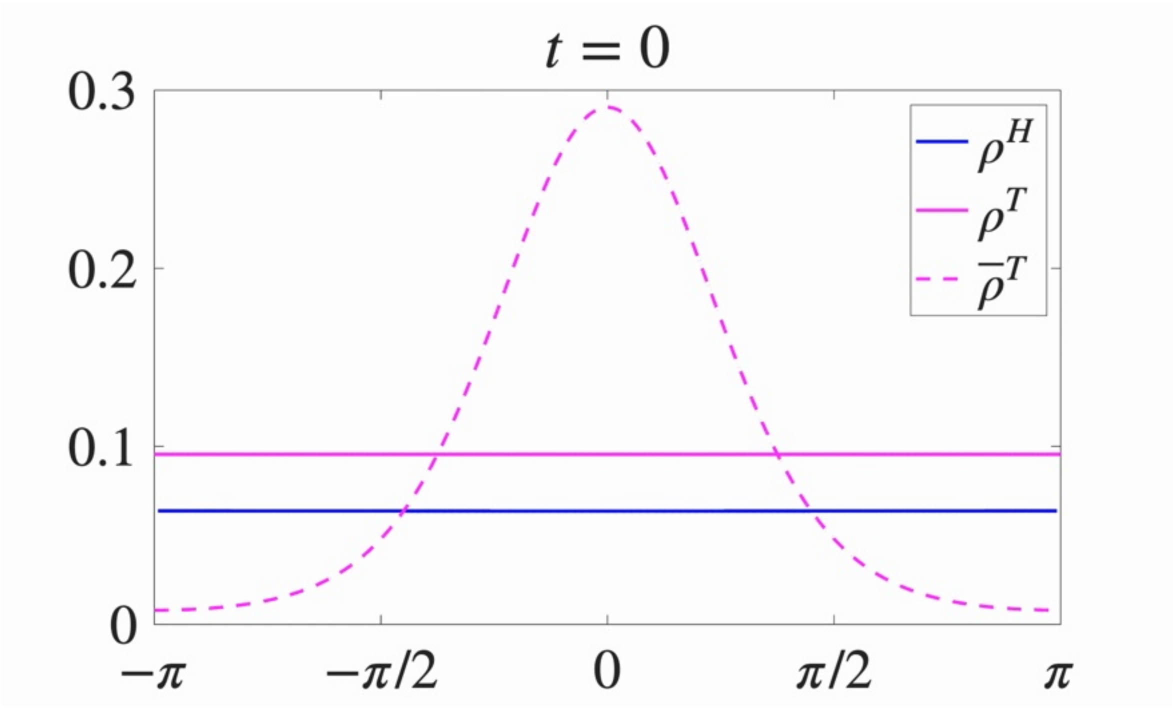
$$\left\| \nabla \cdot \left(\frac{\nabla \bar{\rho}^T(\mathbf{x})}{\bar{\rho}^T(\mathbf{x})} \right) \right\|_\infty < 2$$

the error dynamics globally converges to 0 almost everywhere

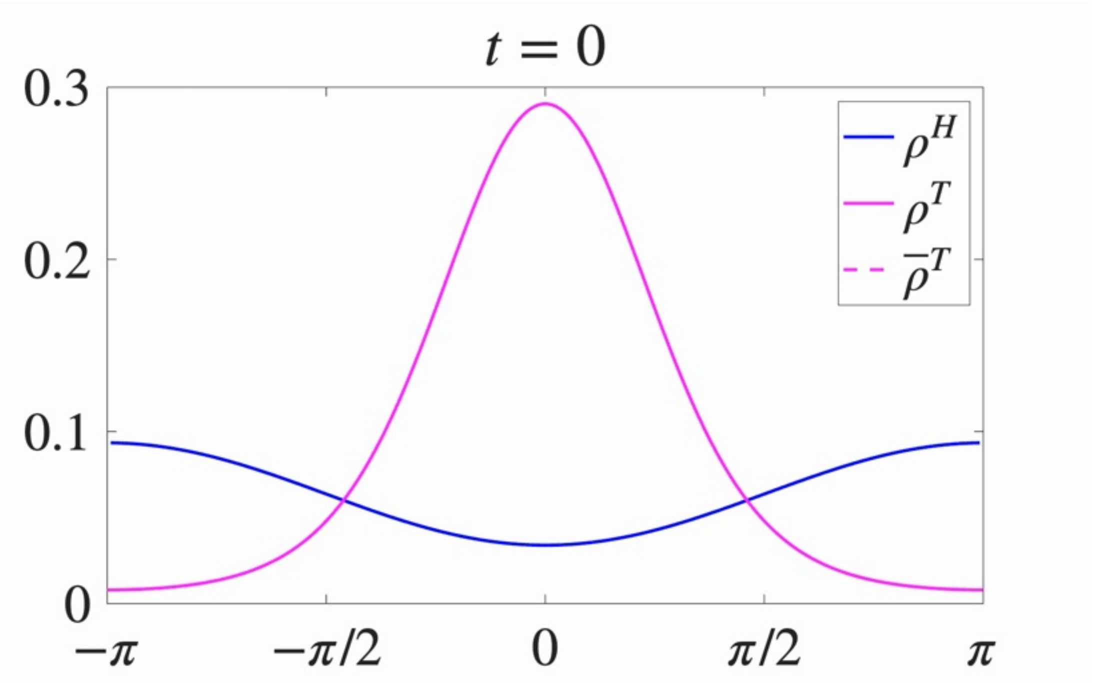
Numerical validation

- *Desired density*: von Mises distribution with zero mean
- *Parameters*: $M^H = 0.4\,, L = \pi\,, D = 0.03$

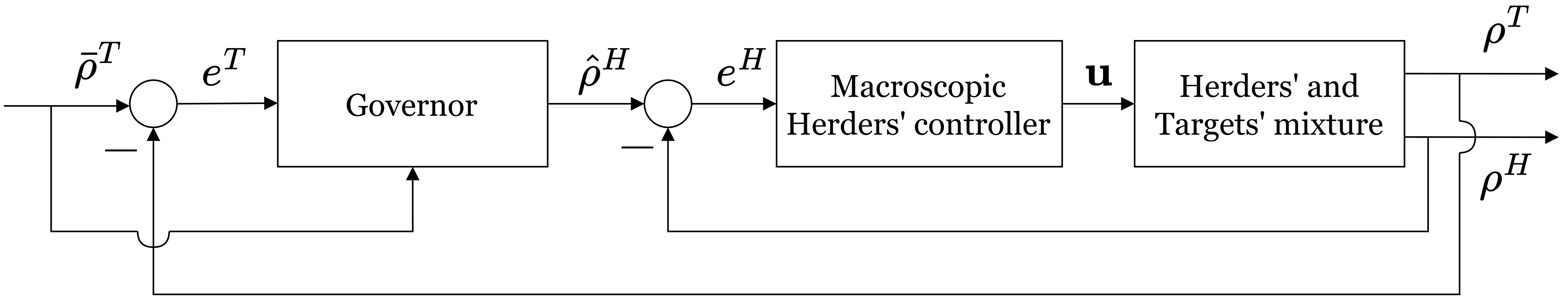
convergence

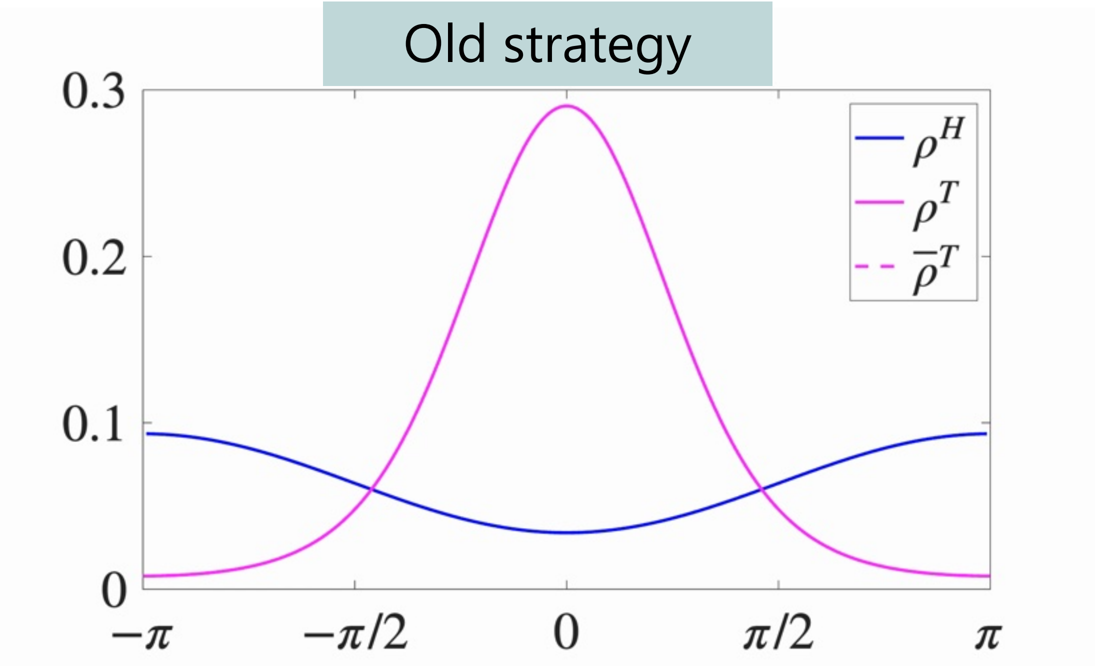


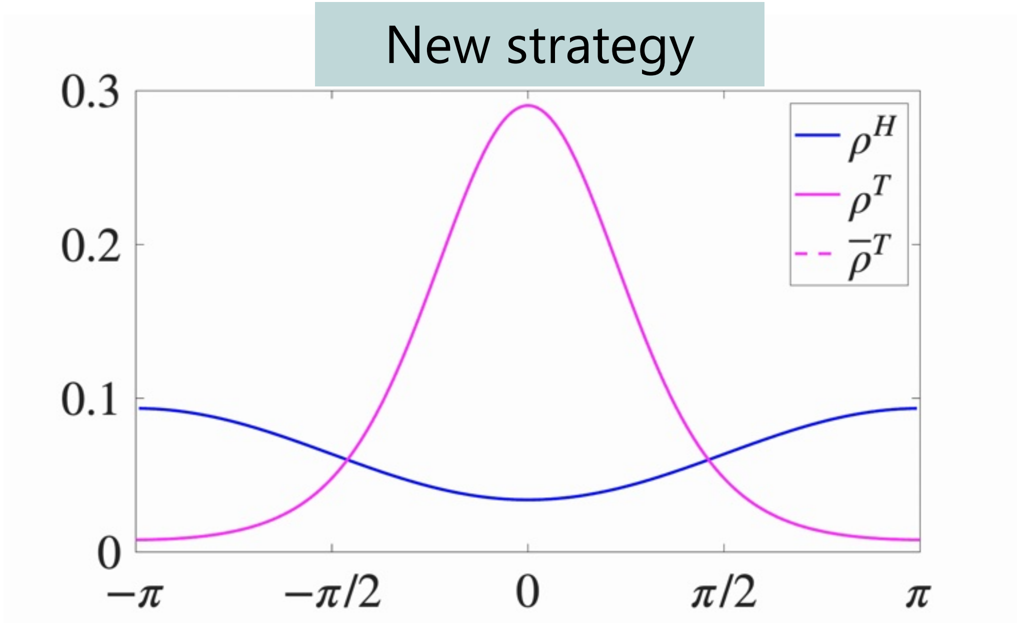
robustness



Reference governor strategy







GC Maffettone et al, *Leader-Follower Density Control of Spatial Dynamics in Large-Scale Multi-Agent Systems*, IEEE TAC, 2025

From macro to micro (discretization)

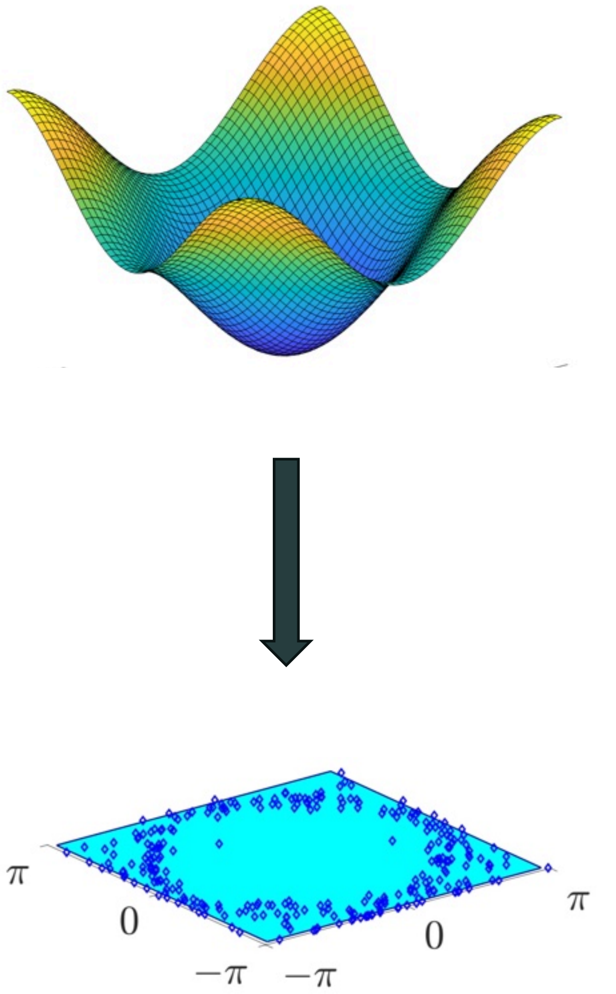
- We now have a *macroscopic controller* but need to implement the strategy at the *microscopic* agent level

$$\nabla \cdot [\rho^H(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)] = -K_H e^H(\mathbf{x},t)$$

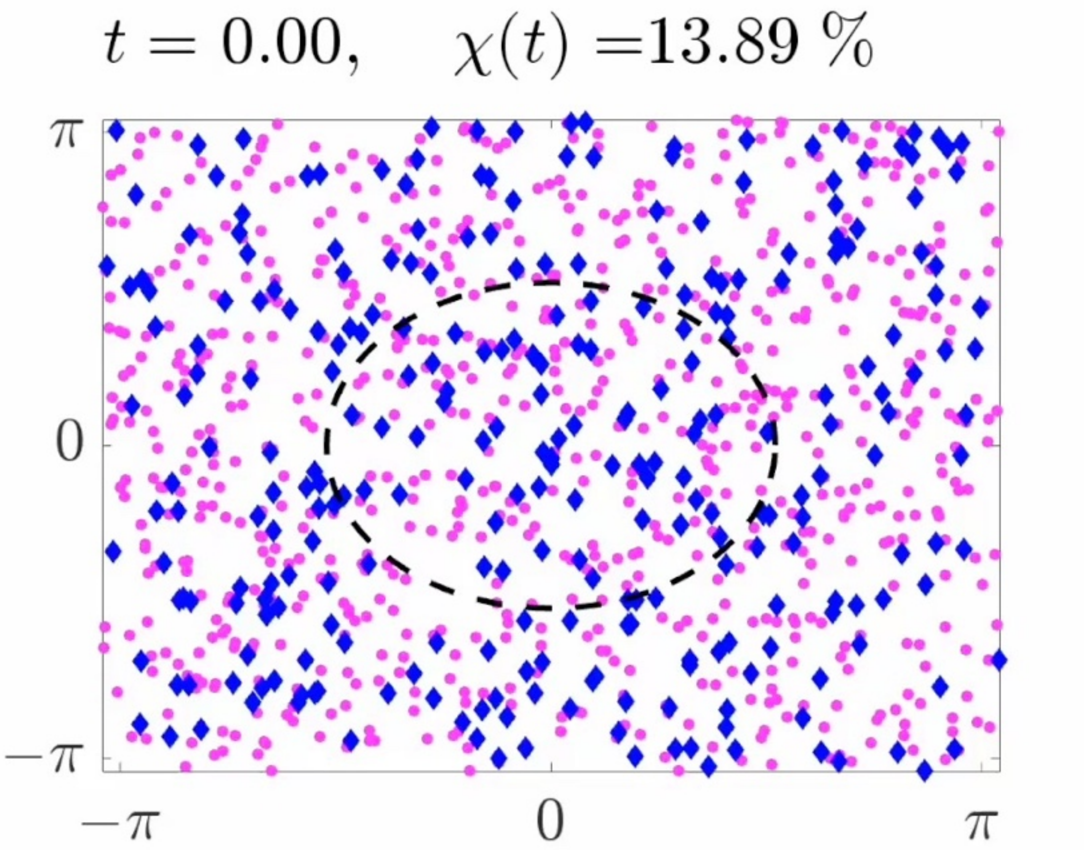
- *Microscopic control inputs* can be computed by the agents via spatial sampling:

$$\mathbf{u}_i(t) = \mathbf{u}(\mathbf{H}_i, t)$$

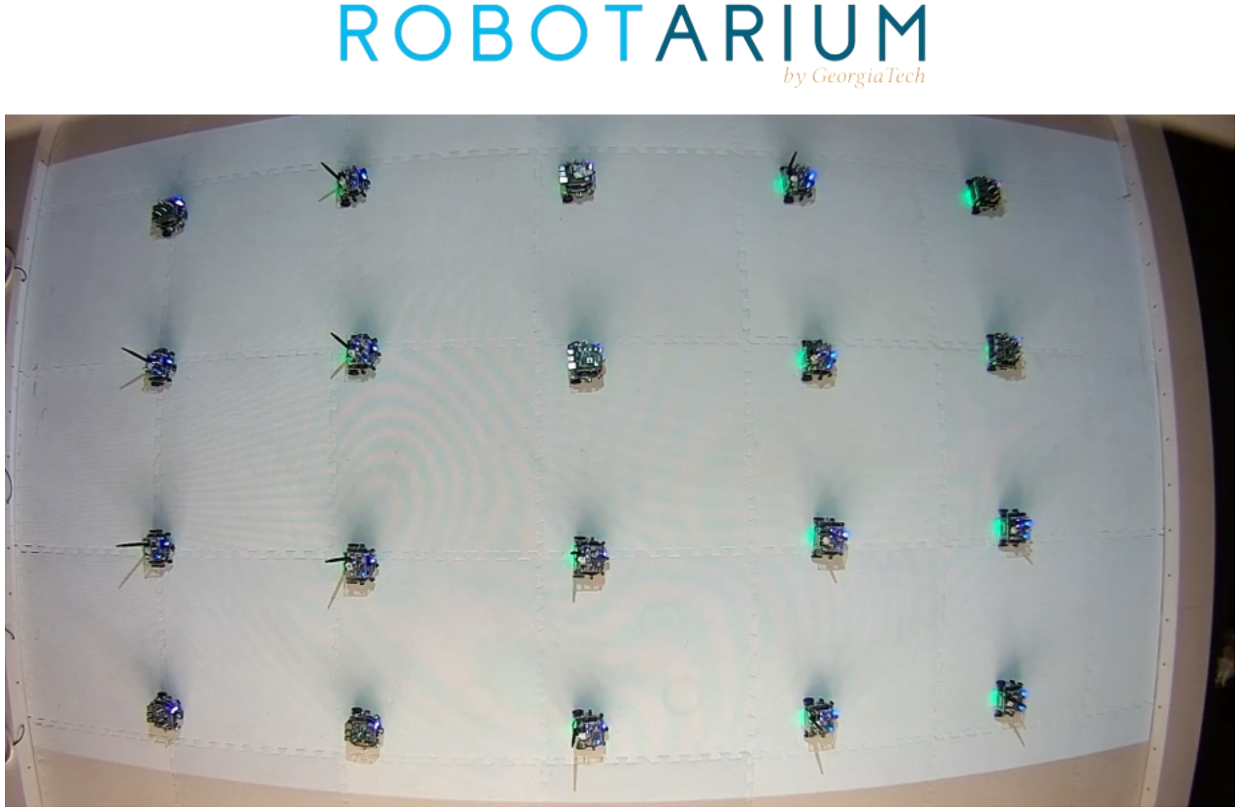
- This can also be extended to higher dimensions



Numerical and experimental validation



280 herders & 720 targets



Mixed reality experiment

B. Di Lorenzo et al, *A Continuification-Based Control Solution for Large-Scale Shepherding*, European Journal of Control, 2025 (in press)
G.C. Maffettone et al, *Mixed Reality Environment and High-Dimensional Continuification Control For Swarm Robotics, IEEE TCST, 2024*

A possible limitation

$$\nabla \cdot [\rho^H(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)] = -K_H e^H(\mathbf{x},t)$$

Macro control

$$\mathbf{u}_i(t) = \mathbf{u}(\mathbf{H}_i, t)$$

Micro control

- To compute $\mathbf{u}_i$ each herder needs to know the positions of all other herders

$$\hat{\rho}^H(\mathbf{x},t) = \sum_{i=1}^{N^H} \mathcal{K}(x - \mathbf{H}_i(t))$$

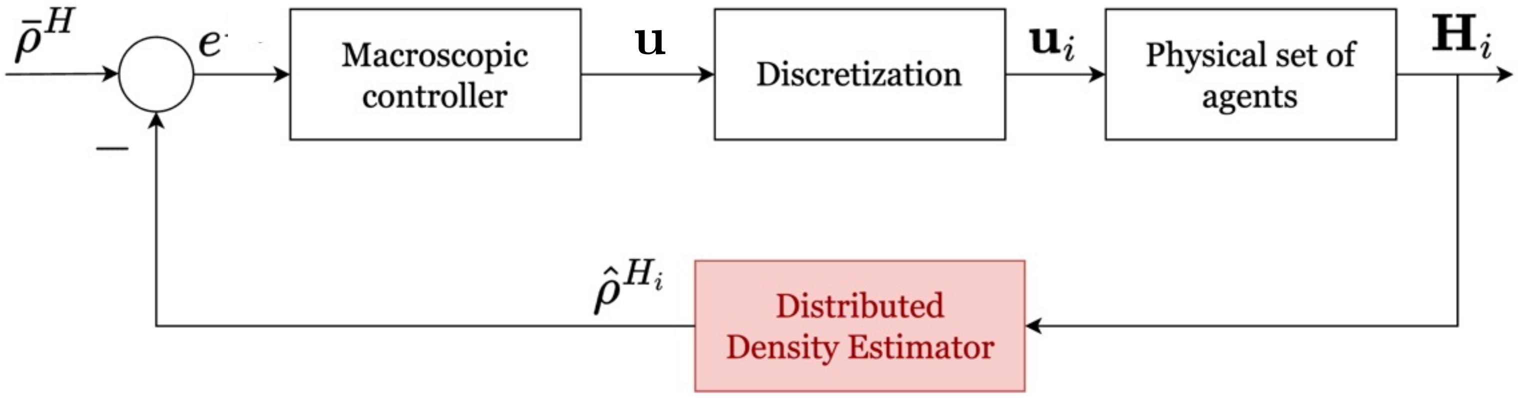
Density computation

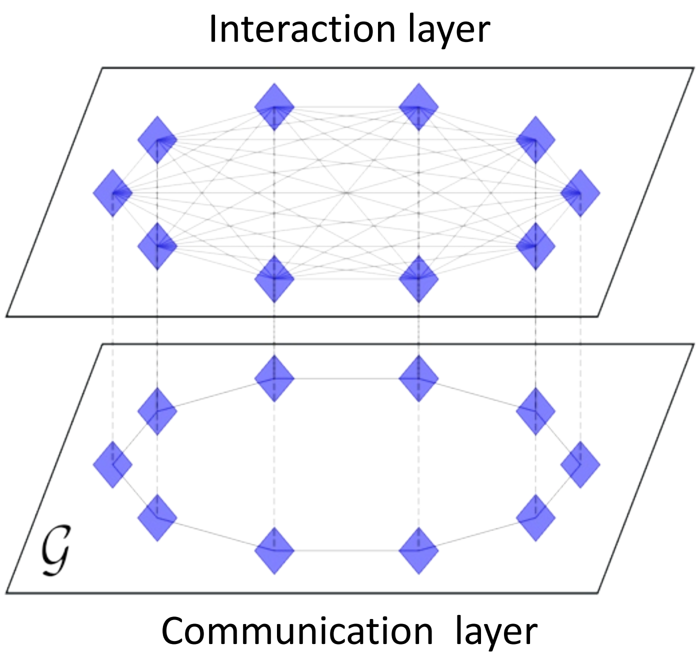
- Solution: estimate the density using a distributed density estimation strategy to compute the microscopic control input

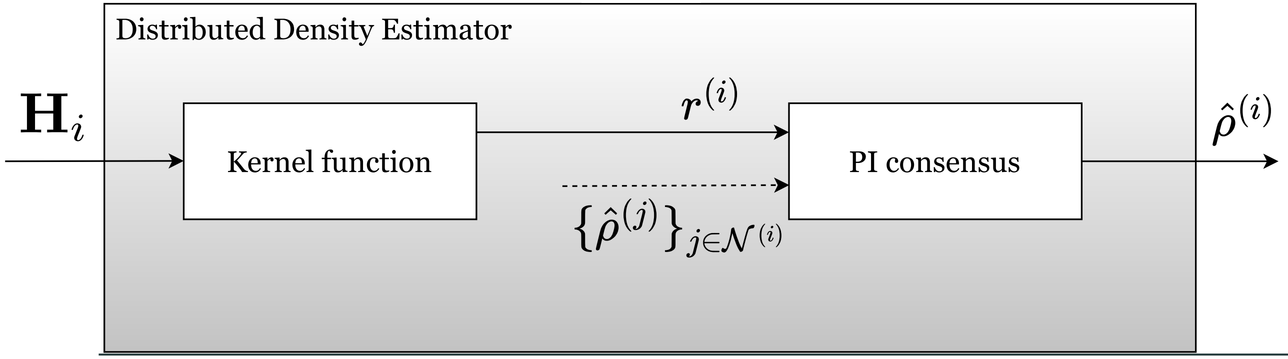
B. Di Lorenzo et al, *Decentralized Continuification Control of Multi-Agent Systems via Distributed Density Estimation*, CSS Letters, 2025

Distributed Density Estimation

- We consider a distributed density estimation scheme based on PI consensus







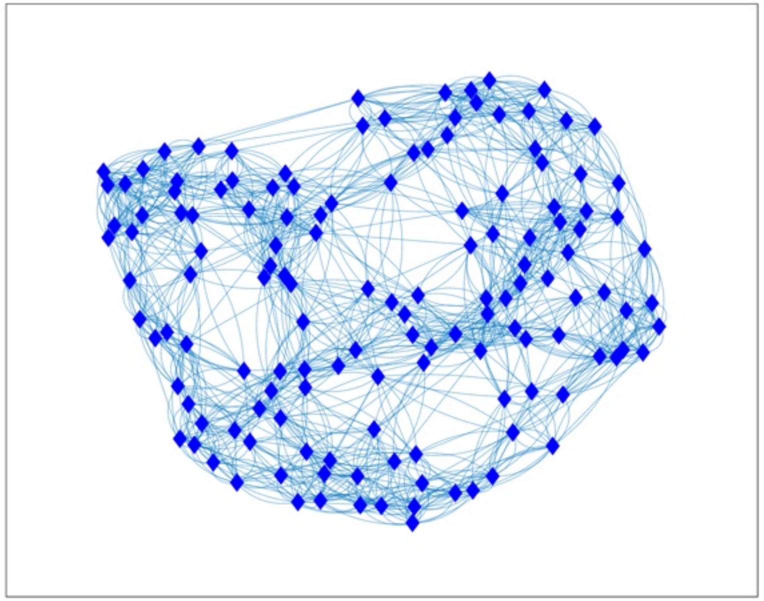
$$\hat{\boldsymbol{\rho}}_t = -\alpha\left(\hat{\boldsymbol{\rho}} - \mathbf{r}\right) - \sigma_P \mathbf{L}\hat{\boldsymbol{\rho}} - \sigma_I \mathbf{L}\int_0^t \hat{\boldsymbol{\rho}}\,\mathrm{d}\tau$$

B. Di Lorenzo, et al, *Decentralized Continuification Control of Multi-Agent Systems via Distributed Density Estimation*, CSS Letters, 2025

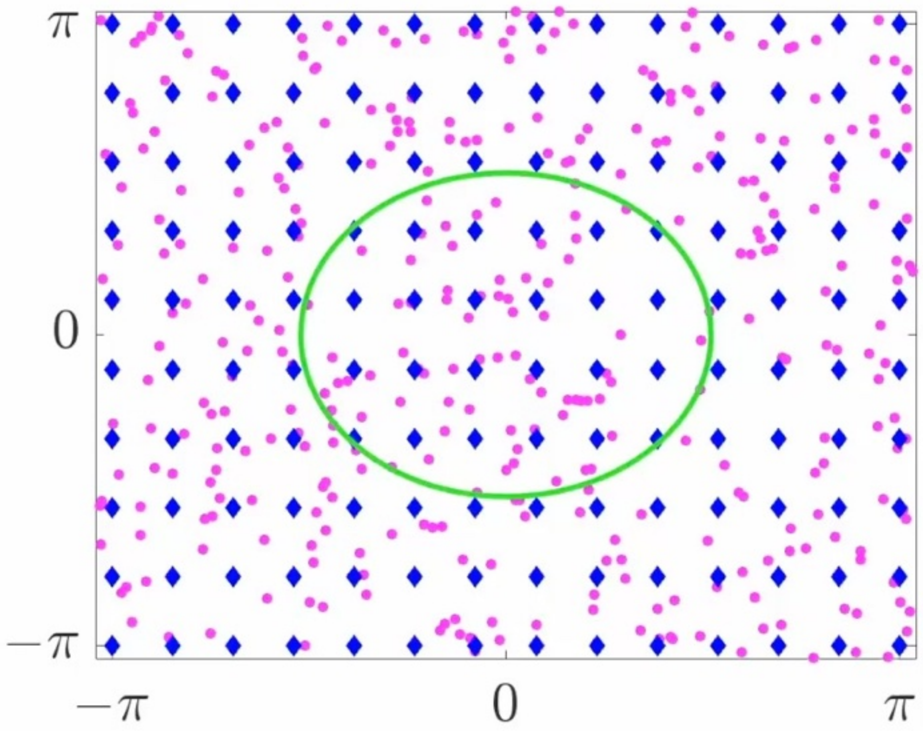
A fully decentralized strategy

- By using the online kernel-based density estimation strategy we can solve the shepherding problem in a fully decentralized manner

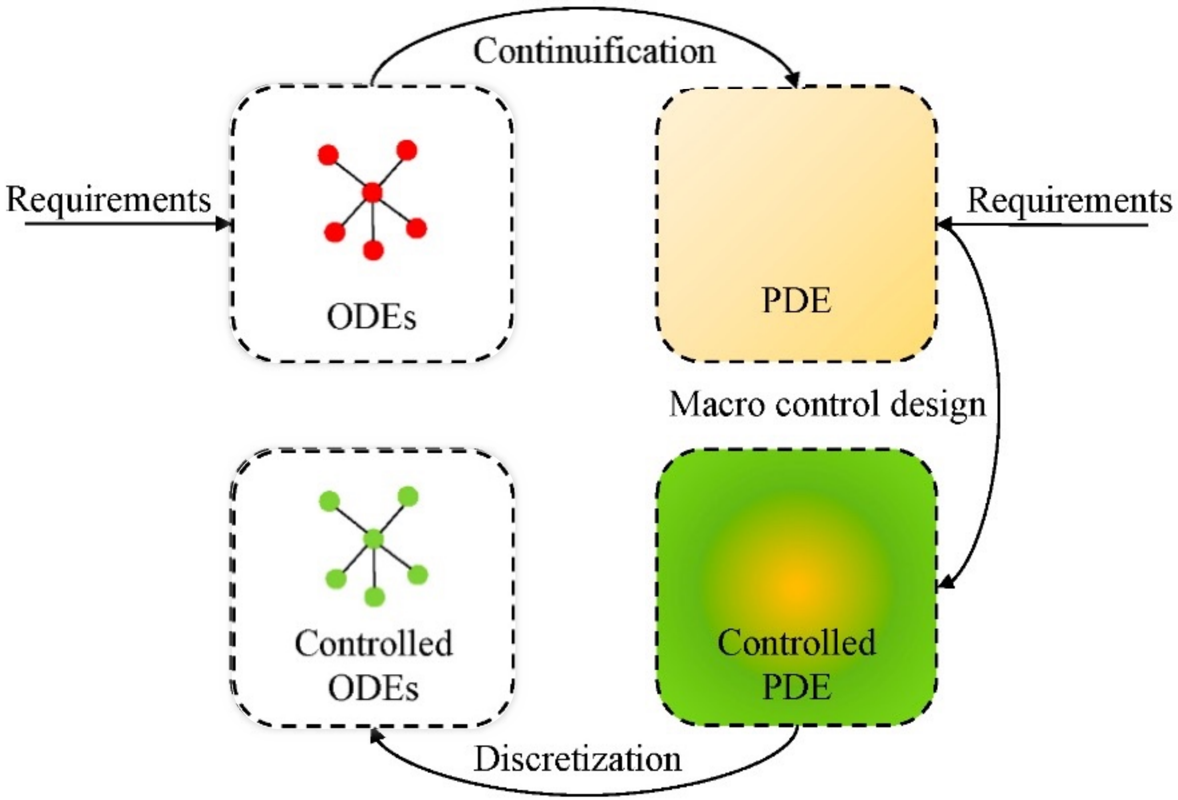
Herders communicate on a
10-nearest neighbors topology



$N^H = 140, \quad N^T = 360$



Continuification-based control



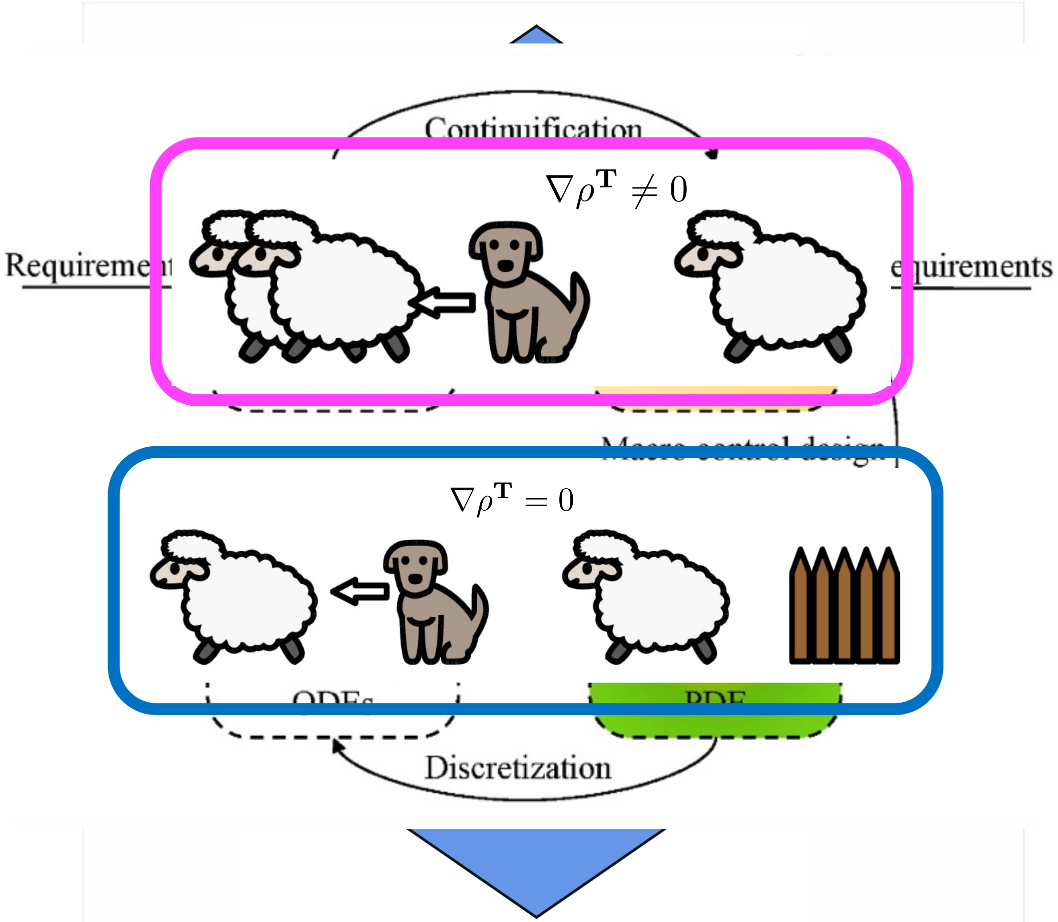
- Continuification
  - Mean field
  - Graphons
- Macroscopic control design
  - in domain
  - boundary
- Discretization
  - Spatial sampling
  - Finite differences schemes for index-dependent PDEs

Nikitin, D., et al, *A continuation method for large-scale modelling and control: from ODEs to PDE, a round trip*, IEEE TAC, 2021
G.C. Maffettone et al. *Continuification control of large-scale multiagent systems in a ring*, IEEE Control Systems Letters, 2022

Conclusions and Open Challenges

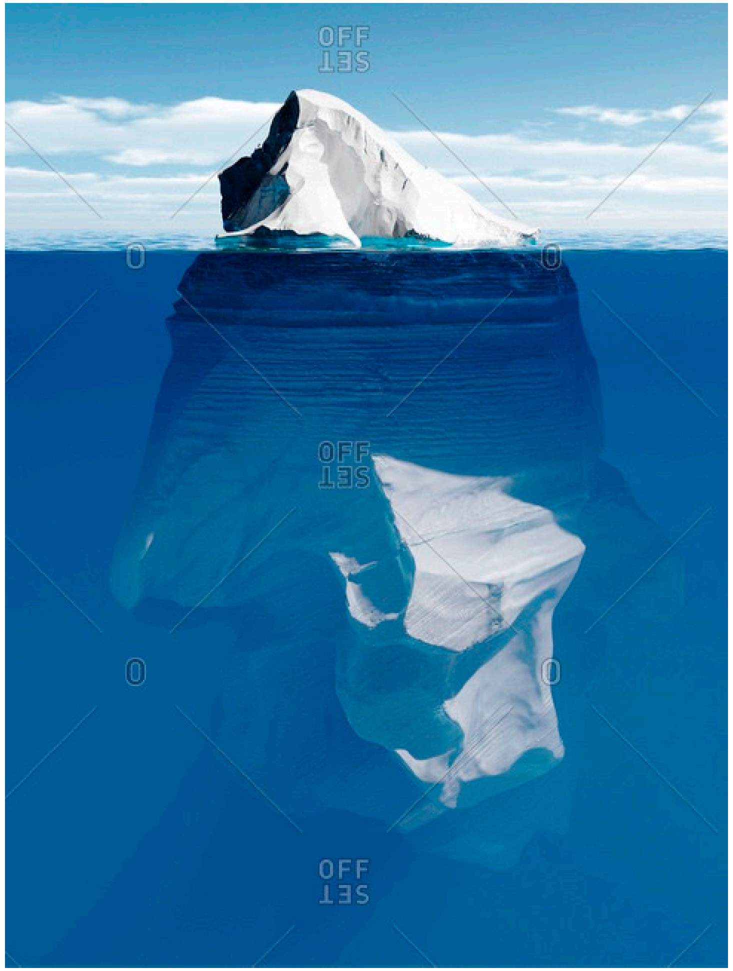
Conclusions

- Controlling complex systems requires bridging the gap between different scales
- *Shepherding* is a great paradigmatic example..
- ..where emerging behaviour needs to arise so that a distributed control task can be solved
- We saw both *microscopic* control solutions and *macroscopic* solution
- *Field equations* where derived in the presence of decision-making controlled agents
- The problem was formulated and solved as a density control problem and then discretized (*continuification-based control*)



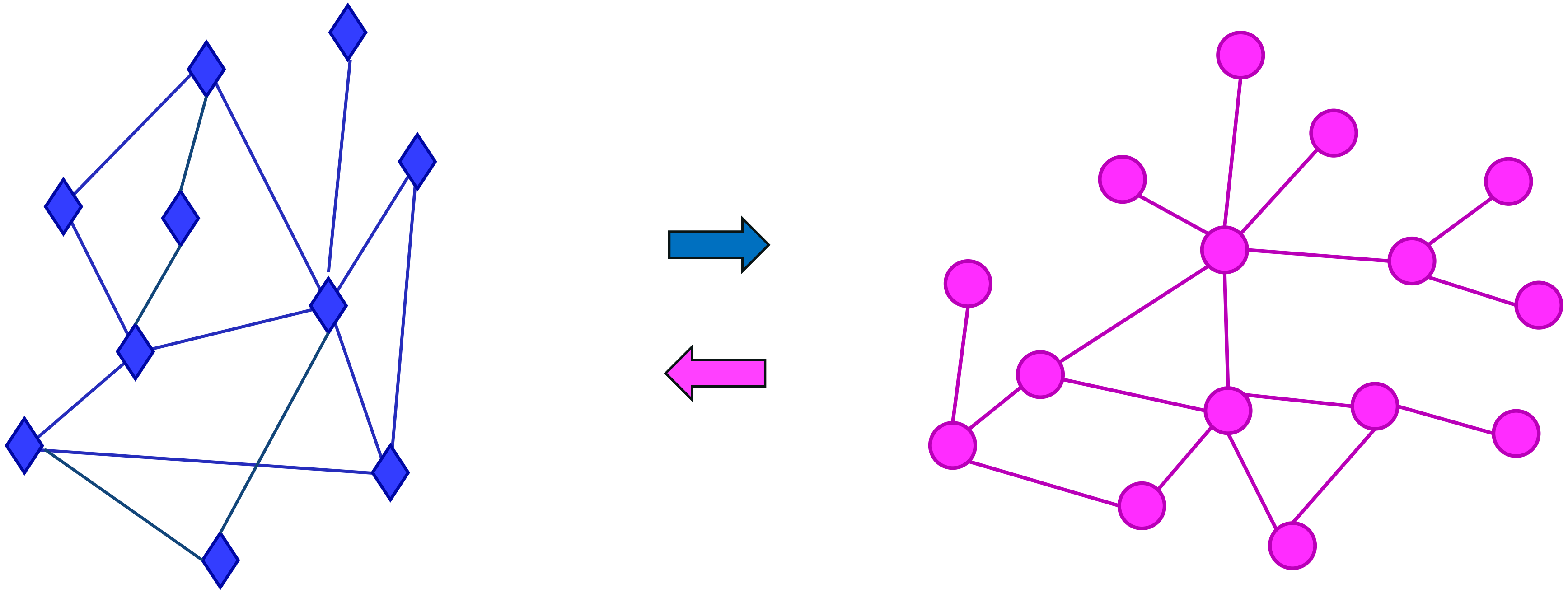
Challenges and open problems

- How do we engineer local rules of interaction for more complex tasks than shepherding?
- What if the targets actively escape from herders?
- What if the herders actively search for targets?
- How to obtain field equations (continuify) more complex microscopic control laws?
- Can we use macroscopic descriptions and continuification for other large-scale systems?
- Are there better ways to discretize macroscopic control laws to obtain microscopic control inputs?



Harnessing complex systems for control

Can we engineer the emerging collective behaviour of a complex multiagent system to solve a distributed control task?



Why this matters now?

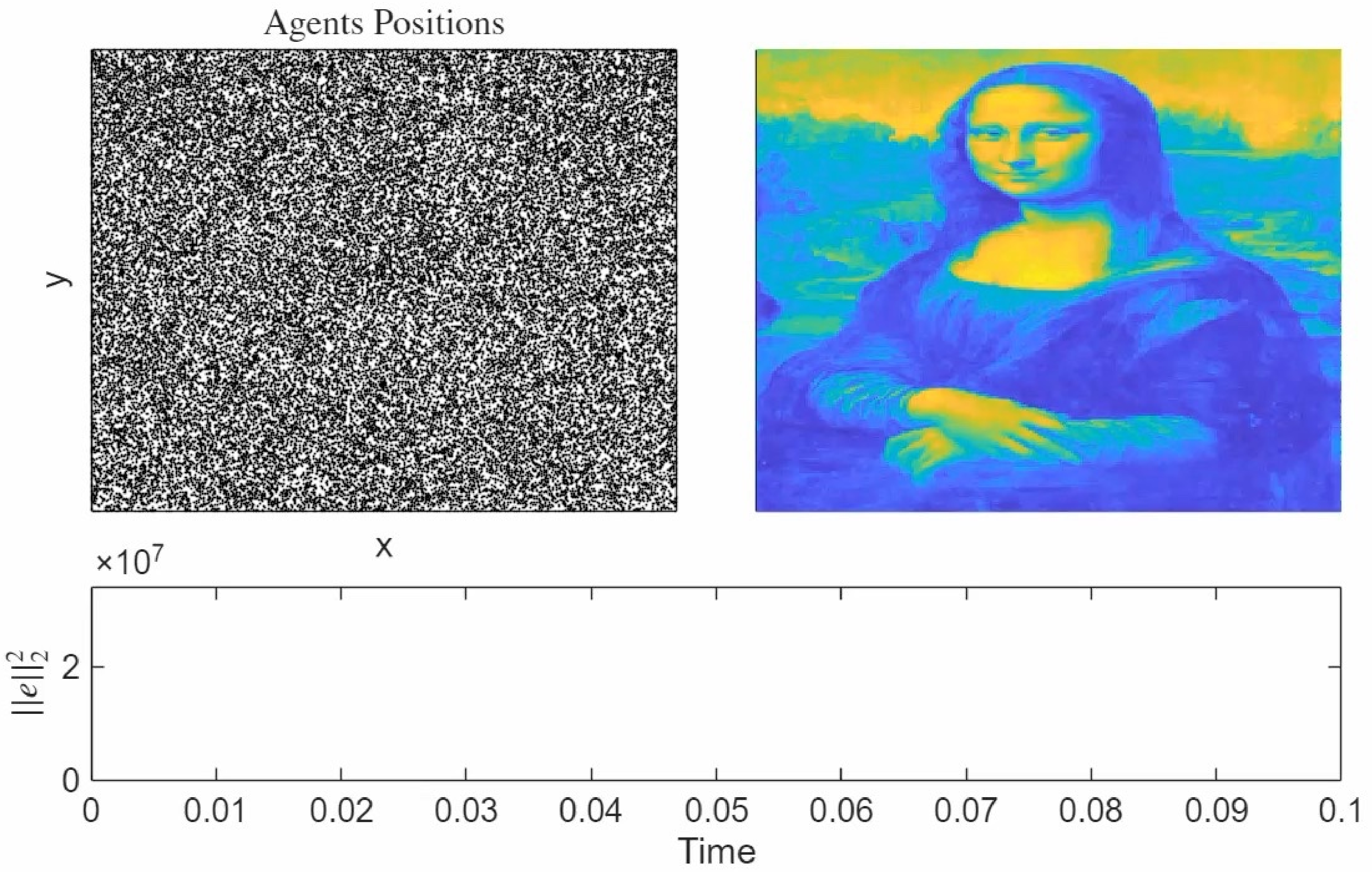
- Large-scale complex systems abound (drone swarms, cells, smart cities etc)
- Traditional methods are too expensive and inefficient to control these systems: *scale, emergence and nonlinearities are uniquely entangled*
- We need complex systems to control their own behaviour and self-organize to solve distributed control tasks



… and maybe one day..

G. C. Maffettone

D. Salzano



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NYU

SSM

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Thank you for your attention

感谢各位的关注

https://www.linkedin.com/in/mario-di-bernardo/

Our publications on which this talk is based

1. M. di Bernardo, "Controlling Collective Behavior in Complex Systems", in J. Baillieul, T. Samad (Eds), Encyclopedia of Systems and Control, Springer, London, 2020
2. F. Auletta, D. Fiore, M. J. Richardson, M. di Bernardo, "Herding stochastic autonomous agents via local control rules and online global target selection strategies", *Autonomous Robots*, 46(3), 469-481, 2022
3. G.C. Maffettone, A. Boldini, M. di Bernardo, M. Porfiri, "Continuification control of large-scale multiagent systems in a ring", *IEEE Control Systems Letters*, vol. 7, pp. 841-846, 2022
4. R. M. D'Souza, M. di Bernardo, and YY. Liu. "Controlling complex networks with complex nodes." *Nature Reviews Physics* 5, 250–262, 2023.
5. F. Auletta, R.W. Kallen, M. di Bernardo, M. J. Richardson. "Predicting and understanding human action decisions during skillful joint-action using supervised machine learning and explainable-AI." *Scientific Reports* 13, 4992, 2023
6. G. C. Maffettone, M. Porfiri, M. di Bernardo, "Continuification control of large-scale multiagent systems under limited sensing and structural perturbations", *IEEE Control Systems Letters*, vol. 7, pp 2425-2430, 2023
7. A. Lama, M. di Bernardo, "Shepherding and herdability in complex multiagent systems", *Physical Review Research*, vol. 6, no. 3, p. L032012, 2024

Our publications (cont.)

8. G.C. Maffettone, L. Liguori, E. Palermo, M. di Bernardo, M. Porfiri, "Mixed Reality Environment and High-Dimensional Continuification Control For Swarm Robotics", *IEEE Trans. on Control Syst. Technology*, vol. 32, no. 6, pp. 2484-2491, 2024
9. G.C. Maffettone, A. Boldini, M. Porfiri, M. di Bernardo, "Leader-Follower Density Control of Spatial Dynamics in Large-Scale Multi-Agent Systems", IEEE Transactions on Automatic Control (early access), 2025
10. B. Di Lorenzo, G.C. Maffettone, M. di Bernardo, "A Continuification-Based Control Solution for Large-Scale Shepherding", European Journal of Control (in press), also accepted for presentation at the European Control Conference 2025
11. A. Lama, M. di Bernardo, Sabine H. L. Klapp, "Nonreciprocal field theory for decision-making in multi-agent control systems", Nature Communication, 2025 (to appear)
12. B. Di Lorenzo, G.C. Maffettone, M. di Bernardo "Decentralized Continuification Control of Multi-Agent Systems via Distributed Density Estimation", IEEE Control Systems Letters, vol. 9, pp. 1580-1585, 2025.
13. I. Napolitano, A. Lama, F. De Lellis, M. di Bernardo, "Emergent Cooperative Strategies for Multi-Agent Shepherding via Reinforcement Learning", 23rd European Control Conference (ECC), 24-27 June, pp. 1809-1814, 2025.
14. S. Covone, I. Napolitano, F. De Lellis, M. di Bernardo, "Hierarchical Policy-Gradient Reinforcement Learning for Multi-Agent Shepherding Control of Non-Cohesive Targets", accepted for presentation at IEEE CDC 2025