Fast Iterative Learning Control Algorithms Based on Heavy Ball with Adaptive Stepsize

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1. Introduction

Iterative Learning Control (ILC):

- Repetitive systems
- High transient performance
- Fast convergence

Heavy ball (HB) strategy:

- First-order method
- Reduce oscillation
- Speed up convergence

Contributions:

- Apply the HB strategy to first-order ILC
- Modify Proportional (P)-type ILC by HB
- Give an adaptive stepsize for HB

First-order ILC + Heavy ball + adaptive stepsize; P-type ILC + Heavy ball
→ Linear discrete-time system
→ Perfect trajectory tracking

2. Method

- Two fast ILC algorithms

$$U_{k+1} = U_k - \alpha L E_k + \boldsymbol{\theta}\,(\boldsymbol{U_k} - \boldsymbol{U_{k-1}})$$

$$Y_k = HU_k + d$$

$$u_{k+1}(t) = u_k(t) - \alpha l e_k(t+1) + \boldsymbol{\theta \Delta u_k(t)}$$

$$\begin{cases} x_k(t+1) = Ax_k(t) + bu_k(t) \\ y_k(t) = cx_k(t) \end{cases}$$

- Convergence conditions

① $\mathrm{real}(\lambda_i(HL)) > 0$; $\alpha,\ \theta < c$

④ $\theta_k = \theta \dfrac{|\Delta E_k^T E_k|}{\|\Delta E_k\| \|E_k\|}$

⇒ $\lim_{k\to\infty} E_k = 0$ ⇔

② $\rho\begin{pmatrix} 1 - cbl + \theta & -\theta \\ 1 & 0 \end{pmatrix} < 1$

③ $|1 - cbl + \theta| + |\theta| < 1$

First-order ILC + Heavy ball:

- Sufficient condition ① for first-order ILC
- The greater $\|I - \alpha HL\|_Q$, the smaller θ should be. If $\|I - \alpha HL\|_Q + \|\alpha HL\|_Q < 1$, $\theta < 1/2$ is enough.
- Suitable for MIMO systems.

P-type ILC + Heavy ball:

- A necessary and sufficient condition ②
- An easily verifiable sufficient condition ③

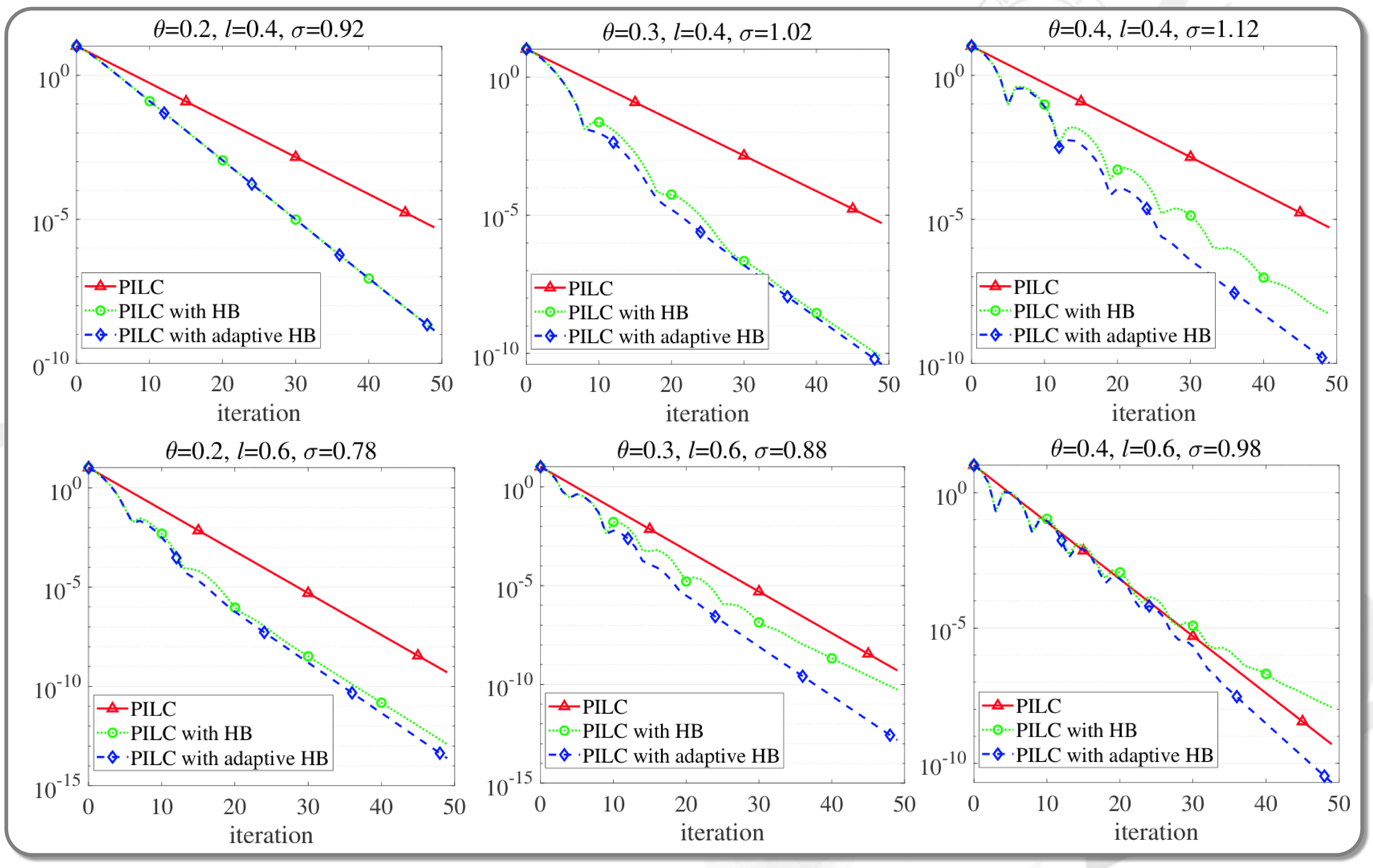
Adaptive stepsize:

- Stepsize θ can be replaced by adaptive stepsize θ_k ④
- Mitigate forward direction rotation

3. Simulation

P-type ILC, P-type ILC + HB, P-type ILC + HB + adaptive stepsize are tested. Each figure corresponds to a gain and a stepsize. Findings:

- The error linearly converges to zero.
- Condition ③ is not necessary for error convergence, nor for faster speed.
- HB can speed up the P-type ILC.
- Adaptive stepsize does not slow down the speed, and enhances HB.



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