RMIT UNIVERSITY

Complex Network Systems:

A Control Science and Engineering Perspective

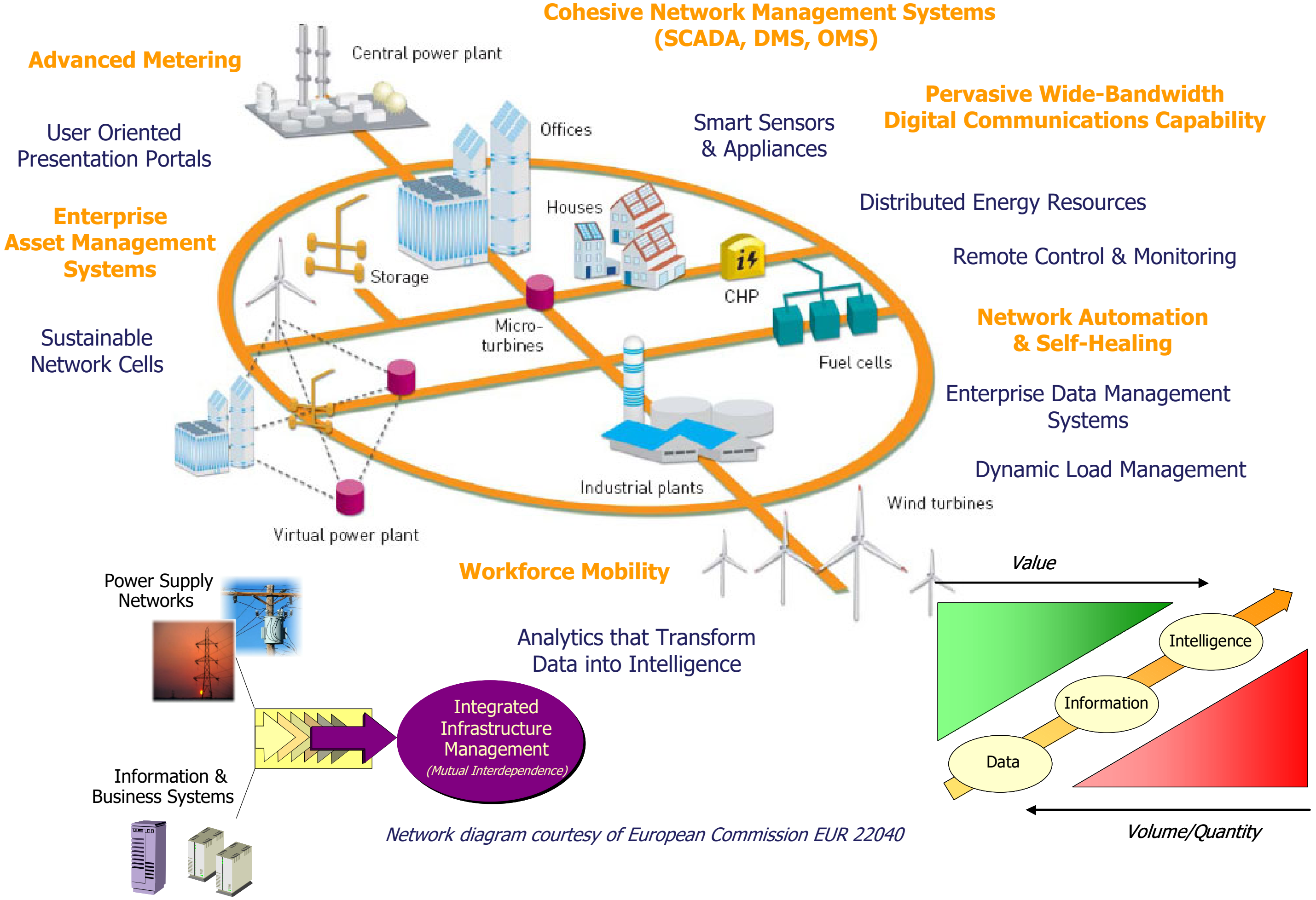
Xinghuo Yu
Royal Melbourne Institute of Technology, Australia

What's next...

Table of Contents

Table of Contents

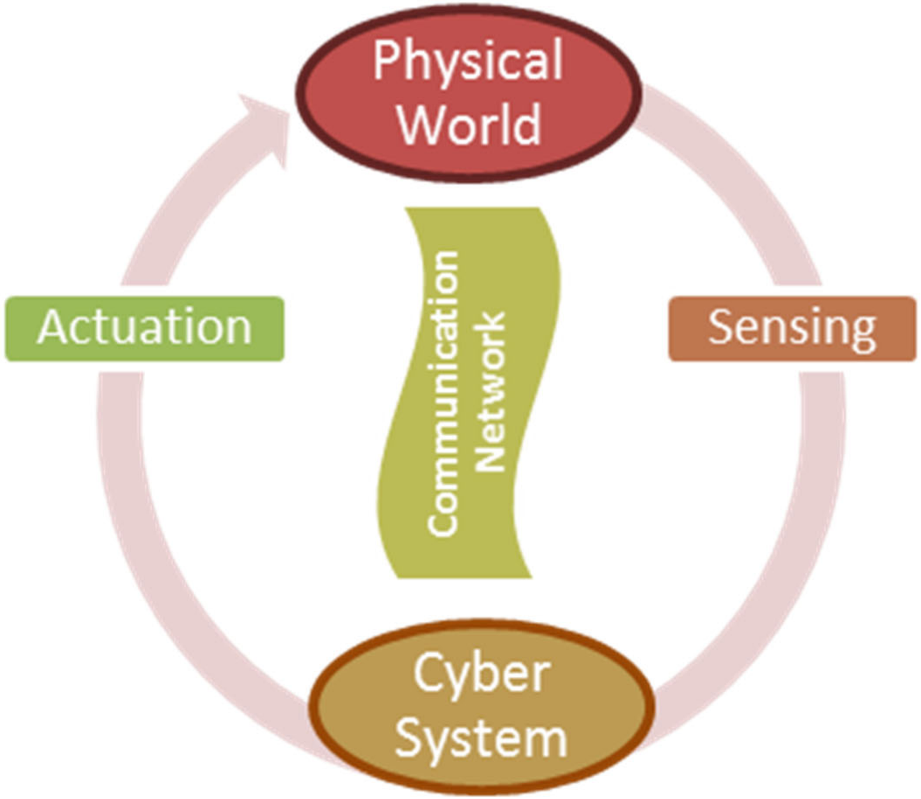
Smart Grid as CNS



Network diagram courtesy of European Commission EUR 22040

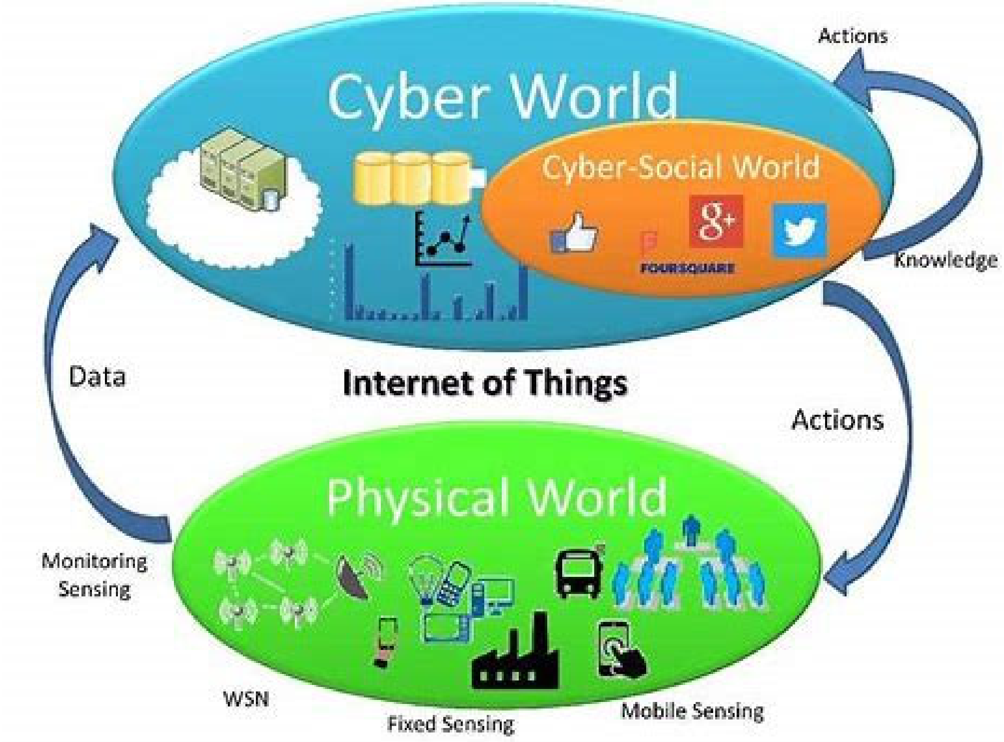
Cyber-Physical World

A broad range of complex, multi-disciplinary, physically-aware next generation engineered systems that integrate embedded computing technologies (cyber part) into the physical world.



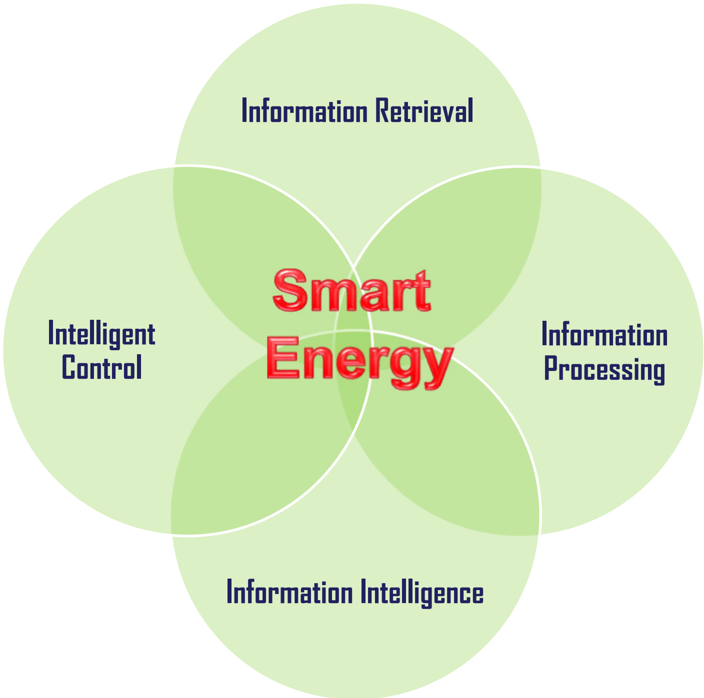
X. Yu, Y. Xue, “Smart Grids: A cyber-physical systems perspective,” Proc IEEE, 104(5): 1058-1070, 2016

Cyber-Physical-Social Systems (CPSS)



S. De, Y. Zhou, A. L. Abad, K. Moessner, "Cyber-Physical-Social Frameworks for Urban Big Data Systems: A survey," Appl Sc, vol. 7, 2017

CPSS in Smart Energy



Underpinned by

- Super high dimensions (spatial scale)
- Real-time (temporal scale) responses
- Intermittency
- High complexity (structured and non-structured, topology, varying conditions)
- Cyber-physical spaces
- Social-economic aspects
- Cybersecurity

The key question: How to handle the exponentially growing size and complexity of Smart Energy Systems effectively and timely?

The "no free lunch theorem" of Wolpert and Macready **…**

Computational complexity for solving a large scale problem cannot be reduced regardless of what algorithms you may use …

We have to have leap of faith … !

The "**simple solutions for complex problems**" problem solving paradigm …

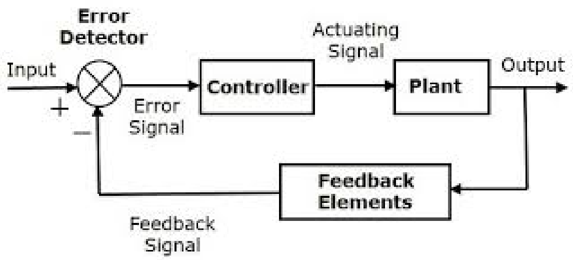
Learning from **NATURE** to deal with future challenges …

D.H. Wolpert, W.G. Macready, "No free lunch theorems for optimization," IEEE T-EC, 1(1): 67-82, 1997.

Table of Contents

Control Theory and Methods

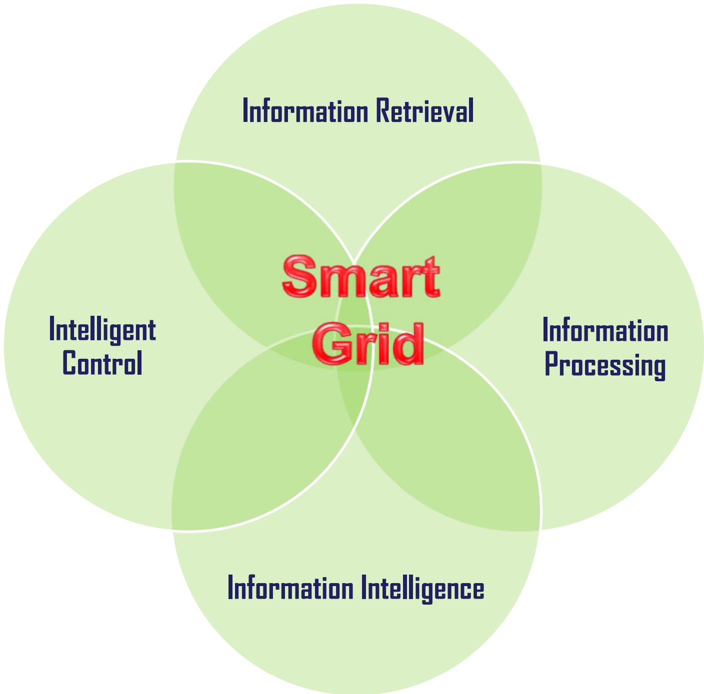
- Open Loop vs Closed Loop
- Linear vs Nonlinear System
- Classic vs Modern Theories
- Frequency vs Time Domains
- SISO vs MIMO
- Centralised vs Distributed
- Deterministic vs Stochastic



Control Systems

- Controllability & Observability
- Stability
- Control Specification
- Model Identification
- Data-driven
- Control Types: Optimal/Adaptive/Robust/ Intelligent/Stochastic/ Switching …

Control in Smart Grid



Underpinned by

- Super high dimensions (spatial scale)
- Real-time (temporal scale) responses
- Intermittency
- High complexity (structured and non-structured, topology, varying conditions)
- Cyber-physical spaces
- Social-economic aspects
- Cybersecurity

How to handle the exponentially growing size and complexity effectively and timely?

An inspiration:
300+ years ago,
Brook Taylor
introduced
Taylor Series
Expansion!

Taylor Series Expansion

Taylor's Theorem : Suppose *f* is continuous on the closed interval [a, b] and has *n + 1* continuous derivatives on the open interval (a, b). If *x* and *c* are points in (a, b), then

The Taylor series expansion of $f(x)$ about c:

$$f(c) + f'(c)(x-c) + \frac{f^{(2)}(c)}{2!}(x-c)^2 + \frac{f^{(3)}(c)}{3!}(x-c)^3 + \dots$$

or

$$Taylor\ Series = \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(c)\ (x-c)^k$$

If the series converge, we can write:

$$f(x) = \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(c)\ (x-c)^k$$

Case 1: Finding minimum set of control nodes

The concept of controllability

- For a canonical linear, time –invariant dynamics

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \qquad [1]$$

$\mathbf{x}(t)$: states of N nodes at time t. A: interaction between nodes. B: control matrix. $\mathbf{u}(t)$: time dependent input vector.

- System is controllable if and only if controllability matrix

$$\mathbf{C} = (\mathbf{B}, \mathbf{AB}, \cdots, \mathbf{A}^{N-1}\mathbf{B}) \qquad [2]$$

has full rank (Kalman's controllability rank condition [1]).

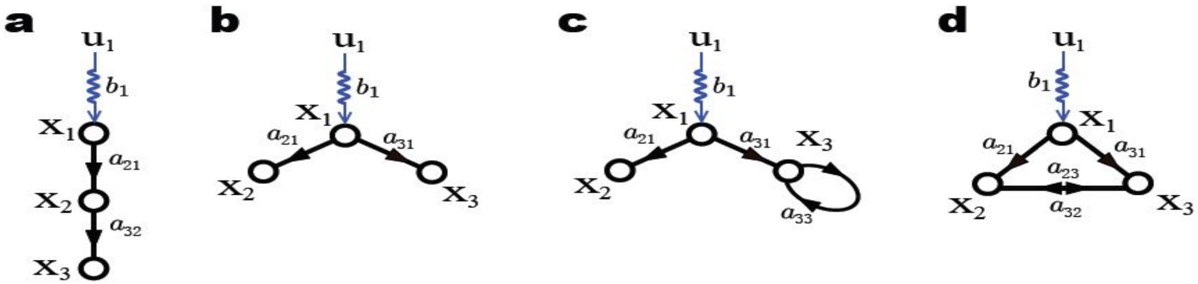
$$\mathrm{rank}(\mathbf{C}) = N \qquad [3]$$

[1] Kalman *et al*, Contrib. Differ. Equ. 1 ,1962

The concept of structural controllability

In the controllability matrix Q: $$Q = [B \;\; AB \;\; \cdots \;\; A^{n-1}B]$$

All 0 are fixed. There is a realization of independent nonzero parameters such that Q has full row rank



$$\mathbf{C} = [\mathbf{B}, \mathbf{A}\cdot\mathbf{B}, \mathbf{A}^2\cdot\mathbf{B}]$$

$$Q = b_1\begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & 0 & a_{32}a_{21} \end{bmatrix}, \quad b_1\begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & 0 \end{bmatrix}, \quad b_1\begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & a_{33}a_{31} \end{bmatrix}, \quad b_1\begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & a_{23}a_{31} \\ 0 & a_{31} & a_{32}a_{21} \end{bmatrix}$$

$\text{rank}\,\mathbf{C} = 3 = n$ — controllable

$\text{rank}\,\mathbf{C} = 2 < n = 3$ — uncontrollable

$\text{rank}\,\mathbf{C} = 3 = n$ — controllable

$\text{rank}\,\mathbf{C} = ?$ — controllable?

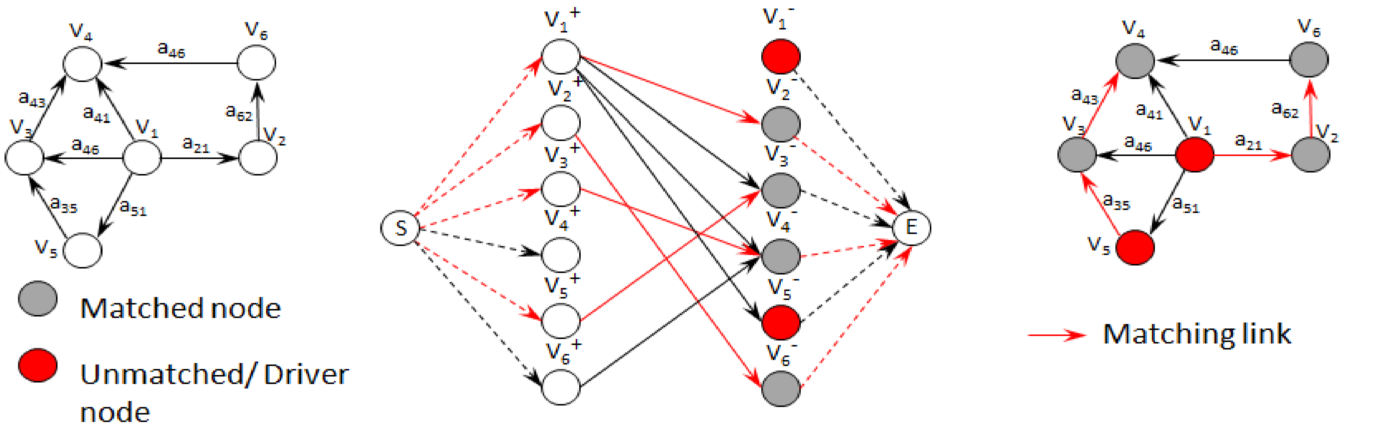
Y. Y. Liu, J. J. Slotine, A. L. Barabasi, Nature, 2011

Structurally controllable

Case 1: Finding minimum set of control nodes

Using structural controllability to find minimum number of driver nodes

- Liu *et al* [2] proved that the minimum number of driver nodes equal to number of unmatched nodes from maximum matching algorithm

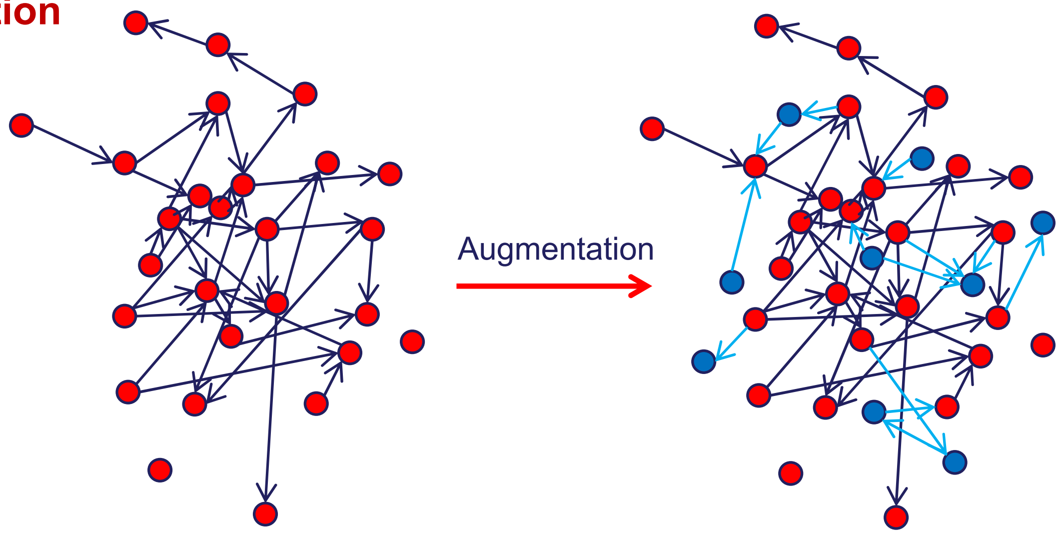


Directed Network **Maximum Matching Algorithm** **Minimum Driver Nodes**

[2] Y. Y. Liu, J. J. Slotine, A. L..Barabasi, Nature, 2011

Case 1: Finding minimum set of control nodes

Network augmentation



Problem: how to maximise system structural controllability with minimum number of driver nodes unchanged? There are a lot of applications.

J. Wang, X. Yu, L. Stone, "Effective augmentation of complex networks," Scientific Reports, vol. 6, 25627, 11 May 2016

Case 2: Selecting the best driver nodes for synchronisability

Network Model

Linearly coupled network:

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N}\beta_{ij}Hx_j \qquad x_i \in R^n \quad i = 1,2,\ldots,N$$

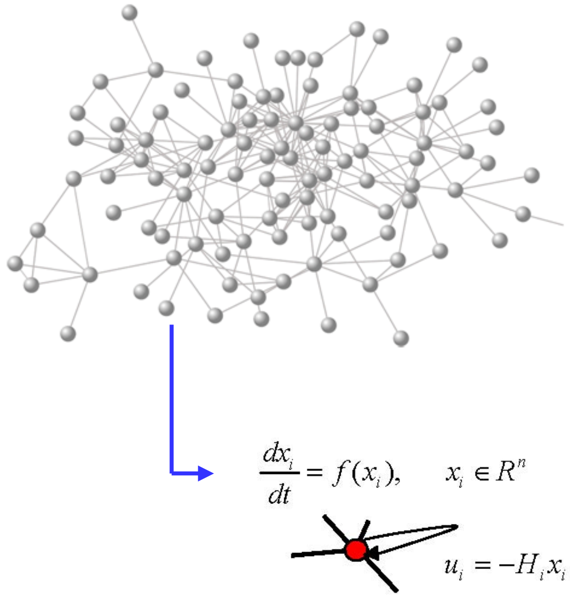
- General assumption: $f(.)$ is Lipschitz. Here, it is linear (or linearized):

$$\dot{x}_i = Ax_i + c\sum_{j=1}^{N}\beta_{ij}Hx_j \qquad x_i \in R^n \quad i = 1,2,\ldots,N$$

- Coupling strength $c > 0$ and H – input coupling matrix

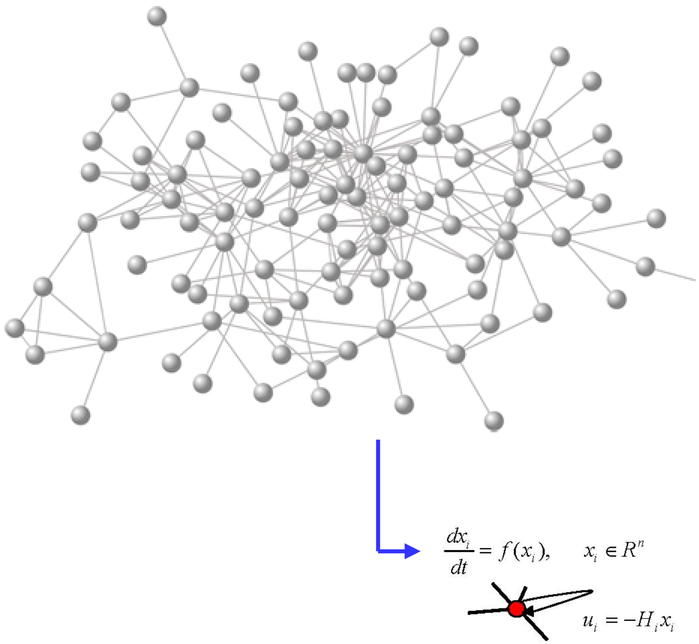
- Adjacency matrix: $\lfloor \beta_{ij} \rfloor_{N\times N}$

If node i points to node j $(j \neq i)$, then $\beta_{ij} = 1$; otherwise $\beta_{ij} = 0$; and $\beta_{ii} = 0$



For undirected networks, $\lfloor \beta_{ij} \rfloor_{N\times N}$ is symmetrical; for directed networks, may not be so

Case 2: Selecting the best driver nodes for synchronisability



Objective: To achieve a certain control goal

Questions:

- How many controllers to use?
- Where to put them?

→ **Pinning Control**

$$\dot{x}_i = Ax_i + c\sum_{j=1}^{N} \beta_{ij} H x_j + \delta_i B u_i$$

$$\delta_i = \begin{cases} 1 & if \quad to-control \\ 0 & if \quad not-control \end{cases}$$

X Li, X F Wang, G Chen, Pinning a complex dynamical network to its equilibrium, IEEE T-CAS I, 51: 2074-2087, 2004

W Yu, G Chen, J Lu, J Kurths, Synchronization via pinning control on general complex networks, SIAM J. Contr. Optim., 51:1395-1416, 2013

Case 2: Selecting the best driver node for synchronisability

Suppose that all nodes of a complex network should be pinned (synchronized) to the following desired state:

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_N(t) = \mathbf{s}(t), \text{ as } t \to \infty,$$

$$\frac{d(s(t))}{dt} = F(s(t))$$

One should design the following state feedback:

$$\frac{dx_i}{dt} = F(x_i) - \sigma \sum_{j=1}^{N} l_{ij} H x_j + u_i, u_i = -\sigma \beta_i k_i (s - x_i)$$

k_i : the feedback gain
β_i=1 for driver nodes, otherwise β_i=0.

- Augmented Laplacian matrix

$$C = \{c_{ij}\} = \begin{bmatrix} l_{11} + k_1\beta_1 & l_{12} & \cdots & l_{1N} \\ l_{21} & l_{22} + k_2\beta_2 & \cdots & l_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ l_{N1} & l_{N2} & \cdots & l_{NN} + k_N\beta_N \end{bmatrix}$$

λ_i: the i^{th} eigenvalue of the augmented Laplacian matrix

- The metric R is defined[2] as a measure of controllability

$$R = \frac{\lambda_N}{\lambda_1}$$

Smaller R results in better synchronizability (i.e. synchronizability over wider coupling strength).

[1] X. F. Wang, G. Chen, “Pinning control of scale-free dynamical networks,” *Physica A,* vol. 310, pp. 521-531, 2002
[2] F. Sorrentino*, et al.*, "Controllability of complex networks via pinning," *Physical Review E,* vol. 75, p. 046103, 2007.

Case 2: Selecting the best driver node for synchronisability

Heuristics:

- **nodes with highest degree**
- **nodes with highest betweenness centrality**
- **nodes with highest closeness centrality**

Using *R*:

- **Calculate** $\Delta R = R_0 - R_i$ for all nodes $i=1,2,\ldots N$; select the node with maximum ΔR.

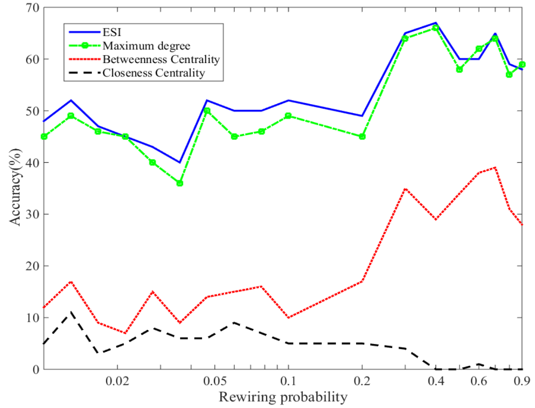
- $R_0 = (\lambda_N/\lambda_1)$ for the main Laplacian matrix.
- $R_i = (\lambda_i/\lambda_1)$ for the augmented Laplacian matrix when node i is controlled.

- **This method is time consuming in large complex networks. Heuristice are also not accurate enough.**

- **Eigenvalue perturbation analysis**
- **Eigen-ratio sensitivity analysis**

$$ESI(i) = \left(x_N^i\right)^2$$

x_n is eigenvectors of L related to λ_n and i shows i^{th} element.



Watts-Strogatz complex network with ***N*=1000** nodes.

A. Moradi Amani, M. Jalili, X. Yu, L. Stone, "Finding the most influential nodes in pinning controllability of complex networks," IEEE T-CAS II, 64(6): 685-689 2017.

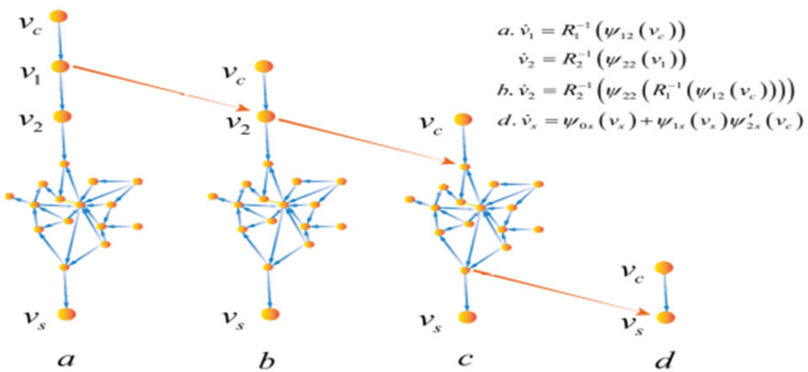
Case 3: Characteristic Modelling Approach (吴宏鑫院士创立)

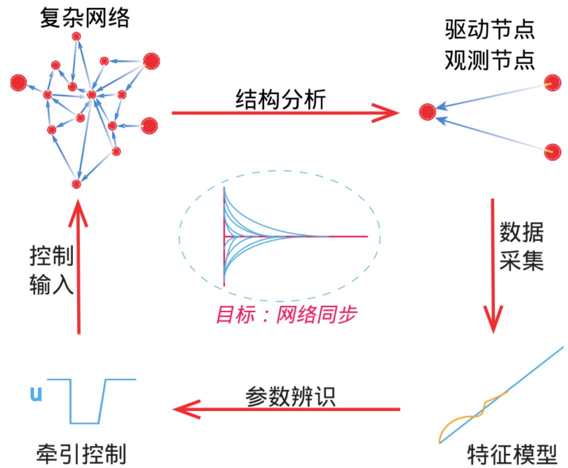
$$\frac{y(s)}{u(s)} = \frac{\sum_{j=0}^{m_0} b_j s^j}{s^{n_0} + \sum_{i=0}^{n_0-1} a_i s^i}, a_i, b_j \in \mathbb{R}$$

Error $O(h^2)$

$$y(k+1) = (2 + w_1(k))y(k) + (-1 + w_2(k))y(k-1) + w_3(k)u(k) + w_4(k)u(k-1)$$

$$\mathcal{V}(k+1) = \hat{\mathcal{A}}_1(k)\mathcal{V}(k) + \hat{\mathcal{A}}_2(k)\mathcal{V}(k-1) + (A \circ \hat{\mathcal{B}}(k))\mathcal{V}(k)$$





L. Chen, X. Yu, C. Sun. Characteristic modeling approach for complex network systems. IEEE T-SMC Syst. 48(8):1383-1388, 2018

Table of Contents

Switching phenomena are everywhere in CNSs

e.g. links on-off intentionally or unintentionally, switching control mechanisms for fast transient responses.

Switching can be classified in two clusters:

Open-loop switching means the switching occurs without influences from any internal system states and external influences

Closed-loop switching means the switching depends on internal systems states and external influences

Switching Patterns

Switching phenomena are everywhere in CNSs.

The frequency influences significantly the behaviours of system dynamics

Low frequency - If it is ‘low’, provided each switching action sustains sufficient ‘long’ time, many existing methodologies for non-switching dynamics can be used by ‘piecing-together’ various ‘smooth’ subsystems – in time or in state.

Medium frequency – Medium frequency is really the area of special interest as piecing-together smooth subsystems may not give accurate picture though methodologies such as Lyapunov theory and Averaging theory may still be applied with caution.

High frequency – The system with high frequency switching tends to violate the theories for usual ‘smooth’ dynamical systems such as the existence and uniqueness of general solutions to smooth ordinary differential equations; drastically different methods such as Filippov theory should be used!

Benefits of Switching

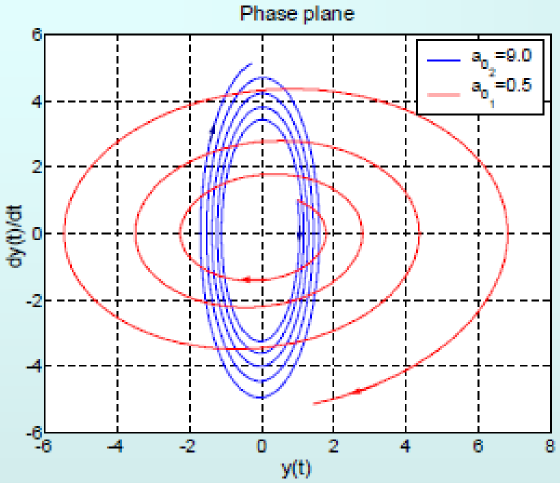
The benefits of switching are enormous … for example,

$$\ddot{y} + a_1\dot{y} + a_{01}y = 0 \qquad \text{sub}\textit{system 1}$$

$$\ddot{y} + a_1\dot{y} + a_{02}y = 0 \qquad \text{sub}\textit{system 2}$$

- $a_1 > 0$ the systems are both asymptotically stable.
- $a_1 = 0$ the systems are both marginally stable.
- $a_1 < 0$ the systems are both unstable.

Switching between unstable systems can yield a stable motion



$$\dot{y}(t)y(t) < 0 \quad system\,1$$

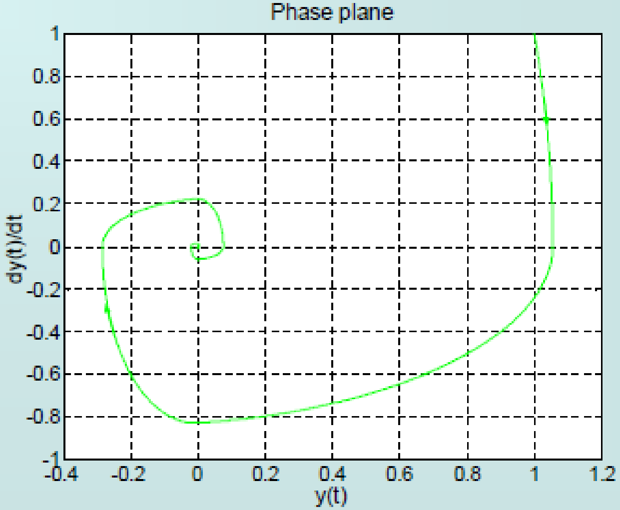
$$\dot{y}(t)y(t) \geq 0 \quad system\,2$$

$$0 < a_{01} < a_{02}$$

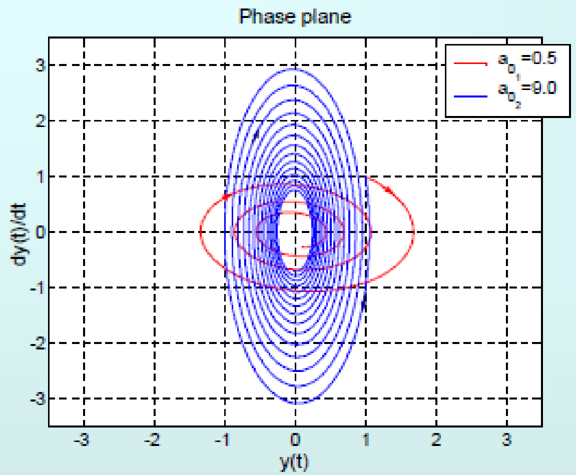
Switched asymptotically stable dynamics

$$a_1 = -0.1$$

Both dynamics are unstable



Switching between stable systems can result in an unstable motion



$$\dot{y}(t)y(t) \geq 0 \quad system\,1$$

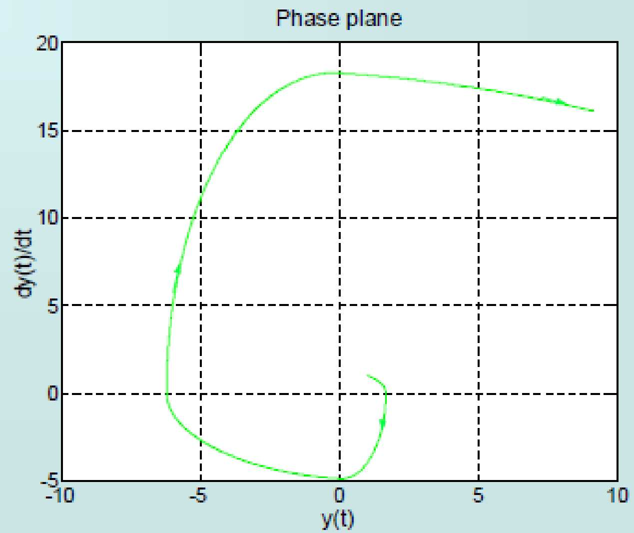
$$\dot{y}(t)y(t) < 0 \quad system\,2$$

$$0 < a_{01} < a_{02}$$

Switched unstable dynamics

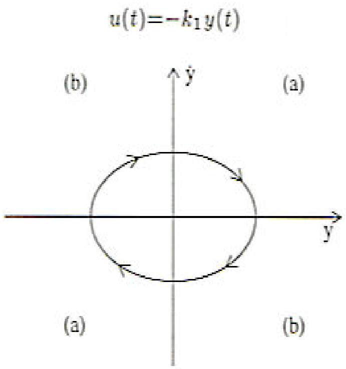
$$a_1 = 0.1$$

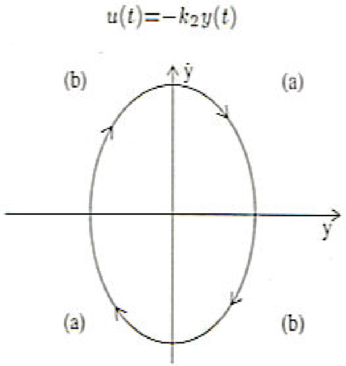
Both dynamics are asymptotically stable



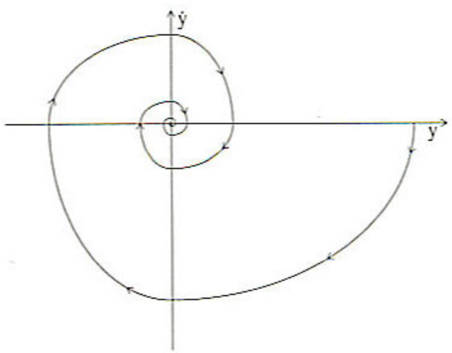
A simple case study

Consider a double integrator given by $\ddot{y} = u(t)$, for $0<k_1<1<k_2$





If we choose switching control $u = \begin{cases} -k_1 y & if \;\; y\dot{y} < 0 \\ -k_2 y & otherwise \end{cases}$



and a new Lyapunov function $V(y) = \frac{1}{2}(\dot{y}^2 + y^2)$

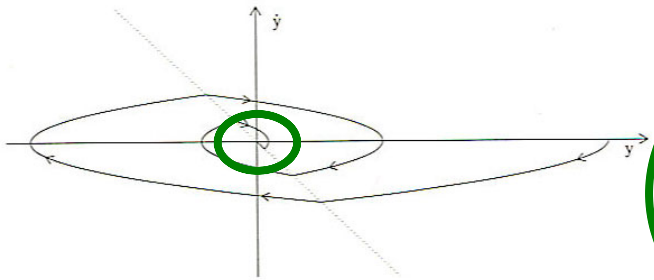
Then $\dot{V} = y\dot{y} + \dot{y}\ddot{y} = \dot{y}(y+u) = \begin{cases} y\dot{y}(1-k_1) & if \;\; y\dot{y} < 0 \\ y\dot{y}(1-k_2) & if \;\; y\dot{y} > 0 \end{cases}$

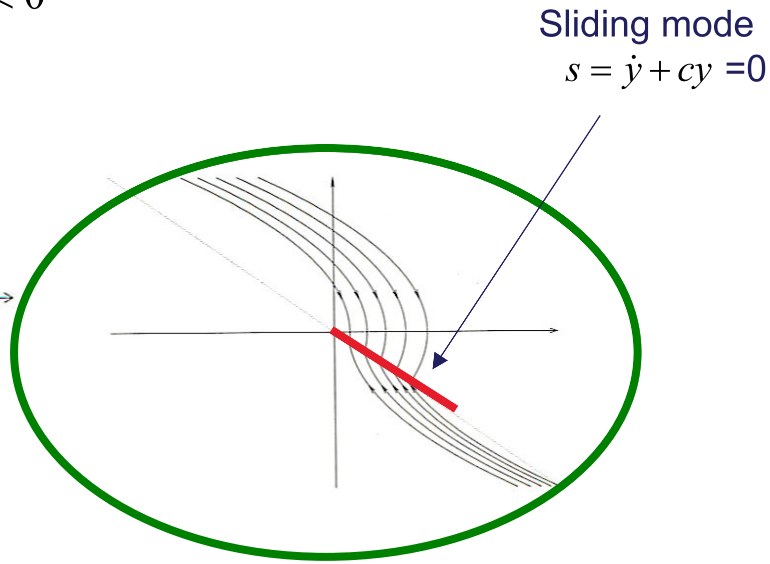
However, if we choose a switching line $s = \dot{y} + cy$

The control is chosen as $u = -k\,\mathrm{sgn}(s) = \begin{cases} -k & if\ \ s > 0 \\ k & if\ \ s < 0 \end{cases}$

Take $V(y) = |s| + \frac{c}{2k}\dot{y}^2$

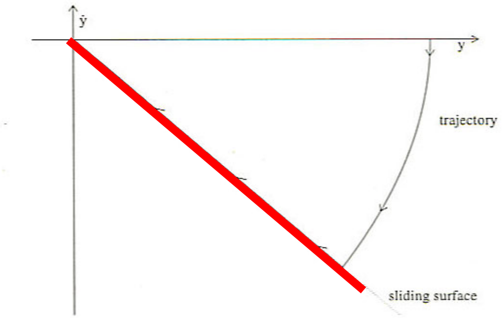
Then $\dot{V} = \mathrm{sgn}(s)(c\dot{y} - k\,\mathrm{sgn}(s)) - c\dot{y}\,\mathrm{sgn}(s) = -k|s| < 0$

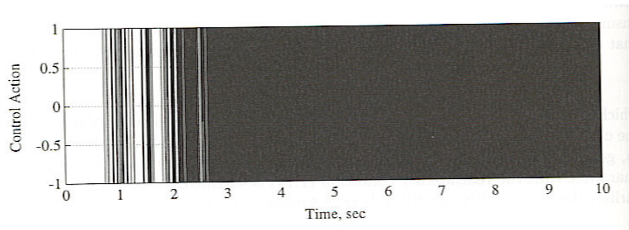




$c|\dot{y}| < 1$, then $s\dot{s} < 0$, sliding mode exists

When $s=0$ is reached, that is, $\dot{y} = -cy,\ c > 0$, $y(t) = \exp(-ct)y(0) \to 0,\ \ for\ \ t \to \infty$





Sliding mode control theory:

For $\dot{x} = f(x) + b(x)u$

1. Define a switching manifold which prescribe the desirable properties $s(x)$
2. Design a discontinuous control $u(x)$, $u = \begin{cases} u^+ & s(x) > 0 \\ u^- & s(x) < 0 \end{cases}$

   such that $\lim\limits_{s \to 0^+} \dot{s} < 0,\ \ and\ \ \lim\limits_{s \to 0^-} \dot{s} > 0$

Vadim Utkin, *Sliding Modes in Control and Optimisation*, Springer, 2013.

Drawbacks of Switching

Sensitivity to switching times leads to many problems in practice, e.g.

Consider a second order system

$$\dot{x}_1 = x_2 \qquad u = \begin{cases} a^+ x_1 & x_1 s > 0 \\ a^- x_1 & x_1 s < 0 \end{cases}$$

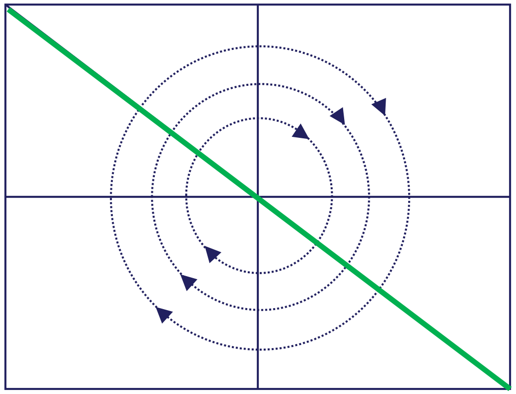
$$\dot{x}_2 = -b x_2 + u$$

where

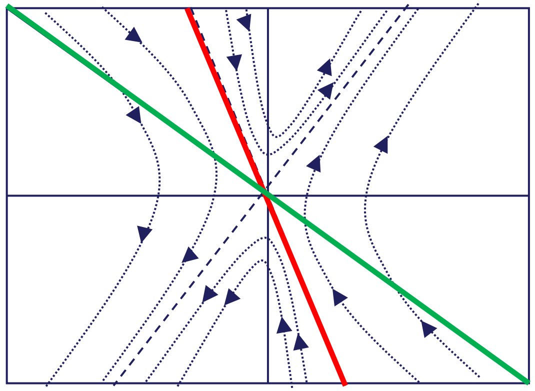
$$b > 0,\ c > 0,\ s = c x_1 + x_2$$

Equivalently,

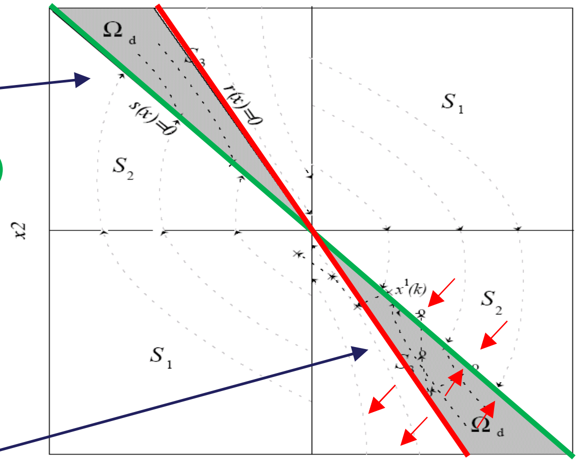
$$\ddot{x} + b\dot{x} + a^{\mp} x = 0$$



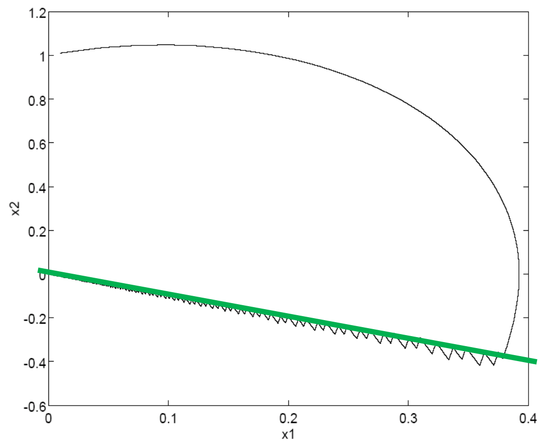
Phase plane portrait for ‘+’ (s>0, a⁺>0)



Phase plane portrait for ‘-’ (s<0, a⁻<0)



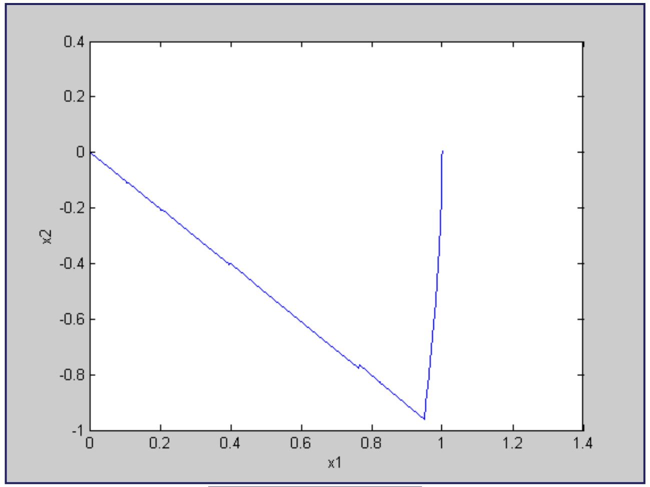
Phase plane portrait



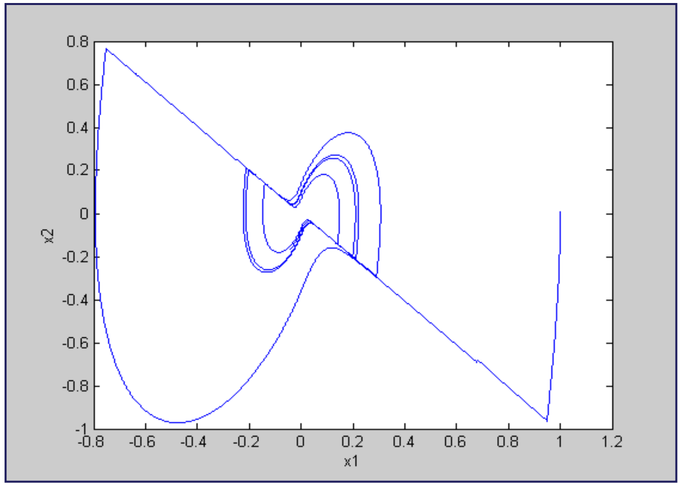
Near continuous-time behavior …

Problem with discretization

Is it true that a 'small enough' sampling period does not cause chaotic motions?



h=0.00158



h=0.00165

With $b=-4.1$, $\alpha=4.1$, $c=1$, according to the upper bound formulae (Potts & Yu, 1991), the maximum H is 0.0016!

R.B. Potts, X. Yu, "Discrete variable structure system with pseudo-sliding modes," J Aust Math Soc B, 32:365-376, 1991.

Another example

$$\dot{x}_1 = x_2 \qquad u = -\operatorname{sgn}(s)$$
$$\dot{x}_2 = u$$

where $\quad c > 0,\ s = cx_1 + x_2$

Equivalently, $\quad \ddot{x} \pm 1 = 0$

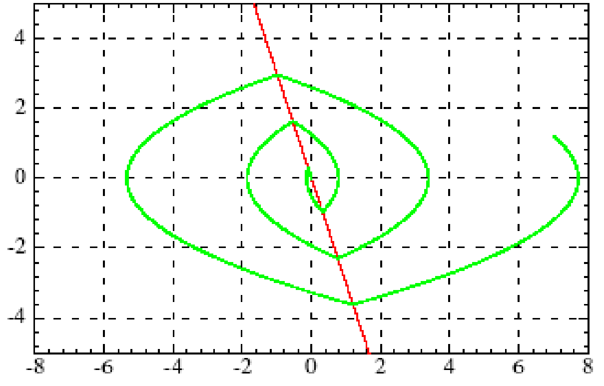


Fig. 1. Continuous time system, $c_1 = 3$, initial point $(x_1, x_2) = (7, 1.2)$,

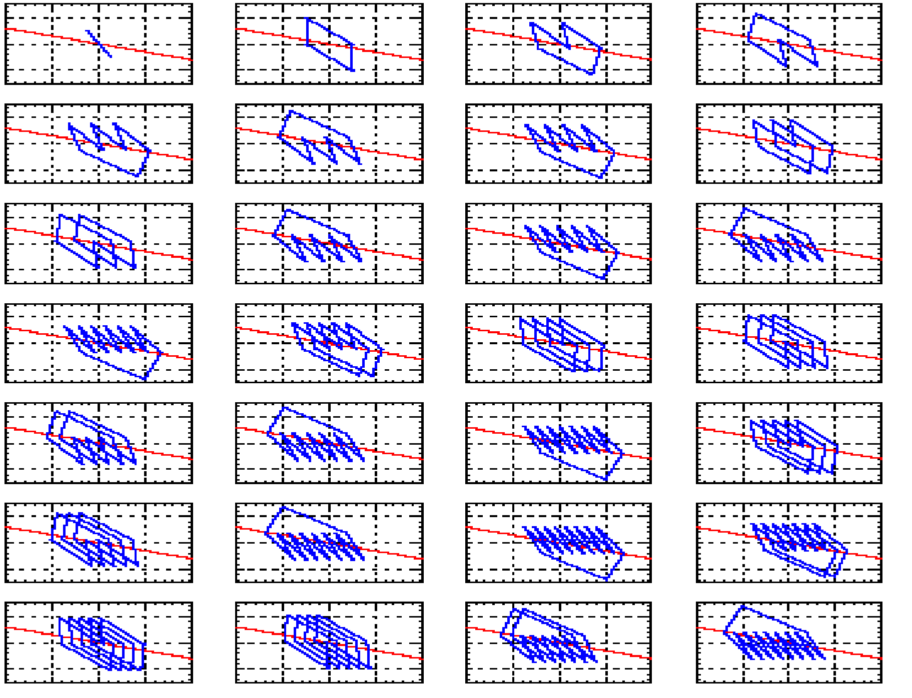


Fig. 2. Short cycles for $h = 0.1$, $c_1 = 3$

Z. Galias, X. Yu, Study of periodic solutions in discretized two dimensional sliding-mode control systems, IEEE T-CAS II, 58(6): 381–385, 2011.

Main Switching Theories

1. Piece-Wise Lyapunov Theory
2. Filippov Theory
3. Averaging Method

1. Piece-wise Lyapunov theory

For $\dot{x} = f(x) + b(x)u(x)$, If there exist Lyapunov function $V_p, p \in P$, two class K_∞ functions α_1 and α_2, and a positive number ρ_0, such that

$$\alpha_1(|x|) \leq V_p \leq \alpha_2(|x|)$$
$$\frac{\partial V_p}{\partial x}(f(x) + b(x)u(x)) \leq -2\rho_0 \, V_p(x)$$
$$V_p(x) \leq \mu V_q(x), \ \forall p, q \in P$$

Then switched control system is globally asymptotically stable for every switching signal with average dwell time

$$\tau > \frac{\log \mu}{2\rho_0}$$

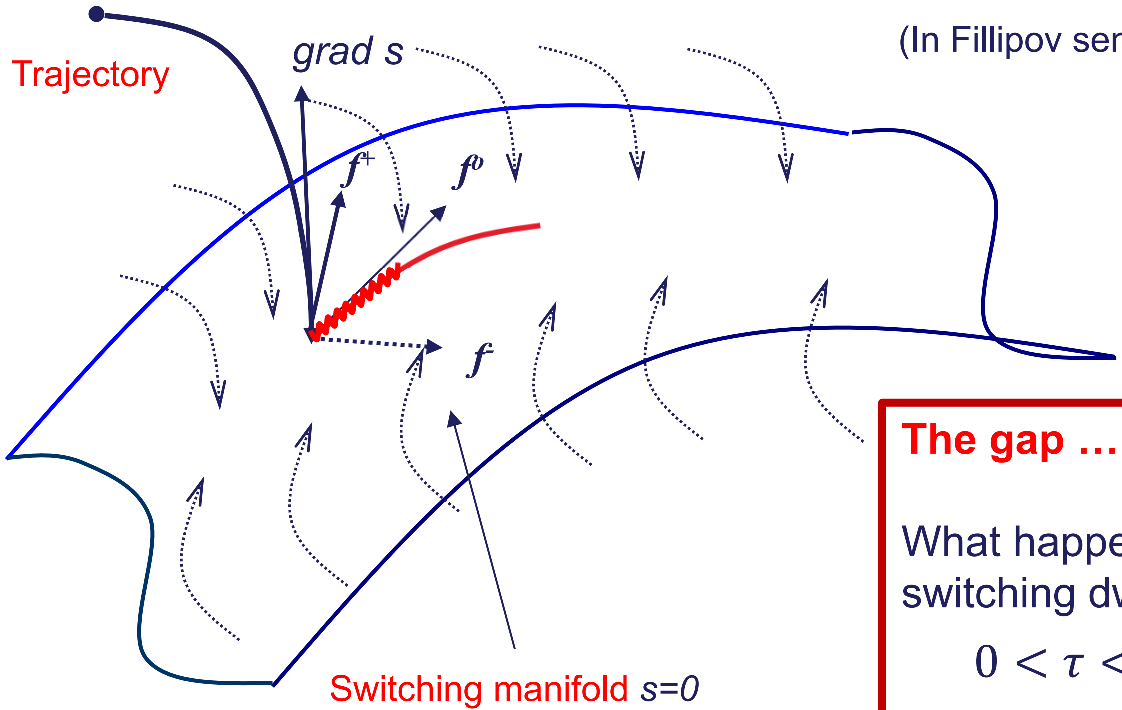
D. Liberzon, *Switching in Systems and Control*, Birkhauser, 2003

2. Filippov Theory

Under infinite switching, $\dot{x} = f(x,t)$ is equivalent to

$$\dot{x} = f_0(x,t) \qquad f^0 = \alpha f^+ + (1-\alpha) f^-,\ 0<\alpha<1$$

(In Fillipov sense)



The gap …

What happens when switching dwell time τ

$$0 < \tau < \frac{\log \mu}{2\rho_0} \text{ ???}$$

A. F. Filippov, *Differential Equations with Discontinuous Righthand Sides, Springer,* 2013.

3. Averaging Method

Consider a general nonlinear dynamical system

$$\dot{x} = \varepsilon f(t, x, \tau)$$

where $f(t, x, \tau)$ is periodic in *t* with period τ , the evolution of the system is said to occur in two timescales: a fast oscillatory one associated with the presence of *t* in *f* and a slow one associated with the presence of ε . The averaged system is expressed as

$$\dot{\tilde{x}} = \varepsilon \frac{1}{\tau} \int_0^\tau f(s, x, 0)\, ds = \varepsilon \widetilde{f}(\tilde{x})$$

where $\tilde{x}$ is the average state over $(0, \tau)$ with respect to ε.

Averaging Theorem:

If $|x(0) - \tilde{x}(0)| = O(\varepsilon)$, then $|x(\varepsilon) - \tilde{x}(\varepsilon)| = O(\varepsilon)$ on a time scale $t \sim \frac{1}{\varepsilon}$

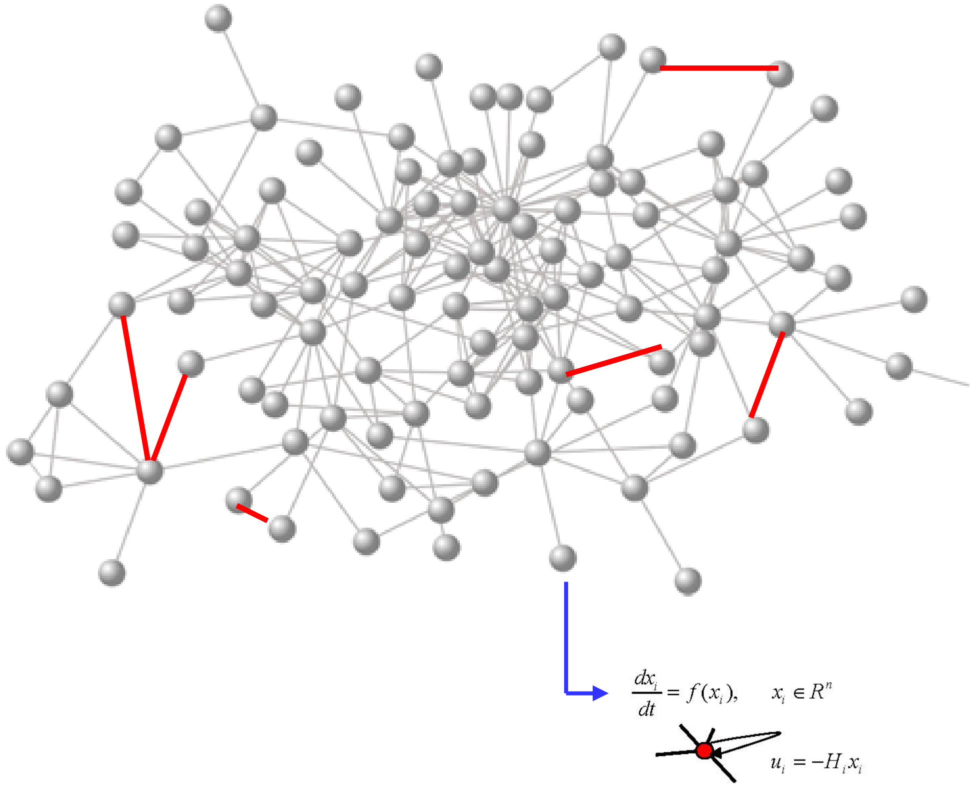
J. A. Sanders, et al, Averaging Methods in Nonlinear Dynamical Systems, Springer, 2007.

Table of Contents

Switching in CNSs

Switching phenomena:

- Links are on or off (switching topology)
- Some dynamical nodes are under discontinuous control e.g. sliding mode



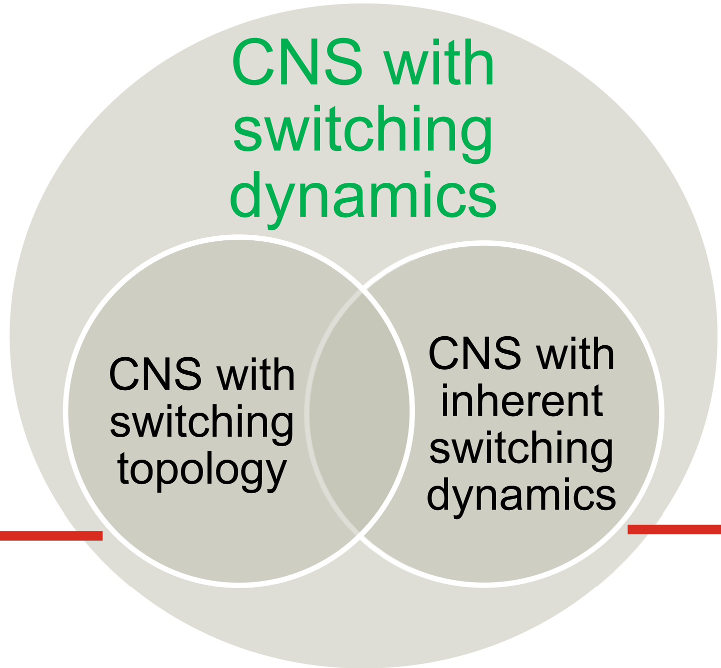
Coordination and Control of CNSs with Switching Topologies

[Switching frequency]:
- **Slowly switching** (CLF, MLFs approaches, stochastic matrices theoy)
- **Fast switching** (Averaging theory, time-varying differential equation theory)

[Triggered mechanism]:
- **Time-driven switching** (CLF, MLFs approaches)
- **State-dependent switching** (Filippov theory)

[Depending on state]:
- **Open loop switching** (CLF, MLFs approaches)
- **Closed-loop switching** (Optimal control, sliding mode theory)

….



[Subsystems' property]:
- **Switching linear systems** (CLF, MLFs approaches)
- **Switching nonlinear systems** (CLF, MLFs approach)

[Triggered mechanism]:
- **Time-driven switching** (CLF, MLFs approaches)
- **State-dependent switching** (Filippov theory)

[Depending on state]:
- **Open loop switching** (CLF, MLFs approaches)
- **Closed-loop switching** (Optimal control, sliding mode theory)

[Hybrid property]:
- **Continuous time switching systems**
- **Discrete time switching systems**
- **Hybrid switching systems**

….

G. Wen, X. Yu, W. Yu, J. Lu, Coordination and control of complex network systems with switching topologies: A survey, IEEE T-SMC Syst, doi:10.1109/TSMC.2019.2961753.

Synchronization with fast switching topology: Averaging method

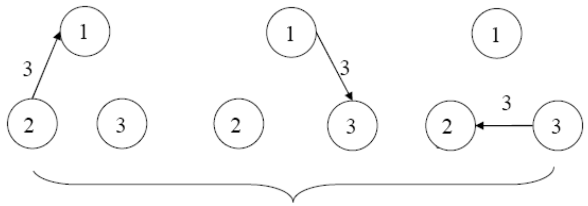
Suppose that there is a constant $T>0$ and a fixed matrix $\bar{L}$ such that

$$\frac{1}{T}\int_{t}^{t+T} L(\tau)d\tau = \bar{L}$$

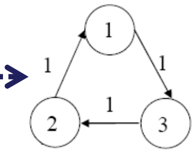
Average Laplacian matrix

for all t, and the networks with linear (nonlinear) nodes can be synchronized under the fixed graph with Laplacian matrix $\bar{L}$. Then, global (local) synchronization in complex networks with fast switching topology can be ensured.

Example: Suppose that the network switches fast among the three network candidates.



The time-average network topology



H. Kim, H. Shim, J. Back, J. H. Seo, Automatica, 47(1):267-272, 2013.

Synchronization with slowly switching topology: An M-matrix based approach

Synchronization criteria
(slowly switching topology)

- Each possible topology should satisfy some connectivity conditions (contains at least a directed spanning tree) ;
- Switching should not be very fast (dwell time constraint condition) .

- **Multiple Lyapunov Functions(MLFs) can be constructed from M-matrix theory! A unified construction approach!**

G. Wen, W. Yu, G. Hu, J. Cao and X. Yu, IEEE Trans. Neural Networks and Learning Systems, 26(12): 3239-3250, 2015. State feedback.

G. Wen, W. Yu, Y. Xia, X. Yu, and J. Hu, IEEE Trans. Systems, Man and Cybernetics, Systems, 47(5): 869-881,2017. Output feedback

How to ensure synchronization in switching complex networks when each possible topology **does not** contain any directed spanning tree?

Synchronization with slowly switching topology: An M-matrix based approach – state feedback

The Laplacian matrix of network topology G^s, $s \in \{1,2,\ldots,p\}$

$$\tilde{L}^s = \begin{pmatrix} \hat{L}^s & c^s \\ \mathbf{0}_N^T & 0 \end{pmatrix} \in \mathbb{R}^{(N+1)\times(N+1)}$$

G^s contains a directed spanning tree rooted at node $N+1$ $\Leftrightarrow$ $\hat{L}^s$ is a **nonsingular M matrix**

Error System: $$\dot{e}_i(t) = f(x_i(t),t) - f(s(t),t) - \alpha \sum_{j=1}^{N} l_{ij}^{\sigma(t)} e_j(t) - \alpha c_i^{\sigma(t)} e_i(t)$$

MLFs: $\mathrm{V(t)} = e(\mathrm{t})^T \left(\Xi^{\sigma(t)} \otimes I_n\right) e(t)$ with $\Xi^{\sigma(t)} \hat{L}^{\sigma(t)} + \left(\hat{L}^{\sigma(t)}\right)^T \Xi^{\sigma(t)} > 0$

G. Wen, W. Yu, G. Hu, J. Cao, X. Yu, Pinning synchronization of directed networks with switching topologies: A multiple Lyapunov functions approach, IEEE T-NNLS, 26(12): 3239-3250, 2015.

Synchronization with slowly switching topology: An M-matrix based approach – output feedback

Node dynamics:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Df(x_i(t), t)$$
$$y_i(t) = Cx_i(t)$$

Control protocol:

$$\dot{\widetilde{x}}_i(t) = A\widetilde{x}_i(t) + Bu_i(t) + \alpha \sum_{j=1}^{N+1} a_{ij}^{(\sigma(t))} F\big(\delta_j(t) - \delta_i(t)\big) + Df(\widetilde{x}_i(t), t)$$
$$u_i(t) = \beta K \sum_{j=1}^{N+1} a_{ij}^{(\sigma(t))} \big(\widetilde{x}_j(t) - \widetilde{x}_i(t)\big)$$

MLFs:

$$V(t) = \widetilde{e}^T(t)\left(\widetilde{\Xi}^{(\sigma(t))} \otimes Q\right)\widetilde{e}(t) + \iota e^T(t)\left(\widetilde{\Xi}^{(\sigma(t))} \otimes P^{-1}\right)e(t)$$

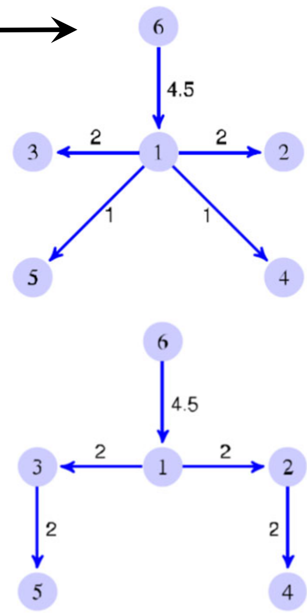
G. Wen, W. Yu, Y. Xia, X. Yu, J. Hu, Distributed tracking of nonlinear multiagent systems under directed switching topology: An observer-based protocol, IEEE T-SMC Syst, 47(5): 869-881,2017.

Pinning synchronization of complex switching networks with a leader of nonzero control inputs

- **Multiple Lyapunov Functions-based approach** (to analysis the dwell time based synchronization criteria)
- **Sliding Mode Control Approach** (to make the followers track the leader asymptotically)

The dynamics of the leader may be subjected to unknown external control inputs

Each possible topology should satisfy some connectivity conditions (contains at least a directed spanning tree)



MLFs-based approach Plus Sliding model control technique!

G. Wen, P. Wang, X. Yu, W. Yu, J. Cao, Pinning synchronization of complex switching networks with a leader of nonzero control inputs, IEEE T-CAS I, 66(8): 3100-3112, 2019.

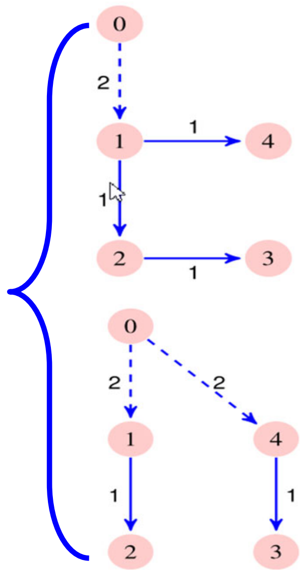
Consensus disturbance rejection for linear multiagent systems with directed switching communication topologies

- **Multiple Lyapunov Functions-based approach** (to analysis the average dwell time based consensus criteria)
- **Unknown Input Observer (UIO) Approach** (to estimate the relative full states' error among neighboring agents)
- **Disturbance Observer (DO) Approach** (for disturbance rejection)

The dynamics of the followers are subjected to non-vanishing external dist

$$\dot{d}_i(t) = W d_i(t)$$

Each possible topology should satisfy some connectivity conditions (contains at least a directed spanning tree)



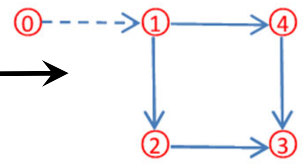
MLFs-based approach Plus UIO Plus DO control technique!

P. Wang, G. Wen, X. Yu, W. Yu, Y. Lv, Consensus disturbance rejection for linear multiagent systems with directed switching communication topologies, IEEE T-CNS, 7(1): 254-265, 2020.

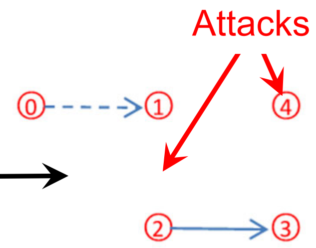
Synchronization of resilient complex networks under attacks

- **Impulsive network model** (modelling the abrupt change of the states as an impulsive disturbance to the synchronization error system)
- **Common Lyapunov Function-based approach** (to analysis synchronization criteria)

The network topology without attacks should satisfy some connectivity conditions (contains at least a directed spanning tree)



The network topology will lose connectivity because of attacks on edges and nodes, and states of the nodes being attacked may change abruptly at some time instants



CLF-based approach Plus Impulsive control technique!

P. Wang, G. Wen, X. Yu, W. Yu, Y. Wan, Synchronization of resilient complex networks under attacks, IEEE T-SMC Syst, doi: 10.1109/TSMC.2019.2895027.

Table of Contents

Concluding remarks …

- Nature has been teaching us many ingenious and ‘easy’ ways to handle huge size and complexity.
- A new methodology, ‘simplexity approach’, is needed to deal with spatio-temporal size and complexity in a timely fashion.
- Balancing between optimality, timeliness, and complexity to deliver performance is a key issue.
- Many switching theories can be used for analysis and synthesis in modelling, control and optimisation of CNSs
- New generation control theories and methodologies for CNSs are emerging – an exciting time ahead!

THANKS

Contact: x.yu@rmit.edu.au
Wechat: xinghuo_yu