Complex Network Systems:
A Control Science and Engineering Perspective

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Smart Grid as CNS

Advanced Metering

User Oriented Presentation Portals

Enterprise Asset Management Systems

Sustainable Network Cells

Central power plant

Offices

Houses

Storage

Micro-turbines

Fuel cells

CHP

Industrial plants

Virtual power plant

Power Supply Networks

Information & Business Systems

Integrated Infrastructure Management (Mutual Interdependence)

Cohesive Network Management Systems (SCADA, DMS, OMS)

Pervasive Wide-Bandwidth Digital Communications Capability

Distributed Energy Resources

Remote Control & Monitoring

Network Automation & Self-Healing

Enterprise Data Management Systems

Dynamic Load Management

Workforce Mobility

Analytics that Transform Data into Intelligence

Value

Information

Intelligence

Data

Volume/Quantity

Network diagram courtesy of European Commission EUR 22040
Cyber-Physical World

A broad range of complex, multi-disciplinary, physically-aware next generation engineered systems that integrate embedded computing technologies (cyber part) into the physical world.

Cyber-Physical-Social Systems (CPSS)

The key question: How to handle the exponentially growing size and complexity of Smart Energy Systems effectively and timely?

Underpinned by

- Super high dimensions (spatial scale)
- Real-time (temporal scale) responses
- Intermittency
- High complexity (structured and non-structured, topology, varying conditions)
- Cyber-physical spaces
- Social-economic aspects
- Cybersecurity
The “no free lunch theorem” of Wolpert and Macready …

Computational complexity for solving a large scale problem cannot be reduced regardless of what algorithms you may use …

We have to have leap of faith …!

The “simple solutions for complex problems” problem solving paradigm …

Learning from NATURE to deal with future challenges …

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Control Theory and Methods

- Open Loop vs Closed Loop
- Linear vs Nonlinear System
- Classic vs Modern Theories
- Frequency vs Time Domains
- SISO vs MIMO
- Centralised vs Distributed
- Deterministic vs Stochastic
- Controllability & Observability
- Stability
- Control Specification
- Model Identification
- Data-driven
- Control Types: Optimal/Adaptive/Robust/Intelligent/Stochastic/ Switching …
Control in Smart Grid

How to handle the **exponentially growing size and complexity** effectively and timely?

Underpinned by

- Super high dimensions (spatial scale)
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An inspiration:
300+ years ago, Brook Taylor introduced Taylor Series Expansion!

Taylor Series Expansion

**Taylor’s Theorem**: Suppose $f$ is continuous on the closed interval $[a, b]$ and has $n + 1$ continuous derivatives on the open interval $(a, b)$. If $x$ and $c$ are points in $(a, b)$, then

The Taylor series expansion of $f(x)$ about $c$:

$$f(c) + f'(c)(x-c) + \frac{f^{(2)}(c)}{2!}(x-c)^2 + \frac{f^{(3)}(c)}{3!}(x-c)^3 + ...$$

or

$$Taylor Series = \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(c) (x-c)^k$$

If the series converge, we can write:

$$f(x) = \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(c) (x-c)^k$$
Case 1: Finding minimum set of control nodes

The concept of controllability

- For a canonical linear, time–invariant dynamics

\[ \dot{x}(t) = Ax(t) + Bu(t) \]  \[1\]

\(x(t)\): states of \(N\) nodes at time \(t\). \(A\): interaction between nodes. \(B\): control matrix. \(u(t)\): time dependent input vector.

- System is controllable if and only if controllability matrix

\[ C = (B, AB, \ldots, A^{N-1}B) \]  \[2\]

has full rank (Kalman’s controllability rank condition \[1\]).

\[ \text{rank}(C) = N \]  \[3\]

The concept of structural controllability

In the controllability matrix $Q$:

$$Q = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

All 0 are fixed. There is a realization of independent nonzero parameters such that $Q$ has full row rank

Case 1: Finding minimum set of control nodes

Using structural controllability to find minimum number of driver nodes

- Liu et al. [2] proved that the minimum number of driver nodes equal to number of unmatched nodes from maximum matching algorithm

Case 1: Finding minimum set of control nodes

Network augmentation

Problem: how to maximise system structural controllability with minimum number of driver nodes unchanged? There are a lot of applications.

Network Model

Linearly coupled network:

\[ \dot{x}_i = f(x_i) + c \sum_{j=1}^{N} \beta_{ij} Hx_j \quad x_i \in R^n \quad i = 1,2,\ldots, N \]

- General assumption: \( f(.) \) is Lipschitz. Here, it is linear (or linearized):

\[ \dot{x}_i = Ax_i + c \sum_{j=1}^{N} \beta_{ij} Hx_j \quad x_i \in R^n \quad i = 1,2,\ldots, N \]

- Coupling strength \( c > 0 \) and \( H \) – input coupling matrix

- Adjacency matrix: \( \left[ \beta_{ij} \right]_{N \times N} \)

If node \( i \) points to node \( j \) \( (j \neq i) \), then \( \beta_{ij} = 1 \); otherwise \( \beta_{ij} = 0 \); and \( \beta_{ii} = 0 \)

For undirected networks, \( \left[ \beta_{ij} \right]_{N \times N} \) is symmetrical; for directed networks, may not be so
Case 2: Selecting the best driver nodes for synchronisability

**Objective:** To achieve a certain control goal

**Questions:**
- How many controllers to use?
- Where to put them?

→ **Pinning Control**

\[ \dot{x}_i = Ax_i + c \sum_{j=1}^{N} \beta_{ij} Hx_j + \delta_i Bu_i \]

\[ \delta_i = \begin{cases} 
1 & \text{if } i \text{ to - control} \\
0 & \text{if } i \text{ not - control} 
\end{cases} \]

X Li, X F Wang, G Chen, Pinning a complex dynamical network to its equilibrium, IEEE T-CAS I, 51: 2074-2087, 2004

Case 2: Selecting the best driver node for synchronisability

Suppose that all nodes of a complex network should be pinned (synchronized) to the following desired state:

\[ x_1(t) = x_2(t) = \cdots = x_N(t) = s(t), \text{ as } t \to \infty, \]

\[ \frac{d(s(t))}{dt} = F(s(t)) \]

One should design the following state feedback:

\[ \frac{dx_i}{dt} = F(x_i) - \sigma \sum_{j=1}^{N} l_{ij} H x_j + u_i, \quad u_i = -\beta_i k_i (s - x_i) \]

\( k_i \) : the feedback gain
\( \beta_i = 1 \) for driver nodes, otherwise \( \beta_i = 0 \).

- Augmented Laplacian matrix

\[ C = \{ c_{ij} \} = \begin{bmatrix} l_{11} + k_1 \beta_1 & l_{12} & \cdots & l_{1N} \\ l_{21} & l_{22} + k_2 \beta_2 & \cdots & l_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ l_{N1} & l_{N2} & \cdots & l_{NN} + k_N \beta_N \end{bmatrix} \]

\( \lambda_i \) : the \( i^{th} \) eigenvalue of the augmented Laplacian matrix

- The metric \( R \) is defined\(^2\) as a measure of controllability

Smaller \( R \) results in better synchronizability (i.e. synchronizability over wider coupling strength).


Case 2: Selecting the best driver node for synchronisability

**Heuristics:**
- nodes with highest degree
- nodes with highest betweenness centrality
- nodes with highest closeness centrality

**Using R:**
- Calculate \( \Delta R = R_0 - R_i \) for all nodes \( i = 1, 2, \ldots, N \); select the node with maximum \( \Delta R \).

\( R_0 = (\lambda_2/\lambda_1) \) for the main Laplacian matrix.
\( R_i = (\lambda_i/\lambda_1) \) for the augmented Laplacian matrix when node \( i \) is controlled.

- This method is time consuming in large complex networks. Heuristics are also not accurate enough.

**Eigenvalue perturbation analysis**
**Eigen-ratio sensitivity analysis**

\[ ESI(i) = \left( x_n^i \right)^2 \]

\( x_n \) is eigenvectors of \( L \) related to \( \lambda_n \) and \( i \) shows \( i^{th} \) element.


**Watts-Strogatz complex network with \( N=1000 \) nodes.**
Case 3: Characteristic Modelling Approach (吴宏鑫院士创立)

\[
\frac{y(s)}{u(s)} = \frac{\sum_{j=0}^{m_0} b_j s^j}{s^{n_0} + \sum_{i=0}^{n_0-1} a_i s^i}, \quad a_i, b_j \in \mathbb{R}
\]

Error $O(h^2)$

\[
y(k + 1) = (2 + w_1(k))y(k) + (-1 + w_2(k))y(k - 1) + w_3(k)u(k) + w_4(k)u(k - 1)
\]

\[
\mathcal{V}(k + 1) = \hat{\mathcal{A}}_1(k)\mathcal{V}(k) + \hat{\mathcal{A}}_2(k)\mathcal{V}(k - 1) + (A \circ \hat{B}(k))\mathcal{V}(k)
\]

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Switching phenomena are everywhere in CNSs
e.g. links on-off intentionally or unintentionally, switching control mechanisms for fast transient responses.

Switching can be classified in two clusters:

Open-loop switching means the switching occurs without influences from any internal system states and external influences

Closed-loop switching means the switching depends on internal systems states and external influences
Switching Patterns

Switching phenomena are everywhere in CNSs.

The frequency influences significantly the behaviours of system dynamics

Low frequency - If it is ‘low’, provided each switching action sustains sufficient ‘long’ time, many existing methodologies for non-switching dynamics can be used by ‘piecing-together’ various ‘smooth’ subsystems – in time or in state.

Medium frequency - Medium frequency is really the area of special interest as piecing-together smooth subsystems may not give accurate picture though methodologies such as Lyapunov theory and Averaging theory may still be applied with caution.

High frequency - The system with high frequency switching tends to violate the theories for usual ‘smooth’ dynamical systems such as the existence and uniqueness of general solutions to smooth ordinary differential equations; drastically different methods such as Filippov theory should be used!
Benefits of Switching

The benefits of switching are enormous … for example,

\[ \dot{y} + a_1 \dot{y} + a_{01} y = 0 \quad \text{subsystem 1} \]
\[ \ddot{y} + a_1 \dot{y} + a_{02} y = 0 \quad \text{subsystem 2} \]

- \( a_1 > 0 \) the systems are both asymptotically stable.
- \( a_1 = 0 \) the systems are both marginally stable.
- \( a_1 < 0 \) the systems are both unstable.
Switching between unstable systems can yield a stable motion.

\[ a_1 = -0.1 \]

Both dynamics are unstable.

Switched asymptotically stable dynamics.
Switching between stable systems can result in an unstable motion.

Switched unstable dynamics
A simple case study

Consider a double integrator given by $\dot{y} = u(t)$, for $0 < k_1 < 1 < k_2$

If we choose switching control

$$u = \begin{cases} 
-k_1y & \text{if } \dot{y} < 0 \\
-k_2y & \text{otherwise}
\end{cases}$$

and a new Lyapunov function

$$V(y) = \frac{1}{2} (y^2 + \dot{y}^2)$$

Then

$$\dot{V} = y\dot{y} + \dot{y}\ddot{y} = \dot{y}(y + u) = \begin{cases} 
\dot{y}(1 - k_1) & \text{if } \dot{y} < 0 \\
\dot{y}(1 - k_2) & \text{if } \dot{y} > 0
\end{cases}$$
However, if we choose a switching line $s = \dot{y} + cy$

The control is chosen as $u = -k \text{sgn}(s) = \begin{cases} -k & \text{if } s > 0 \\ k & \text{if } s < 0 \end{cases}$

Take $V(y) = |s| + \frac{c}{2k} \dot{y}^2$

Then $\dot{V} = \text{sgn}(s)(c\dot{y} - k \text{sgn}(s)) - c\dot{y} \text{sgn}(s) = -k|s| < 0$

$c|\dot{y}| < 1$, then $ss < 0$, sliding mode exists
When \( s=0 \) is reached, that is, \( \dot{y} = -cy, \ c > 0, \ y(t) = \exp(-ct)y(0) \to 0, \ for \ t \to \infty \)

**Sliding mode control theory:**

For \( \dot{x} = f(x) + b(x)u \)

1. Define a switching manifold which prescribe the desirable properties \( s(x) \)
2. Design a discontinuous control \( u(x) \),

\[
\begin{cases}
  u^+ & s(x) > 0 \\
  u^- & s(x) < 0
\end{cases}
\]

such that \( \lim_{s \to 0^+} \dot{s} < 0 \), and \( \lim_{s \to 0^-} \dot{s} > 0 \)

Drawbacks of Switching

Sensitivity to switching times leads to many problems in practice, e.g.

Consider a second order system

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -bx_2 + u \\
u &= \begin{cases} 
a^+ x_1 & x_1 s > 0 \\
a^- x_1 & x_1 s < 0 \end{cases}
\end{align*}
\]

where

\[b > 0, \ c > 0, \ s = cx_1 + x_2\]

Equivalently,

\[\dot{x} + b\dot{x} + a^\top x = 0\]
Near continuous-time behavior ...

Phase plane portrait for '+' (s>0, a+>0)

Phase plane portrait for '-' (s<0, a<0)

Switching line (sliding mode)

Unstable asymptote
Problem with discretization

Is it true that a ‘small enough’ sampling period does not cause chaotic motions?

With \( b = -4.1, \, \alpha = 4.1, \, c = 1 \), according to the upper bound formulae (Potts & Yu, 1991), the maximum \( H \) is 0.0016!

Another example

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = u \\
\text{where} \quad c > 0, \ s = cx_1 + x_2
\]

Equivalently, \( \dot{x} \pm 1 = 0 \)

Fig. 1. Continuous time system, \( c_1 = 3 \), initial point \((x_1, x_2) = (7, 1.2)\).

Fig. 2. Short cycles for \( h = 0.1, \ c_1 = 3 \)

Main Switching Theories

1. Piece-Wise Lyapunov Theory
2. Filippov Theory
3. Averaging Method
1. Piece-wise Lyapunov theory

For $\dot{x} = f(x) + b(x)u(x)$, if there exist Lyapunov function $V_p, p \in P$, two class $K_\infty$ functions $\alpha_1$ and $\alpha_2$, and a positive number $\rho_0$, such that

$$\alpha_1(|x|) \leq V_p \leq \alpha_2(|x|)$$
$$\frac{\partial V_p}{\partial x}(f(x) + b(x)u(x)) \leq -2\rho_0 V_p(x)$$
$$V_p(x) \leq \mu V_q(x), \forall p, q \in P$$

Then switched control system is globally asymptotically stable for every switching signal with average dwell time

$$\tau > \frac{\log \mu}{2\rho_0}$$

2. Filippov Theory

Under infinite switching, \( \dot{x} = f(x,t) \) is equivalent to

\[
\dot{x} = f_0(x,t) \quad f^0 = \alpha f^+ + (1-\alpha)f^-, \ 0<\alpha<1
\]

(In Fillipov sense)

3. Averaging Method

Consider a general nonlinear dynamical system

\[ \dot{x} = \varepsilon f(t, x, \tau) \]

where \( f(t, x, \tau) \) is periodic in \( t \) with period \( \tau \), the evolution of the system is said to occur in two timescales: a fast oscillatory one associated with the presence of \( t \) in \( f \) and a slow one associated with the presence of \( \varepsilon \). The averaged system is expressed as

\[ \dot{x} = \varepsilon \frac{1}{\tau} \int_0^\tau f(s, x, 0) \, ds = \varepsilon \overline{f}(\bar{x}) \]

where \( \bar{x} \) is the average state over \((0, \tau)\) with respect to \( \varepsilon \).

**Averaging Theorem:**

If \( |x(0) - \bar{x}(0)| = O(\varepsilon) \), then \( |x(\varepsilon) - \bar{x}(\varepsilon)| = O(\varepsilon) \) on a time scale \( t \sim \frac{1}{\varepsilon} \)

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Switching in CNSs

Switching phenomena:

- Links are on or off (switching topology)
- Some dynamical nodes are under discontinuous control e.g. sliding mode
Coordination and Control of CNSs with Switching Topologies

[Switching frequency]:
- **Slowly switching** (CLF, MLFs approaches, stochastic matrices theory)
- **Fast switching** (Averaging theory, time-varying differential equation theory)

[Triggered mechanism]:
- **Time-driven switching** (CLF, MLFs approaches)
- **State-dependent switching** (Filippov theory)

[Depending on state]:
- **Open loop switching** (CLF, MLFs approaches)
- **Closed-loop switching** (Optimal control, sliding mode theory)

[Subsystems’ property]:
- **Switching linear systems** (CLF, MLFs approaches)
- **Switching nonlinear systems** (CLF, MLFs approach)

[Triggered mechanism]:
- **Time-driven switching** (CLF, MLFs approaches)
- **State-dependent switching** (Filippov theory)

[Depending on state]:
- **Open loop switching** (CLF, MLFs approaches)
- **Closed-loop switching** (Optimal control, sliding mode theory)

[Hybrid property]:
- **Continuous time switching systems**
- **Discrete time switching systems**
- **Hybrid switching systems**

Synchronization with fast switching topology: Averaging method

Suppose that there is a constant $T > 0$ and a fixed matrix $\bar{L}$ such that

$$\frac{1}{T} \int_{t}^{t+T} L(\tau) d\tau = \bar{L}$$

for all $t$, and the networks with linear (nonlinear) nodes can be synchronized under the fixed graph with Laplacian matrix $\bar{L}$. Then, global (local) synchronization in complex networks with fast switching topology can be ensured.

**Example:** Suppose that the network switches fast among the three network candidates.

Synchronization with slowly switching topology: An M-matrix based approach

**Synchronization criteria**
*(slowly switching topology)*

- Each possible topology should satisfy some connectivity conditions (contains at least a directed spanning tree);
- Switching should not be very fast (dwell time constraint condition).

- Multiple Lyapunov Functions (MLFs) can be constructed from M-matrix theory! A unified construction approach!


How to ensure synchronization in switching complex networks when each possible topology does not contain any directed spanning tree?
Synchronization with slowly switching topology: An M-matrix based approach – state feedback

The Laplacian matrix of network topology $G^s$, $s \in \{1, 2, \ldots, p\}$

$G^s$ contains a directed spanning tree rooted at node $N + 1$

$\hat{L}^s$ is a nonsingular M matrix

Error System: $\dot{e}_i(t) = f(x_i(t), t) - f(s(t), t) - \alpha \sum_{j=1}^{N} l_{ij}^{\sigma(t)} e_j(t) - \alpha c_i^{\sigma(t)} e_i(t)$

MLFs: $V(t) = e(t)^T (\Xi^{\sigma(t)} \otimes I_n) e(t)$ with $\Xi^{\sigma(t)} \hat{L}^{\sigma(t)} + (\hat{L}^{\sigma(t)})^T \Xi^{\sigma(t)} > 0$

Synchronization with slowly switching topology: An M-matrix based approach – output feedback

Node dynamics:

\[ \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Df(x_i(t), t) \]
\[ y_i(t) = Cx_i(t) \]

Control protocol:

\[ \dot{\tilde{x}}_i(t) = A\tilde{x}_i(t) + Bu_i(t) + \alpha \sum_{j=1}^{N+1} a_{ij}^{(\sigma(t))} F(\delta_j(t) - \delta_i(t)) \]
\[ + Df(\tilde{x}_i(t), t) \]
\[ u_i(t) = \beta K \sum_{j=1}^{N+1} a_{ij}^{(\sigma(t))} (\tilde{x}_j(t) - \tilde{x}_i(t)) \]

MLFs:

\[ V(t) = \tilde{e}^T(t) (\tilde{\Xi}^{(\sigma(t))} \otimes Q) \tilde{e}(t) + \iota e^T(t) (\tilde{\Xi}^{(\sigma(t))} \otimes P^{-1}) e(t) \]

Pinning synchronization of complex switching networks with a leader of nonzero control inputs

- Multiple Lyapunov Functions-based approach (to analysis the dwell time based synchronization criteria)
- Sliding Mode Control Approach (to make the followers track the leader asymptotically)

The dynamics of the leader may be subjected to unknown external control inputs

Each possible topology should satisfy some connectivity conditions (contains at least a directed spanning tree)

MLFs-based approach Plus Sliding model control technique!

Consensus disturbance rejection for linear multiagent systems with directed switching communication topologies

- **Multiple Lyapunov Functions-based approach** (to analyze the average dwell time based consensus criteria)
- **Unknown Input Observer (UIO) Approach** (to estimate the relative full states' error among neighboring agents)
- **Disturbance Observer (DO) Approach** (for disturbance rejection)

The dynamics of the followers are subjected to non-vanishing external dist:

\[
\dot{d}_i(t) = W d_i(t)
\]

Each possible topology should satisfy some connectivity conditions (contains at least a directed spanning tree)

MLFs-based approach Plus UIO Plus DO control technique!

Synchronization of resilient complex networks under attacks

- **Impulsive network model**
  (modelling the abrupt change of the states as an impulsive disturbance to the synchronization error system)

- **Common Lyapunov Function-based approach** (to analysis synchronization criteria)

The network topology without attacks should satisfy some connectivity conditions (contains at least a directed spanning tree)

The network topology will lose connectivity because of attacks on edges and nodes, and states of the nodes being attacked may change abruptly at some time instants

CLF-based approach Plus Impulsive control technique!

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Concluding remarks …

• Nature has been teaching us many ingenious and ‘easy’ ways to handle huge size and complexity.
• A new methodology, ‘simplicity approach’, is needed to deal with spatio-temporal size and complexity in a timely fashion.
• Balancing between optimality, timeliness, and complexity to deliver performance is a key issue.
• Many switching theories can be used for analysis and synthesis in modelling, control and optimisation of CNSs.
• New generation control theories and methodologies for CNSs are emerging – an exciting time ahead!
THANKS

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