Obstacle-Avoidance Distributed Optimal Coordination of Multiple Euler-Lagrangian Systems

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Abstract: This paper investigates the problem of obstacle avoidance in the distributed optimal coordination for multiple uncertain Euler-Lagrangian (EL) systems. The main challenge focuses on the co-design of obstacle avoidance mechanism and distributed optimization strategy. To address it, a novel safety barrier function in the closed form of path integrals is constructed. By combining distributed optimization algorithm and adaptive tracking control, a distributed coordination algorithm is proposed. Based on Lyapunov method and boundedness analysis for the barrier function, it is proven that the global convergence and collision avoidance of the EL systems can be guaranteed in the presence of parametric uncertainties.

Key Words: Distributed optimization, obstacle avoidance, Euler-Lagrangian systems, multi-agent systems

1 Introduction

Distributed optimization over multi-agent networks has attracted much attention due to various practical applications in convex computation, resource allocation, and localization [1, 2]. The aim is to solve an optimization problem cooperatively in a distributed manner where the team performance function is composed of a sum of local objective functions. With potential applications of cyber-physical systems, the distributed optimization with physical dynamics as executive bodies, termed *distributed optimal coordination (DOC)* [3], can be completed based on effective combination of the (cyber) computation/communication and the (physical) dynamics/ control in many applications, such as the cooperative search of signal sources [4, 5], the motion coordination [8], the distributed state estimation [6] and the distributed optimal power flow [7]. Recently, many important results on DOC for dynamical multi-agent networks are developed. In [9], the problem is fundamentally investigated for a class of continuous-time multi-agent systems with single-integrator dynamics, by explicitly taking into account nonuniform gradient gains, finite-time convergence, and a common convex constraint set. Further, Zhang and Hong [10] and Xie and Lin [11] extend the traditional distributed optimization algorithm to high-order multi-agent systems. In [3], Zhang et al. propose a gradient-based DOC algorithm with adaptive mechanism for multiple uncertain heterogeneous EL systems. Considering that many physical agents are usually modeled by general linear dynamics, the distributed optimization algorithms are developed for general continuous-time linear multi-agent systems in [12, 13]. However, in these results some safety issues such as collisions with obstacles are not considered.

Obstacle avoidance as an important safety objective has been widely investigated. Many important obstacle avoidance methods have been developed in various multi-agent coordination controls, such as objective tracking [20], trajectory tracking [21], formation control [17], navigation control [19], leader-follower [18] and general coordination controls [16]. However, few results have been obtained for multi-agent collision avoidance within the DOC problem framework. Note that the proposed obstacle-avoidance method in [16] is possibly applicable to the current DOC problem. However, the obtained approximate solution may not realize the original control objective. In this paper we focus on simultaneously achieving two objectives: the DOC and the obstacle avoidance.

In this paper, the obstacle-avoidance DOC problem is studied for a group of uncertain EL systems which move on the n-dimensional Euclidean space, possibly populating static obstacles. The objective is to guarantee all the EL agents to achieve consensus while minimizing a given team performance function and avoiding collisions with the obstacles. The technical contributions are summarized as follows:

- By extending the pervious DOC problem [3] and obstacle avoidance problem [20], the obstacle-avoidance DOC is considered, intrinsically a multi-objective problem. To the best of our knowledge, this is the first trial on DOC protocol design with collision-free guarantees.
- A fully distributed algorithm is developed which consists of the secure virtual distributed optimization algorithm and adaptive tracking control structure. Safety barrier certificates in the closed form of path integrals into adaptive nonlinear control can guarantee the obstacle-avoidance behaviors.
- By combining Lyapunov method and boundedness analysis for the barrier functions, it is proved that the proposed protocol can guarantee the global convergence of the optimal coordination while avoiding collisions with obstacles, even in the presence of parametric uncertainties.

Notations: The symbols $\mathbb{N}$ and $\mathbb{R}$ denote the sets of natural and real numbers, respectively. The subset of $\mathbb{N}$, $\{1, \cdots, N\}$ is denoted by $[N]$. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The notation $\|\cdot\|$ refers to the Euclidean vector norm. Further, for a positive-define matrix $W \in \mathbb{R}^{n\times n}$ and a vector $y \in \mathbb{R}^n$, denote

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$\|y\|_W = \sqrt{y^TWy}$. I_n represents the n-dimension identity matrix. Denote $\mathbf{1}_N = [1,\cdots,1]^T \in \mathbb{R}^N$. Given a set of matrices $X_i \in \mathbb{R}^{n_i\times m}$, $i = 1,\cdots,N$, we denote by $\mathrm{col}(X_1,\cdots,X_N) \in \mathbb{R}^{(\sum_{i=1}^N n_i)\times m}$ the block matrix $[X_1^T,\cdots,X_N^T]^T$.

2 Problem Formulation and Preliminaries

Consider a network system composed of N heterogeneous EL agents with an associated undirected communication graph $\mathcal{G} = (\mathcal{V},\mathcal{E})$, moving on the n-dimensional Euclidean space, possibly populating static obstacles. The dynamics of each agent $v_i \in \mathcal{E}$ is described as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i \tag{1}$$

where $q_i, \dot{q}_i \in \mathbb{R}^n$ denote the generalized position and velocity vectors, respectively; $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix; $C_i(q_i,\dot{q}_i)\dot{q}_i \in \mathbb{R}^n$ is the vector for Coriolis and centripetal forces; $G_i(q_i) \in \mathbb{R}^n$ is the gravity vector; and $\tau_i \in \mathbb{R}^n$ is the control force.

The dynamics (1) satisfies the following properties [15]:

Property 1. The inertia matrix $M_i(q_i)$ is symmetric, positive definite, and satisfies that $\|M_i(q_i)\| \le m$, where m is an unknown constant.

Property 2. $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ is skew symmetric.

Property 3. For any $x,y \in \mathbb{R}^n$, $M_i(q_i)x + C_i(q_i,\dot{q}_i)y + G_i(q_i) = \Omega_i(q_i,\dot{q}_i,x,y)\varphi_i^*$, where $\varphi_i^* \in \mathbb{R}^p$ is a constant vector consisting of the uncertain parameters of EL system (1), and $\Omega_i(q_i,\dot{q}_i,x,y) \in \mathbb{R}^{n\times p}$ is a known regression matrix that is only dependent on state variables.

Remark 1. The EL system is a class of important physical systems which can be used to describe many mechanical systems, such as mobile robots, rigid bodies, and autonomous vehicles [15].

Suppose that there are $M \ge 0$ *static* obstacles and let $d_j^c \in \mathbb{R}^n$ and $\mathcal{D}_j \subset \mathbb{R}^n$ denote the center of mass of the jth obstacle and the region of the Euclidean space that it occupies, respectively. In what follows elliptical obstacles are considered, i.e.,

$$\mathcal{D}_j = \{q \in \mathbb{R}^n : \|q - d_j^c\|_{D_j}^2 \le \rho_j^2\}$$

where $\rho_j > 0$ and $D_j = D_j^T > 0$. Note that if a static obstacle is not elliptical, possibly even in the presence of nonsmooth edges, it is possible to enclose the obstacle within an ellipse, thus smoothing the obstacle. This can be achieved by exploiting the notion of geometric moments of the portion of the Euclidean space that constitutes the obstacle [22, 23]. In fact, the moments up to order 2 are related to the geometric parameters of the smallest ellipse that contains the region of interest, see e.g., [23].

From the safety considerations of the EL systems, it is required that all agents can bypass the obstacles. An obstacle-avoidance trajectory for the multi-agent EL system (1) is defined as follows.

Definition 1 [20]. The EL multi-agent system (1) is said to be obstacle-avoidance if $\|q_i(t)-d_j^c\|_{D_j} > \rho_j$ for all $t \ge 0$, $i \in [N]$ and $j \in [M]$.

It should be pointed out that, in this paper we ignore the critical condition that the running trajectories of the agents are tangent with the boundaries of the obstacle regions.

In this EL multi-agent system, agent $v_i \in \mathcal{V}$ is associated with a local cost function $f_i : \mathbb{R}^n \to \mathbb{R}$, which is privately known by this agent. The global team performance function of the whole system is defined as $f(\boldsymbol{s}) = \sum_{i=1}^N f_i(s_i)$, where $\boldsymbol{s} = \mathrm{col}(s_1,...,s_n)$. The mission of the EL multi-agent system (1) is to achieve consensus at the optimal generalized position $q^* = \arg\min_{s\in\mathbb{R}^n}\sum_{i=1}^N f_i(s)$. Moreover, we assume that each EL agent can sense all the obstacles and sufficient time is provided to accomplish the optimization task [20].

Assumption 1 [24]. The function f_i is differentiable and convex for all $i = 1,\cdots,N$.

Assumption 2 [24]. The graph $\mathcal{G}$ is undirected and connected.

Assumption 3 [20]. The initial position and target position of each agent do not cross the obstacle region, i.e., $\|q_i(0) - d_j^c\|_{D_j} > \rho_j$ and $\|q^* - d_j^c\|_{D_j} > \rho_j$ for all $i \in [N]$ and $j \in [M]$.

Assumptions 1 and 2 are common in the existing literature on distributed convex optimization [3, 24]. Assumption 3 guarantees that the initial and target positions are outside the obstacle range. Finally, to guarantee the feasibility of problem, it is assumed that the static elliptical obstacles do not form an impermeable boundary about initial and target positions of one or more agents.

Denote $q = \mathrm{col}(q_1,\cdots,q_N)$ and $L = \mathcal{L}\otimes I_n$. Under Assumption 2, the primary DOC objective is equivalent to

$$\min\ f(q) = \sum_{i=1}^N f_i(q_i),\ \text{subject to}\ Lq = 0 \tag{2}$$

Since $f(q)$ is convex and then the equality constraint is linear, the constrained optimization problem is feasible. The following lemma gives the analysis on the optimal solution of (2).

Lemma 1 [24]. Under Assumptions 1 and 2, define

$$F(q,\chi) = f(q) + q^TL\chi + \frac{1}{2}q^TLq.$$

Then F is differentiable and convex in its first argument and linear in its second, and:

(i) if $(q^\star,\chi^\star)$ is a saddle point of F, then $q^\star$ is a solution of (2);

(ii) if $q^\star$ is a solution of (2), there exists $\chi^\star$ with $L\chi^\star = \nabla F(q^\star,\chi^\star)$ such that $(q^\star,\chi^\star)$ is a saddle point of F.

3 Obstacle-Avoidance DOC Strategy Design

3.1 Algorithm

To derive the secure DOC strategy in presence of the obstacles, we first construct a virtual control command q_i^r as the solution of the following dynamic equation:

$$\begin{cases} \dot{q}_i^r = -\nabla f_i(q_i^r) - \sum_{j\in\mathcal{N}_i}[\Delta q_{ij}^r + \Delta\chi_{ij}] \\ \dot{\chi}_i = \sum_{j\in\mathcal{N}_i}\Delta q_{ij}^r \end{cases} \tag{3}$$

where $\Delta q_{ij}^r = q_i^r - q_j^r$ and $\Delta\chi_{ij} = \chi_i - \chi_j$.

Lemma 2 [24]. The virtual command derived by dynamics (3) satisfies $\lim_{t\to\infty} q_i^r(t) = q^*$ for all $i \in [N]$.

In fact, Proposition 1 shows that the design protocol of DOC for single-integrator systems moving in an obstacle-free environment. However, in an obstacle-populated environment the virtual command trajectory $q_i^r(t)$ derived by (3) may still collide with obstacle $\mathcal{D}_j$, i.e., there may exist a time $t \geq 0$ such that $\|q_i^r(t) - d_j^c\|_{D_j} \leq \rho_j$. To derive a secure command trajectory, we should choose an appropriate initial value $q_i^r(0)$ such that

$$\|q_i^r(t) - d_j^c\|_{D_j} \geq \rho_j + \delta_i, \; \forall t \geq 0$$

where δ_i is a positive constant. The following trajectory initialization algorithm provides a selection method.

Algorithm 1. Trajectory initialization of agent v_i

1. Select any initial value $\hat{q}_i^r(0)$, and simulate the overall trajectory $\hat{q}_i^r(t)$ according to dynamics (3)
2. Set $T_i = \arg\min_j \{T : \|\hat{q}_i^r(t) - d_j^c\|_{D_j} \geq \rho_j + \delta_i, \forall t \geq T\}$
3. Compute $T = \max_{i\in[N]} T_i$ as Maximum Consensus Algorithm [25]
4. Set $q_i^r(0) = \hat{q}_i^r(T)$

According to the definition of T in Algorithm 1, it can be seen that $\|q_i^r(t) - d_j^c\|_{D_j} \geq \rho_j + \delta_i$, $\forall t \geq 0$ is ensured along with the trajectory with $q_i^r(0)$ as the initial state. Under Assumption 3, the existence of T can be guaranteed.

Based on Lemma 2, the DOC problem can be recast in the following framework of tracking problem.

Problem 1. Consider a multi-agent EL system consisting of N agents with dynamics (1). Problem 1 can be recast into finding control inputs τ_i, $i = 1, \cdots, N$ that steer agent v_i asymptotically tracks q_i^r while bypassing the obstacles.

To address Problem 1, we define the barrier function $B_{\mathcal{D}_j}(q_i) = 2/\mathbf{N}_{\mathcal{D}_j}(q_i)$ with

$$\mathbf{N}_{\mathcal{D}_j}(q_i) = \begin{cases} 0, & \text{if } \|q_i - d_j^c\|_{D_j} \leq \rho_j \\ \|q_i - d_j^c\|_{D_j}^2 - \rho_j^2, & \text{otherwise} \end{cases}$$

Using the auxiliary dynamics (3), we design the adaptive tracking controller

$$\alpha_i = -[k_{i,1} + \mathcal{B}_{\mathcal{D}}(q_i)](q_i - q_i^r) - \nabla f_i(q_i^r) - \sum_{j\in\mathcal{N}_i} [\Delta q_{ij}^r + \Delta \chi_{ij}] \tag{4}$$

$$\dot{\mathcal{B}}_{\mathcal{D}}(q_i) = -\sum_{j=1}^{M} B_{\mathcal{D}_j}(q_i)(q_i - d_j^c)^T \dot{q}_i \tag{5}$$

$$\tau_i = \Omega_i(q_i, \dot{q}_i, -\Lambda_i, \alpha_i)\hat{\varphi}_i - \hat{k}_i(\dot{q}_i - \alpha_i) + q_i^r - q_i \tag{6}$$

$$\dot{\hat{\varphi}}_i = \Gamma_{i,1}\Omega_i(q_i, \dot{q}_i, -\Lambda_i, \alpha_i)(\alpha_i - \dot{q}_i) \tag{7}$$

$$\dot{\hat{k}}_i = \gamma_i \|\dot{q}_i - \alpha_i\|^2 \tag{8}$$

where $k_{i,1} > 1$ is the design parameter and $\hat{\varphi}_i$ and $\hat{k}_i$ are the estimates of φ_i^* and k_i, respectively, k_i are some constants such that $K = \text{diag}\{k_1, \cdots, k_N\}$ such that $K > M^T(q)(L^4 + L^3)M(q)/4$ and

$$\Lambda_i = \frac{\partial \nabla f_i(q_i^r)}{\partial q_i^r}\dot{q}_i^r + \sum_{j\in\mathcal{N}_i}(\dot{q}_i - \dot{q}_j) + [k_{i,1} + \mathcal{B}_{\mathcal{D}}(q_i)](\dot{q}_i - \dot{q}_i^r) - \left[\sum_{j=1}^{M} B_{\mathcal{D}_j}(q_i)(q_i - d_j^c)^T \dot{q}_i\right](q_i - q_i^r).$$

3.2 Convergence analysis

The following theorem gives the convergence analysis of the closed-loop system..

Theorem 1. Consider the multi-agent EL system (1) in an obstacle-populated environment. Under Assumptions 1–3, the adaptive distributed controller (3)–(8) globally steers the multi-agent EL system (1) to achieve consensus with obstacle avoidance guarantee. Moreover, all the closed-loop systems are uniformly ultimately bounded.

Before beginning the proof of Theorem 1, we introduce the following notations. Let $q^\star = \mathbf{1}_N \otimes q^*$, where $q^* = \arg\min_{s\in\mathbb{R}^n} \sum_{i=1}^N f_i(s)$. By Lemma 1, there exists $\chi^\star$ such that $(q^\star, \chi^\star)$ is the saddle point of F. We denote

$$\begin{aligned}
\dot{q} &= \text{col}(\dot{q}_1, \cdots, \dot{q}_N), \dot{q}_r = \text{col}(\dot{q}_1^r, \cdots, \dot{q}_N^r), \\
\nabla f(q) &= \text{col}(\nabla f_1(q_1), \cdots, \nabla f_N(q_N)), \\
\chi &= \text{col}(\chi_1, \cdots, \chi_N), q = \text{col}(q_1, \cdots, q_N), \\
\dot{q} &= \text{col}(\dot{q}_1, \cdots, \dot{q}_N), \tau = \text{col}(\tau_1, \cdots, \tau_N), \\
\varphi^* &= \text{col}(\varphi_1^*, \cdots, \varphi_N^*), \hat{\varphi} = \text{col}(\hat{\varphi}_1, \cdots, \hat{\varphi}_N), \\
G(q) &= \text{col}(G_1(q_1), \cdots, G_N(q_N)), \alpha = \text{col}(\alpha_1, \cdots, \alpha_N) \\
\Gamma_1 &= \text{diag}(\Gamma_{1,1}, \cdots, \Gamma_{1,N}), \Gamma_2 = \text{diag}(\Gamma_{2,1}, \cdots, \Gamma_{2,N}), \\
B(q) &= \text{diag}(\mathcal{B}_{\mathcal{D}}(q_1), \cdots, \mathcal{B}_{\mathcal{D}}(q_N)), \\
K_1 &= \text{diag}(k_{1,1}, \cdots, k_{1,N}), \hat{K} = \text{diag}(\hat{k}_1, \cdots, \hat{k}_N), \\
M(q) &= \text{diag}(M_1(q_1), \cdots, M_N(q_N)), \\
C(q, \dot{q}) &= \text{diag}(C_1(q_1, \dot{q}_1), \cdots, C_N(q_N, \dot{q}_N)), \\
\Omega(q, \dot{q}, -\Lambda, \alpha) &= \text{diag}(\Omega_1(q_1, \dot{q}_1, -\Lambda_1, \alpha_1), \\
&\quad \cdots, \Omega_N(q_N, \dot{q}_N, -\Lambda_N, \alpha_N)).
\end{aligned}$$

Then state equations (1) and controller (3)–(8) can be rewritten as the compact form, respectively,

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \tag{9}$$

and

$$\begin{cases}
\dot{q}_r = -\nabla f(q_r) - L(q_r + \chi) \\
\dot{\chi} = Lq_r \\
\alpha = -[K_1 + B(q)](q - q_r) - \nabla f(q_r) - L(q_r + \chi) \\
\tau = \Omega(q, \dot{q}, -\Lambda, \dot{q})\hat{\varphi} - \hat{K}(\dot{q} - \alpha) + (q_r - q) \\
\dot{\hat{\varphi}} = \Gamma_1\Omega(q, \dot{q}, -\Lambda, \dot{q}_r)(\dot{q} - \alpha) \\
\dot{\hat{K}} = \Gamma\|\dot{q} - \alpha\|^2
\end{cases} \tag{10}$$

where

$$\Lambda = \frac{\partial \nabla f(q)}{\partial q}\dot{q} + L\dot{q} + [K_1 + B(q)](\dot{q} - \dot{q}_r) + \dot{B}(q)(q - q_r).$$

Next, we show the convergence of the closed-loop system (9)–(10). The convergence analysis is established based on Lyapunov method and boundedness analysis for barrier functions $\mathcal{B}_{\mathcal{D}}(q_i)$ for all $i = 1, \cdots, N$, where the Lyapunov function candidate is selected as $V = V_0 + V_1 + V_2$ with

$$\begin{aligned}
V_0 &= \frac{1}{2}\|q_r - q^\star\|^2 + \frac{1}{2}\|\chi - \chi^\star\|^2 \\
V_1 &= \frac{1}{2}\|q - q_r\|^2, \\
V_2 &= \frac{1}{2}z^T M(q) z + \frac{1}{2}\tilde{\varphi}^T \Gamma_1^{-1}\tilde{\varphi} + \sum_{i=1}^{N} \frac{1}{2\gamma_i}\tilde{k}_i^2,
\end{aligned}$$

where $z = \dot{q} - \alpha$ and $\tilde{\varphi} = \varphi^* - \hat{\varphi}$, and $\tilde{k}_i = k_i - \hat{k}_i$ for $i = 1, \cdots, N$ are the corresponding estimate errors.

The proof is divided into three steps. First, we establish a Lyapunov analysis on V. Then, we prove the boundedness of barrier functions $\mathcal{B}_{\mathcal{D}}(q_i)$ for $i = 1, \cdots, N$ by contradiction. Finally, we show the convergence of the overall algorithm and the boundedness of all the closed-loop signals.

Proposition 1. The following inequality holds:

$$\dot{V} \leq -\varepsilon(\|z\|^2 + \|q - q_r\|^2) - \sum_{i=1}^{N} \mathcal{B}_{\mathcal{D}}(q_i)\|q_i - q_i^r\|^2 \tag{11}$$

where ε is a sufficiently small positive constant.

Proof. We show this result by three steps:

Step 1. The time derivative of V_0 along with (10) can be expressed as

$$\begin{aligned}\dot{V}_0 =&(q_r - q^\star)^T\dot{q}_r + (\chi - \chi^\star)^T Lq_r \\ =&(q_r - q^\star)^T[-\nabla f(q_r) - 2Lq_r - L\chi] + (\chi - \chi^\star)^T Lq_r \\ =&(q_r - q^\star)^T[-\nabla f(q_r) - L(q_r + \chi)] \\ &+ (\chi - \chi^\star)^T Lq_r - q_r^T Lq_r \\ =&-(q_r - q^\star)^T\frac{\partial F(q_r, \chi)}{\partial q_r} + (\chi - \chi^\star)^T Lq_r - q_r^T Lq_r \\ \overset{(a)}{\leq}& F(q^\star, \chi) - F(q_r, \chi) + (\chi - \chi^\star)^T Lq_r - q_r^T Lq_r \\ =&F(q^\star, \chi) - F(q, \chi) + F(q, \chi) - F(q, \chi^\star) - q_r^T Lq_r \\ =&F(q^\star, \chi) - F(q^\star, \chi^\star) + F(q^\star, \chi^\star) - F(q, \chi^\star) - q_r^T Lq_r \\ \overset{(b)}{\leq}& -q_r^T Lq_r\end{aligned} \tag{12}$$

where inequality: (a) follows the fact that $F(q, \chi)$ is convex in its first argument q; (b) follows from the fact that $(q^\star, \chi^\star)$ is the saddle point of $F(q, \chi)$.

Step 2. The time derivative of V_1 along with (10) can be expressed as

$$\begin{aligned}\dot{V}_1 =&(q - q_r)^T[\dot{q} + \nabla f(q_r) + Lq_r + L\chi] \\ =&-(q - q_r)^T[K_1 + B(q)](q - q_r) + (q - q_r)^T z\end{aligned} \tag{13}$$

Step 3. Consider the dynamics of $z = \dot{q} - \alpha$. Its time derivative can be expressed as

$$\begin{aligned}\dot{z} =&M^{-1}(q)[\tau - C(q, \dot{q})\dot{q} - G(q)] + K_1(\dot{q} - \dot{q}_r) + \frac{\partial \nabla f(q)}{\partial q}\dot{q} \\ &+ L\dot{q} + L^2 q + B(q)(\dot{q} - \dot{q}_r) + \dot{B}(q)(q - q_r)\end{aligned} \tag{14}$$

where $M^{-1}(q)$ is well-defined by Property 1.

Then, the time derivative of V_2 along with (14) can be computed by

$$\begin{aligned}\dot{V}_2 =&z^T M(q)\dot{z} + \frac{1}{2}z^T \dot{M}(q)z - \tilde{\varphi}^T\Gamma_1^{-1}\dot{\hat{\varphi}} - \sum_{i=1}^{N}\frac{1}{\gamma_i}\tilde{k}_i\dot{\hat{k}}_i \\ \overset{(a)}{=}&z^T\Big[\tau - C(q, \dot{q})\dot{q} - G(q) + M(q)\Big(\frac{\partial \nabla f(q)}{\partial q}\dot{q} + L\dot{q} \\ &+L^2(q - q_r) + L^2 q_r + B(q)(\dot{q} - \dot{q}_r) + \dot{B}(q)(q - q_r)\Big)\Big] \\ &+ z^T C(q, \mu)z - \tilde{\varphi}^T\Gamma_1^{-1}\dot{\hat{\varphi}} - \sum_{i=1}^{N}\frac{1}{\gamma_i}\tilde{k}_i\dot{\hat{k}}_i \\ \overset{(b)}{\leq}&z^T\Big[\tau - C(q, \dot{q})\dot{q} - G(q) + M(q)\Big(\frac{\partial \nabla f(q)}{\partial q}\dot{q} + L\dot{q} \\ &+B(q)(\dot{q} - \dot{q}_r) + \dot{B}(q)(q - q_r)\Big) \\ &+M^T(q)(L^4 + L^3)M(q)z/4\Big] + z^T C(q, \dot{q})z \\ &- \tilde{\varphi}^T\Gamma_1^{-1}\dot{\hat{\varphi}} - \sum_{i=1}^{N}\frac{1}{\gamma_i}\tilde{k}_i\dot{\hat{k}}_i + q_r^T Lq_r \\ \overset{(c)}{=}&z^T[\tau + (K - \varepsilon I)z - \Omega(q, \dot{q}, -\Lambda, \dot{q})\varphi] + q_r^T Lq_r \\ &- \tilde{\varphi}^T\Gamma_1^{-1}\dot{\hat{\varphi}} + \sum_{i=1}^{N}\frac{1}{\gamma_i}\tilde{k}_i(\gamma_i z_i^2 - \dot{\hat{k}}_i)\end{aligned} \tag{15}$$

where equality: (a) follows from Property 2; (b) follows from the Young's inequality

$$z^T M(q)L^2(q - q_r) \leq \|q - q_r\|^2 + \frac{1}{4}z^T M^T(q)L^4 M(q)z;$$

$$z^T M(q)L^2 q_r \leq q_r^T Lq_r + \frac{1}{4}z^T M^T(q)L^3 M(q)z;$$

(c) follows from Property 3 and $M^T(q)(L^4 + L^3)M(q)/4 \leq K - \varepsilon I$ with ε being a sufficiently small positive constant.

Substituting the adaptive controller and update laws in (10) into (15) yields that

$$\begin{aligned}\dot{V}_2 \leq& -\varepsilon\|z\|^2 - \tilde{\varphi}^T\Gamma_1^{-1}[\Gamma_1\Omega(q, \dot{q}, -\Lambda, \dot{q}_r)z + \dot{\hat{\varphi}}] \\ &+ q_r^T Lq_r - (q - q_r)^T z + \|q - q_r\|^2 \\ =& -\varepsilon\|z\|^2 + q_r^T Lq_r - (q - q_r)^T z + \|q - q_r\|^2\end{aligned} \tag{16}$$

which follows from the design $\dot{\hat{\varphi}} = -\Gamma_1\Omega(q, \dot{q}, -\Lambda, \dot{q}_r)z$.

Finally, consider the overall Lyapunov function V. Based on (12), (13) and (16), the inequality (11) is guaranteed. ■

Next, we prove the boundedness of the barrier function.

Proposition 2. The barrier functions $B_{\mathcal{D}_j}(q_i)$ for all $i = 1, \cdots, N$ and $j = 1, \cdots, M$ are bounded, and the EL multi-agent system can avoid colliding with the obstacles.

Proof. We show that the boundedness of $B_{\mathcal{D}_j}(q_i)$ for all $i = 1, \cdots, N$ and $j = 1, \cdots, M$ based on (11) by contradiction. To this end, we assume there exists a time instant T, an agent $v_{i^*} \in \mathcal{V}$ and obstacle region $\mathcal{D}_{j^*}$ such that $\lim_{t\to T}\mathcal{B}_{\mathcal{D}_{j^*}}(q_{i^*}(t)) = +\infty$ or equivalently $\|q_{i^*}(T) - d_{j^*}^c\|_{D_j} = \rho_{j^*}$ and $\|q_{i^*}(t) - d_{j^*}^c\|_{D_j} > \rho_{j^*}, \forall t < T$. Since $q_{i^*}(t)$ is continuous, then there exists a time $T_\delta \geq T$ such that $\|q_{i^*}(t) - d_{j^*}^c\|_{D_j} \leq \rho_{j^*}$ for all $t \in [T, T_\delta]$[1]. Recalling (11), one has

$$\begin{aligned}\|q_{i^*}(t) - q_{i^*}^r(t)\|_{D_j} =& \|q_{i^*}(t) - q_{j^*}^c + q_{j^*}^c - q_{i^*}^r(t)\|_{D_j} \\ \geq& \big|\|q_{j^*}^c - q_{i^*}^r(t)\|_{D_j} - \|q_{i^*}(t) - q_{j^*}^c\|_{D_j}\big| \\ \geq&\delta_i, \ \forall t \in [T, T_\delta]\end{aligned} \tag{17}$$

Then by computation, we have

$$\begin{aligned}&\mathcal{B}_{\mathcal{D}}(q_{i^*}(t)) \\ =& -\sum_{j=1}^{M}\int_{\alpha=0}^{t} B_{\mathcal{D}_j}(q_{i^*}(\alpha))(q_{i^*}(\alpha) - d_j^c)^T\dot{q}_{i^*}(\alpha)d\alpha \\ =&\sum_{j=1}^{M}\int_{\|q_{i^*}(t) - d_j^c\|_{D_j}^2}^{\|q_{i^*}(0) - d_j^c\|_{D_j}^2}\frac{1}{2}B_{\mathcal{D}_j}(q_{i^*}(\alpha))d\|q_{i^*} - d_j^c\|_{D_j}^2\end{aligned}$$

[1]Note that here we have ignored the case where the running trajectory of agent v_{i^*} is tangent with the obstacle boundary $\partial\mathcal{D}_j$.

$$
\begin{aligned}
&\geq \int_{\|q_{i^*}(t)-d^c_{j^*}\|^2_{D_j}}^{\|q_{i^*}(0)-d^c_{j^*}\|^2_{D_j}} \frac{1}{2} B_{D_{j^*}}(q_{i^*}) d\|q_{i^*} - d^c_{j^*}\|^2_{D_{j^*}} - \xi \\
&\geq \int_{\|q_{i^*}(T)-d^c_{j^*}\|^2_{D_j}}^{\|q_{i^*}(0)-d^c_{j^*}\|^2_{D_j}} \frac{1}{\|q_{i^*} - d^c_{j^*}\|^2_{D_{j^*}} - \rho^2_{j^*}} d\|q_{i^*} - d^c_{j^*}\|^2_{D_{j^*}} - \xi \\
&= \log(\|q_{i^*}(0) - d^c_{j^*}\|^2_{D_j} - \rho^2_{j^*}) \\
&\quad - \log(\|q_{i^*}(T) - d^c_{j^*}\|^2_{D_j} - \rho^2_{j^*}) - \xi \\
&= \log \frac{\|q_{i^*}(0) - d^c_{j^*}\|^2_{D_j} - \rho^2_{j^*}}{\|q_{i^*}(T) - d^c_{j^*}\|^2_{D_j} - \rho^2_{j^*}} - \xi \\
&= +\infty, \ \forall t \in [T, T_\delta] \qquad (18)
\end{aligned}
$$

where constant $\xi \geq \left| \sum_{j\in\Psi} \log \frac{\|q_{i^*}(0)-d^c_j\|^2_{D_j} - \rho^2_j}{\|q_{i^*}(t)-d^c_j\|^2_{D_j} - \rho^2_j} \right|$, with $\Psi = \{j \in [M] : \|q_{i^*}(t) - d^c_j\|_{D_j} > \|q_{i^*}(0) - d^c_j\|_{D_j}\}$.

On the other hand, taking integral for two sides of (11) from $t = T$ to T_δ yields that

$$
\int_{t=T}^{T_\delta} \mathcal{B}_{\mathcal{D}}(q_{i^*}(t)) \|q_{i^*}(t) - q^r_{i^*}(t)\|^2 dt < V(T) \qquad (19)
$$

Combining (18) and (19), we obtain that $q_{i^*}(t) - q^r_{i^*}(t) = 0$ for all $t \in [T, T_\delta]$, a contradiction with (17). Therefore, we have proved that $B_{\mathcal{D}_j}(q_i)$ is bounded. ■

Now based on the above two propositions, we finish the proof of Theorem 1.

Proof of Theorem 1. From (11), we have that q_r, q, χ, z, $\hat{\varphi}$ and $\hat{k}_i$ are all bounded. Since $B_{\mathcal{D}_j}(q_i)$ for all $i = 1, \cdots, N$ are bounded, i.e., there exists a positive constant b_M such that

$$
|B_{\mathcal{D}_j}(q_i)| \leq 2b_M, \ \forall t \geq 0,
$$

then we have

$$
\begin{aligned}
&|B_{\mathcal{D}_j}(q_i)| \\
&= \sum_{j=1}^{M} \int_{\|q_i(t)-d^c_j\|^2_{D_j}}^{\|q_i(0)-d^c_j\|^2_{D_j}} \frac{1}{2} B_{D_j}(q_i(\alpha)) d\|q_i - d^c_j\|^2_{D_j} \\
&\leq b_M \left| \sum_{j=1}^{M} \int_{\|q_i(t)-d^c_j\|^2_{D_j}}^{\|q_i(0)-d^c_j\|^2_{D_j}} d\|q_i - d^c_j\|^2_{D_j} \right| \\
&\leq b_M \sum_{j=1}^{M} (\|q_i(t) - d^c_j\|^2_{D_j} + \|q_i(0) - d^c_j\|^2_{D_j}).
\end{aligned}
$$

Since $q(t)$ is bounded, then $\mathcal{B}_M(\Delta q(t))$ is bounded. Then according to (4), α is also bounded. Further, $\dot{q} = z + \alpha$ is also bounded. As a result, control signal $\tau(t)$ is bounded, and all the signals in the closed-loop are bounded. From (11), we have $q(t) - q_r(t), z(t) \in L_\infty$ and $q(t) - q_r(t), z(t) \in L_2$. By Barbalat's Lemma, we can derive that $\lim_{t\to\infty} q(t) - q_r(t) = 0$ and $\lim_{t\to\infty} z(t) = 0$. Also, $\lim_{t\to\infty} q_r(t) = q^*$ from Lemma 2. Hence, we have $\lim_{t\to\infty} q_i(t) = q^*$, $i = 1, \cdots, N$. ■

Remark 2. Compared with the existing DOC strategy in [3], the proposed DOC strategy has the following advantages:

- The collisions with obstacles can be eliminated.

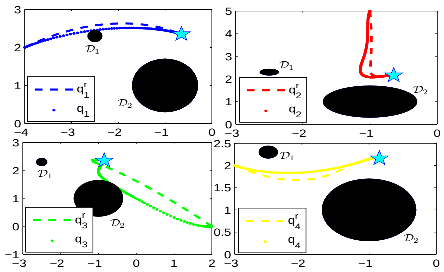

Fig. 1: Motion trajectories of these four EL systems.

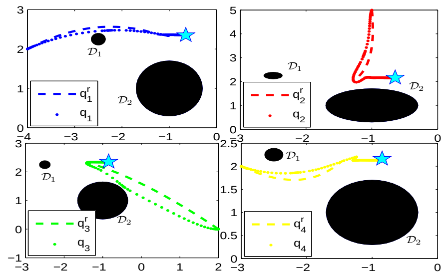

Fig. 2: Motion trajectories of these four EL systems.

- The local cost function f_i is only required to be convex and differentiate. The assumptions that f_i is ω-strongly convex and ∇f_i is θ-Lipschitz are unnecessary.
- By introducing an adaptive gain $\hat{k}_i$ given in (8), no additional algorithm is required to solve the maximum eigenvalue of the Laplacian matrix L in distribution.

4 Simulations

In this section, we apply our algorithm to the problem of optimal coordination of multiple EL models which moves on the obstacle-populated Euclidean plane. In the example, the specific objective is to steer four EL systems which lie in different positions to achieve consensus at the location which is optimal for the whole team. The dynamics of each EL system [3] is described by (1) with

$$
M_i(q_i) = \begin{bmatrix} b_{i1} + 2b_{i2}\cos q_{iy} & b_{i3} + b_{i2}\cos q_{iy} \\ b_{i3} + b_{i2}\cos q_{iy} & b_{i3} \end{bmatrix}
$$

$$
C_i(q_i, \dot{q}_i) = \begin{bmatrix} -b_{i2}\sin q_{iy}\dot{q}_{iy} & -b_{i2}\sin q_{iy}(\dot{q}_{ix} + \dot{q}_{iy}) \\ b_{i2}\sin q_{iy}\dot{q}_{ix} & 0 \end{bmatrix}
$$

and $G_i(q_i) = [0, 0]^T$, where $q_i = \mathrm{col}(q_{ix}, q_{iy})$ and $\varphi_i = \mathrm{col}(b_{i1}, b_{i2}, b_{i3})$ for $i = 1, \cdots, 4$. Note that φ_i are the unknown system parameters. Assume that these four agents communicate with each other over a 2-regular graph given by Fig. 1. It is easily verified that Assumption 2 is satisfied.

In our simulations, the parameters assoicated with agents are $\varphi_1 = [5, 1, 1]^T$, $\varphi_2 = [5, 1, 2]^T$, $\varphi_3 = [6, 1, 1]^T$ and $\varphi_4 = [6, 1, 2]^T$. Assume that the initial states of the EL systems are $[q^T_1(0), \dot{q}^T_1(0)]^T = [-4, 2, 0, 0]^T$, $[q^T_2(0), \dot{q}^T_2(0)]^T = [-1, 5, 0, 0]^T$, $[q^T_3(0), \dot{q}^T_3(0)]^T = [2, 0, 0, 0]^T$ and $[q^T_4(0), \dot{q}^T_4(0)]^T = [-3, 2, 0, 0]^T$. The problem of optimal coordination consists in finding a distributed control that is able to drive each EL system from the different initial positions to achieve consensus at the target position which is nearest from these initial positions. This problem can be formulated as the following optimization problem:

$$\min \sum_{i=1}^{4} \|q_i - q_i(0)\|^2, s.t.\ q_1 = q_2 = q_3 = q_4.$$

In the simulation, the paths of these four agents are blocked by two circular obstacles, i.e. $\mathcal{D}_1 = \{q \in \mathbb{R}^2 : \|q - [-2.5, 2.3]^T\|^2 \leq 0.15^2\}$ and $\mathcal{D}_2 = \{q \in \mathbb{R}^2 : \|q - [-1, 1]^T\|^2 \leq 0.65^2\}$.

Now we apply the proposed secure DOC strategy to complete the motion coordination task. In Algorithm 1, we choose $\hat{q}_i^r(0) = q_i(0)$. The simulated trajectory $\hat{q}_i^r(t)$ derived by (18) does not collide with the obstacles, and resultantly $T = 0$. To illustrate the advantage of the use of the safety barrier function, we also simulate the unsafe algorithm by directly letting $\mathcal{B}_{\mathcal{D}}(q_i) = 0$ in (4), (6) and (7). The corresponding simulation results are given in Figs. 1 and 2, where the "blue star" represents the target position. From Fig. 1, it can be seen that ELs #1 and #3 will collide with the obstacles by using the existing distributed control strategy, while the proposed DOC strategy steers the four EL agents to achieve consensus at the optimal position while avoiding collision with the obstacles, as illustrated by Fig. 2.

5 Conclusion

In this paper, the problem of DOC with obstacle avoidance for multiple EL systems has been investigated. With the safety barrier certificates, distributed optimization and adaptive nonlinear control, we proposed a fully distributed protocol for uncertain EL systems to ensure the optimal performance with collision avoidance with obstacles, and gave the corresponding global convergence and collision-free guarantee results. It is important to note that the collisions between inner agents are also another safety threat in the multi-agent coordination [26, 27]; the DOC with avoiding collisions between two agents will be investigated in the future work.

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