Learning-Based Control:

A Theory based on Robust Adaptive Dynamic Programming

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Small-Gain Theory: Robust Nonlinear Control Design

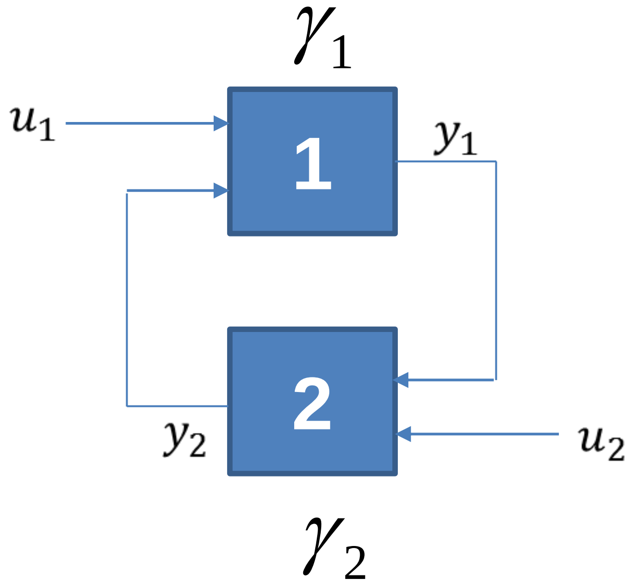
Plenary lecture @2003 Chinese Control Conference

Tolstoy(托尔斯泰):
All happy families are alike.

Why so?

because they all satisfy the small-gain condition!

$\gamma_1 \circ \gamma_2 < Id$ implies "network stability".



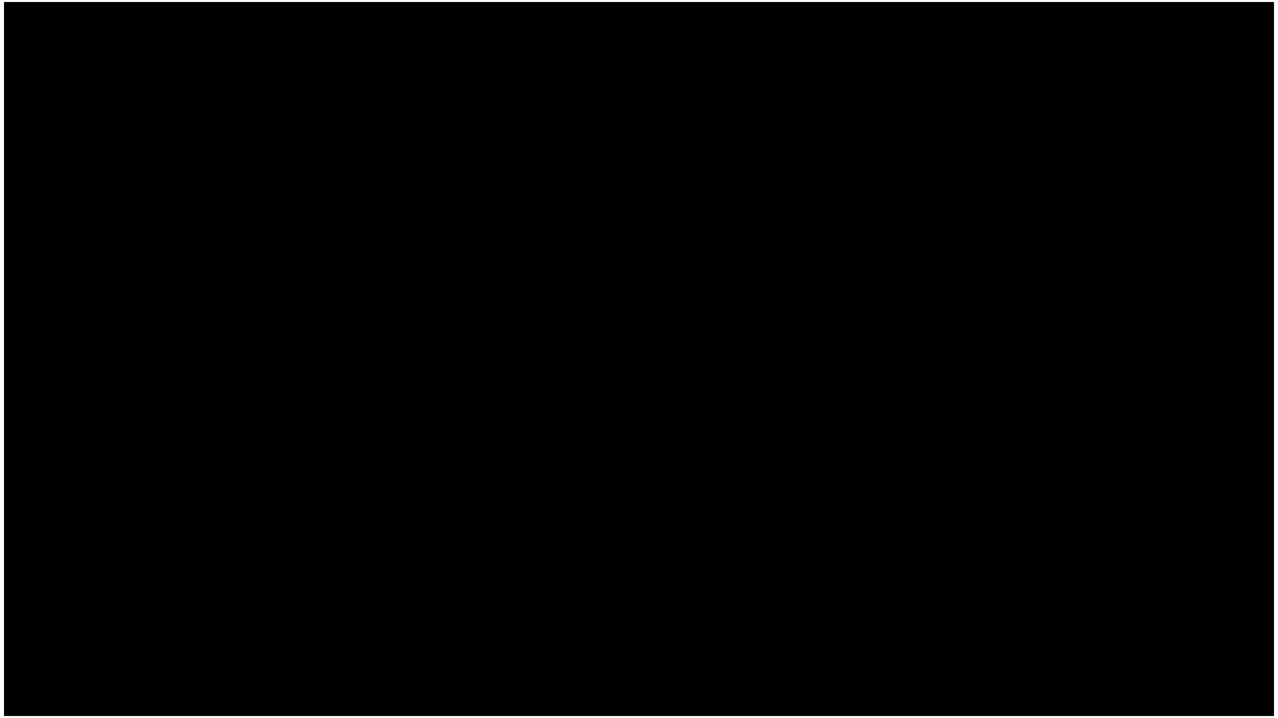
where γ_1 : gain from y_2 to y_1 and γ_2 : gain from y_1 to y_2

Small-Gain Theory 2

第二集

Learning-Based Control

Why Learning-Based Control Theory?



Data and Learning-Based Control

Use i/o data to learn better (adaptive/optimal) controllers in the absence of exact model knowledge

- **Rapid response**
- **Stability/robustness/safety guarantee**
- **Optimality for reduced energy consumption**

Adaptive Optimal Control Problem

How to solve $\min J(x_0;u)=\int_{t_0}^{\infty} r(x,u)dt$

subject to

$$\dot{x}=f\left(x,u\right),\ \text{with unknown}\ f$$

Model-based approach

- **For linear systems, many published papers by several authors:**

  Guo/Duncan/Pasik-Duncan, Bitmead, Kumar, HF Chen, etc

- **For nonlinear systems,**
  "Almost" None

Limitations of Dynamic Programming (DP)

How to solve $\min J(x_0;u) = \int_{t_0}^{\infty} r(x,u)\,dt$

subject to

$\dot{x} = f\left(x,u\right)$, with unknown f

Bellman's Dynamic Programming is <u>not</u> applicable, because of

- Curse of dimensionality (Bellman, 1959)
- Curse of modeling (Bertsekas, 1996)

Outline

- Data-Driven Learning-based Control Theory: Why?
- Robust Adaptive Dynamic Programming
  - Adaptive LQR for continuous-time linear systems
  - Extensions: nonlinear and robust
- Application:

  Connected and Autonomous Vehicles
- Conclusions and Future Work

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Why Data-driven Learning-based Control?

- "System modeling is expensive, time consuming, and inaccurate." **(Frank Lewis @ASCC'09)**

- **Brought together "stability" and "reinforcement learning" (for c-t systems)**

- **"Adaptive Dynamic Programming" (ADP):**

An active research area, integrating reinforcement learning (RL) and controls to remove the curses of dimensionality and of modeling.

Reinforcement Learning (RL)

Model-free approach

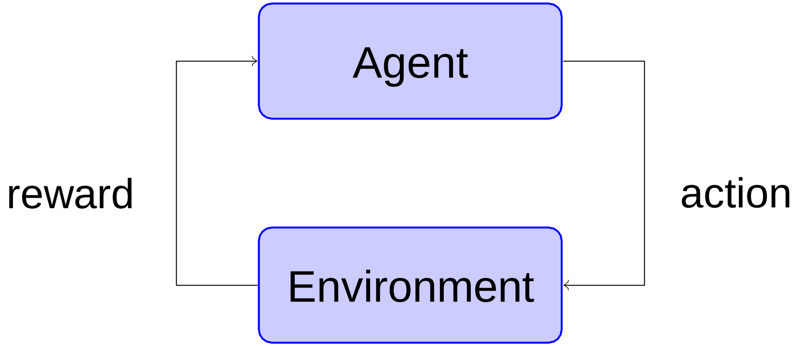

Figure: Reinforcement Learning (Minsky, 1954).

Maximizing the cumulative reward, through

1) Exploration (finding better policies).

2) Agent-environment interaction.

Many papers

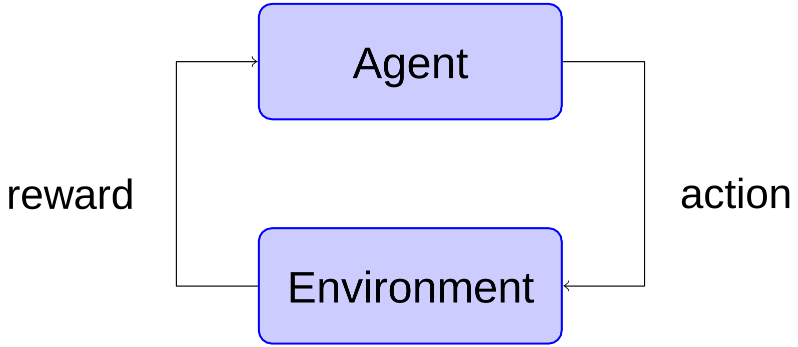

Figure: Reinforcement Learning (Minsky, 1954).

1. CS perspective (Barto, Dayan, Sutton, Watkins, Kaelbling, Littman, Doya).
   Finite state/action space; early ideas.

2. OR perspective (Bertsekas, Tsitsiklis, Van Roy, Nedic, Borkar, Powell).
   Countable state/action space; advanced convergence analysis (stochastic approximation, function approximation).

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The history of DP and RL

1952	Dynamic programming	(Bellman)
1954	Reinforcement learning	(Minsky)
1960	DP algorithms (VI, PI) for MDPs	(Bellman; Howard)
1960s	Positive & Negative DP	(Blackwell; Strauch)
1968	RL + approximate DP	(Werbos)
1983	Actor-critic algorithm	(Barto)
1984	TD-learning	(Sutton)
1989	Q-learning	(Watkins)
1990s	Neuro-DP	(Bertsekas)
2010s	Adaptive DP	(Lewis)
2013	Abstract DP	(Bertsekas)

2012 – Robust Adaptive DP (here)

Continuous-time ADP: still in its infancy

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LTI system $\dot{x} = Ax + Bu,\ x(0) = x_0$

When (A, B) are **unknown**, find an “i/s data-driven” linear control policy

$u = -Kx$

that minimizes $J = \int_0^\infty (x^T Q x + u^T R u) dt = x_0^T P x_0$

where $Q = Q^T \geq 0,\ R = R^T > 0,\ (A, B)$ is controllable, and $(A, Q^{1/2})$ is observable.

Linear-quadratic regulator (LQR)

Optimal Controller:

- $u^* = -K^* x,$
- $\mathcal{J}(x_0; u^*) = x_0^T P^* x_0,\ with\ P^* = P^{*T} > 0$

Algebraic Riccati equation: $A^T P^* + P^* A - P^* B R^{-1} B^T P^* + Q = 0, \quad K^* = R^{-1} B^T P^*.$

Question 1:

How to learn suboptimal controllers, from i/s data, that converge to the (unknown) optimal controller?

1. Policy Iteration (PI)
2. Value Iteration (VI)

Learning-based Policy Iteration

Assume the knowledge of an initial stabilizing policy K_0

From

$$\int_t^{\infty}\left(x^T Q x+u^T R u\right)d\tau=\int_t^{t+\delta t}\left(x^T Q x+u^T R u\right)d\tau+\int_{t+\delta t}^{\infty}\left(x^T Q x+u^T R u\right)d\tau$$

Integral RL equation.

ADP for partially unknown linear systems (Lewis et al., 2009)

$$x^T(t)P_j x(t)=\int_t^{t+\delta t} x^T\left(Q+K_j^T R K_j\right)x d\tau+x^T(t+\delta t)P_j x(t+\delta t),$$

$$K_{j+1}=R^{-1}B^T P_j.$$

B is required. $x(t)$ is generated by $u=-K_j x$ (on-policy).

Under mild conditions,

$$P_j \to P^*,\quad K_j \to K^*.$$

Learning-based Policy Iteration

ADP for partially unknown linear systems (Lewis et al., 2009)

$$x^T(t)P_jx(t) = \int_t^{t+\delta t} x^T(Q + K_j^T RK_j)x d\tau + x^T(t+\delta t)P_jx(t+\delta t),$$

$$K_{j+1} = R^{-1}B^T P_j.$$

B is required. $x(t)$ is generated by $u = -K_j x$ (on-policy).

Integral RL equation.

ADP for fully unknown linear systems (Jiang & ZPJ, 2012)

$$x^T(t)P_jx(t) = \int_t^{t+\delta t} \left(x^T(Q + K_j^T RK_j)x - 2(K_{j+1}x)^T R(u' + K_jx)\right) d\tau + x^T(t+\delta t)P_jx(t+\delta t),$$

B is not required. x is generated by $u = u'$ (off-policy). Usually, we choose

$u' = -K_jx + \xi$ or $u' = -K_0x + \xi$

On-line Off-policy PI-based ADP algorithm

Jiang & ZPJ, 2012

Collecting i/s data over $[t_i, t_{i+1}]$, $i = 0, \ldots, l-1$,

$$\Theta_k \begin{bmatrix} \hat{P}_k \\ vec(K_{k+1}) \end{bmatrix} = \Xi_k \qquad (**)$$

$$\Theta_k = \left[\delta_{xx} - 2I_{xx}\left(I_n \otimes K_k^T R\right) - 2I_{xu}\left(I_n \otimes R\right)\right] \in \mathbb{R}^{l \times \left(\frac{n(n+1)}{2} + nm\right)},$$

$$\Xi_k = -I_{xx} vec\left(Q + K_k^T R K_k\right),$$

For $P \in \mathbb{R}^{n \times n}$ and $x \in \mathbb{R}^n$,

$$\delta_{xx} = [\bar{x}(t_1) - \bar{x}(t_0), \bar{x}(t_2) - \bar{x}(t_1), \cdots, \bar{x}(t_l) - \bar{x}(t_{l-1})]^T \in \mathbb{R}^{l \times \frac{n(n+1)}{2}},$$

$$I_{xx} = \left[\int_{t_0}^{t_1} x \otimes x d\tau, \int_{t_1}^{t_2} x \otimes x d\tau, \cdots, \int_{t_{l-1}}^{t_l} x \otimes x d\tau\right]^T \in \mathbb{R}^{l \times n^2},$$

$$I_{xu} = \left[\int_{t_0}^{t_1} x \otimes u d\tau, \int_{t_1}^{t_2} x \otimes u d\tau, \cdots, \int_{t_{l-1}}^{t_l} x \otimes u d\tau\right]^T \in \mathbb{R}^{l \times nm},$$

$$\bar{x} = [x_1^2, x_1 x_2, \cdots, x_1 x_n, x_2^2, x_2 x_3, \cdots, x_{n-1} x_n, x_n^2,]^T \in \mathbb{R}^{\frac{n(n+1)}{2}},$$

$$\hat{P} = [p_{11}, 2p_{12}, \cdots, 2p_{1n}, p_{22}, 2p_{23}, \cdots, 2p_{2n}, p_{nn}]^T \in \mathbb{R}^{\frac{n(n+1)}{2}}.$$

1) Full rank of Θ_k

$\Rightarrow$ unique solution of (**)

(due to exploration noise ξ)

2) $P_k \to P^*$, $K_k \to K^*$ as $k \to \infty$.

3) Stability + suboptimality

without ξ.

Learning-based Value Iteration

Question:

Can we <u>remove</u> the assumption on the knowledge of an initial, stabilizing policy K_0, when the system dynamics are <u>not</u> known?

Yes!

Generalize & apply the “Value Iteration” (VI) method to continuous-time dynamical systems.

History of Value Iteration (VI)

Value iteration:

- 1959 — VI for MDPs (Bellman)
- 1960 — The name of Value iteration was introduced (Howard)
- 1995 — VI for DT linear systems. (Lancaster & Rodman)
- 2015 — VI for DT nonlinear systems (Bertsekas, Lewis, …)

Policy iteration:

- 1960 — PI for MDPs (Howard)
- 1969 — PI for CT linear systems. (Kleinman)
- 1976 — PI for DT linear systems. (Bertsekas)
- 1995 — PI for CT affine nonlinear systems (Beard & Saridis)
- 2014 — PI for CT nonaffine nonlinear systems (Bian, ZPJ, etc)
- 2015 — PI for DT nonlinear systems (Bertsekas, D. Liu, Lewis, …)

- ➢ VI is more difficult.
- ➢ It is still an open problem to develop VI for continuous-time systems.
- ➢ We give a VI by combining DMRE and stochastic approximation theory.

VI for Linear-quadratic regulator (LQR)

Bian & ZPJ, Automatica, 2016

Continuous-time VI: $\lim_{t\to\infty} M(t) = P^*$, where

$$\dot{M} = A^T M + MA - MBR^{-1}B^T M + Q, \qquad M(0) = M^T(0) > 0$$

Stochastic Approximation:

$$\theta_{t+1} = \theta_t + \epsilon_t(g(\theta_t) + \delta M_t) + Z_t$$

where

- Z_t is a projection term;
- ϵ_t is the step size;
- $\{\delta M_t\}$ is a sequence of i.i.d random variables, $E[\delta M_t] = 0, Var[\delta M_t] < \infty$;
- $g(\cdot)$ is measurable and locally Lipschitz.

Convergence: $\theta_t \to \theta^*$ with probability 1,

$\dot{\theta} = g(\theta)$ is asymptotically stable at θ^*. *(Kushner-Yin, 2003)*

VI for Linear-quadratic regulator (LQR)

$\{B_p\}_{q=0}^{\infty}$: $B_q \subseteq B_{q+1}, \lim_{q\to\infty} B_q = \{P \in \mathbb{R}^{n\times n}: P^T = P \geq 0\}$

$\{\epsilon_k\}_{k=0}^{\infty}$: $\epsilon_k > 0, \lim_{k\to\infty} \epsilon_k = 0, \sum_{k=0}^{\infty} \epsilon_k = \infty$. $\varepsilon > 0$ is a threshold

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Algorithm 1 SA-based continuous-time VI algorithm (Bian & ZPJ, 2016):

Choose $P_0 = P_0^T > 0.\, k, q \leftarrow 0.$

Loop

$\tilde{P}_{k+1} \leftarrow P_k + \epsilon_k(A^T P_k + P_k A - P_k B R^{-1} B^T P_k + Q)$

if $\tilde{P}_{k+1} \notin B_q$ **then**

$P_{k+1} \leftarrow P_0.\, q \leftarrow q + 1$

else if $\frac{|\tilde{P}_{k+1} - P_k|}{\epsilon_k} < \varepsilon$ **then return** $\hat{P}^* = P_k$

else $P_{k+1} \leftarrow \hat{P}_{k+1}$

$k \leftarrow k + 1$

- Convergence: $P_k \to P^*$
- Stability: If $Q > \varepsilon I_n$, then $A - B\hat{K}^*$ is Hurwitz, where $\hat{K}^* = R^{-1} B^T \hat{P}^*$, and $\hat{P}^*$ is obtained from the VI algorithm.

On-line Off-policy ADP algorithm (Bian & ZPJ, 2016):

Solve H_k and K_k from

$$x^T(t+\delta t)P_k(t+\delta t) = \int_t^{t+\delta t} x^T H_k x ds + 2\int_t^{t+\delta t} u^T R K_k x ds + x^T(t) P_k x(t). \quad \textbf{(1)}$$

where $H_k = A^T P_k + P_k A$.

Continuous-time VI-based ADP algorithm

Choose $P_0 = P_0^T > 0. k, q \leftarrow 0.$
Apply a locally bounded input u to the system.
Loop

Solve (H_k, K_k) from (1)

$\tilde{P}_{k+1} \leftarrow P_k + \epsilon_k(A^T P_k + P_k A - P_k B R^{-1} B^T P_k + Q)$

$= P_k + \epsilon_k(H_k - K_k^T R K_k + Q)$

if $\tilde{P}_{k+1} \notin B_q$ **then**

$P_{k+1} \leftarrow P_0. q \leftarrow q + 1$

else if $\frac{|\tilde{P}_{k+1} - P_k|}{\epsilon_k} < \varepsilon$ **then return** $\hat{P}^* = P_k$

else $P_{k+1} \leftarrow \hat{P}_{k+1}$

$k \leftarrow k + 1$

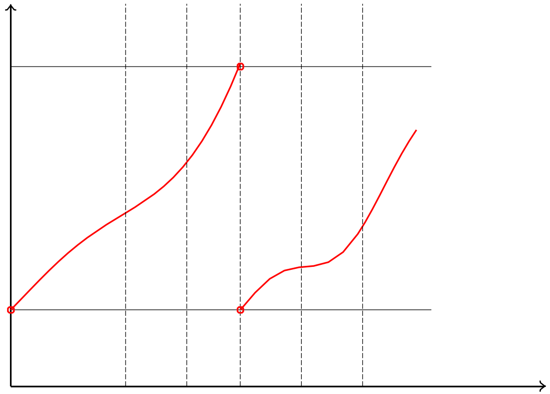

Figure: Bounded learning trajectory.

How about general nonlinear systems?

$$\dot{x} = f(x,u), \qquad \min \quad \mathcal{J}(x_0;u) = \int_0^\infty r(x,u)dt\,.$$

with unknown dynamics *f* ?

- **No distinction between linear and nonlinear problems**
- **First solution to adaptive/nonlinear optimal control**

Policy iteration for nonaffine systems

(Bian, Jiang & ZPJ, 2014)

Nonaffine system:

$$\dot{x} = f(x,u), \qquad \mathcal{J}(x_0;u) = \int_0^\infty r(x,u)dt\,.$$

HJB equation:

$$0 = \min_{v\in\mathbb{R}^m}\{\partial_x V^*(x) f(x,v) + r(x,v)\}, \qquad V^*(0) = 0,$$

$$\mu^*(x) = \arg\min_{v\in\mathbb{R}^m}\{\partial_x V^*(x) f(x,v) + r(x,v)\}$$

Policy iteration

1. Policy evaluation: $\partial_x V_j(x) f\left(x, \mu_j(x)\right) + r\left(x, \mu_j(x)\right) = 0, V_j(0) = 0.$
2. Policy improvement: $\mu_{j+1}(x) = \arg\min_{v\in\mathbb{R}^m}\{\partial_x V^*(x) f(x,v) + r(x,v)\}, \forall x \in \mathbb{R}^n$

- Convergence: If μ_0 is admissible, $V_j \to V^*, \mu_j \to \mu^*$.
- Stability: μ_j is stabilizing and, $\mathcal{J}(x_0; u_j(x)) < \infty$

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Basis function approximation

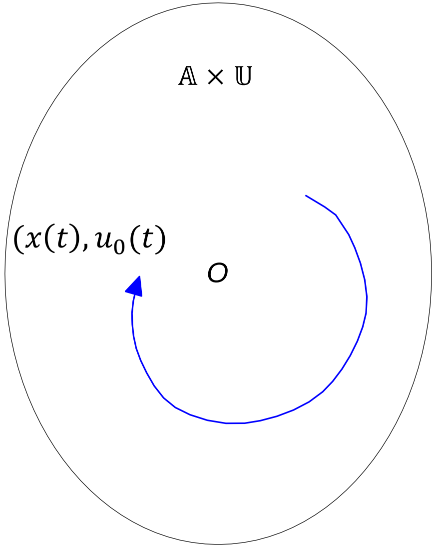
Using basis function approximation, we have for all $x \in \mathbb{A}$ and $v \in \mathbb{U}$,

$$V_j(x) = \sum_{i=1}^{N} \hat{w}_i^j \phi_i(x) + e_\phi^j(x),$$

$$\partial_x V_j(x) f(x,v) = \sum_{i=1}^{N} \hat{c}_i^j \psi_i(x,v) + e_\psi^j(x,v),$$

$$\mu_j(x) = \sum_{i=1}^{N} \hat{l}_i^j \theta_i(x) + e_\theta^j(x)$$

Linear-like problem



- $\{\phi_i\}_{i=1}^N, \{\psi_i\}_{i=1}^N, and\ \{\theta_i\}_{i=1}^N,$
  with $\phi_i: \mathbb{R}^n \to \mathbb{R}, \psi_i: \mathbb{R}^n \times \mathbb{R}^m \to R,\ and\ \theta_i: \mathbb{R}^n \to \mathbb{R}^m,$
  are three sets of linearly independent and continuous functions;
- $e_\phi^j, e_\psi^j, and\ e_\theta^j$ are the approximation errors.

ADP algorithm: Semiglobal stabilization

Assumption (Persistent excitation (PE))

For all $\{\hat{\mu}_j\}_{j=0}^{\infty}$, there exist $\bar{M} > 0$ and $\gamma > 0$, such that for all $M \geq \bar{M}$,

$\frac{1}{M}\sum_{k=1}^{M} {\Theta_k^j}^T \Theta_k^j \geq \gamma I_{2N}$, $\Theta_k^j \in \mathbb{R}^{1\times 2N}$ is the vector of input-state data

The ADP algorithm:

1. Apply $u_0(t)$ to the system. $j \leftarrow 0$.
2. Policy evaluation: $$[\hat{w}^j, \hat{c}^j]^T = -\left(\sum_{k=1}^{M} {\Theta_k^j}^T \Theta_k^j\right)^{-1} \sum_{k=1}^{M} {\Theta_k^j}^T \int_{t_{k-1}}^{t_k} r\left(x, \hat{\mu}_j(x)\right) dt.$$
3. Policy update: $$\hat{l}^{j+1} = \arg\min_{\{l \mid l\theta(x)\in\mathbb{U}\}} \{\hat{c}^j \psi(x, l\theta(x)) + r(x, l\theta(x))\}, \hat{\mu}_j = \hat{l}^j\theta.$$

Convergence on $\mathbb{A}$:

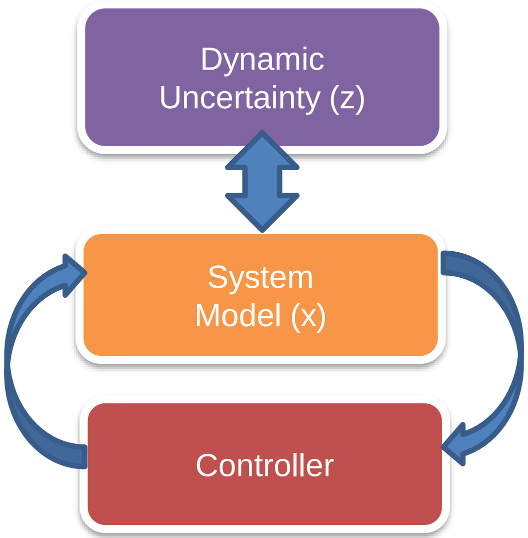
$$\lim_{N\to\infty} \left|\sum_{i=1}^{N} \hat{l}_i^j \theta_i(x) - \mu_j(x)\right| = 0, \quad \lim_{N\to\infty} \left|\sum_{i=1}^{N} \hat{w}_i^j \phi_i(x) - V_j(x)\right| = 0.$$

Bian, Jiang & ZPJ, 2014

Question 2:

How to learn suboptimal controllers with guaranteed robustness to dynamic uncertainties?

Robust Adaptive Dynamic Programming



dim(z, x) unknown,

with possibly huge dim(z)

Dynamic uncertainties:

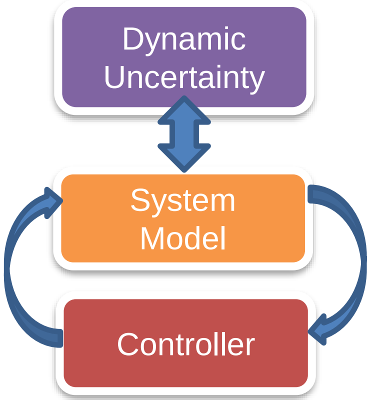
- Mismatch between model and plant
- Observation errors
- Subsystems in large-scale networks
- Model reduction

Note: Previous ADP algorithms assume the system order is known!

RADP: Robust Adaptive Dynamic Programming

Jiang & ZPJ; TNNLS, 2013

For illustration, consider partially linear composite systems with "**dynamic uncertainty**".

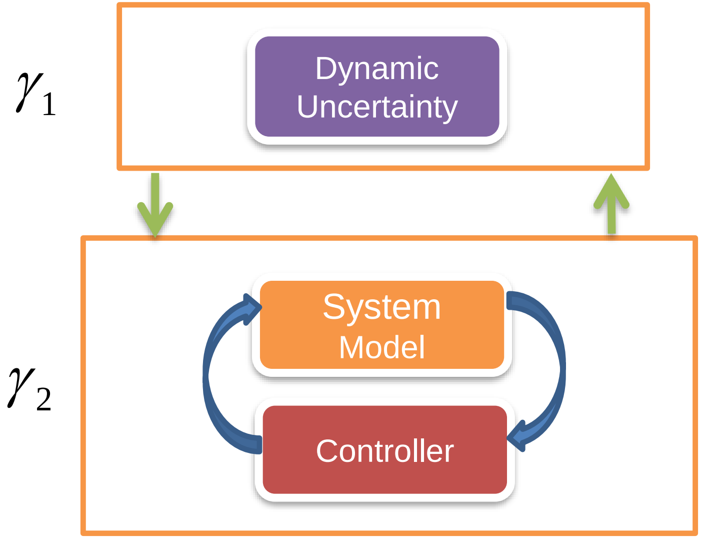


$$\dot{w} = q(w, y)$$
$$\dot{x} = Ax + B[u + E\Delta(w, y)]$$
$$y = Cx$$

where A, B, C, E are unknown matrices, q and Δ are unknown locally Lipschitz functions vanishing at the origin.

Challenge: How to learn robust/adaptive nonlinear optimal controllers via real-time and partial-state information?

RADP for partially linear composite systems



Input-to-state stability (ISS) and *Input-to-output stability* (IOS) [Sontag 1989], [Sontag & Wang 1995].

The state-space *nonlinear small-gain theorem* proposed in [Jiang, Teel, & Praly 1994] is an important tool for network stability and control.

A simplified version of the small-gain theorem: If $\gamma_1 \circ \gamma_2 < \mathrm{Id}$, then, the overall system is globally asymptotically stable at the origin.

Challenge

How to achieve gain assignment γ_2 *via* i/o data and ADP?

Special Case: Linear Gain Assignment

Jiang & ZPJ; TNNLS, 2013

$$\text{S1}: \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \qquad \text{S2}: \begin{cases} \dot{x} = Ax + B(u + Ew) \\ y = Cx \end{cases}$$

Lemma (Gain assignment): Let $u = -K^* x$ be the optimal control policy of system S1 and assume the weighting matrices satisfying $Q > \gamma C^T C$ and $R^{-1} > EE^T$. Then, there exists a continuously differentiable, positive definite and radially unbounded function $V(x)$, such that along the solutions of S2, we have

$$\dot{V} \le -\gamma |y|^2 + |w|^2$$

Remark: The constant $\gamma > 0$ can be arbitrarily assigned by choosing appropriate weighting matrices Q and R, without knowing A and B.

Linear Gain Assignment

Jiang & ZPJ; TNNLS, 2013

Assumption: There exist a continuously differentiable, positive definite and radially unbounded function W and two constants $c_1, c_2 \geq 0$, such that

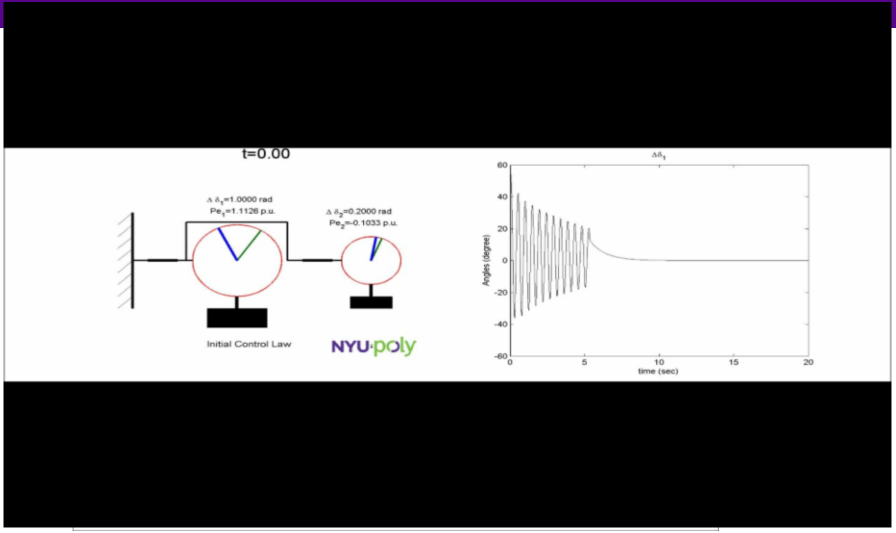
$$\frac{\partial W}{\partial z} q(w, y) \leq -c_1 |\Delta(w, y)|^2 + c_2 |y|^2$$

Lemma (Global Stabilization): Under mild assumptions, the overall system is globally asymptotically sable under the control policy $u = -K^* x$

if the following small-gain condition holds:

$$\frac{1}{\gamma} \frac{c_2}{c_1} < 1$$

Application to a Power System



Control Challenges:
1. Unknown dynamics
2. Locally available state variables
3. Prevent oscillation

Robust-ADP Approach:
1. Online learning
2. Partial state feedback
3. Stability and Suboptimality

Mechanical Dynamics: [P. Kundur et al. 1994]

$$\frac{d^2\delta_i}{dt^2} = -\frac{D_i}{2H_i}\frac{d\delta_i}{dt} + \frac{\omega_0}{2H_i}\left(P_{mi} - P_{e_i}\right) \qquad i = 1, 2$$

Governor Dynamics:

$$\frac{dP_{mi}}{dt} = \frac{1}{T_i}[-P_{mi} + u_i] \qquad i = 1, 2$$

Active Power:

$$P_{e1} = E_1 E_2 \left(B_{12}\sin\delta_{12} + G_{12}\cos\delta_{12}\right) + E_1\frac{V_s}{x_{ds}}\sin\delta_1$$

$$P_{e2} = E_1 E_2 \left(B_{21}\sin\delta_{21} + G_{21}\cos\delta_{21}\right)$$

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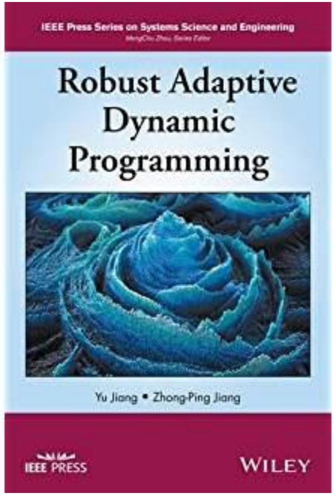
RADP: Robust Adaptive Dynamic Programming



Recent extensions:

- **Value iteration (c-t)**
- **Output feedback ADP**
- **Adaptive/optimal output regulation via ADP**
- **ADP for multi-agent systems**
- **Stochastic systems**

Tools:

Semiglobal ADP, Global ADP, Decentralized ADP,
with applications in electric power systems, human motor control

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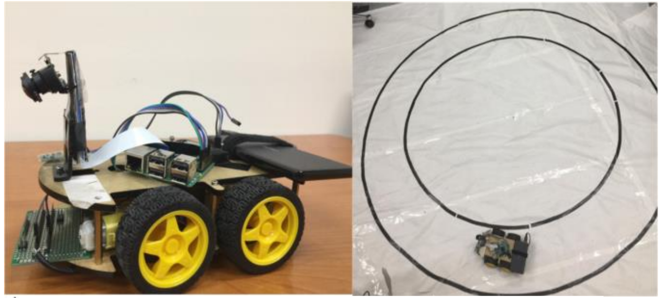
1. Reinforcement Learning for Vision-Based Lateral Control



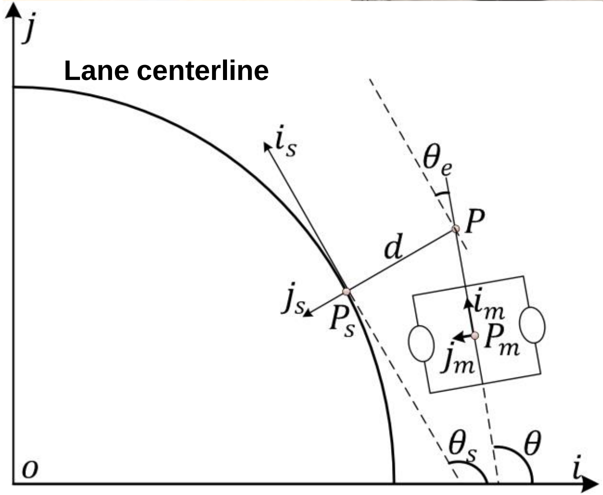
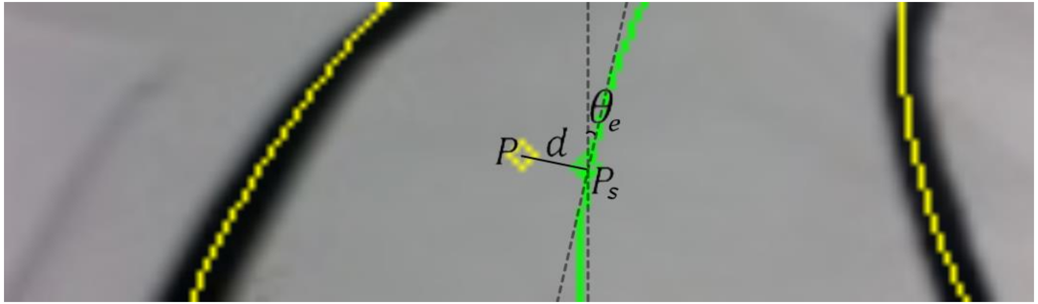


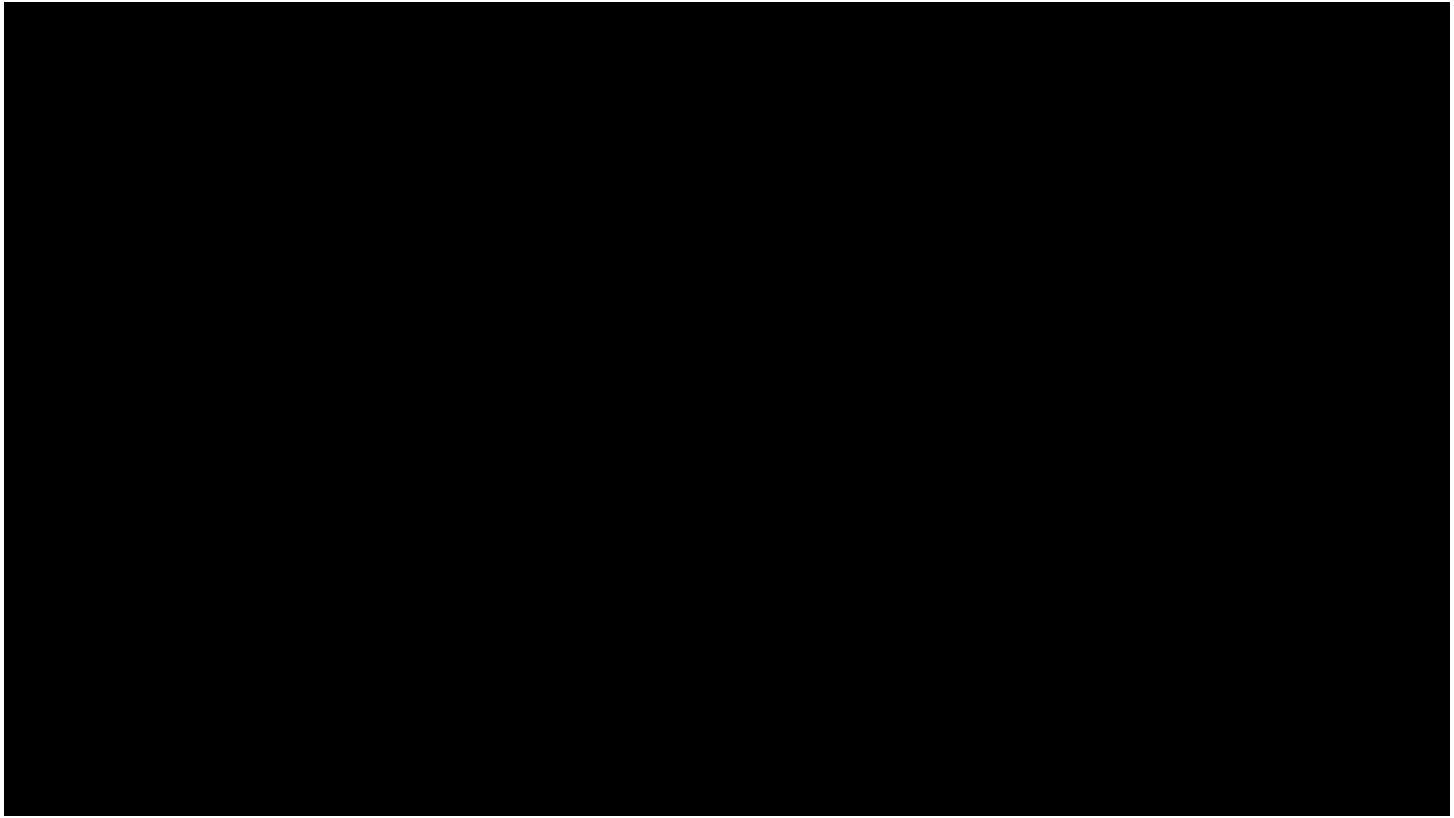
Image before and after processing

(a) Raw image

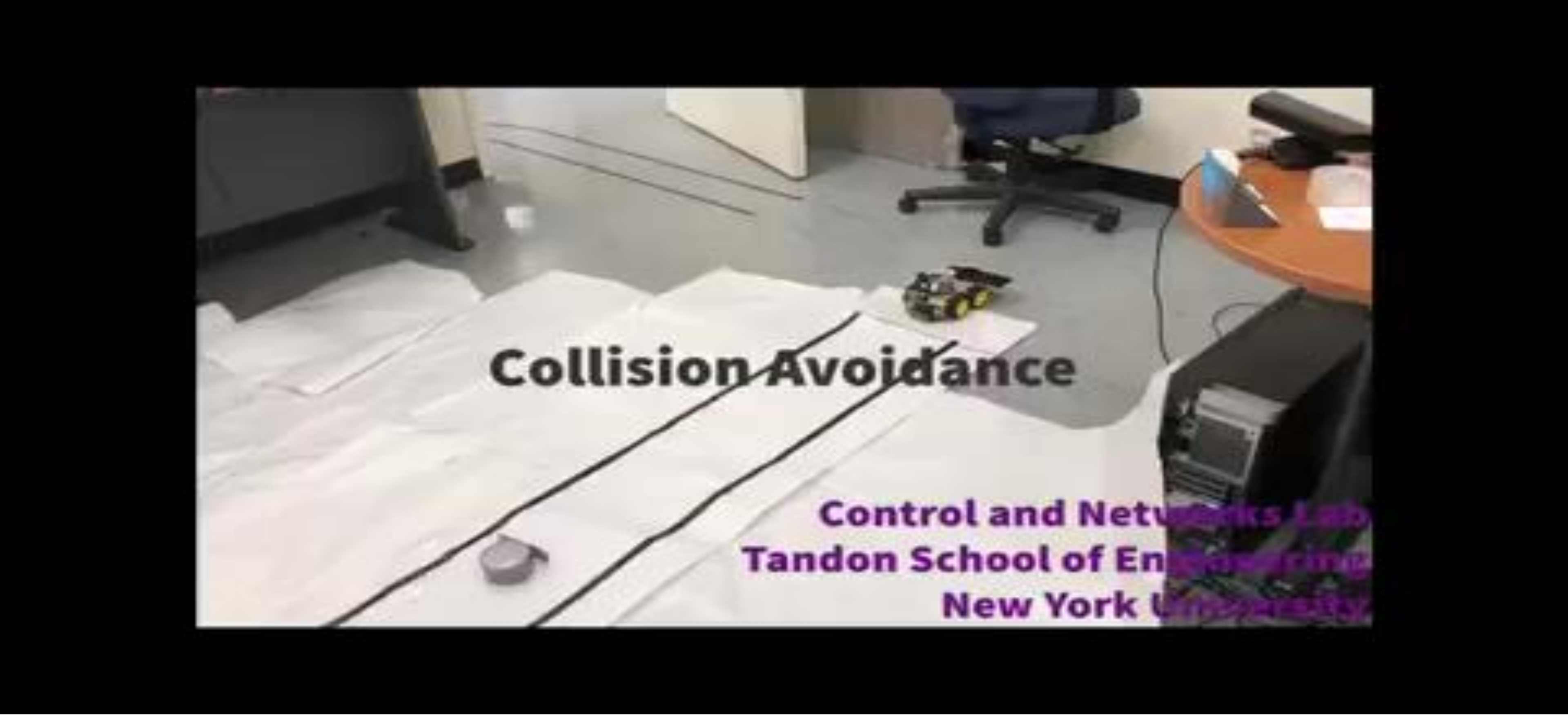


(b) Processed image including detected lane boundaries, lane centerline and $\xi = [d,\ \theta_e]^T$.

Learning behavior



Car Following and Lane Keeping



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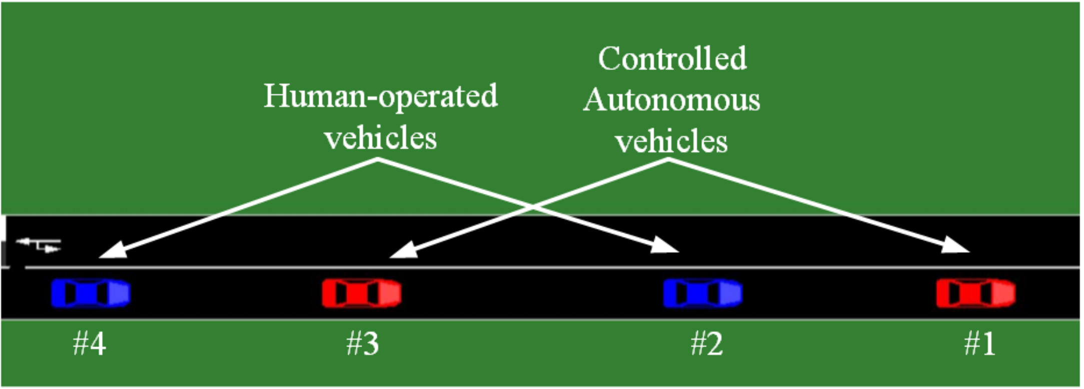
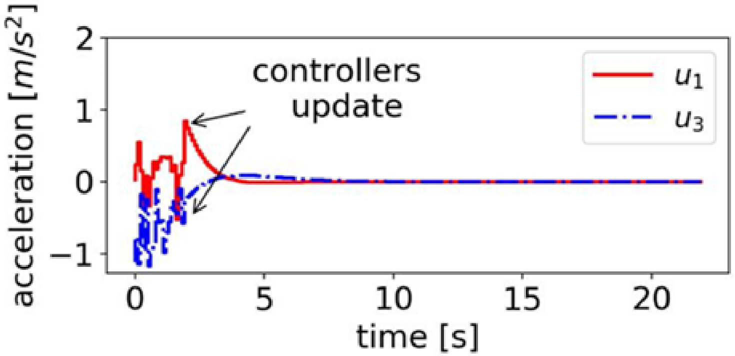
Microscopic Simulation in SUMO

2. Robust autonomous driving with humans in the loop



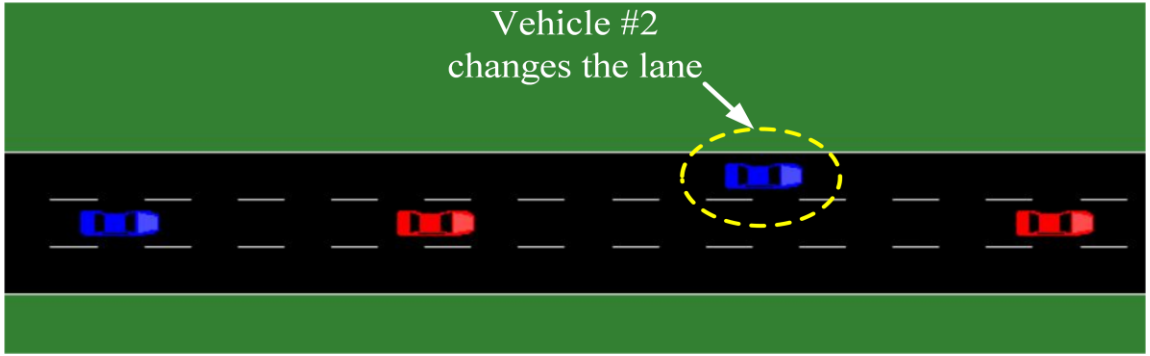


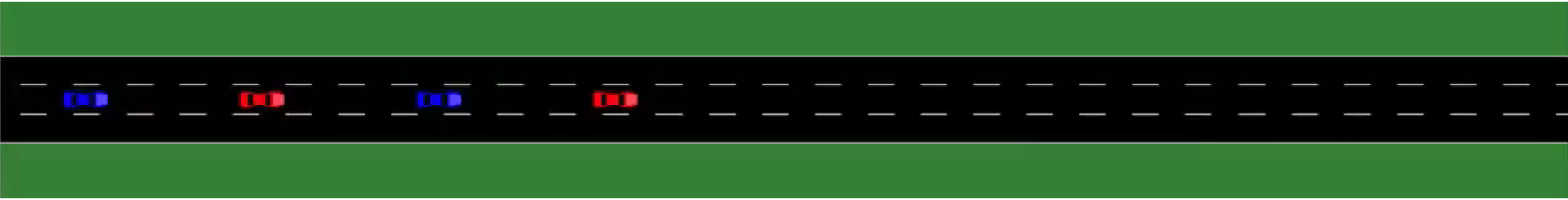
Robustness Evaluation

The speed of the leading vehicle is $v_0 = v^* + v_0^{\mathrm{amp}} \sin(\omega_f t)$ with amplitude $v_0^{\mathrm{amp}} = 5\ [m/s]$, frequency $\omega_f = 1\ [rad/s]$ and $v^* = 15\ [m/s]$.

Other scenarios: merging and splitting

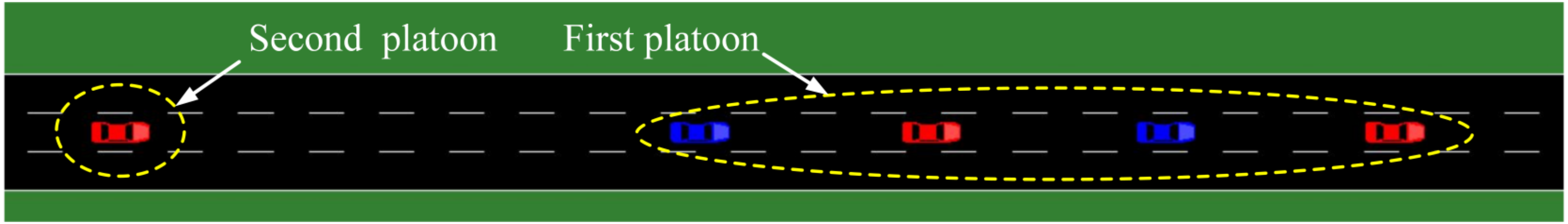
We also test the cut-out scenario:

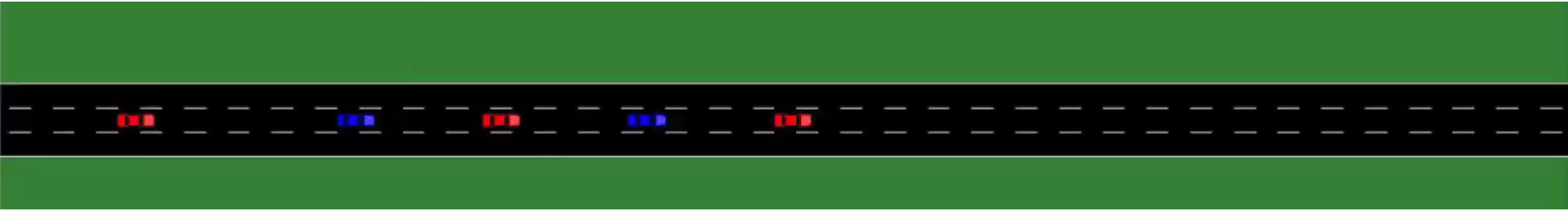




Other scenarios: merging and splitting

The merging of two platoons

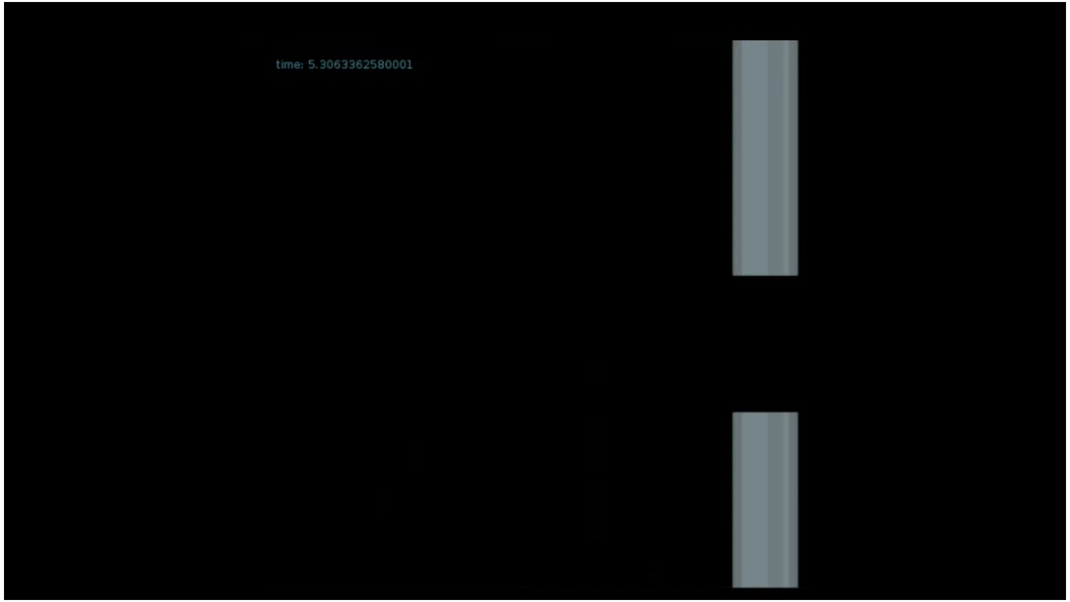




In both cases, after learning, the learned controllers can stabilize the new platoon as wanted.

3. Data-efficient Reinforcement Learning

“Flappy Bird” with RL using RADP-based Learning Controller



Conclusions

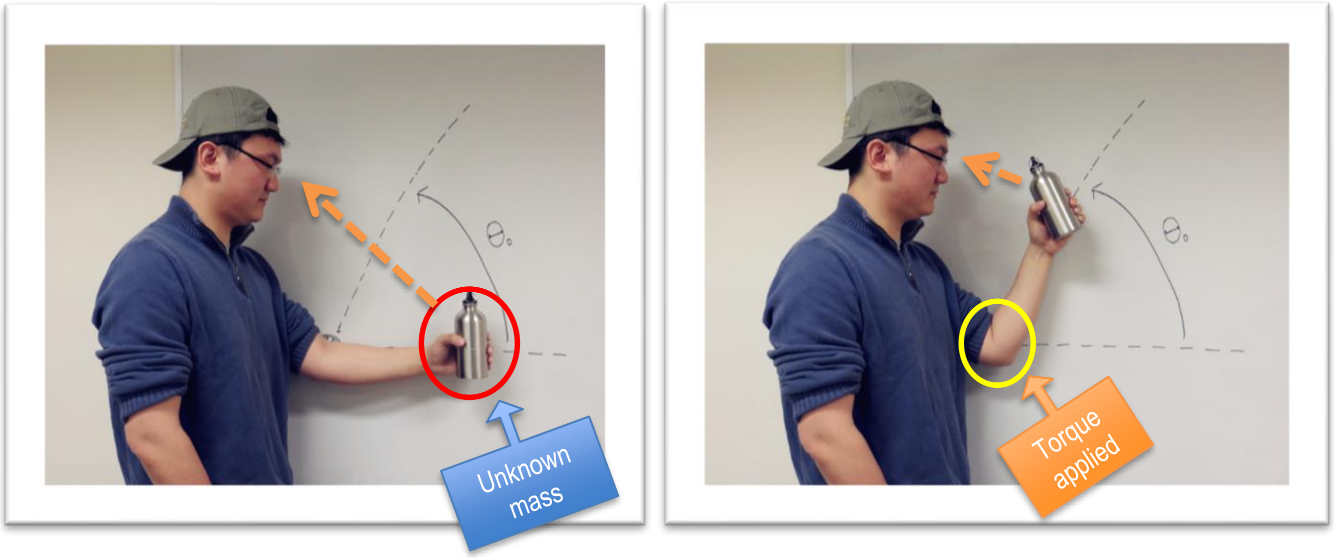
- **Learning-based nonlinear control is a promising field, yet still in its infancy.**
- **RADP (Robust Adaptive Dynamic Programming) for data-driven, learning-based robust/adaptive optimal control design.**
- **Validations via applications to power systems and CAVs.**

下集预告

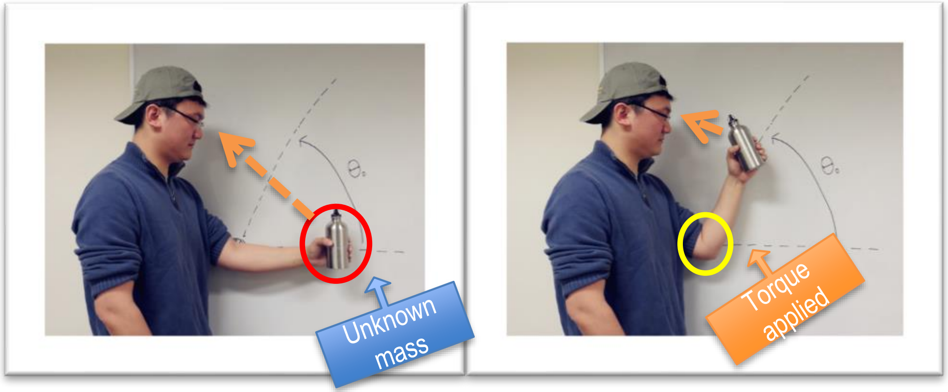
Future Work: Human Motor Control

- **Is RADP a computational mechanism of human motor control?**

- **A case study: reaching problem**



- **Is RADP a computational mechanism of human motor control?**
- **A case study: reaching problem**



Sensorimotor Control:

- One of the most common activities in daily lives
- Highly stereotyped trajectories have been reported
- Still unclear how the trajectories are formulated
- Research in this area may be helpful for better understanding related diseases.

$$\dot{\eta} = -\frac{1}{\tau_N}\eta + T_m$$

$$I\ddot{\theta} = -mgl\cos\theta + \eta + T_m$$

Original System Model

$$\dot{\eta} = -\frac{1}{\tau_N}\eta + T_m$$
$$I\ddot{\theta} = -mgl\cos\theta + \eta + T_m$$

State and input transformation

$$x_1 = \theta - \theta_0$$
$$x_2 = \dot{\theta}$$
$$w = \eta - \frac{\tau\_Nmgl\cos\theta_0}{\tau_N + 1}\sin - Ix_2$$

Transformed system

$$\dot{w} = -\frac{1+\tau_N}{\tau_N}w + Ix_2$$
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{2mgl}{I}\sin\left(\frac{x_1}{2}\right)\sin\left(\frac{x_1}{2} + \theta_0\right) + \frac{1}{I}(u + Ix_2 + w)$$

Cost

$$J = \int_0^\infty (100x_1^2(t) + x_2^2(t) + u^2(t))dt$$

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A sensorimotor control problem (Cont'd)

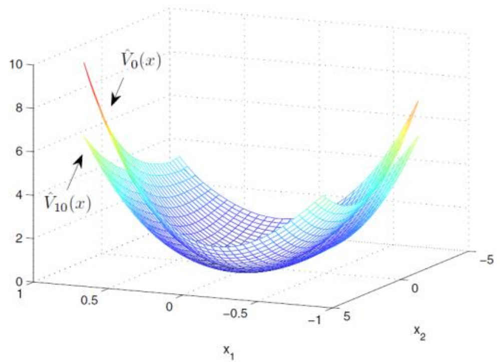


Figure: Cost functions

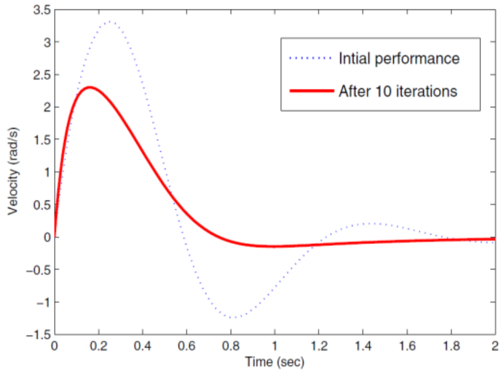
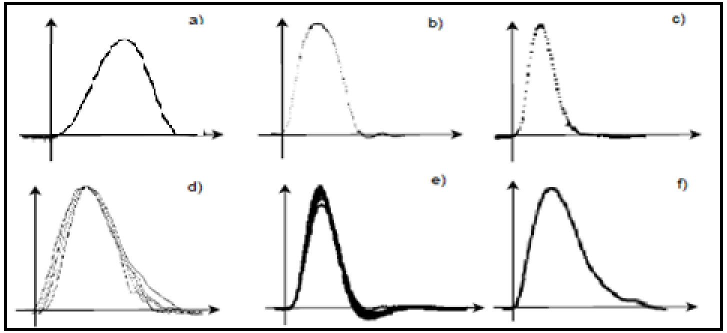


Figure: Velocity profiles



Experimental observations

a) [Harris & Wolpert, Nature, 1998]

b) [Morasso, Exp Brain Research, 1981]

c) [Abend et al, Brain: A Journal of Neurology, 1982]

d) [Atkeson et al. J of Neuroscience, 1985]

e) [Cooke, Neurobiology of Aging, 1989]

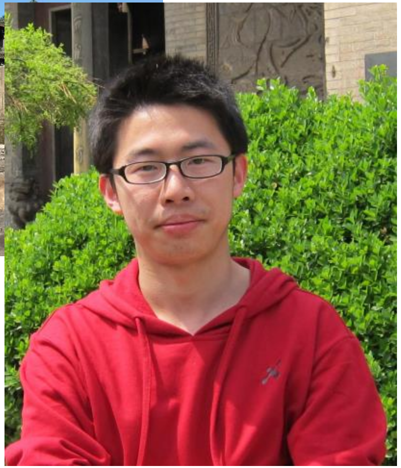
f) [Flash et al, J of Neuroscience, 1985]

NYU

Acknowledgements

- **Frank Lewis and his students for collaboration on ADP**

- **My students:**



Thank YOU for your attention!

Please send me your comments and feedback:
zjiang@nyu.edu