An approach to linear active disturbance rejection controller design with a linear quadratic regulator for a non–minimum phase system

Ryo Tanaka[1], Tetsunori Koga[1]
[1]Dept. of Control and Information Systems Engineering, Kurume College, 1–1–1, Komorino, Kurume, Fukuoka, 830–8555 Japan
E-mail: rtanaka@kurume–nct.ac.jp

Abstract

In a linear active disturbance rejection control (LADRC)[1] theory, this study proposes an approach to determining the control low based on the linear quadratic regulator (LQR) method[2] for a non–minimum phase (NMP) plant. The stability of the whole closed–loop control system is evaluated using the Routh–Hurwitz stability criterion. The effectiveness of the proposed method is confirmed by performing numerical simulations and comparing their results with those of the conventional I–PD control. In addition, the proposed LADRC with an LQR is confirmed to demonstrate a performance that is approximately the same as that of the conventional LADRC.

Block diagrams

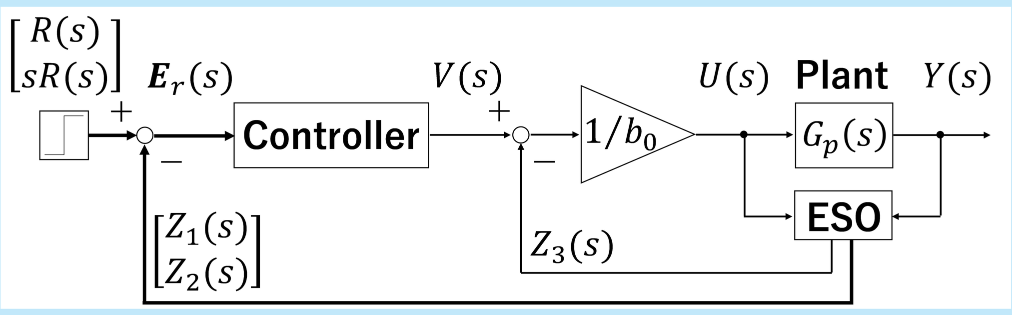

Fig.1 LADRC system

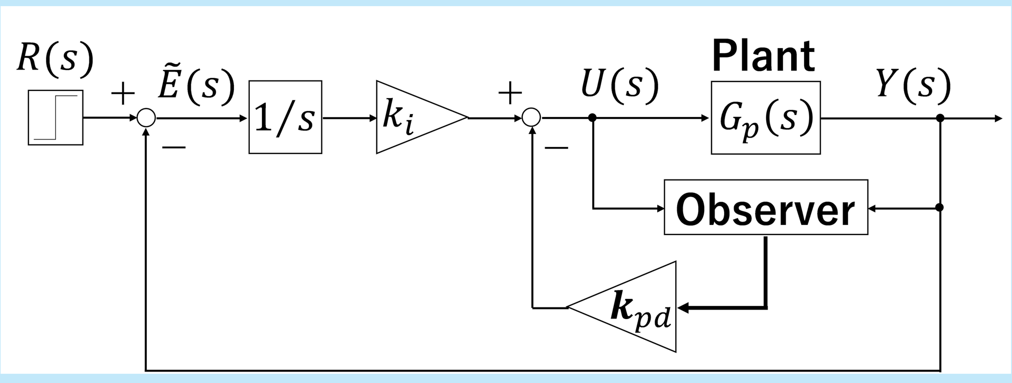

Fig.2 I–PD system with a state observer

Stability analysis

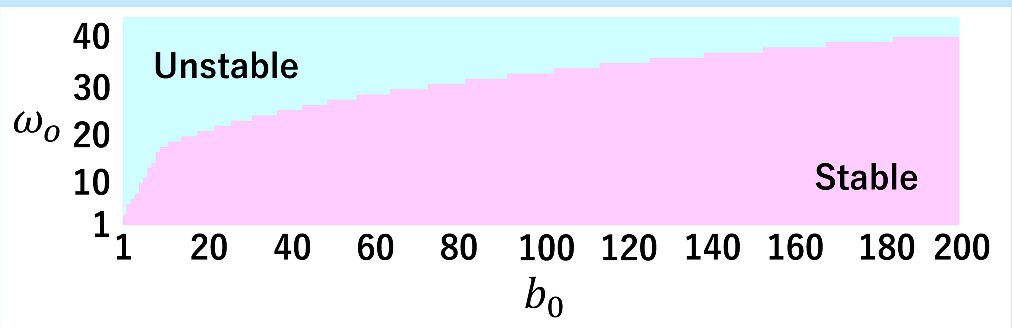

Fig.3 Stable and unstable area search result[3]

Simulation setups

Table 1 Parameters of the proposed LADRC (Case 1)

Variable meaning	Value
Observer bandwidth ω_o	10^3
Weight matrix $\boldsymbol{Q}$	diag(0, 1.0, 10^4)
Weight coefficient R	10
Controller gain k_d	120.3185
Controller gain k_p	31.6228
Scaling factor b_0	10^4

Table 2 Parameters of the I–PD control system

Variable meaning	Value
Weight matrix $\tilde{\boldsymbol{Q}}$	diag(1.0, 1.0, 1.0)
Weight coefficient $\tilde{R}$	1.0
State–feedback gain $\boldsymbol{k}_{pd}$	[0.3595 0.5616]
Integral gain k_i	1.0
Observer gain vector $\tilde{\boldsymbol{L}}$	$[0.3821\ -0.4270]^{\mathsf{T}}$

Table 3 Parameters of the proposed LADRC (Case 2)

Variable meaning	Value
Observer bandwidth ω_o	10
Weight matrix $\boldsymbol{Q}$	diag(0, 1.0, 10^2)
Weight coefficient R	0.1
Controller gain k_d	53.9194
Controller gain k_p	31.6228
Scaling factor b_0	10

Proposed LADRC with an LQR for a NMP plant

A 2nd order non–minimum phase plant $G_p(s)$:

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{b(c-s)}{s^2 + (a_1 + \Delta a_1)s + a_2 + \Delta a_2} \quad (1)$$

The extended state observer (ESO) :

$$\dot{\boldsymbol{z}}(t) = (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})\boldsymbol{z}(t) + \boldsymbol{B}u(t) + \boldsymbol{L}y(t) \quad (2)$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \boldsymbol{B} = [0\ b_0\ 0]^{\mathsf{T}}, \boldsymbol{C} = [1\ 0\ 0],$$

$\boldsymbol{L} = [\beta_1\ \beta_2\ \beta_3]^{\mathsf{T}}$: The ESO gain vector

$\boldsymbol{z}(t) = [z_1(t)\ z_2(t)\ z_3(t)]$: The ESO state vector

The control low :

$$u(t) = \frac{v(t) - z_3(t)}{b_0} \quad (3a)$$

$$v(t) = k_p(r(t) - z_1(t)) - k_d z_2(t) - z_3(t) \quad (3b)$$

The approximated transfer function $\tilde{G}_{vy}(s)$ between $V(s)$ and $Y(s)$:

$$\tilde{G}_{vy}(s) = \frac{n_1 s + n_2}{s^2 + d_1 s} \quad (4)$$

$$n_1 = -\frac{b(\beta_3 - c\beta_2)}{a_1 b_0 \beta_2 + a_2 b_0 \beta_1 + bc\beta_3}$$

$$n_2 = \frac{bc\beta_3}{a_1 b_0 \beta_2 + a_2 b_0 \beta_1 + bc\beta_3}$$

$$d_1 = \frac{a_2 b_0 \beta_2}{a_1 b_0 \beta_2 + a_2 b_0 \beta_1 + bc\beta_3}$$

The quadratic cost function J :

$$J = \int_0^\infty \left\{ \boldsymbol{x}_e(t)^{\mathsf{T}} \boldsymbol{Q} \boldsymbol{x}_e(t) + Rv(t)^2 \right\} dt \quad (5)$$

The optimal gain vector $\boldsymbol{K}_e$:

$$\boldsymbol{K}_e = R^{-1}\boldsymbol{B}_e^{\mathsf{T}}\boldsymbol{S} \quad (6)$$

$\boldsymbol{B}_e$: The augmented constant vector

$\boldsymbol{S}$: The solution of the Riccati equation

The Riccati equation :

$$\boldsymbol{A}_e^{\mathsf{T}}\boldsymbol{S} + \boldsymbol{S}\boldsymbol{A}_e - R^{\mathsf{T}}\boldsymbol{S}\boldsymbol{B}_e\boldsymbol{B}_e^{\mathsf{T}}\boldsymbol{S} + \boldsymbol{Q} = \boldsymbol{0} \quad (7)$$

Simulation results

A reference input $r(t)$ is assumed to be a step signal. The step set–point is introduced at $t = 10$ [s]. The nominal plant parameters for the plant transfer function $G_p(s)$ are $a_1 = 7$, $a_2 = 10$, and $b = c = 1$. In this simulation, the performance of the proposed LADRC with the LQR method is compared with that of an I–PD control and a conventional LADRC method. In the conventional LADRC, the controller bandwidth ω_c is chosen to be $\omega_o/4$, i.e., $k_p = \omega_c^2$, and $k_d = 2\omega_c$. We verify the robustness of the plant against modeling errors, as shown in Figs. 4, 5, 6 and 7.

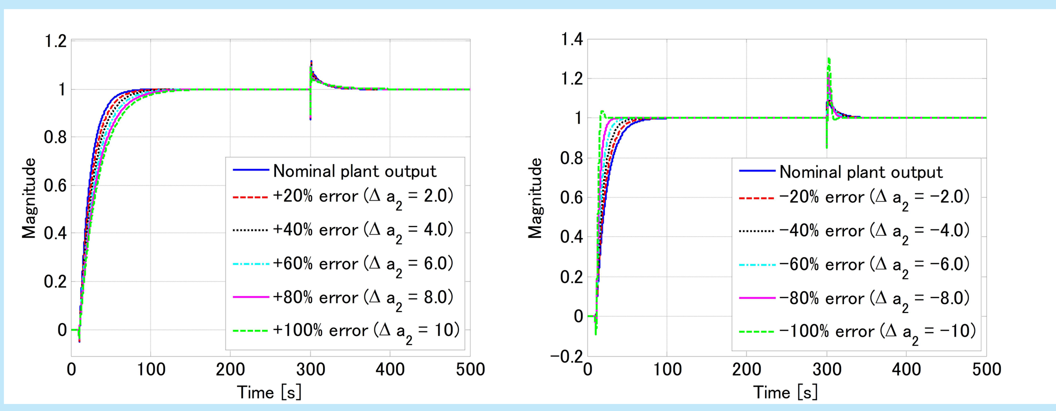

Fig.4 Results of the proposed LADRC (Case 1)

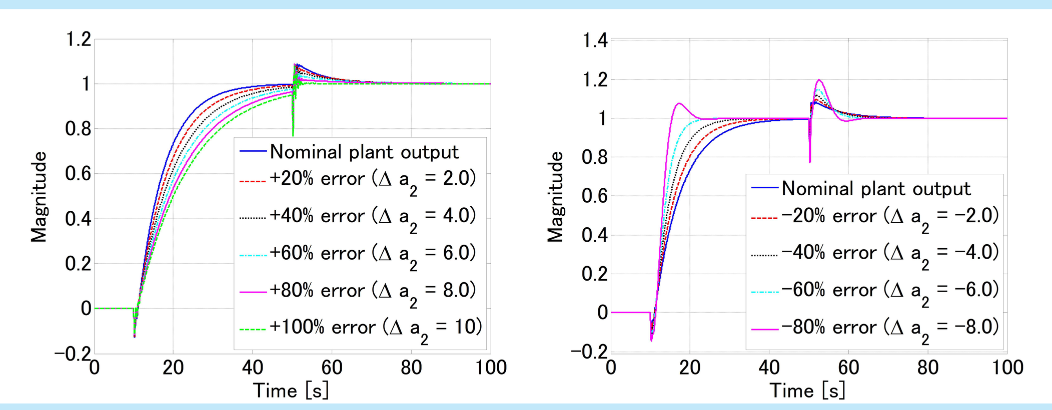

Fig.6 Results of the proposed LADRC (Case 2)

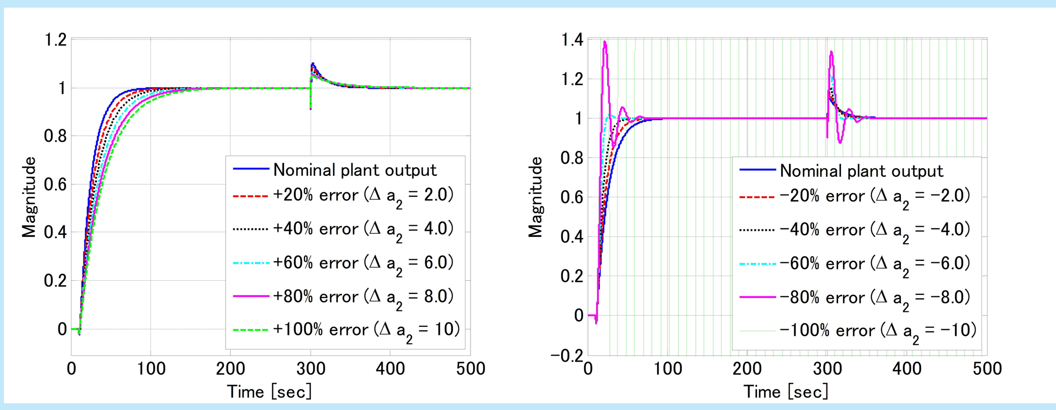

Fig.5 Results of the I–PD control

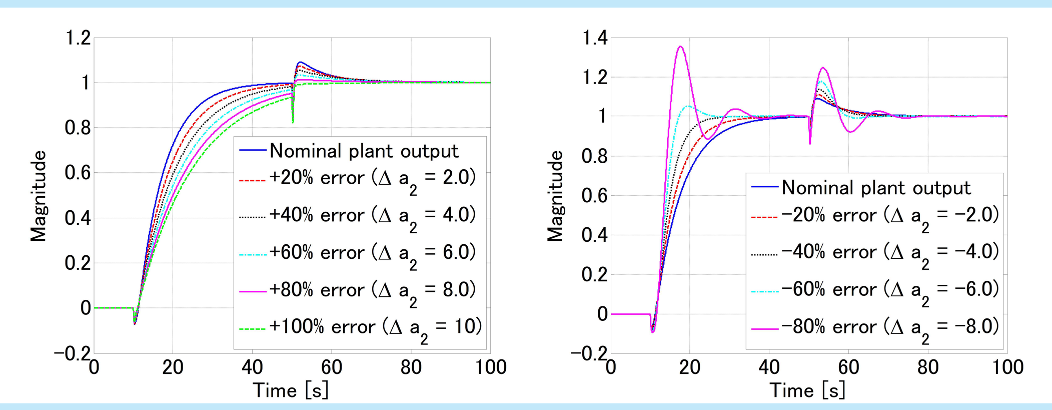

Fig.7 Results of the conventional LADRC

Conclusion

This study proposed an approach to designing a controller based on the LQR method for the non–minimum phase plant. It was verified that the proposed LADRC with the LQR method exhibited an excellent robustness of the plant with a modeling error compared with that of the I–PD control. In addition, it was confirmed that the proposed method can achieve approximately the same performance as the conventional LADRC.

References

[1] J. Han. From PID to Active Disturbance Rejection Control, *IEEE Trans. on Industrial Electronics*, 56(3): 900–906, 2009.
[2] R. T. Stefani, C. J. Savant, B. Shahian and G. H. Hostetter. *Design of Feedback Control Systems third edition*, Saunders College Publishing, 1994.
[3] S. Zhao and Z. Gao. Active Disturbance Rejection Control for Non–minimum Phase Systems, pages 6066–6070. in Proc. of the 29th CCC2010, 2010.