Global Consensus of Multi-Agent Systems with Intermittent Directed Communication in the Presence of Actuator Saturation

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Abstract: This paper studies the global leader-following consensus problem for a multi-agent system with intermittent directed communication in the presence of actuator saturation. Both the follower agents and the leader agent are described by a chain of integrators of an arbitrary length. A bounded consensus algorithm is constructed for each follower agent which utilizes the information obtained from the communication network intermittently. Global leader-following consensus is achieved by the consensus algorithms when the communication topology among all agents, when active, contains a spanning tree rooted at the leader agent. Simulation results illustrate the theoretical conclusions.

Key Words: Multi-agent systems, intermittent directed communication, actuator saturation, consensus

1 Introduction

As a fundamental problem in the study of cooperative control of multi-agent systems, consensus of multi-agent systems has drawn much attention over the past decades. Numerous results have been obtained pertaining to various agent dynamics and communication topologies [1–7].

More specifically, consensus for single-integrator systems was achieved under a jointly connected switching undirected communication topology [1]. Reference [2] relaxed the assumption on the communication topology in [1] to a weaker assumption that the union of the switching directed graphs contains a spanning tree frequently enough. Reference [3] further extended the single-integrator results of [2] to agents represented by higher-order systems under the same network connectivity assumption in [2]. For agents with general linear dynamics, the leader-following consensus problem was achieved under a fixed connected undirected topology in [4]. Reference [4] also achieved consensus of marginally stable linear systems under a jointly connected switching undirected graph. Reference [5] studied the discrete-time counterpart of [4]. Reference [6] relaxed the assumption on the communication topology in [5] to a jointly connected directed communication topology. Reference [7] designed fully distributed consensus protocols for general linear systems under the assumption that the communication topology contains a directed spanning tree.

Actuator saturation is ubiquitous in real world control systems and degrades the performance of the closed-loop system. In a severe case, it may even destabilize the system. Over the past decades, much research effort has been devoted to solving the consensus problem for multi-agent systems in the presence of actuator saturation [8]. It was established that global stabilization of linear system subject to actuator saturation can be achieved only when the linear system is asymptotically null controllable with bounded controls (ANCBC), that is, when it is stabilizable and all its poles are in the closed left-half plane [9]. In view of this fundamental result, the study of the consensus problem, in the presence of actuator saturation and under various communication topologies, has mainly focused on multi-agent systems where the agent dynamics are ANCBC [10–14].

For a group of single-integrator systems, global consensus was achieved when the communication topology contains a directed spanning tree [10]. For a group of double-integrator systems or a group of neutrally stable linear systems, global consensus was achieved by linear feedback under a directed topology that is strongly connected and detailed balanced [11]. Reference [11] also established results for these two classes of systems under a switching undirected topology. Reference [12] considered the discrete-time counterpart of [11] under a connected undirected communication topology. For general ANCBC agent dynamics, linear feedback was designed using the low gain feedback design technique [15] to achieve semi-global consensus under either a connected undirected or a jointly connected undirected topology [13]. Later, global consensus for general ANCBC agent dynamics was achieved by nonlinear control laws when the communication topology is strongly connected and detailed balanced [14].

A common constraint that exists in the communication network is that the information cannot be transmitted continuously under some circumstances such as sensor failures. Some works have been carried out on solving the consensus problem for multi-agent systems under intermittent communication [16–21]. Most of these works focused on first-order systems [16] or second-order systems [17–19] under either a connected undirected or strongly connected directed communication topology. Reference [20] studied the consensus problem for a group of general linear systems with periodic intermittent communication through a directed topology containing a spanning tree. In [21], actuator saturation was taken into consideration when solving the semi-global consensus problem for a group of ANCBC systems with periodic intermittent communication through a connected undirected communication topology by using the low gain feedback design technique [15].

Differently from the aforementioned papers, where the

This work is supported by the US Army Research Office under grant W911NF-17-1-0535.

agents are either in simple dynamics or the assumption on the communication topology is strong, we aim to solve the problem of global leader-follower consensus of a group of multiple-integrator systems in the presence of actuator saturation via aperiodically intermittent communication. The communication topology, when active, contains a directed spanning tree rooted at the leader agent. The consensus algorithm for each agent utilizes the information of other agents obtained through multi-hop paths in the communication network intermittently. Global consensus is achieved by these consensus algorithms.

The contributions of this paper are summarized as follows. Compared with the existing results on the consensus problem for multi-agent systems in the presence of actuator saturation, where either a connected undirected graph [12, 13, 21] or a strongly connected detailed balanced directed graph [11, 14] is assumed, our assumption on the communication topology is relaxed to a directed graph containing a spanning tree. Such a relaxation is nontrivial and is made possible by a judicious choice of nonquadratic Lyapunov functions quite different from those in the aforementioned papers. The second contribution is the consideration of actuator saturation while the communication is intermittent. Unlike in the situation when actuator saturation is absent [16–20], actuator saturation limits our ability to intensify the control effort while the communication is active. The third contribution lies in the relaxation on the intermittent property of the communication. A common limitation of the existing works is the restriction on the communication ratio, the infimum of the ratio between the time period of a connection and the time period of a disconnection that follows. In [17, 18, 20, 21], consensus can be achieved if the communication ratio is greater than a threshold value, which is determined by the agent dynamics and the communication topology. In our work, the ratio can be any positive number, independent of the agent dynamics and the communication topology. Finally, compared with [21], where semi-global consensus is achieved over an undirected topology, we achieve global consensus over a directed communication topology.

Organization. In Section 2, we recall some preliminaries in graph theory. We then state the problem of global leader-following consensus with intermittent directed communication in the presence of actuator saturation in Section 3. In Section 4, a consensus algorithm is constructed for each follower agent. These consensus algorithms use the information of other agents obtained through multi-hop paths in the communication network intermittently. The number of hops is no greater than the length of the chain of integrators. Global leader-following consensus is then established. Simulation results are presented in Section 5. A brief conclusion is drawn in Section 6.

Notation. $\mathbf{1}_N = [1\ 1\ \cdots\ 1]^{\mathrm{T}} \in \mathbb{R}^N$. For a symmetric matrix $A \in \mathbb{R}^{n\times n}$, $\overline{\lambda}(A)$ and $\underline{\lambda}(A)$ represent its maximum and minimum eigenvalues, respectively. For two matrices A and B, $A \otimes B$ denotes their Kronecker product. A positive matrix $A \in \mathbb{R}^{n\times m}$, where all its entries are positive, is denoted as $A \succ 0$. For two integers k_1 and k_2, $I[k_1, k_2] = \{k_1, k_1+1, \cdots, k_2\}$, if $k_2 \geq k_1$, and $I[k_1, k_2] = \{k_1, k_1 - 1, \cdots, k_2\}$, if $k_1 > k_2$.

2 Preliminaries

Let the communication topology among a network of N follower agents be represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is a finite nonempty set of N nodes, each representing a follower agent, and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges of the graph. An edge (v_i, v_j) indicates that agent j has access to the information of agent i. Agent i is a neighbor of agent j if $(v_i, v_j) \in \mathcal{E}$. Let $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ be the adjacency matrix associated with $\mathcal{G}$, where $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Here we assume that $a_{ii} = 0$ for all $i \in I[1, N]$. Let $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ be the Laplacian matrix associated with $\mathcal{A}$, where $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$.

Besides the N follower agents, there exists a leader agent, labeled as agent v_0. The communication between follower agent i and the leader agent is denoted as a_{i0}, where $a_{i0} > 0$ if follower agent i has access to the information of the leader agent and $a_{i0} = 0$ otherwise. Let $\bar{\mathcal{G}}$ be a graph which consists of graph $\mathcal{G}$, node v_0 and the edges between the leader agent and its neighbors. Let the Laplacian matrix associated with $\bar{\mathcal{G}}$ be denoted as $\bar{\mathcal{L}}$. Then, $\bar{\mathcal{L}}$ can be partitioned as

$$\bar{\mathcal{L}} = \begin{bmatrix} 0 & 0_N^{\mathrm{T}} \\ -a_0 & \mathcal{M} \end{bmatrix},$$

where $a_0 = [a_{10}\ a_{20}\ \cdots, a_{N0}]^{\mathrm{T}}$ and $\mathcal{M} = \mathcal{L} + \mathrm{diag}\{a_{10}, a_{20}, \cdots, a_{N0}\}$.

Lemma 2.1 *[2] 0 is an eigenvalue of $\bar{\mathcal{L}}$ and all of the nonzero eigenvalues of $\bar{\mathcal{L}}$ are in the open left half plane. Furthermore, 0 is a simple eigenvalue of $\bar{\mathcal{L}}$ if and only if $\bar{\mathcal{G}}$ contains a directed spanning tree.*

Lemma 2.2 *[7] If $\bar{\mathcal{G}}$ contains a directed spanning tree rooted at the leader, $\Lambda = \Psi\mathcal{M} + \mathcal{M}^{\mathrm{T}}\Psi > 0$, where $\Psi = \mathrm{diag}\{\psi_1, \psi_2, \cdots, \psi_N\}$ with $[\psi_1, \psi_2, \cdots, \psi_N]^{\mathrm{T}} = (\mathcal{M}^{\mathrm{T}})^{-1}\mathbf{1}_N \succ 0$.*

3 Problem Statement

Consider a group of N follower agents, each being described by a chain of integrators of length n,

$$\dot{x}_i = Ax_i + Bu_i, \quad i \in I[1, N], \tag{1}$$

where $x_i = [x_{i1}\ x_{i2}\ \cdots\ x_{in}]^{\mathrm{T}} \in \mathbb{R}^n$ and $u_i \in \mathbb{R}$, with $|u_i| \leq u_{\max}$, for some positive scalar $u_{\max}$, are respectively the state and the control input of agent i, and

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{n\times n}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{n\times 1}.$$

The leader agent is described by,

$$\dot{x}_0 = Ax_0, \tag{2}$$

where $x_0 = [x_{01}\ x_{02}\ \cdots\ x_{0n}]^{\mathrm{T}} \in \mathbb{R}^n$ is the state.

In this paper, we consider a multi-agent system consisting of the group of follower agents (1) and the leader agent (2) and operating on an underlying communication network. The communication network is active intermittently as shown in Fig. 1, where $[t_k, t_{k+1})$, $k = 0, 1, 2\cdots$, are nonempty, non-overlapping time intervals with $t_0 = 0$. For $k = 0, 1, 2, \cdots$, t_k is the beginning of a connection, $\tau_k \geq \varepsilon_0$, for any positive scalar ε_0, is the time period when communication network is active and $\bar{\tau}_k = t_{k+1} - t_k - \tau_k \leq$

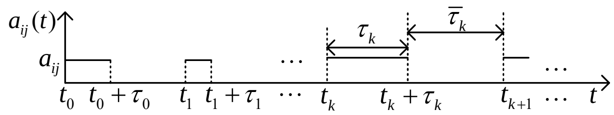

Fig. 1: The intermittent communication illustrated by $a_{ij}(t)$, $i \in I[1,N]$, $j \in I[0,N]$.

θ, for any positive scalar θ, is the following time period when communication network is inactive. The intermittent communication is illustrated by $a_{ij}(t)$, $i \in I[1,N], j \in I[0,N]$, where $a_{ij}(t) = a_{ij}$ if the communication network is active and $a_{ij}(t) = 0$ if the communication network is inactive. The communication topology, when active, satisfies the following assumption.

Assumption 3.1 *The directed graph $\bar{\mathcal{G}}$ contains a spanning tree rooted at the leader agent v_0.*

The problem we are to solve is to design a bounded consensus algorithm u_i for each follower agent i, which uses only the information obtained from the communication network intermittently, such that, for all initial conditions $x_i(0) \in \mathbb{R}^n, i \in I[0,N], \lim_{t\to\infty}(x_i(t) - x_0(t)) = 0, i \in I[1,N]$.

4 Main Results

Denote the difference between a state of a follower agent and the corresponding state of the leader as $\hat{x}_{il} = x_{il} - x_{0l}$, $l \in I[1,n], i \in I[1,N]$, and let $\hat{x}_i = [\hat{x}_{i1}\ \hat{x}_{i2}\ \cdots\ \hat{x}_{in}]^{\mathrm{T}}$, $i \in I[1,N]$. It then follows from (1) and (2) that

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu_i, \quad i \in I[1,N]. \tag{3}$$

Let $\tilde{x} = [\tilde{x}_1^{\mathrm{T}}\ \tilde{x}_2^{\mathrm{T}}\ \cdots\ \tilde{x}_n^{\mathrm{T}}]^{\mathrm{T}}$ with $\tilde{x}_l = [\hat{x}_{1l}\ \hat{x}_{2l}\ \cdots\ \hat{x}_{Nl}]^{\mathrm{T}}, l \in I[1,n]$, and $u = [u_1 u_2 \cdots u_N]^{\mathrm{T}}$. It then follows from (3) that

$$\dot{\tilde{x}} = (A \otimes I_N)\tilde{x} + (B \otimes I_N)u. \tag{4}$$

To construct the consensus algorithm for each follower agent, we carry out a state transformation on system (4). Let κ be a positive scalar whose value is to be determined later and $T = [T_{ij}]$, where, for $i \in I[1,N]$, $T_{ij} = 0_{N\times N}$, $j \in I[1,i]$, and $T_{ij} = \mathrm{C}_{n-i}^{n-j}(\kappa\mathcal{M})^{n-j}$, $j \in I[i+1,n]$ with $\mathrm{C}_p^q = \frac{p!}{q!(p-q)!}$. The non-singularity of T follows from the non-singularity of $\mathcal{M}$, which is implied by Lemma 2.1. On the new state $\bar{x} = T\tilde{x}$, system (4) takes the following form

$$\dot{\bar{x}} = \left(\bar{A} \otimes \mathcal{M}\right)\bar{x} + \left(\bar{B} \otimes I_N\right)u, \tag{5}$$

where $\bar{x} = [\bar{x}_1^{\mathrm{T}}\ \bar{x}_2^{\mathrm{T}}\ \cdots\ \bar{x}_n^{\mathrm{T}}]^{\mathrm{T}}$ with $\bar{x}_l = [\bar{x}_{1l}\ \bar{x}_{2l}\ \cdots\ \bar{x}_{Nl}]^{\mathrm{T}}$, $l \in I[1,n]$, and

$$\bar{A} = \begin{bmatrix} 0 & \kappa & \kappa & \cdots & \kappa \\ 0 & 0 & \kappa & \cdots & \kappa \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \kappa \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{n\times n}, \quad \bar{B} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix}_{n\times 1}.$$

With the consideration of intermittent communication, we construct a bounded consensus algorithm for each agent i, $i \in I[1,N]$, as follows,

$$u_i(t) = -\sum_{l=1}^{n}\sigma_l\left(\kappa\left(\sum_{j=1}^{N}a_{ij}(t)(\bar{x}_{il}(t) - \bar{x}_{jl}(t)) + a_{i0}(t)\bar{x}_{il}(t)\right)\right), \tag{6}$$

where $a_{ij}(t)$, $i \in I[1,N]$, $j \in I[0,N]$, are as defined in Section 3 and $\sigma_l : \mathbb{R} \to \mathbb{R}$ is a saturation function defined as $\sigma_l(s) = \mathrm{sign}(s)\min\{|s|, \Delta_l\}$. It is noted that the control input of agent $i, i \in I[1,N]$, is bounded by $|u_i| \le \sum_{l=1}^{n}\Delta_l$, where $\Delta_l, l \in I[1,n]$, satisfy the following conditions,

$$\begin{cases} \sqrt{\frac{\underline{\lambda}(\Psi)}{2\overline{\lambda}(\Psi)}}\Delta_l \ge \frac{2\overline{\lambda}(\Psi)}{\underline{\lambda}(\Lambda)}\|\mathcal{M}\|\sum\limits_{j=0}^{l-1}\Delta_j,\ l \in I[1,n], \\ \sum\limits_{l=1}^{n}\Delta_l \le u_{\max}, \end{cases} \tag{7}$$

for some positive scalar Δ_0.

Remark 4.1 *There exist Δ_l, $l \in I[0,n]$, that satisfy (7). Let Δ be a positive scalar satisfying $\Delta \in (0, u_{\max})$ and $\rho^* \in (0,1)$ be a positive scalar satisfying $\rho^* \le \min\left(1 - \frac{\Delta}{u_{\max}}, \frac{\sqrt{\underline{\lambda}(\Psi)}\underline{\lambda}(\Lambda)}{\sqrt{\underline{\lambda}(\Psi)}\underline{\lambda}(\Lambda) + 2\sqrt{2\overline{\lambda}^3(\Psi)}\|\mathcal{M}\|}\right)$. Choose $\Delta_l = \rho^{n-l}\Delta$, $l \in I[0,n]$. Then (7) will be satisfied for any $\rho \in (0,\rho^*)$.*

Let $P_1 > 0$ be such that

$$\left((\bar{A} - \kappa\bar{B}\bar{B}^{\mathrm{T}}) \otimes \mathcal{M}\right)^{\mathrm{T}}P_1 + P_1\left((\bar{A} - \kappa\bar{B}\bar{B}^{\mathrm{T}}) \otimes \mathcal{M}\right) = -I, \tag{8}$$

and $P_2 > 0$ be such that

$$\left(\bar{A} \otimes \mathcal{M}\right)^{\mathrm{T}}P_2 + P_2\left(\bar{A} \otimes \mathcal{M}\right) - \gamma_2 P_2 = -\gamma_2 I, \tag{9}$$

where $\gamma_2 = \mu\kappa$ with $\mu > 0$, whose values is independent of κ and to be determined. In (8), (9), and later in the paper, I is an identity matrix of compatible dimensions. We first present the following lemmas.

Lemma 4.1 *There exist positive matrices $P_1 > 0$ satisfying (8) and $P_2 > 0$ satisfying (9). Also, $P_1 = \frac{1}{\kappa}\bar{P}_1$ and $P_2 = \bar{P}_2$, where $\bar{P}_1 > 0$, independent of κ, is the solution to*

$$\left((\hat{A} - \bar{B}\bar{B}^{\mathrm{T}}) \otimes \mathcal{M}\right)^{\mathrm{T}}\bar{P}_1 + \bar{P}_1\left((\hat{A} - \bar{B}\bar{B}^{\mathrm{T}}) \otimes \mathcal{M}\right) = -I, \tag{10}$$

and $\bar{P}_2 > 0$, independent of κ, is the solution to

$$\left(\hat{A} \otimes \mathcal{M}\right)\bar{P}_2 + \bar{P}_2\left(\hat{A} \otimes \mathcal{M}\right) - \mu\bar{P}_2 = -\mu I, \tag{11}$$

with $\hat{A} = \frac{1}{\kappa}\bar{A}$.

Proof: Since

$$(\bar{A} - \kappa\bar{B}\bar{B}^{\mathrm{T}}) \otimes \mathcal{M} = \begin{bmatrix} -\kappa\mathcal{M} & 0 & \cdots & 0 \\ -\kappa\mathcal{M} & -\kappa\mathcal{M} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\kappa\mathcal{M} & -\kappa\mathcal{M} & \cdots & -\kappa\mathcal{M} \end{bmatrix}$$

and all the eigenvalues of $\mathcal{M}$ have positive real part, the matrix $((\bar{A} - \kappa\bar{B}\bar{B}^{\mathrm{T}}) \otimes \mathcal{M})$ is Hurwitz and therefore P_1 exists.

Equation (9) can be written as

$$\left(\bar{A} \otimes \mathcal{M} - \frac{\gamma_2}{2}I\right)^{\mathrm{T}}P_2 + P_2\left(\bar{A} \otimes \mathcal{M} - \frac{\gamma_2}{2}I\right) = -\gamma_2 I.$$

Since

$$\bar{A} \otimes \mathcal{M} - \frac{\gamma_2}{2}I = \begin{bmatrix} -\frac{\gamma_2}{2}I & \kappa\mathcal{M} & \cdots & \kappa\mathcal{M} \\ 0 & -\frac{\gamma_2}{2}I & \cdots & \kappa\mathcal{M} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\frac{\gamma_2}{2}I \end{bmatrix},$$

the matrix $(\bar{A} \otimes \mathcal{M} - \frac{\gamma_2}{2}I)$ is Hurwitz and therefore P_2 exists.

Since $\hat{A} = \frac{1}{\kappa}\bar{A}$, $\hat{A} - \bar{B}\bar{B}^{\mathrm{T}} = \frac{1}{\kappa}(\bar{A} - \kappa\bar{B}\bar{B}^{\mathrm{T}})$. Equation (10) can be written as

$$\left((\bar{A} - \kappa\bar{B}\bar{B}^{\mathrm{T}}) \otimes \mathcal{M}\right)^{\mathrm{T}}\frac{1}{\kappa}\bar{P}_1 + \frac{1}{\kappa}\bar{P}_1\left((\bar{A} - \kappa\bar{B}\bar{B}^{\mathrm{T}}) \otimes \mathcal{M}\right) = -I,$$

By (8), we get $P_1 = \frac{1}{\kappa}\bar{P}_1$. Equation (11) can be written as

$$\left(\bar{A} \otimes \mathcal{M}\right)^{\mathrm{T}}\bar{P}_2 + \bar{P}_2\left(\bar{A} \otimes \mathcal{M}\right) - \gamma_2\bar{P}_2 = -\gamma_2 I.$$

By (9), we get $P_2=\bar{P}_2$. $\square$

Lemma 4.2 *There exists a sufficiently large κ^* such that, for any $\kappa\geq\kappa^*$, $\alpha_l-2\ln\rho>\delta,\ \ l=0,1,2,\cdots$, where δ is a sufficiently small positive scalar, $\rho=\max\left\{\frac{\overline{\lambda}(P_2)}{\underline{\lambda}(P_1)},\frac{\overline{\lambda}(P_1)}{\underline{\lambda}(P_2)}\right\}$, and $\alpha_l=\tau_l\gamma_1-\bar{\tau}_l\gamma_2,\ \ l=0,1,2,\cdots$, with $\gamma_1=\frac{1}{\overline{\lambda}(P_1)}$.*

Proof: Let $\mu\in(0,\frac{\varepsilon_0}{2\overline{\lambda}(\bar{P}_1)\theta}]$. By Lemma 4.1, we have $\gamma_1=\frac{\kappa}{\overline{\lambda}(\bar{P}_1)}$ and $\rho=\max\left\{\frac{\kappa\overline{\lambda}(\bar{P}_2)}{\underline{\lambda}(\bar{P}_1)},\frac{\overline{\lambda}(\bar{P}_1)}{\kappa\underline{\lambda}(\bar{P}_2)}\right\}$. Let $\kappa^*\geq\kappa_1^*=\sqrt{\frac{\overline{\lambda}(\bar{P}_1)\underline{\lambda}(\bar{P}_1)}{\overline{\lambda}(\bar{P}_2)\underline{\lambda}(\bar{P}_2)}}$. Then $\rho=\frac{\kappa\overline{\lambda}(\bar{P}_2)}{\underline{\lambda}(\bar{P}_1)}$. Since $\bar{\tau}_l\leq\theta$ and $\tau_l\geq\varepsilon_0, l=0,1,2\cdots$, for any positive scalars ε_0 and θ,

$$\begin{aligned}\alpha_l-2\ln\rho&=\tau_l\gamma_1-\bar{\tau}_l\gamma_2-2\ln\rho\\&\geq\Big(\frac{\varepsilon_0}{\overline{\lambda}(\bar{P}_1)}-\theta\mu\Big)\kappa-2\ln\kappa-2\ln\frac{\overline{\lambda}(\bar{P}_2)}{\underline{\lambda}(\bar{P}_1)},\\&\geq 2\Big(\frac{\varepsilon_0\kappa}{4\overline{\lambda}(\bar{P}_1)}-\ln\kappa-\ln\frac{\overline{\lambda}(\bar{P}_2)}{\underline{\lambda}(\bar{P}_1)}\Big).\end{aligned}$$

Since

$$\lim_{\kappa\to\infty}\frac{\frac{\varepsilon_0\kappa}{4\overline{\lambda}(\bar{P}_1)}}{\ln\kappa+\ln\frac{\overline{\lambda}(\bar{P}_2)}{\underline{\lambda}(\bar{P}_1)}}=\lim_{\kappa\to\infty}\frac{\varepsilon_0\kappa}{4\overline{\lambda}(\bar{P}_1)}=+\infty,$$

there exists a sufficiently large $\kappa^*\geq\kappa_1^*$ such that, for $\kappa\geq\kappa^*$, $\frac{\varepsilon_0\kappa}{4\overline{\lambda}(\bar{P}_1)}-\ln\kappa-\ln\frac{\overline{\lambda}(\bar{P}_2)}{\underline{\lambda}(\bar{P}_1)}>\frac{\delta}{2}$, i.e., $\alpha_l-2\ln\rho>\delta, l=0,1,2,\cdots$. $\square$

The following theorem establishes that global leader-following consensus of the multi-agent system is achieved under the consensus algorithms (6).

Theorem 4.1 *Consider the group of follower agents (1) and the leader agent (2). Let the communication topology, when active, satisfies Assumption 3.1. There exists a sufficiently large κ^* such that, for any $\kappa\geq\kappa^*$, global leader-following consensus can be achieved by the consensus algorithms (6).*

Proof: Let $z_{il}=\sum_{j=1}^N a_{ij}(\bar{x}_{il}-\bar{x}_{jl})+a_{i0}\bar{x}_{il},\ l\in I[1,n], i\in I[1,N],\ z_l=[z_{1l}\ z_{2l}\ \cdots\ z_{Nl}]^{\mathrm{T}}, l\in I[1,n]$, and $z=[z_1^{\mathrm{T}}\ z_2^{\mathrm{T}}\ \cdots\ z_n^{\mathrm{T}}]^{\mathrm{T}}$. Then $z_l=\mathcal{M}\bar{x}_l, l\in I[1,n]$, and $z=(I\otimes\mathcal{M})\bar{x}$. From (5) and (6), we have

$$\dot{z}=\left(\bar{A}\otimes\mathcal{M}\right)z+\left(\bar{B}\otimes\mathcal{M}\right)u, \tag{12}$$

or

$$\begin{cases}\dot{z}_1&=\mathcal{M}(\kappa z_2+\kappa z_3+\cdots+\kappa z_n+u),\\ &\vdots\\ \dot{z}_{n-1}&=\mathcal{M}(\kappa z_n+u),\\ \dot{z}_n&=\mathcal{M}u,\end{cases}$$

with

$$u(t)=\begin{cases}-\sum\limits_{l=1}^n\sigma_l(\kappa z_l(t)), t\in[t_k,t_k+\tau_k),\\ 0,\ t\in[t_k+\tau_k,t_{k+1}),\ k=0,1,2,\cdots.\end{cases} \tag{13}$$

Here, we abuse the notation by using $\sigma_l,\ l\in I[1,n]$, to denote both a scalar valued and a vector valued saturation function.

We first consider the evolution of z_n, which is governed by

$$\dot{z}_n(t)=\begin{cases}-\mathcal{M}\sigma_n(\kappa z_n(t))+w_n(t),\ t\in[t_k,t_k+\tau_k),\\ 0,\ t\in[t_k+\tau_k,t_{k+1}), k=0,1,2,\cdots,\end{cases} \tag{14}$$

with $w_n(t)=-\mathcal{M}\sum_{l=1}^{n-1}\sigma_l(\kappa z_l(t))$. Choose a Lyapunov function candidate

$$V_n(z_n)=2\kappa z_n^{\mathrm{T}}\Psi\sigma_n(\kappa z_n)-\sigma_n^{\mathrm{T}}(\kappa z_n)\Psi\sigma_n(\kappa z_n),$$

where Ψ is as defined in Lemma 2.2. It is noted that $V_n(z_n)\geq\kappa z_n^{\mathrm{T}}\Psi\sigma_n(\kappa z_n)>0,\ \ z_n\neq0$. Then V_n is positive definite and radially unbounded with respect to z_n. Note that $V_n(z_n)$ is not differentiable everywhere. For each $i\in I[1,N]$, let $z_{0in}\in\{\frac{\Delta_n}{\kappa},-\frac{\Delta_n}{\kappa}\}$. The directional derivative of $\sigma_n(\kappa z_{in})$ at z_{0in} along $\dot{z}_{0in}$ is given by

$$\begin{aligned}&\lim_{t\to0^+}\frac{\sigma(\kappa z_{0in}+t\kappa\dot{z}_{in})-\sigma_n(\kappa z_{0in})}{t}\\&=:\zeta_{in}=\begin{cases}0, & |\kappa z_{0in}+t\kappa\dot{z}_{in}|>\Delta_n,\\ \kappa\dot{z}_{in}, & |\kappa z_{0in}+t\kappa\dot{z}_{in}|\leq\Delta_n.\end{cases}\end{aligned}$$

Let $\frac{\mathrm{d}\sigma(\kappa z_{in})}{\mathrm{d}t}$ be the directional derivative of $\sigma(\kappa z_{in})$ at z_{in} along $\dot{z}_{in}$. Then, we have

$$\frac{\mathrm{d}\sigma(\kappa z_{in})}{\mathrm{d}t}=\begin{cases}0, & |\kappa z_{in}|>\Delta_n,\\ \zeta_{in}, & |\kappa z_{in}|=\Delta_n,\\ \kappa\dot{z}_{in}, & |\kappa z_{in}|<\Delta_n.\end{cases}$$

The derivative of V_n along the trajectory of z_n can be evaluated as follows,

$$\begin{aligned}\dot{V}_n(t)&=2\kappa\dot{z}_n^{\mathrm{T}}(t)\Psi\sigma(\kappa z_n(t))+2\kappa z_n^{\mathrm{T}}(t)\Psi\frac{\mathrm{d}\sigma_n(\kappa z_n(t))}{\mathrm{d}t}\\&\quad-2\sigma_n^{\mathrm{T}}(\kappa z_n(t))\Psi\frac{\mathrm{d}\sigma_n(\kappa z_n(t))}{\mathrm{d}t},\end{aligned} \tag{15}$$

where $\frac{\mathrm{d}\sigma_n(\kappa z_n(t))}{\mathrm{d}t}=\left[\frac{\mathrm{d}\sigma(\kappa z_{1n})}{\mathrm{d}t}\ \frac{\mathrm{d}\sigma(\kappa z_{2n})}{\mathrm{d}t}\ \cdots\ \frac{\mathrm{d}\sigma(\kappa z_{Nn})}{\mathrm{d}t}\right]^{\mathrm{T}}$. For $t\in[t_k,t_k+\tau_k), k=0,1,2,\cdots$, (15) can be continued as

$$\begin{aligned}\dot{V}_n(t)=&2\kappa(-\mathcal{M}\sigma_n(\kappa z_n(t))+w_n(t))^{\mathrm{T}}\Psi\sigma(\kappa z_n(t))\\&+2(\kappa z_n(t)-\sigma_n(\kappa z_n(t)))^{\mathrm{T}}\Psi\frac{\mathrm{d}\sigma_n(\kappa z_n(t))}{\mathrm{d}t}\\=&-\kappa\sigma_n^{\mathrm{T}}(\kappa z_n(t))\left(\mathcal{M}^{\mathrm{T}}\Psi+\Psi\mathcal{M}\right)\sigma_n(\kappa z_n(t))\\&+2\kappa\sigma_n^{\mathrm{T}}(\kappa z_n(t))\Psi w_n(t)\\&+2\sum_{i=1}^N\psi_i(\kappa z_{in}(t)-\sigma_n(\kappa z_{in}(t)))\frac{\mathrm{d}\sigma_n(\kappa z_{in})}{\mathrm{d}t}\\=&-\kappa\sigma_n^{\mathrm{T}}(\kappa z_n(t))\Lambda\sigma_n(\kappa z_n(t))+2\kappa\sigma_n^{\mathrm{T}}(z_n(t))\Psi w_n(t)\\&+2\sum_{|\kappa z_{in}(t)|\leq\Delta_n}\psi_i\times0\times\frac{\mathrm{d}\sigma_n(\kappa z_{in})}{\mathrm{d}t}\\&+2\sum_{|\kappa z_{in}(t)|>\Delta_n}\psi_i(\kappa z_{in}(t)-\sigma_n(\kappa z_{in}(t)))\times0\\\leq&-\kappa\underline{\lambda}(\Lambda)\|\sigma_n(\kappa z_n(t))\|^2+2\kappa\overline{\lambda}(\Psi)\|\sigma_n(\kappa z_n(t))\|\|w_n(t)\|\\=&-\kappa\underline{\lambda}(\Lambda)\|\sigma_n(\kappa z_n(t))\|\Big(\|\sigma_n(\kappa z_n(t))\|-\frac{2\overline{\lambda}(\Psi)}{\underline{\lambda}(\Lambda)}\|w_n(t)\|\Big)\\\leq&-\kappa\underline{\lambda}(\Lambda)\|\sigma_n(\kappa z_n(t))\|\\&\times\Big(\|\sigma_n(\kappa z_n(t))\|-\frac{2\overline{\lambda}(\Psi)\|\mathcal{M}\|}{\underline{\lambda}(\Lambda)}\sum_{j=1}^{n-1}\Delta_j\Big).\end{aligned} \tag{16}$$

For $t\in[t_k+\tau_k,t_{k+1}), k=0,1,2,\cdots$, (15) can be continued as $\dot{V}_n(t)=0$.

We next show that there exists an integer $k_n>0$ such that $\|\kappa z_n(t)\|_\infty\leq\Delta_n$ for $t\in[t_k,t_{k+1}),\ k\geq k_n$. Let $c_n=\underline{\lambda}(\Psi)\Delta_n^2$ and $L_{V_n}(c_n)=\{V_n(z_n):V_n\leq c_n\}$. For any $z_n\notin L_{V_n}(c_n)$, we have $V_n>c_n$ and hence $2\kappa^2z_n^{\mathrm{T}}\Psi z_n\geq V_n>c_n$. Then, we have $\|\kappa z_n\|>\sqrt{\frac{c_n}{2\overline{\lambda}(\Psi)}}=\sqrt{\frac{\underline{\lambda}(\Psi)}{2\overline{\lambda}(\Psi)}}\Delta_n$ and $\|\sigma(\kappa z_n)\|>\sqrt{\frac{\underline{\lambda}(\Psi)}{2\overline{\lambda}(\Psi)}}\Delta_n$. We claim that there exists a finite integer $k_n>0$ and a time instant $t_n\in[t_{k_n-1},t_{k_n})$, such that $z_n(t_n)\in L_{V_n}(c_n)$. We will show this claim by contradiction. Suppose that $z_n(t)\notin L_{V_n}(c_n)$ for all $t\geq t_n$. For $t\in[t_k,t_k+\tau_k),\ k\geq k_n$, (16) can be continued as follows,

$$\dot{V} \le -\kappa\underline{\lambda}(\Lambda)\sqrt{\frac{\underline{\lambda}(\Psi)}{2\overline{\lambda}(\Psi)}}\Delta_n\left(\sqrt{\frac{\underline{\lambda}(\Psi)}{2\overline{\lambda}(\Psi)}}\Delta_n - \frac{2\overline{\lambda}(\Psi)}{\underline{\lambda}(\Lambda)}\|\mathcal{M}\|\sum_{j=1}^{n-1}\Delta_j\right),$$

which, by (7), can be continued as

$$\dot{V}_n \le -\kappa\sqrt{2\underline{\lambda}(\Psi)\overline{\lambda}(\Psi)}\|\mathcal{M}\|\Delta_n\Delta_0 := -\varepsilon_n.$$

Then, we obtain that

$$\begin{cases} V_n(t) \le -\varepsilon_n(t-t_{k_n}) + V_n(t_{k_n}),\ t\in[t_{k_n}, t_{k_n}+\tau_{k_n}), \\ V_n(t) = V_n(t_{k_n}+\tau_{k_n}), t\in[t_{k_n}+\tau_{k_n}, t_{k_n+1}). \end{cases}$$

By recursion, we have

$$V_n(t_k) \le -\varepsilon_n\sum_{l=k_n}^{k-1}\tau_l + V_n(t_{k_n}),\ k > k_n.$$

Since $\lim_{k\to+\infty}\sum_{l=k_n}^{k-1}\tau_l \ge \lim_{k\to+\infty}(k-k_n)\varepsilon_0 = +\infty$, we have $\lim_{k\to+\infty}V_n(t_k) = -\infty$, which contradicts the fact that $V_n(t) \ge 0$ for any $t \ge 0$. Thus, there exists a finite integer $k_n > 0$ and a time instant $t_n \in [t_{k_n-1}, t_{k_n})$, such that $z_n(t_n) \in L_{V_n}(c_n)$.

For $t \ge t_n$, $\dot{V}_n(z_n) \le 0$ for any $z_n(t)$ on the boundary of $L_{V_n}(c_n)$. Thus, $z_n(t)$ will remain in $L_{V_n}(c_n)$ for $t \ge t_n$. For any $z_n \in L_{V_n}(c_n)$, we have $V_n \le c_n$ and hence $\kappa z_n^{\mathrm{T}}\Psi\sigma_n(\kappa z_n) \le V_n \le c_n$. That is, $\sum_{i=1}^{N}\kappa z_{in}\sigma_n(\kappa z_{in}) \le \frac{c_n}{\underline{\lambda}(\Psi)} = \Delta_n^2$, which indicates that $|\kappa z_{in}| \le \Delta_n$, $i\in I[1,N]$. Thus, we have proven that there exists an integer k_n such that $\|\kappa z_n(t)\|_\infty \le \Delta_n$ for $t\in[t_k, t_{k+1})$, $k \ge k_n$.

For $k \ge k_n$, the consensus algorithm (13) simplifies to

$$u(t) = \begin{cases} -\kappa z_n(t) - \sum\limits_{l=1}^{n-1}\sigma_l(\kappa z_l(t)), t\in[t_k, t_k+\tau_k), \\ 0,\ t\in[t_k+\tau_k, t_{k+1}), \end{cases}$$

and z_{n-1} is governed by

$$\dot{z}_{n-1}(t) = \begin{cases} -\mathcal{M}\sigma_{n-1}(\kappa z_{n-1}(t)) + w_{n-1}(t), t\in[t_k, t_k+\tau_k), \\ 0,\ t\in[t_k+\tau_k, t_{k+1}), \end{cases}$$

with $w_{n-1}(t) = -\mathcal{M}\sum_{l=1}^{n-2}\sigma_l(\kappa z_l(t))$. Following a similar analysis as that of the evolution of z_n, we can recursively show that, for $l\in I[n-1,1]$, there exists an integer $k_l \ge k_{l+1}$ such that $\|\kappa z_l(t)\|_\infty \le \Delta_l$ for $t\in[t_k, t_{k+1})$, $k \ge k_l$.

Finally, for $k \ge k_1$, the consensus algorithm (13) simplifies to

$$u(t) = \begin{cases} -(\kappa\bar{B}^{\mathrm{T}}\otimes I)z(t),\ t\in[t_k, t_k+\tau_k), \\ 0,\ t\in[t_k+\tau_k, t_{k+1}). \end{cases}$$

Consequently, system (12) can be written as

$$\dot{z}(t) = \begin{cases} \left((\bar{A}-\kappa\bar{B}\bar{B}^{\mathrm{T}})\otimes\mathcal{M}\right)z(t), t\in[t_k, t_k+\tau_k), \\ (\bar{A}\otimes\mathcal{M})z(t),\ t\in[t_k+\tau_k, t_{k+1}). \end{cases} \tag{17}$$

We next establish asymptotic stability of the closed-loop system (17). Consider the following multiple Lyapunov function candidate

$$V(t) = \begin{cases} z^{\mathrm{T}}P_1 z,\ \ t\in[t_k, t_k+\tau_k), \\ z^{\mathrm{T}}P_2 z,\ \ t\in[t_k+\tau_k, t_{k+1}), \end{cases}$$

where $P_1 > 0$ is the solution of (8) and $P_2 > 0$ is the solution of (9). For $t\in[t_k, t_k+\tau_k)$, $k \ge k_1$, the derivative of V along the trajectory of (17) can be evaluated as follows,

$$\begin{aligned}\dot{V} &= z\left(\left((\bar{A}-\kappa\bar{B}\bar{B})^{\mathrm{T}}\otimes\mathcal{M}\right)^{\mathrm{T}}P_1 + P_1\left((\bar{A}-\kappa\bar{B}\bar{B}^{\mathrm{T}})\otimes\mathcal{M}\right)\right)z \\ &= -z^{\mathrm{T}}z \le -\gamma_1 V(t).\end{aligned}$$

For $t\in[t_k+\tau_k, t_{k+1})$, $k \ge k_1$, the derivative of V along the trajectory of (17) can be evaluated as follows,

$$\dot{V} = z^{\mathrm{T}}\left((\bar{A}\otimes\mathcal{M})^{\mathrm{T}}P_2 + P_2(\bar{A}\otimes\mathcal{M})\right)z \le \gamma_2 V(t).$$

Then, we obtain that, for $k \ge k_1$,

$$V(t) \le \begin{cases} \mathrm{e}^{-(t-t_k)\gamma_1}V(t_k),\ \ t\in[t_k, t_k+\tau_k), \\ \mathrm{e}^{(t-t_k-\tau_k)\gamma_2}V(t_k+\tau_k),\ \ t\in[t_k+\tau_k, t_{k+1}). \end{cases}$$

Thus,

$$\begin{aligned} V(t_{k_1+1}) &= z^{\mathrm{T}}(t_{k_1+1})P_1 z(t_{k_1+1}) \\ &\le \frac{\overline{\lambda}(P_1)}{\underline{\lambda}(P_2)}z^{\mathrm{T}}(t_{k_1+1})P_2 z(t_{k_1+1}) \\ &\le \frac{\overline{\lambda}(P_1)}{\underline{\lambda}(P_2)}\mathrm{e}^{\bar{\tau}_{k_1}\gamma_2}V(t_{k_1}+\tau_{k_1}) \\ &\le \frac{\overline{\lambda}(P_1)\overline{\lambda}(P_2)}{\underline{\lambda}(P_2)}\mathrm{e}^{\bar{\tau}_{k_1}\gamma_2}\|z(t_{k_1}+\tau_{k_1})\|^2 \\ &\le \frac{\overline{\lambda}(P_1)\overline{\lambda}(P_2)}{\underline{\lambda}(P_1)\underline{\lambda}(P_2)}\mathrm{e}^{\bar{\tau}_{k_1}\gamma_2-\tau_{k_1}\gamma_1}V(t_{k_1}) \\ &\le \rho^2\mathrm{e}^{-\alpha_{k_1}}V(t_{k_1}). \end{aligned}$$

By recursion, for $k \ge k_1$, we obtain that

$$V(t_k) \le \rho^{2(k-k_1)}\mathrm{e}^{-\sum\limits_{l=k_1}^{k-1}\alpha_l}V(t_{k_1}) = \mathrm{e}^{-\sum\limits_{l=k_1}^{k-1}(\alpha_l-2\ln\rho)}V(t_{k_1}). \tag{18}$$

For $t\in[t_k, t_k+\tau_k)$, by (18),

$$V(t) \le \mathrm{e}^{-(t-t_k)\gamma_1}V(t_k) \le \mathrm{e}^{-\sum\limits_{l=k_1}^{k-1}(\alpha_l-2\ln\rho)}V(t_{k_1}). \tag{19}$$

For $t\in[t_k+\tau_k, t_{k+1})$, by (18),

$$\begin{aligned} V(t) &\le \mathrm{e}^{(t-t_k-\tau_k)\gamma_2}V(t_k+\tau_k) \\ &\le \mathrm{e}^{\bar{\tau}_k\gamma_2}\overline{\lambda}(P_2)\|z(t_k+\tau_k)\|^2 \\ &\le \rho\mathrm{e}^{\bar{\tau}_k\gamma_2}z^{\mathrm{T}}(t_k+\tau_k)P_1 z(t_k+\tau_k) \\ &\le \rho\mathrm{e}^{\bar{\tau}_k\gamma_2-\tau_k\gamma_1}V(t_k) \\ &\le \rho\mathrm{e}^{-\alpha_k-\sum\limits_{l=k_1}^{k-1}(\alpha_l-2\ln\rho)}V(t_{k_1}) \\ &\le \mathrm{e}^{-(\alpha_k-\ln\rho)}\mathrm{e}^{-\sum\limits_{l=k_1}^{k-1}(\alpha_l-2\ln\rho)}V(t_{k_1}). \end{aligned} \tag{20}$$

Let κ^* be defined as in Lemma 4.2. Then, $\alpha_l - 2\ln\rho > \delta$. Combining (19) and (20), we have,

$$V(t) \le \mathrm{e}^{-\sum\limits_{l=k_1}^{k-1}(\alpha_l-2\ln\rho)}V(t_{k_1}), t\in[t_k, t_{k+1}), k \ge k_1.$$

Thus,

$$0 \le \lim_{t\to\infty}V(t) \le \lim_{k\to\infty}\mathrm{e}^{-\lim\limits_{k\to\infty}\sum\limits_{l=k_1}^{k-1}(\alpha_l-2\ln\rho)}V(t_{k_1}) = 0.$$

Hence, $\lim_{t\to\infty}z(t) = 0$ and $\lim\limits_{t\to\infty}\tilde{x}(t) = \lim_{t\to\infty}T^{-1}(I\otimes\mathcal{M}^{-1})z(t) = 0$, which indicates that global leader-following consensus is achieved. □

5 Simulation Results

Consider a group of three follower agents, each of which is described by (1) with $n = 3$ and $u_{\max} = 50$. The leader is described by (2) with $n = 3$. The communication topology among agents is shown in Fig. 2, where $a_{10} = a_{21} = a_{32} = 1$. The intermittent communication follows the pattern of $t_{k+1} = t_k + 50$s with $\tau_k = 40$s, $k = 0, 1, 2, \cdots$.

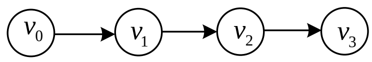

Fig. 2: The communication topology.

Let $\kappa = 2.7$ and $\mu = 0.3$. We get $\gamma_2 = 0.81$. Solving equations (8) and (9), we obtain that $\overline{\lambda}(P_1) = 1.189$, $\underline{\lambda}(P_1) = 0.06$, $\overline{\lambda}(P_2) = 8818.5$ and $\underline{\lambda}(P_2) = 0.3$. Then, we

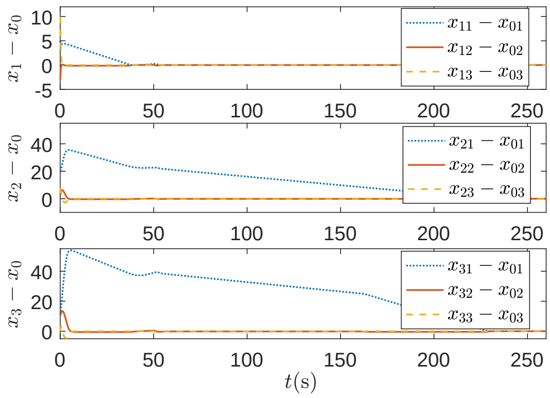

Fig. 3: Evolutions of $\hat{x}_{il}$, $l, i \in I[1, 3]$, the difference between x_i and x_0, for $i \in I[1, 3]$.

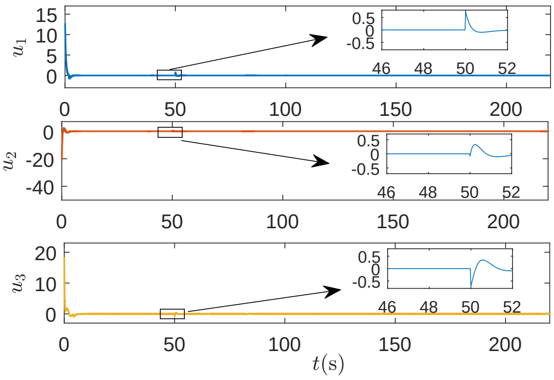

Fig. 4: Evolutions of the control inputs $u_i, i \in I[1, 3]$.

have $\gamma_1 = 0.841$, $\ln(\rho) = 11.9$ and $\alpha_l - 2 \ln \rho > 1$. Choose $\Delta_0 = 0.005$, $\Delta_1 = 0.1$, $\Delta_2 = 2$ and $\Delta_3 = 40$, which satisfy (7). Under the consensus algorithms (6), we simulate the closed-loop system with $x_0 = [-1\ 2\ 0]^{\mathrm{T}}$, $x_1 = [0\ 2\ \ -4]^{\mathrm{T}}$, $x_2 = [-1\ 0\ 3]^{\mathrm{T}}$ and $x_3 = [1\ \ -2\ \ -3]^{\mathrm{T}}$. Shown in Fig. 3 are the evolutions of the differences between the state of the follower agents and the state of the leader. It is clear that these differences converge to 0 as time goes by, indicating that global leader-following consensus is achieved. Shown in Fig. 4 are the bounded control inputs of the follower agents. The control inputs are 0 during the time periods of 40s to 50s, 90s to 100s, 140s to 150s, and 190s to 200s.

6 Conclusions

In this paper, we studied the global leader-following consensus problem for a group of a chain of integrators with intermittent directed communication in the presence of actuator saturation. A bounded control algorithm was constructed for each follower agent that achieved global leader-following consensus under a certain communication topology. This consensus algorithm for each agent utilized information of other agents obtained through the communication network intermittently.

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