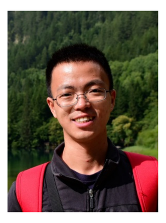
Bounded Consensus of Linear Multi-Agent Systems with External Disturbances Through a Reduced-Order Adaptive Feedback Protocol

CCC2018



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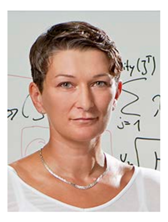

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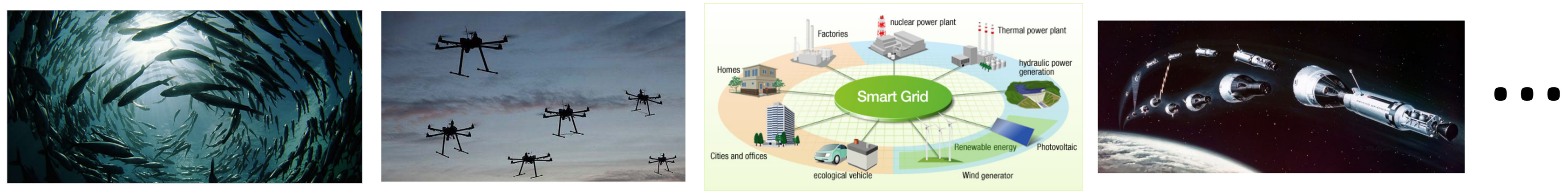
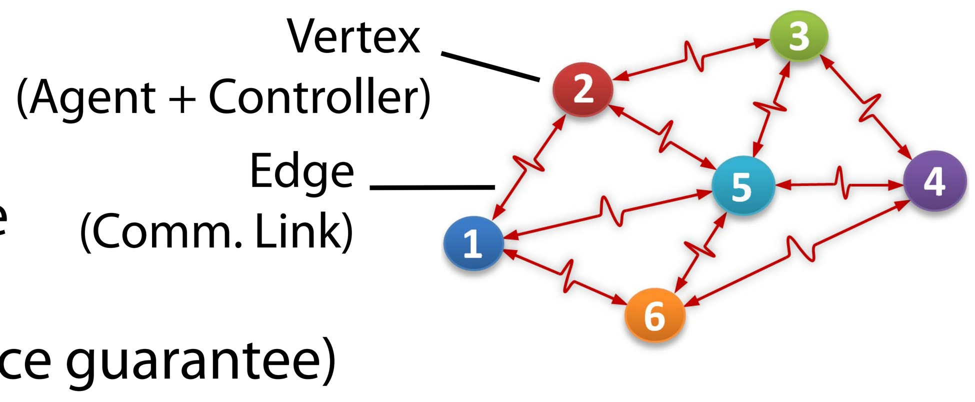
Background and motivation

- Applications of multi-agent systems (MASs)



- Basic properties
  - Distributed control
  - Limited energy storage
- Requirements
  - Easy to design (existence guarantee)
  - Easy to implement (no global info., reduced-order, etc.)



☞ **Reduced-order adaptive output-feedback protocol**

Problem statement

- System setup

Adaptive law

$$\dot{c}_{ij} = \alpha_{ij}\left(\|\tilde{y}_{ij} + C_y B_r \tilde{r}_{ij}\|_2^2 - \phi_{ij} c_{ij}\right)$$

$$c_{ij}(0) = c_{ji}(0),\ \alpha_{ij} = \alpha_{ji} > 0,\ \phi_{ij} > 0$$

c_{ij}

Controller (n_rth-order)

$$\dot{r}_i = H r_i + F_r \sum_{j\in\mathcal{N}_i} c_{ij} a_{ij}\left(\tilde{y}_{ij} + C_y B_r \tilde{r}_{ij}\right)$$

$$u_i = G r_i + F_u \sum_{j\in\mathcal{N}_i} c_{ij} a_{ij}\left(\tilde{y}_{ij} + C_y B_r \tilde{r}_{ij}\right)$$

u_i

Agent i (n_x th-order)

$$\dot{x}_i = A x_i + B_u u_i + w_i$$

$$y_i = C_y x_i$$

$\tilde{r}_{ij} = r_i - r_j$

$\tilde{y}_{ij} = y_i - y_j$

Neighbors $\mathcal{N}_i$

$w_i \in \{w : \|w(t)\|_2 \le \bar{w}, t \ge 0\}$

- Consensus error $s_{ei} \triangleq \begin{bmatrix} x_i \\ r_i \end{bmatrix} - \frac{1}{N}\sum_{j=1}^{N}\begin{bmatrix} x_j \\ r_j \end{bmatrix}$, $s_e = \mathrm{col}\{s_{e1}, \ldots, s_{eN}\}$

☞ **Objective:** s_e **and** c_{ij} **are bounded**

Solvability condition

Theorem 1 Both s_e and c_{ij} are uniformly ultimately bounded if

1. Matrices H, G and B_r are such that

$$H \text{ is Hurwitz},\ AB_r - B_r H = B_u G,$$
$$\mathrm{rank}(B_r) = n_r \text{ and } \mathrm{rank}([B_u, B_r]) = n_x; \quad (1)$$

2. Matrices F_u and F_r are given by

$$\begin{bmatrix} F_u \\ F_r \end{bmatrix} = -\begin{bmatrix} R_u \\ R_r \end{bmatrix} P C_y^{\mathrm{T}}, \quad (2)$$

where R_u and R_r are matrices such that $R_r B_r = \mathbf{I}_{n_r}$ and $B_u R_u + B_r R_r = \mathbf{I}_{n_x}$, and for a given $Q > 0$, $P > 0$ is such such that

$$PA^{\mathrm{T}} + AP - PC_y^{\mathrm{T}} C_y P + Q = 0.$$

Comparisons with existing results

- A general parametrization for both full-order ($n_r = n_x$) and reduced-order ($n_r < n_x$) protocols
- Reduced-order **vs.** Full-order ([1])
- Relative info. **vs.** Absolute info. ([2])
- No relative input **vs.** Relative input ([3])
- Adaptive gain **vs.** Fixed gain from known Laplacians ([2, 3])

Error analysis

Corollary 1 With the parameterization in Theorem 1, s_e and c_{ij} exponentially converge to the domain

$$\mathbb{D}_e \triangleq \left\{ s_e, c_{ij} \middle| 0 < V_e \le \frac{\bar{V}}{\delta} \right\},$$

where

$$\bar{V} \triangleq \theta^{-1}\lambda_{\min}^{-1}(P)\sum_{i=1}^{N}\bar{w}_i^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} a_{ij}\phi_{ij}\bar{c}^2$$

$$V_e \triangleq s_e^{\mathrm{T}}\left(\mathbf{I} \otimes \begin{bmatrix} P^{-1} + \varepsilon R_r^{\mathrm{T}} M R_r & P^{-1} B_r \\ B_r^{\mathrm{T}} P^{-1} & B_r^{\mathrm{T}} P^{-1} B_r \end{bmatrix}\right) s_e + \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\frac{a_{ij}}{2\alpha_{ij}}(c_{ij} - \bar{c})^2$$

and positive scalars δ, ε, θ and $\bar{c}$ are such that

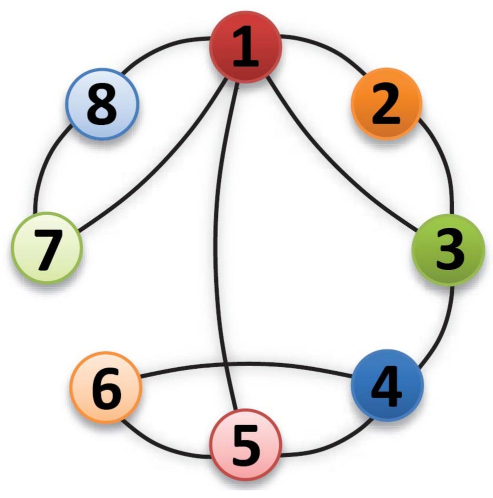
$$0 < \delta \le \alpha_{ij}\phi_{ij}, \bar{c} \ge \frac{1}{2\lambda_2(\mathcal{L})}, \begin{bmatrix} \Phi_{11} & \varepsilon\Phi_{21}^{\mathrm{T}} \\ \varepsilon\Phi_{21} & \varepsilon\Phi_{22} \end{bmatrix} < 0$$

with

$$\Phi_{11} = P^{-1}A + A^{\mathrm{T}}P^{-1} + (\theta + \delta)P^{-1} - 2\bar{c}\lambda_2(\mathcal{L})C_y^{\mathrm{T}}C_y$$

$$\Phi_{21} = MHR_r - MR_rA, \Phi_{22} = MH + H^{\mathrm{T}}M + \delta M.$$

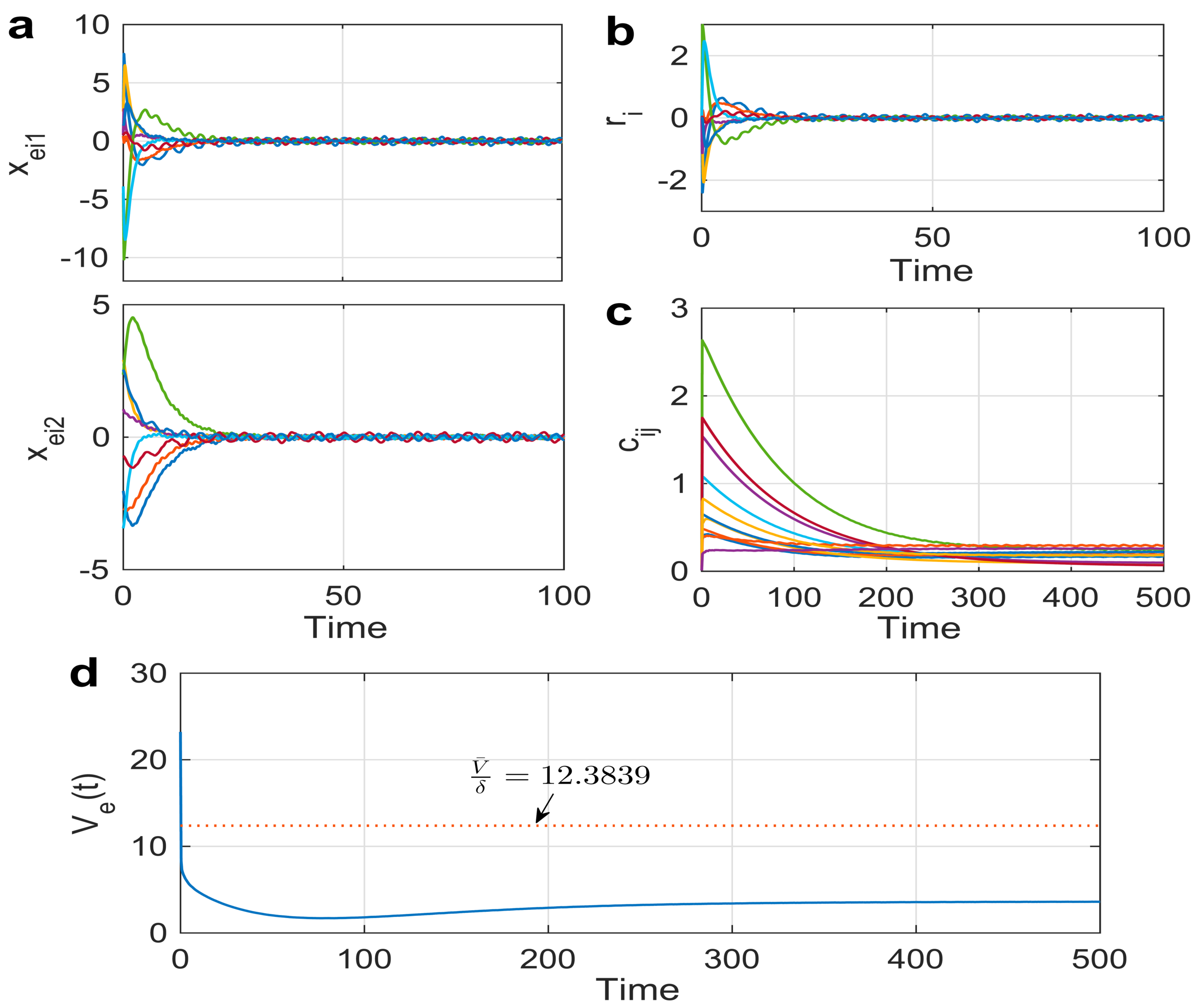
Numerical example



$$A = \begin{bmatrix} 0 & 0.2 \\ -0.2 & 0 \end{bmatrix},\ B_u = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ C_y = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$H = -0.6325,\ B_r = \begin{bmatrix} 3.1623 \\ 1 \end{bmatrix},\ G = 2.2$$

$$F_u = 6.0428,\ F_r = -2.9686.$$



Conclusion

- A new reduced-order adaptive protocol is proposed
- A tractable parametrization condition is derived
- Estimation of attraction domain of consensus error is presented

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References

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