第九届控制科学与工程前沿论坛 杭州

迭代学习控制研究现状及其挑战性问题

孙明轩

浙江工业大学

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控制器设计的主流方法

- 极点配置方法
- 优化设计方法
- **Lyapunov**方法 等等

- 压缩映射方法(见于迭代算法分析)
  - ✓ 调节压缩率改变收敛速率
  - ✓ 收敛域：本质鲁棒收敛

学习控制

重复学习原理
(Learning Through Trials, LTT)

- 迭代学习控制 ILC （1978, 1984）
- 重复控制 RC （1981）

提纲

一、重复系统：工业背景及算例

二、ILC六要素 (Arimoto’s Postulates)

三、内模原理与学习不变量原理

四、压缩映射ILC(CMILC)的研究问题与现状

五、关于迭代学习方法

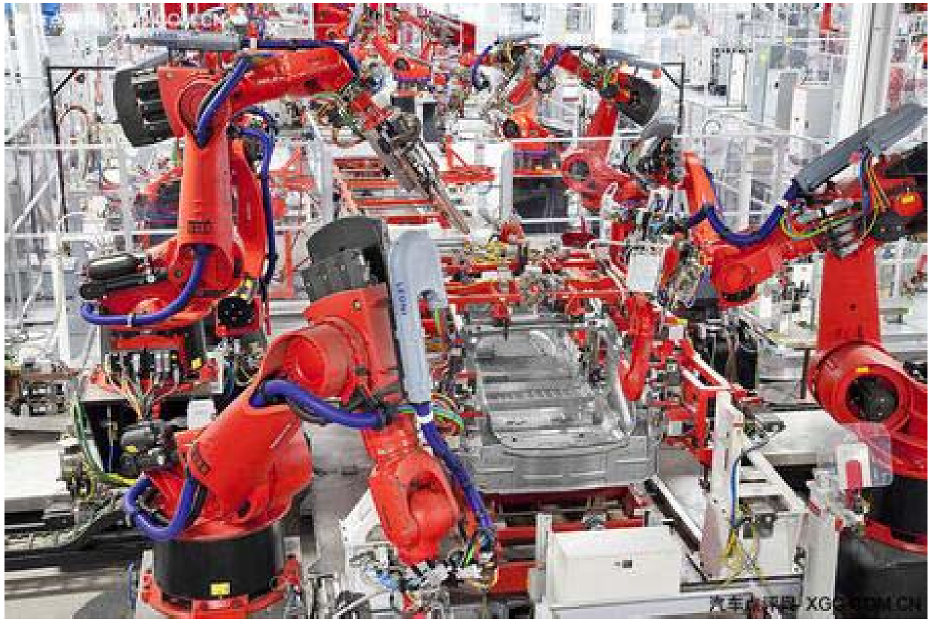
一、重复系统：工业背景及算例

重复系统

- 重复运行：期望轨迹是重复(迭代)一致的
- 周期运行：参考信号为周期的

工业机器人

相比人的操作水平：高品质控制
高速高性能



高精度作业：医疗、微纳操作等

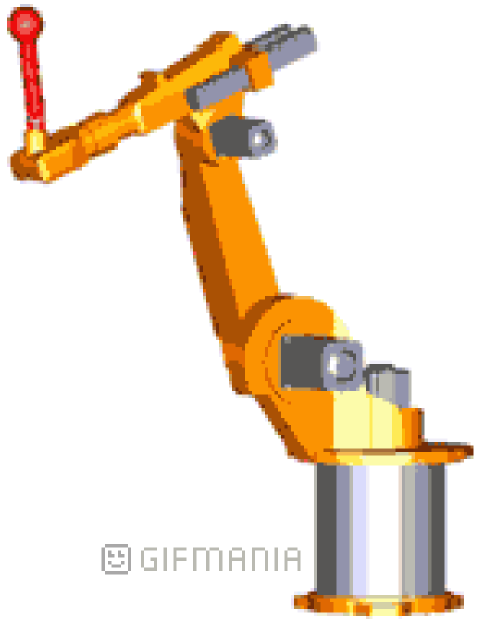
最适应的场合是重复作业
(在流水线作业中不怕枯燥、不知疲倦)

流水线作业的装配机器人需要快速响应

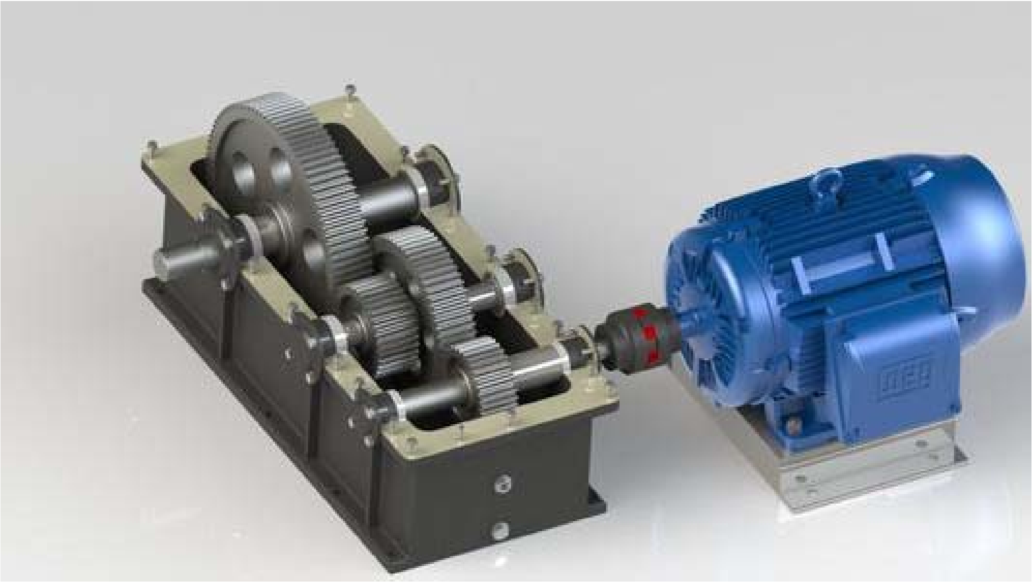
核心零部件：伺服电机、减速器、控制器

工业机器人的运行特点

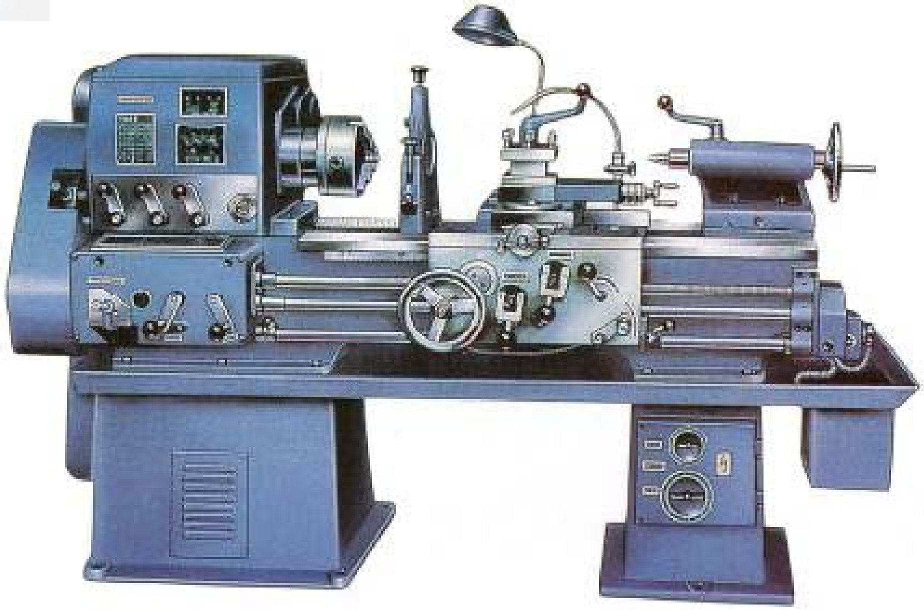
- ✓ 有限作业区间
- ✓ 重复作业
- ✓ 在整个作业区间上的高性能控制



电机：一般动力源



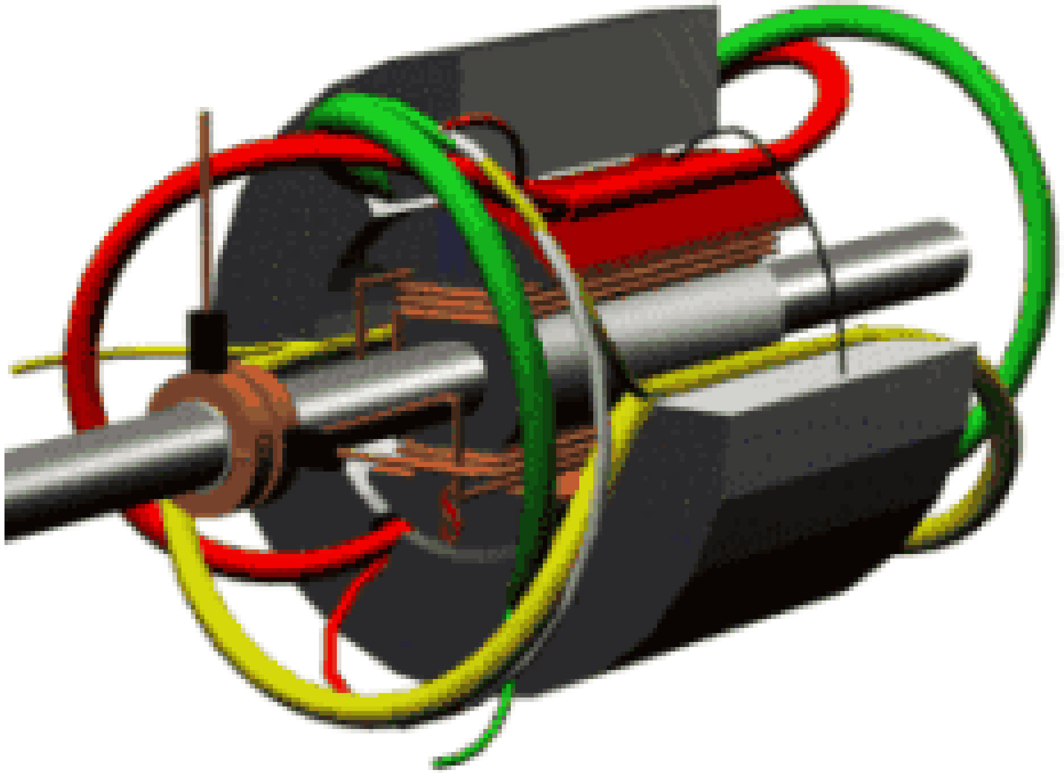
齿轮减速箱

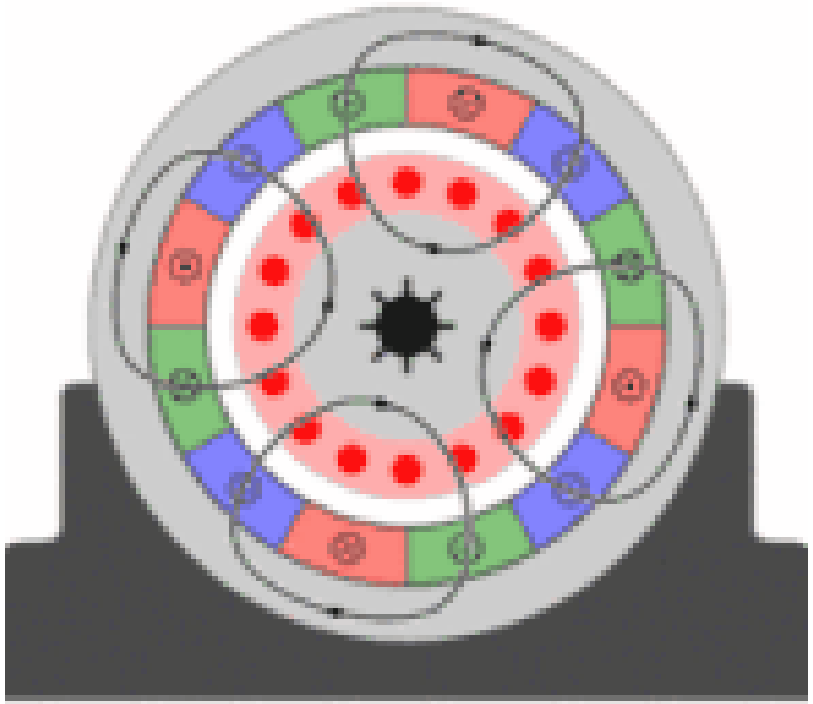


滚珠丝杠

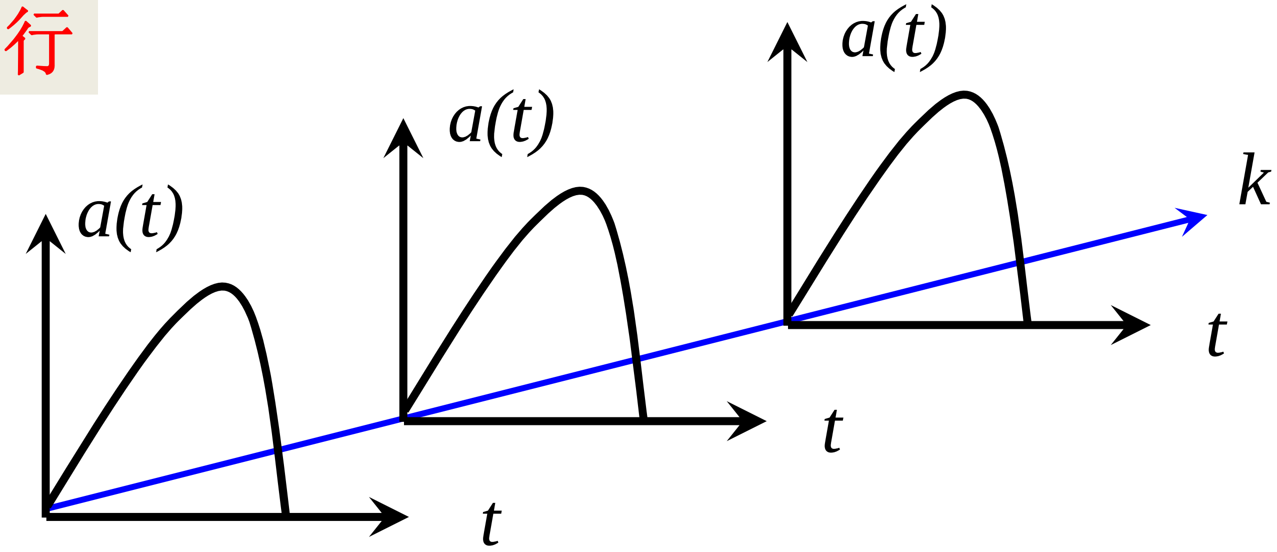
电机的运行特点

作业区间[0,2π]
转子每转一圈，即循环一次，
高速周期运行系统

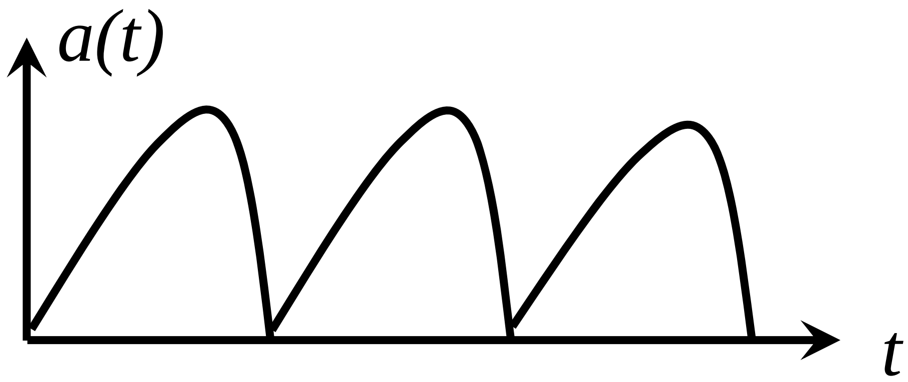




重复运行



周期运行



重复系统(装置、过程)

- 电子封装
- 自动焊接(切割)过程
- 数控加工(模具加工)
- 晶圆制备(Wafer Fabrication)
- 间歇反应釜
- 电力电子线路：逆变器
- 硬盘驱动器
- VCD/DVD
- 飞行器起飞、降落
- 火炮射表编制
- 火箭复用

多段/分段重复系统

考虑如下系统

$$x(t+1)=x(t)+hf(x(t))+hbu(t),\ y(t)=cx(t)$$

采用学习律

$$u_{k+1}(t)=u_k(t)+\gamma(e_k(t+1)-e_k(t))$$

其收敛性条件为

$$|1-\gamma cbh|<1,\quad 0<\gamma cbh<2\quad (与f(x)无关、与h有关)$$

置$h=0.001$

$$0<\gamma cb<2000$$

取$\gamma=1/\hat{c}\hat{b}$

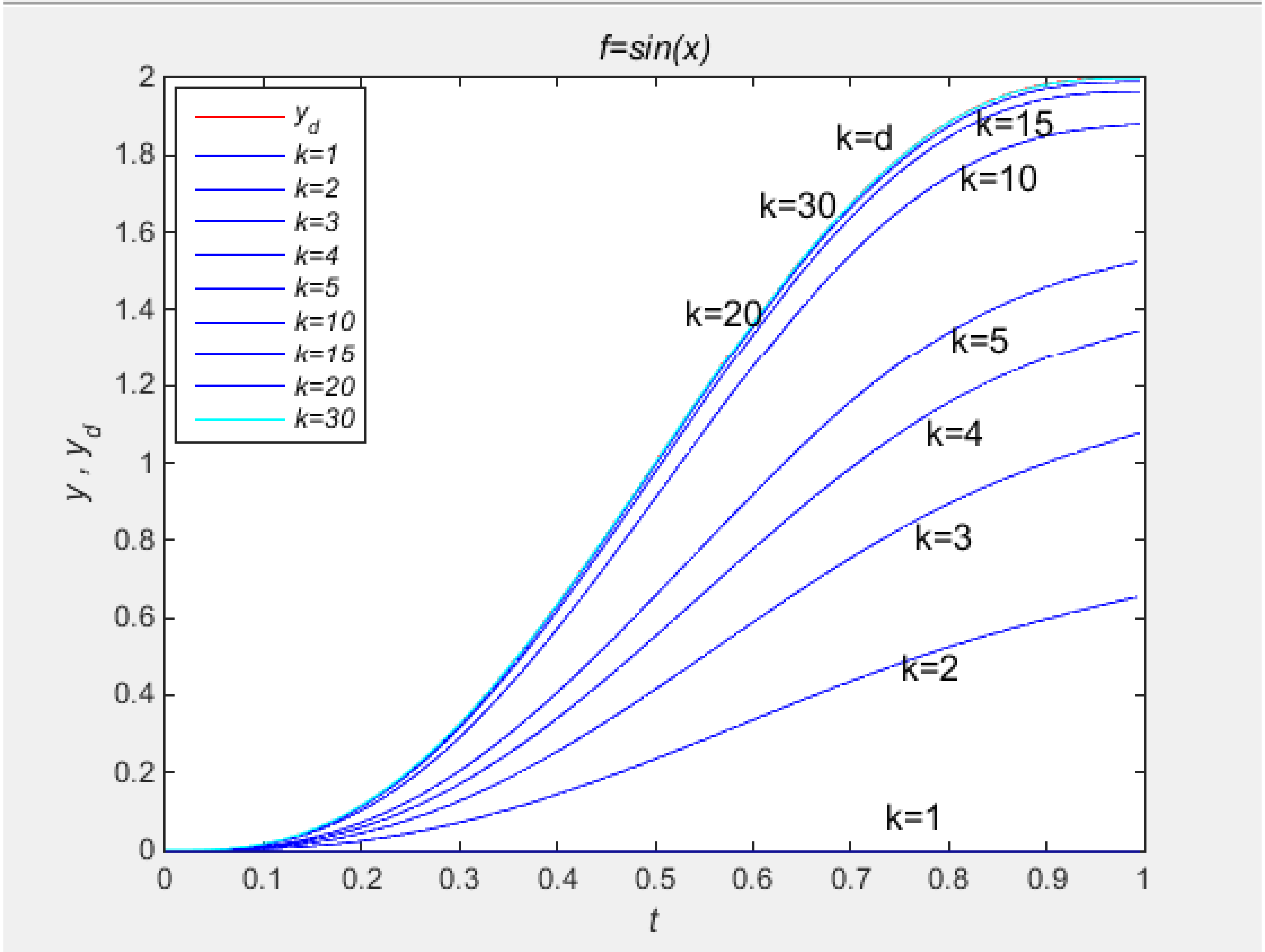
$$0<cb/\hat{c}\hat{b}<2000$$

假设$cb=1$

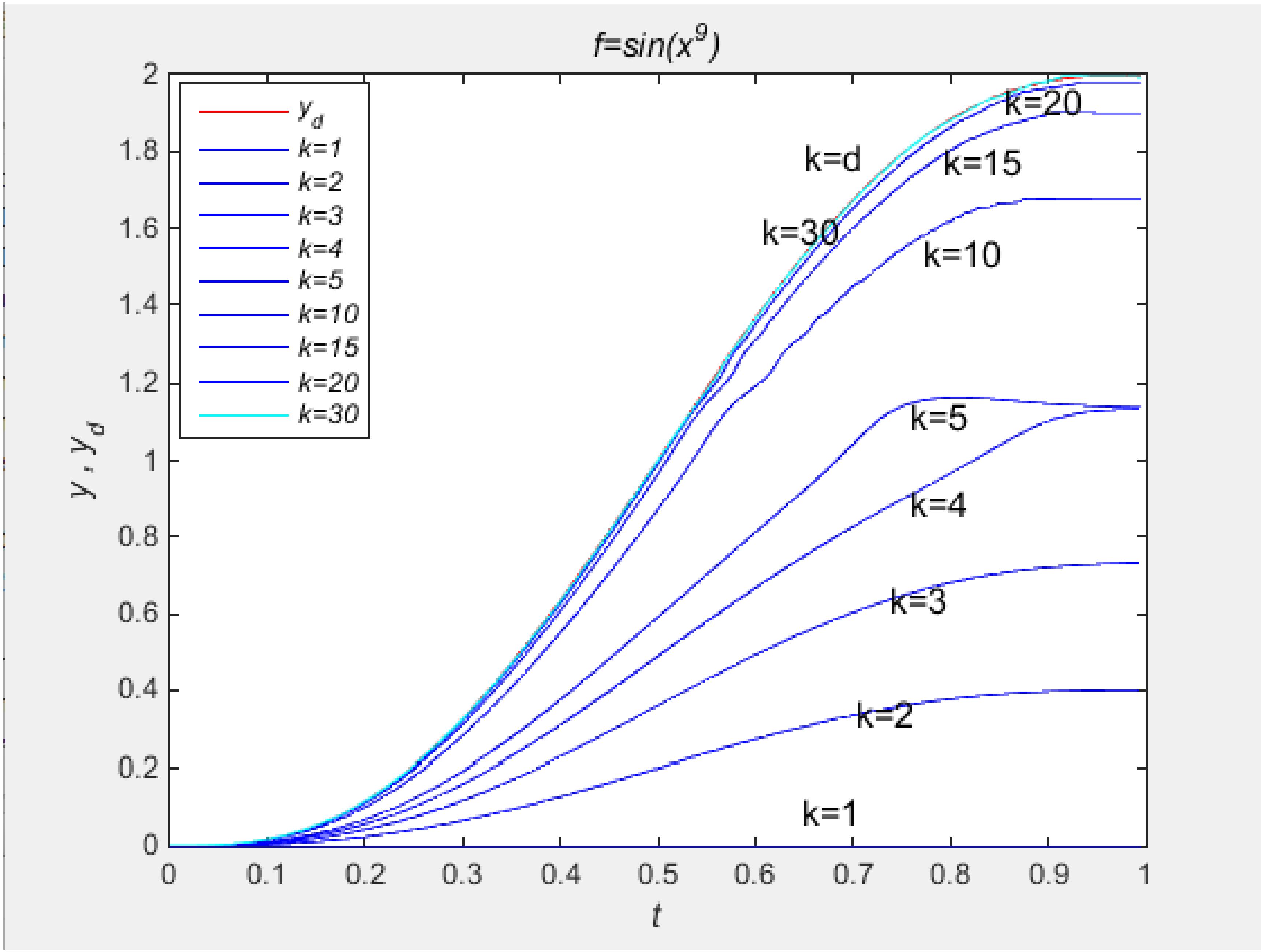
$$0.0005<\hat{c}\hat{b}<\infty$$

取cb的估计为0.005，即$\gamma=200$

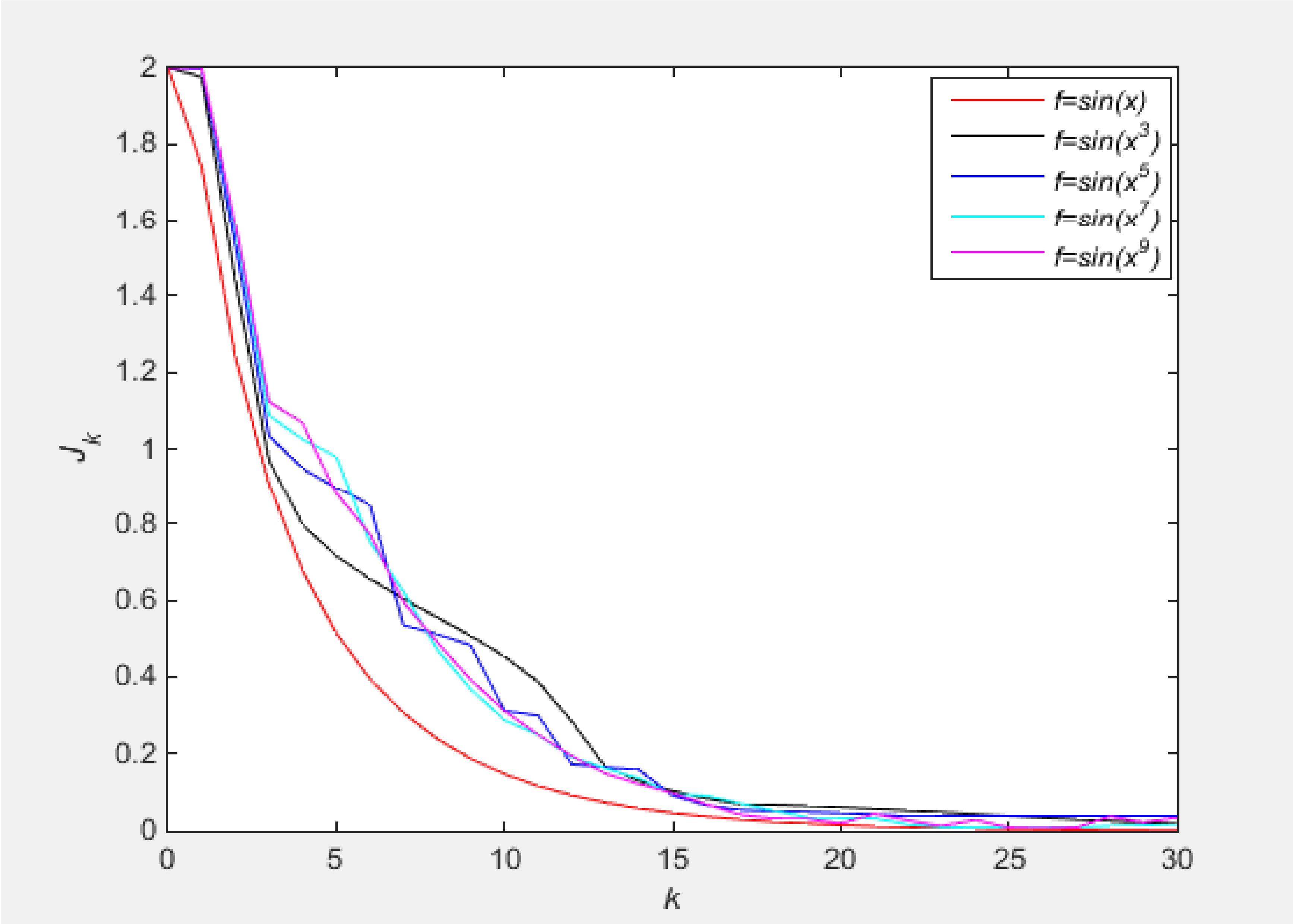
期望轨迹：$y_d(t)=2\times[6(t)^3-15(t)^4+10(t)^5],t\in[0,1]$



f=sin(x)
y_d
k=1
k=2
k=3
k=4
k=5
k=10
k=15
k=20
k=30
k=d
k=15
k=10
k=30
k=20
k=5
k=4
k=3
k=2
k=1
y , y_d
t
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



f=sin(x^9)
y_d
k=1
k=2
k=3
k=4
k=5
k=10
k=15
k=20
k=30
k=d
k=20
k=15
k=30
k=10
k=5
k=4
k=3
k=2
k=1
y , y_d
t



f=sin(x)
f=sin(x^3)
f=sin(x^5)
f=sin(x^7)
f=sin(x^9)
J_k
k
0
5
10
15
20
25
30
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0

“粗模型”下实现完全跟踪效果

迭代学习控制的两大优点（可应用场合）

- ✓ 算法简单 适于快速作业对象
- ✓ 前馈补偿作用 有益于控制精度的提高

重复系统的迭代学习控制理论

- 重复系统的应用背景是较为广泛的；
- 迭代学习控制利用系统重复性设计控制器；
- 现有反馈控制没有利用这类受控对象的重复特征，重复运行时控制性能得不到提高；
- 反馈控制能够有效镇定受控对象；
- 在反馈控制基础上，形成“反馈控制+迭代学习控制”的双环控制结构。

二、ILC六要素(Arimoto's Postulates)

(A_1) Each operation ends in a fixed finite time $T > 0$.

(A_2) A desired output $\mathbf{y}_d(t)$ is given *a priori* over the time duration $t \in [0, T]$.

(A_3) Repeatability of the initial setting is satisfied, i.e. the initial state $\mathbf{x}_k(0)$ of the objective system can be set the same at the beginning of each operation in the following way:

$$\mathbf{x}_k(0) = \mathbf{x}^0 \quad \text{for} \quad k = 1, 2, \ldots$$

where k denotes the trial number of operation.

(A_4) Invariance of the system dynamics is assured throughout repeated trainings.

(A_5) Each output trajectory $\mathbf{y}_k(t)$ can be measured without noise and hence the error signal

$$\mathbf{e}_k(t) = \mathbf{y}_d(t) - \mathbf{y}_k(t)$$

can be utilized in construction of the next command input.

(A_6) The next command input $\mathbf{u}_{k+1}(t)$ must be composed of a simple and fixed recursive law as follows:

$$\mathbf{u}_{k+1}(t) = \mathbf{F}(\mathbf{u}_k(t), \mathbf{e}_k(t))$$

三、内模原理与学习不变量原理

零输入、特定初始条件下的信号模型

(Goodwin et al., Control System Design, 2001)

阶跃： $$\frac{1}{s}$$

周期： $$\frac{1}{1-e^{-Ts}},\ \frac{e^{-Ts}}{1-e^{-Ts}}$$

学习模型

定常量学习模型:

$$U(s)=\frac{1}{s}E(s)$$

$$\dot{u}(t)=e(t)$$

$$\therefore \quad u(t)\to 0,\ \ u(t)\to \text{Const.},\ \ \text{as } t\to\infty, e(t)\to 0$$

周期量学习模型:

$$U(s)=\frac{1}{1-e^{-Ts}}E(s)$$

$$u(t)-u(t-T)=e(t)$$

$$u(t)=u(t-T)+e(t)$$

$\therefore\ u(t)-u(t-T)\to 0,$ as $t\to\infty,\ e(t)\to 0$

i.e., $u(t)\to$ a periodic signal, as $t\to\infty,\ e(t)\to 0$

时变量模型:

$$u(t') = u(t' - T) + e(t')$$

令$t = t' + kT$, $u(t') = u_k(t)$, $t \in [0, T], k = 0, 1, 2, \cdots$

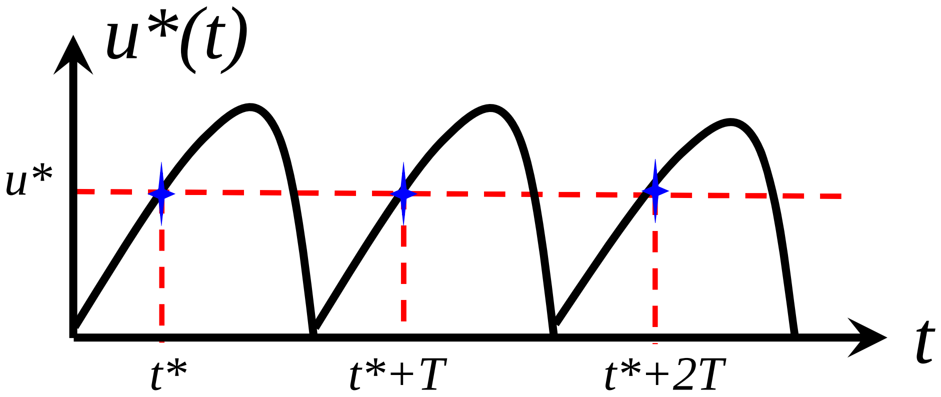
$$u_k(t) = u_{k-1}(t) + e_k(t)$$

$\therefore$ $u_k(t) \to u_{k-1}(t)$, as $k \to \infty$, $t \in [0, T]$, $e_k(t) \to 0$

i.e., $u_k(t) \to$ a time-variant signal, as $k \to \infty, t \in [0, T]$, $e_k(t) \to 0$

周期学习：$u(t)=u(t-T)+e(t)$

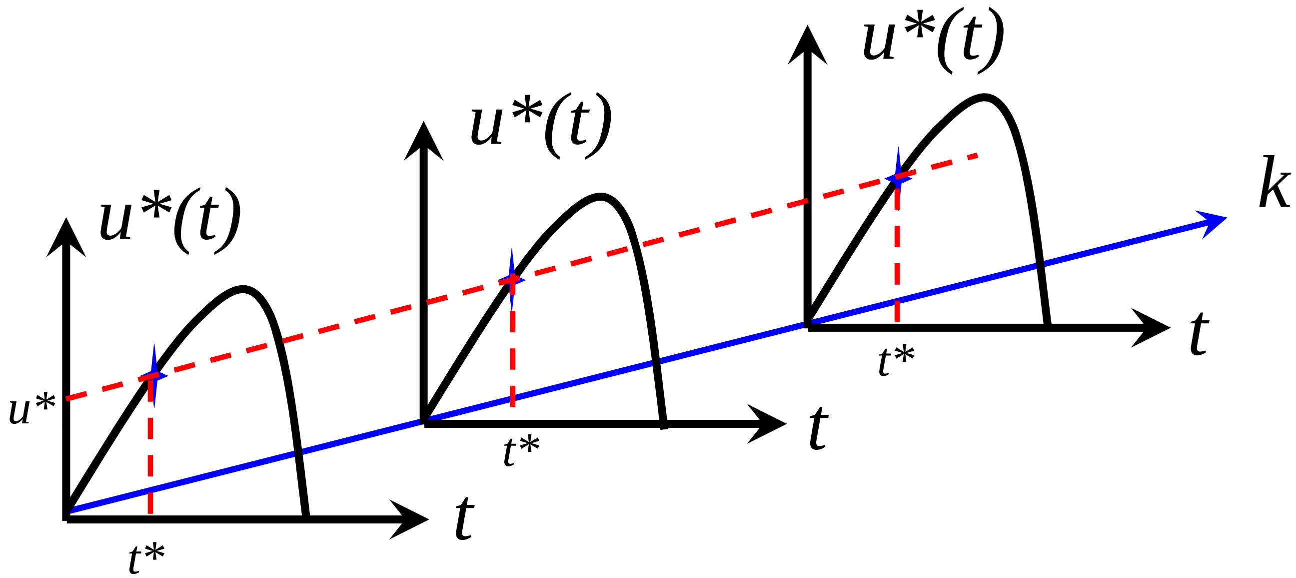
学习不变量原理



如果有重复，
寻找不改变的东西！

[德] 恩格尔
《解决问题的策略》

迭代学习：$u_k(t)=u_{k-1}(t)+e_k(t)$



迭代学习机制

$$u_k(t) = u_{k-1}(t) + e_k(t)$$

$$t \in [0, T],\ k = 0, 1, 2, \cdots$$

修正项： $e_k(t)$

被修正项： $u_{k-1}(t)$

前馈控制：补偿作用 $u_{k-1}(t)$

累加作用：沿迭代轴的积分

$$u_k(t) = u_0(t) + \sum_{i=1}^{k} e_i(t)$$

当 $\lim\limits_{k\to\infty} u_k(t) = u^*(t)$ 时，

$$\sum_{i=1}^{\infty} e_i(t) = u^*(t) - u_0(t)$$

四、CMILC的研究问题与现状

依据Arimoto's 六要素描述ILC根本问题

(A_1) Each operation ends in a fixed finite time $T > 0$.

(A_2) A desired output $\mathbf{y}_d(t)$ is given *a priori* over the time duration $t \in [0, T]$.

(A_3) Repeatability of the initial setting is satisfied, i.e. the initial state $\mathbf{x}_k(0)$ of the objective system can be set the same at the beginning of each operation in the following way:

$$\mathbf{x}_k(0) = \mathbf{x}^0 \quad \text{for} \quad k = 1, 2, \ldots$$

where k denotes the trial number of operation.

(A_4) Invariance of the system dynamics is assured throughout repeated trainings.

(A_5) Each output trajectory $\mathbf{y}_k(t)$ can be measured without noise and hence the error signal

$$\mathbf{e}_k(t) = \mathbf{y}_d(t) - \mathbf{y}_k(t)$$

can be utilized in construction of the next command input.

(A_6) The next command input $\mathbf{u}_{k+1}(t)$ must be composed of a simple and fixed recursive law as follows:

$$\mathbf{u}_{k+1}(t) = \mathbf{F}(\mathbf{u}_k(t), \mathbf{e}_k(t))$$

迭代学习算法分析

- **收敛性**

  满足六要素的情形
- **鲁棒性**

  关于六要素不确定性的鲁棒性问题
- **鲁棒收敛性**

  六要素中的要求不满足时的收敛性问题

P1. 学习算法：收敛性与收敛速度

(A6) The next command input $u_{k+l}(t)$ must be composed of a simple and fixed recursive law as follows:

$$u_{k+1}(t) = F(u_k(t), e_k(t))$$

D-type learning law and its variants:

D-type: $u_{k+1}(t) = u_k(t) + \Gamma \dot{e}_k(t)$

PD-type: $u_{k+1}(t) = u_k(t) + \Gamma(\dot{e}_k(t) + Le_k(t))$

PID-type: $u_{k+1}(t) = u_k(t) + \Gamma(\dot{e}_k(t) + L_I \int_0^t e_k(s)ds + L_P e_k(t))$

P-type learning for systems with relative degree one:

$$u_{k+1}(t) = u_k(t) + \Gamma e_k(t)$$

ILC学习系统的因果性

Higher-order learning law:

$$F(u_k(t), e_k(t)) \Rightarrow F(u_k(t), \cdots, u_{k-M}(t), e_k(t), \cdots, e_{k-M}(t))$$

$$u_{k+1}(t) = P_1 u_k(t) + P_2 u_{k-1}(t) + Q_1 e_k(t) + +Q_2 e_{k-1}(t)$$

Varying-order learning law:

$$F(u_k(t), e_k(t)) \Rightarrow F(u_{k-j}(t), e_{k-j}(t))$$

$$u_{k+1}(t) = u_{k-j}(t) + \Gamma e_{k-j}(t)$$

利用反馈信息的学习算法

Closed-loop learning law:

$$F(u_k(t), e_k(t)) \Rightarrow F(u_k(t), e_{k+1}(t))$$

$$u_k(t) = u_{k-1}(t) + \Gamma e_k(t)$$

学习算法的收敛速度

跟踪误差的压缩映射

$$\|e_{k+1}(\cdot)\| \le \rho \|e_k(\cdot)\|, \quad 0 < \rho < 1$$

学习算法A: $\|e_{k+1}(\cdot)\| \le \rho_1 \|e_k(\cdot)\|$

学习算法B: $\|e_{k+1}(\cdot)\| \le \rho_2 \|e_k(\cdot)\|$

P. 若$\rho_1 < \rho_2$，则学习算法A $\succ$ 学习算法B

设存在Δ_1、Δ_2使得

$$\|e_{k+1}(\cdot)\| = \rho_1 \|e_k(\cdot)\| + \Delta_1$$

$$\|e_{k+1}(\cdot)\| = \rho_2 \|e_k(\cdot)\| + \Delta_2$$

若$\rho_1 < \rho_2$，欲使

$$\rho_1 \|e_k(\cdot)\| + \Delta_1 \le \rho_2 \|e_k(\cdot)\| + \Delta_1 \le \rho_2 \|e_k(\cdot)\| + \Delta_2$$

应使

$$\Delta_1 \le \Delta_2$$

P2. 考虑量测噪声的学习算法

(A5) Each output trajectory $y_k(t)$ can be measured without noise

带滤波器的学习算法

随机系统框架下的学习算法理论

P3. ILC系统的鲁棒性

(A4) Invariance of the system dynamics is assured throughout repeated trainings

Iteration-indendent disturbances:

$$\dot{x}_k(t) = f(x_k(t)) + B(x_k(t))u_k(t) + w(t)$$

$$y_k(t) = g(x_k(t)) + v(t)$$

Iteration-dendent disturbances:

$$\dot{x}_k(t) = f(x_k(t)) + B(x_k(t))u_k(t) + w_k(t)$$

$$y_k(t) = g(x_k(t)) + v_k(t)$$

P-type learning for systems with relative degree one:

$$u_{k+1}(t) = u_k(t) + \Gamma e_k(t)$$

Its convergence dependents on the passivity.

Not robust with respect to the uncertainties:

$$u_k(0) = u_0(0) + \Gamma \sum_{i=0}^{k-1} e_i(0)$$

Closed-loop P-type learning:

$$u_k(t) = u_{k-1}(t) + \Gamma e_k(t)$$

$$u_k(0) = u_{-1}(0) + \Gamma \sum_{i=0}^{k} e_i(0)$$

P-type learning law with a forgetting factor, $0<\alpha<1$

$$u_{k+1}(t) = (1-\alpha)u_k(t) + \Gamma e_k(t)$$
$$= (1-\alpha)^{k+1}u_0(t) + \Gamma\sum_{i=0}^{k}(1-\alpha)^{k-i}e_i(t)$$
$$\le (1-\alpha)^{k+1}u_0(t) + \frac{1-(1-\alpha)^{k+1}}{1-(1-\alpha)}\Gamma\overline{e}$$

as $|e_k(t)| \le \overline{e}$. Hence,

$$\lim_{k\to\infty}\sup u_k(t) \le \frac{1}{\alpha}\Gamma\overline{e}$$

As $e_k(t) \to 0,\ u_k(t) \to 0.$

P4. ILC系统的初值问题

(A3) Repeatability of the initial setting is satisfied, i.e., the initial state $x_k(0)$ of the objective system can be set the same at the beginning of each operation in the following way:

$$x_k(0) = x^0$$

for $k = 1, 2, \cdots$

IIC) $x_k(0) = x_d(0)$

ISC) $x_k(0) = x^0 \; (\neq x_d(0))$

ISC) $x_k(0) = x^0 \ (\neq x_d(0))$

D-type learning: $u_{k+1}(t) = u_k(t) + \Gamma \dot{e}_k(t)$

$$y^*(t) = y_d(t) - C, \quad C = e_0(0)$$

PD-type learning: $u_{k+1}(t) = u_k(t) + \Gamma(\dot{e}_k(t) + Le_k(t))$

$$y^*(t) = y_d(t) - e^{-Lt}C, \quad C = e_0(0)$$

Initial rectified D-type learning:

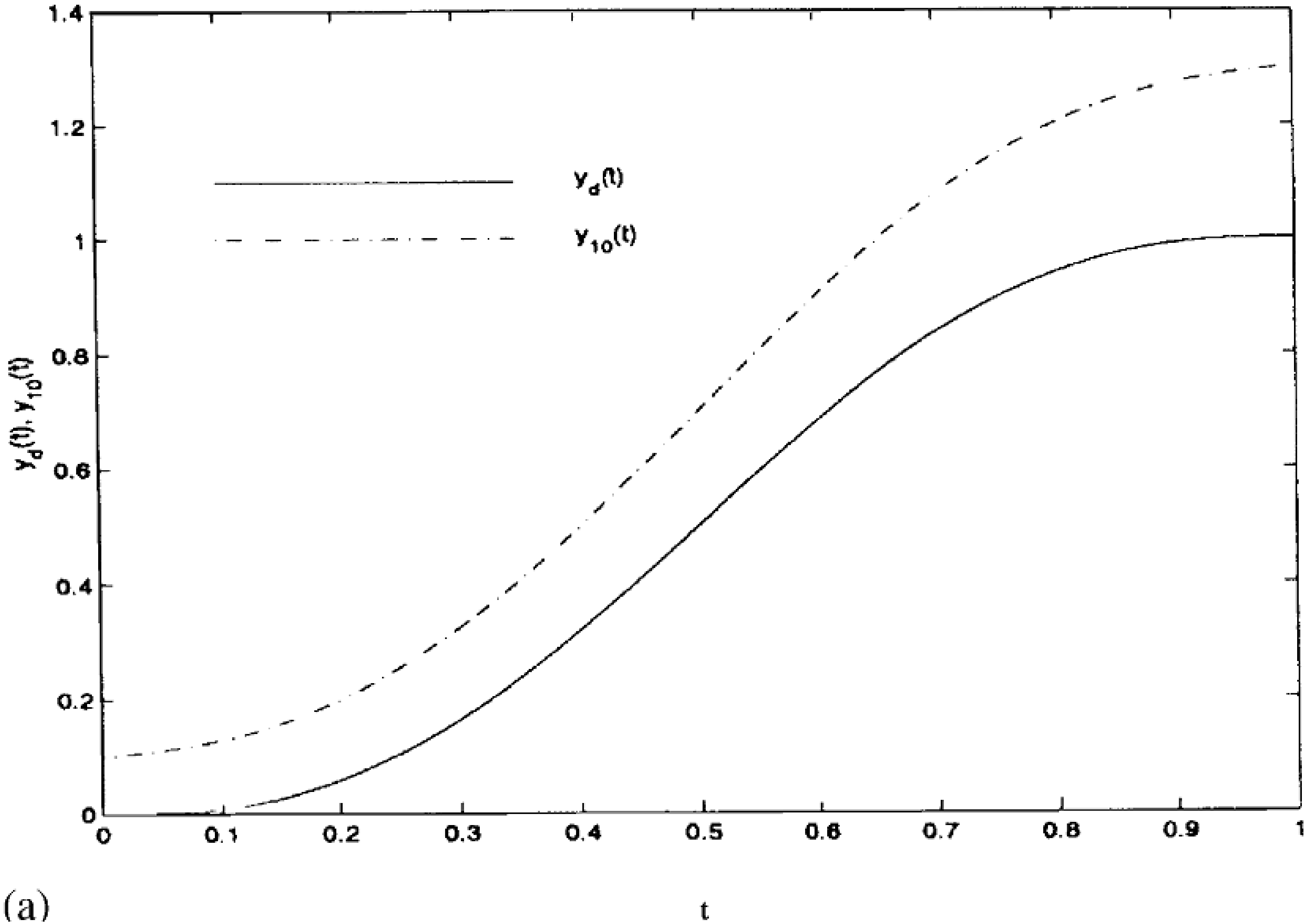
$$u_{k+1}(t) = u_k(t) + \Gamma(\dot{e}_k(t) + \theta_h(t)C)$$

$$y^*(t) = y_d(t) + C\int_h^t \theta_h(\tau)d\tau$$

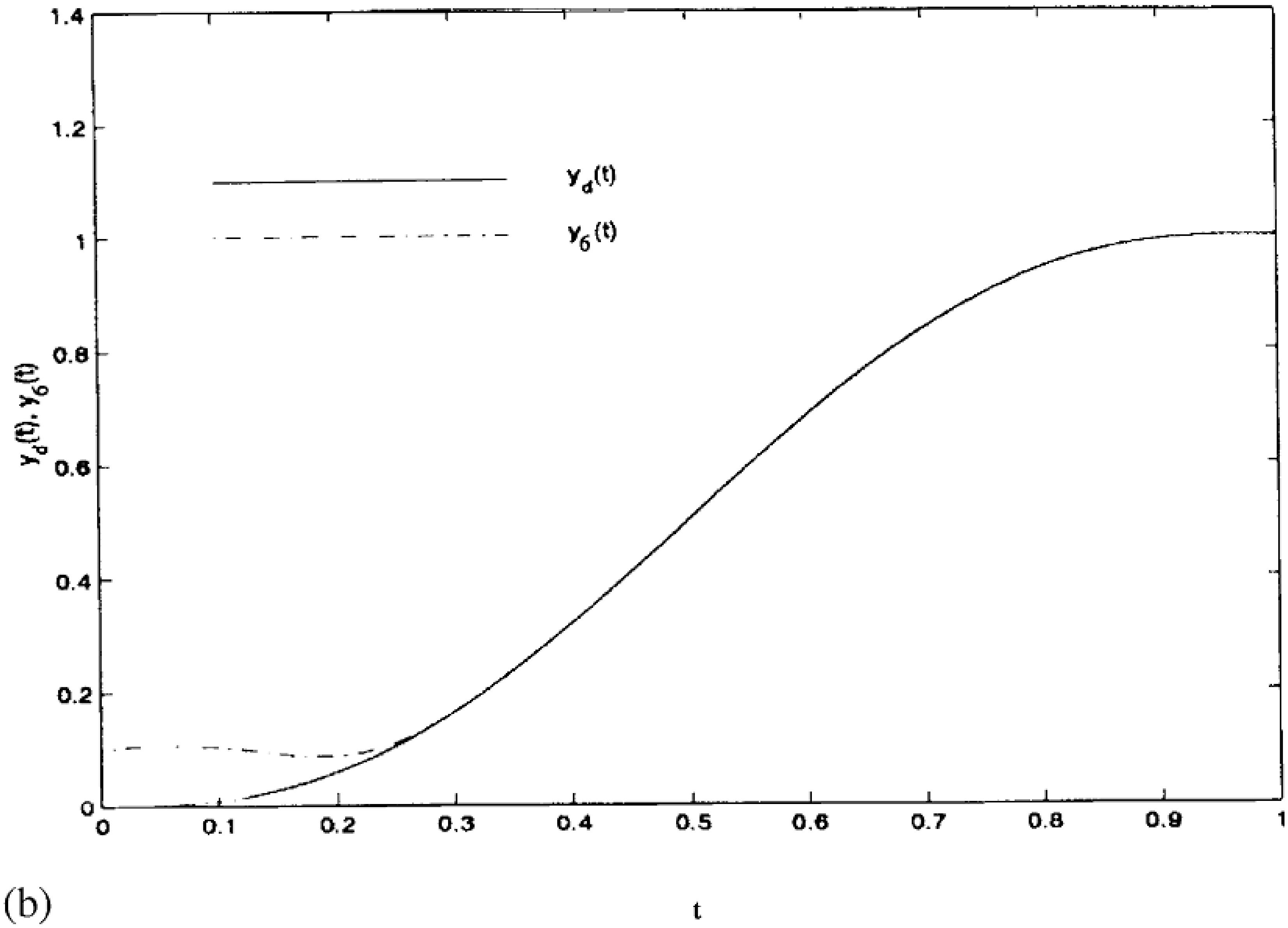
Initial rectified PD-type learning:

$$u_{k+1}(t) = u_k(t) + \Gamma(\dot{e}_k(t) + Le_k(t) + e^{-Lt}\theta_h(t)C)$$

$$y^*(t) = y_d(t) - e^{-Lt}C\int_h^t \theta_h(\tau)d\tau$$



(a)



(b)

P5. 非一致轨迹问题

(A2) A desired output $y_d(t)$ is given a priori over the time duration, $t \in [0, T]$

Two cases:

i) The desied output with measurement noise; and

ii) The iteration-dependent desired output, i.e., $y_{d,k}(t)$

P6. 变区间问题

(A1) Each operation ends in a fixed finite time $T > 0$, i.e.,

$$t \in [0, T]$$

Two cases:

i) randomly varying duration; and

ii) variant duration, $T_k > 0$, i.e., $t \in [0, T_k]$

CMILC for systems with well-defined relative degree

Continuous-time ILC, for system $\{f(x), b(x), g(x)\}$

$$y^{(r)} = L_f^r g(x) + L_b L_f^{r-1} g(x) u$$

$$u_{k+1}(t) = u_k(t) + \Gamma e_k^{(r)}(t)$$

Discrete-time ILC, for system $\{f(x,u), g(x)\}$

$$y(t+r) = g \circ \overline{f}^{\,r-1}(f(x(t),u(t)))$$

$$u_{k+1}(t) = u_k(t) + \Gamma e_k(t+r)$$

CMILC for systems with well-defined relative degree

Anticipatory ILC

$$
\begin{aligned}
y(t+\Delta) = {} & g(x(t)) + \Delta L_f g(x(t)) + \cdots \\
& + \frac{\Delta^{r-1}}{(r-1)!} L_f^{r-1} g(x(t)) \\
& + \int_t^{t+\Delta} \int_t^{t_1} \cdots \int_t^{t_{r-1}} [L_f^r g(x(t_r)) + L_b L_f^{r-1} g(x(t_r)) u(t_r)] dt_r \cdots dt_1
\end{aligned}
$$

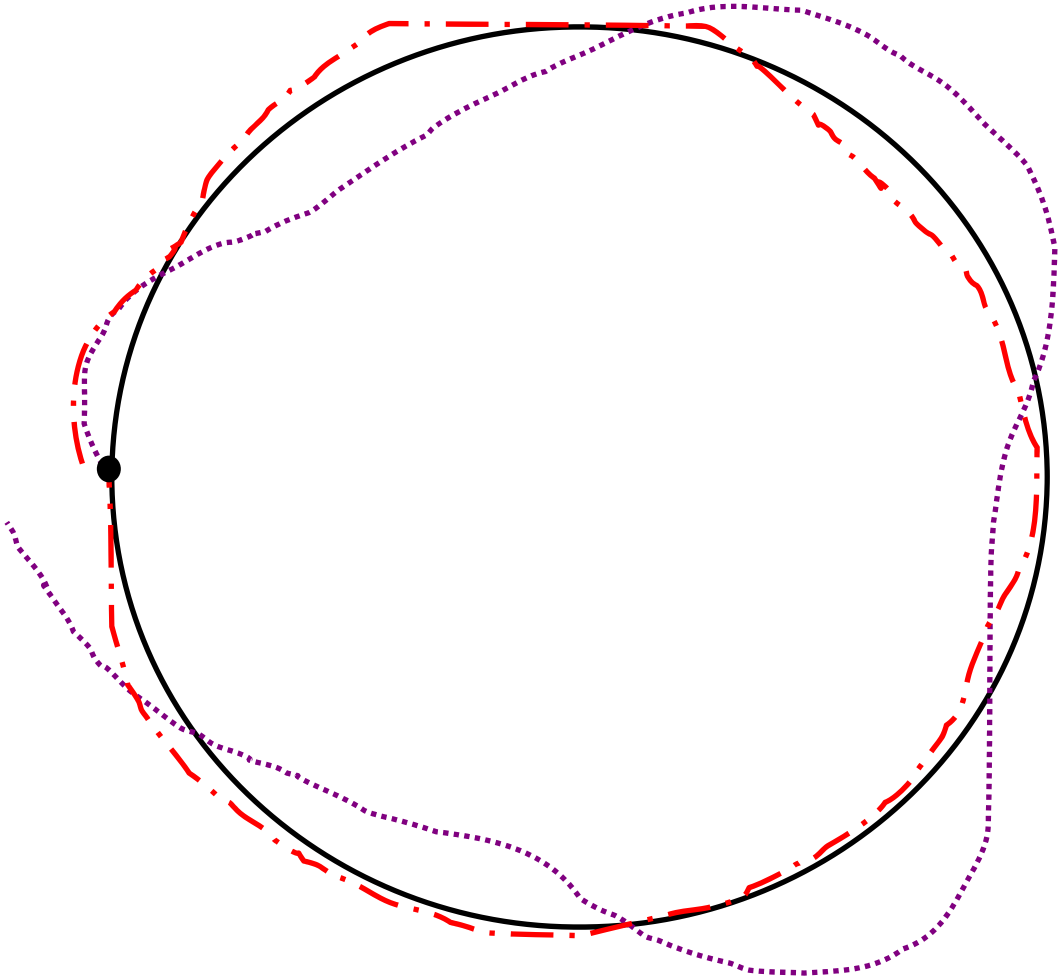
$$
u_{k+1}(t) = u_k(t) + \Gamma e_k(t+\Delta)
$$

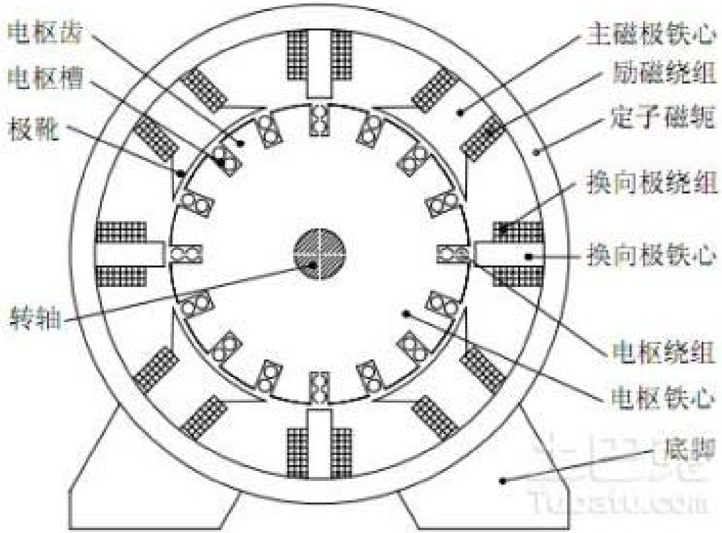
Sampled-data ILC

$$
\begin{aligned}
y(jh+h) = {} & g(x(jh)) + h L_f g(x(jh)) + \cdots \\
& + \frac{h^{r-1}}{(r-1)!} L_f^{r-1} g(x(jh)) \\
& + \int_{jh}^{jh+h} \int_{jh}^{t_1} \cdots \int_{jh}^{t_{r-1}} L_f^r g(x(t_r)) dt_r \cdots dt_1 \\
& + \int_{jh}^{jh+h} \int_{jh}^{t_1} \cdots \int_{jh}^{t_{r-1}} L_b L_f^{r-1} g(x(t_r)) dt_r \cdots dt_1 \, u(jh)
\end{aligned}
$$

$$
u_{k+1}(t) = u_k(t) + \Gamma e_k(jh+h)
$$

五、关于迭代学习方法





Temporal-ILC vs. Spacial-ILC

推广ILC： ILC ⟶ RC

以**ILC**的分析与设计方法解决**RC**的分析与设计问题

Y. Wang, D. Wang, B. Zhang, Y. Ye, From iterative learning control to robust repetitive learning control
Proc. 2005 IEEE/ASME Int. Conf. Advanced Intelligent Mechatronics, Monterey, California, USA, 24-28 July, 2005: 969-974
G. Pipeleers, K. L. Moore, Unified analysis of iterative learning and repetitive controllers in trial domain, IEEE Transactions on Automatic Control : 2014,59(4): 953-965

RLC：结合ILC与RC

RC

IC: $x_{k+1}(0) = x_k(T)$

Learnability: $\theta_k(T) = \theta_k(0) \quad (\theta(t+T) = \theta(t))$

ILC

IC: $x_k(0) = x^0$

Learnability: $\theta_{k+1}(t) = \theta_k(t) \quad (\theta_k(T) \neq \theta_k(0))$

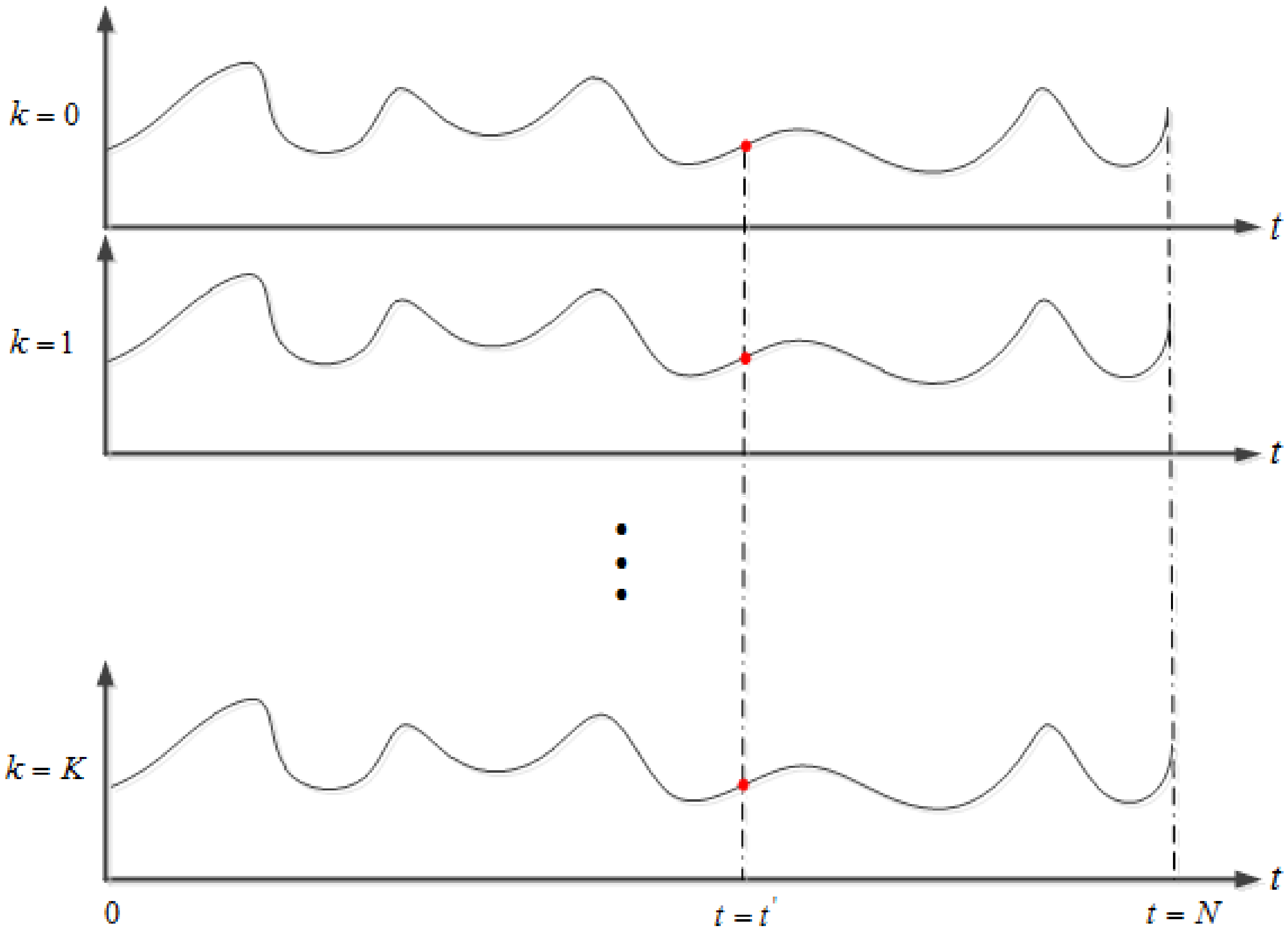
RLC

IC: $x_{k+1}(0) = x_k(T)$

Learnability: $\theta_{k+1}(t) = \theta_k(t)$

ILI：有限区间时变参数估计

重复运行下的有限区间时变系统



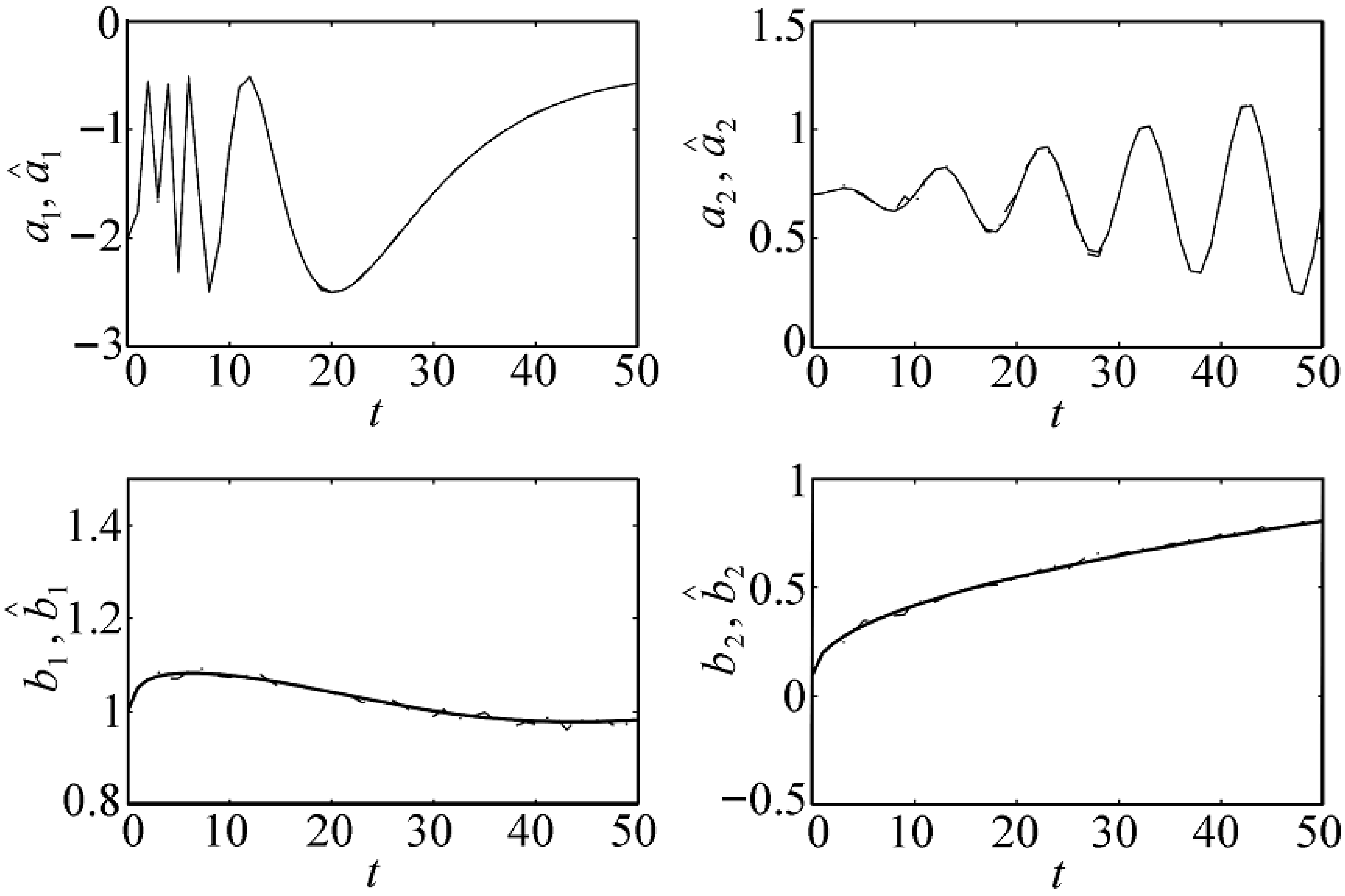
递推算法与学习算法

被辨识系统：沿时间方向行进
递推算法：沿时间方向进行
学习算法：沿迭代方向进行

递推算法实现（定常参数）渐近一致估计
学习算法实现（时变参数）完全估计
它是整个区间上的一致估计

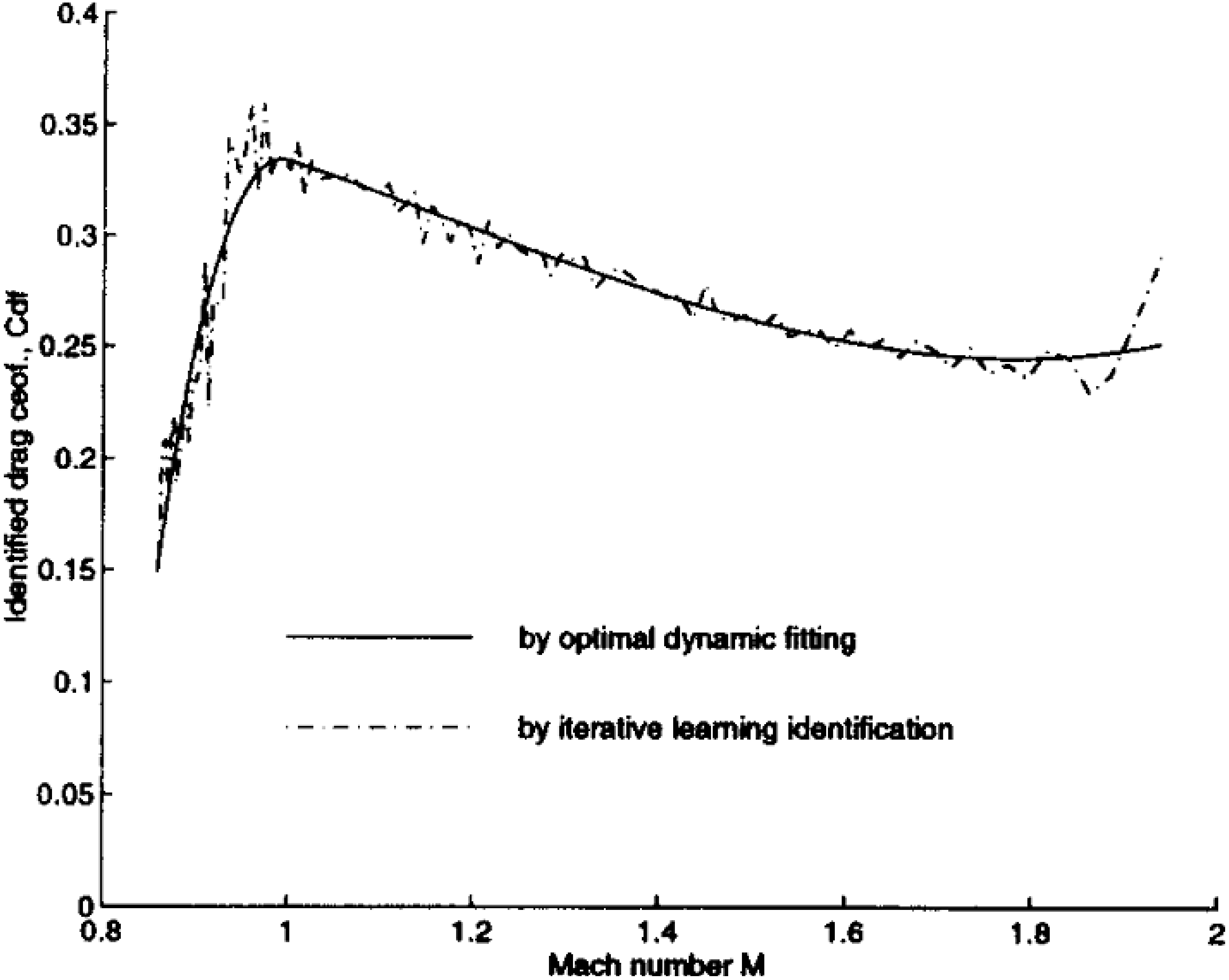
精确辨识与补偿，有益于精确控制

最小二乘学习算法



0
−1
−2
−3
$a_1, \hat{a}_1$
0 10 20 30 40 50
t
1.5
1
0.5
0
$a_2, \hat{a}_2$
0 10 20 30 40 50
t
1.4
1.2
1
0.8
$b_1, \hat{b}_1$
0 10 20 30 40 50
t
1
0.5
0
−0.5
$b_2, \hat{b}_2$
0 10 20 30 40 50
t

Drag coefficient curve identification of projectiles from flight tests



ILO：迭代学习观测器

被估计系统：沿时间方向行进

Lunberger观测器/Kalman滤波器：

沿时间方向进行

ILO：沿迭代方向进行

Lunberger观测器/Kalman滤波器：

实现渐近估计（“点”估计）

ILO：实现完全估计（“区间”估计）

精确估计有益于精确控制

谢谢！