Pinning Control and Controllability of Complex Networks

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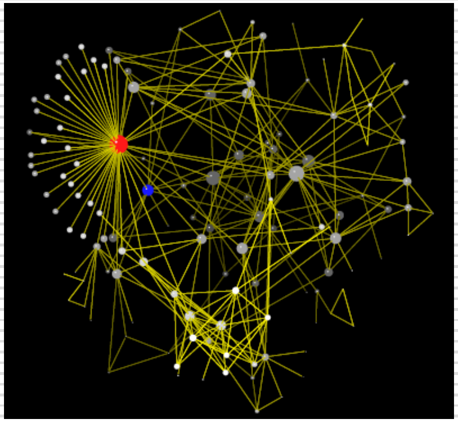
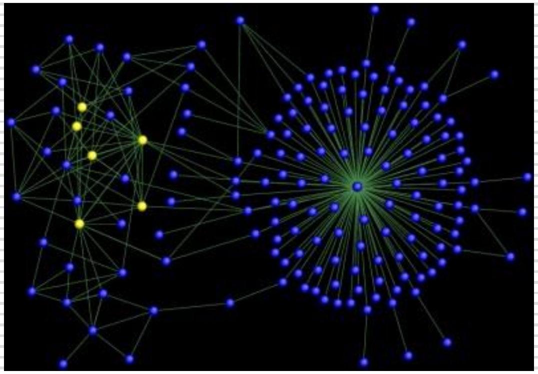


35th Chinese Control Conference, Chengdu, July 27-29, 2016

Dedicated to the Memory of

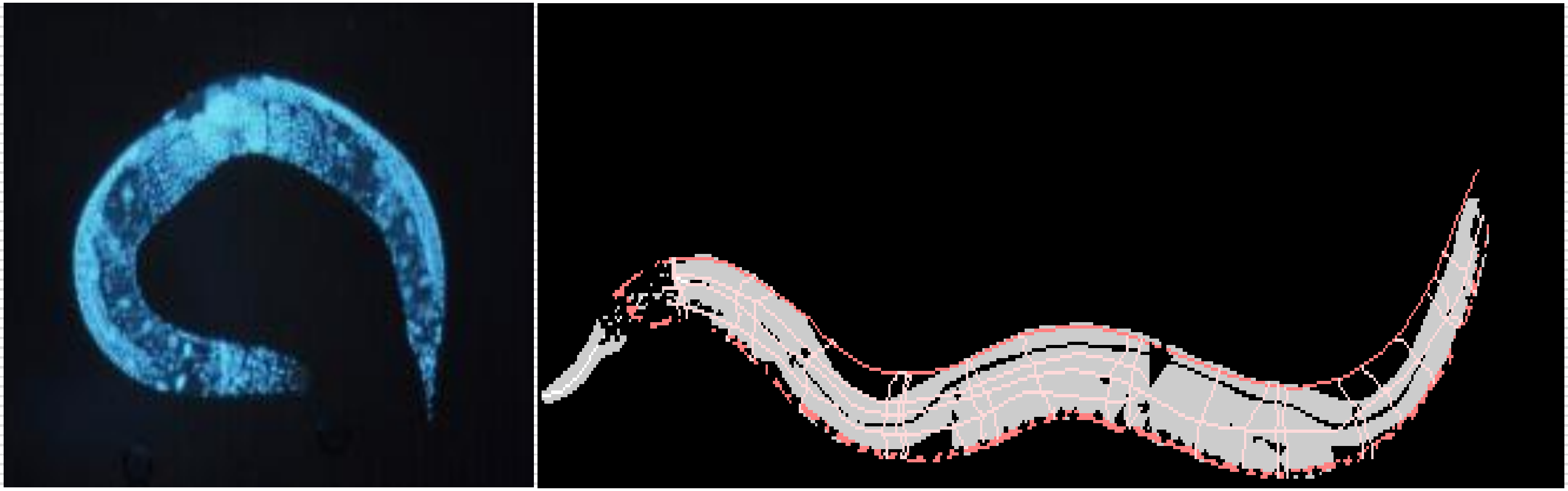
Rudolf E Kalman
(1930-5-19 ---- 2016-7-3)

Motivational Examples



Example:

C. elegans



In its Neural Network:

Neurons: 300~500 Synapses: 2500~7000

Excerpt

"The worm Caenorhabditis elegans has 297 nerve cells. The neurons switch one another on or off, and, making 2345 connections among themselves. They form a network that stretches through the nematode's millimeter-long body."

"**How many neurons would you have to commandeer to control the network with complete precision?**"

The answer is, on avergae: 49

-- Adrian Cho, ***Science***, 13 May **2011**, vol. 332, p 777

Here, control = stimuli

Another Example

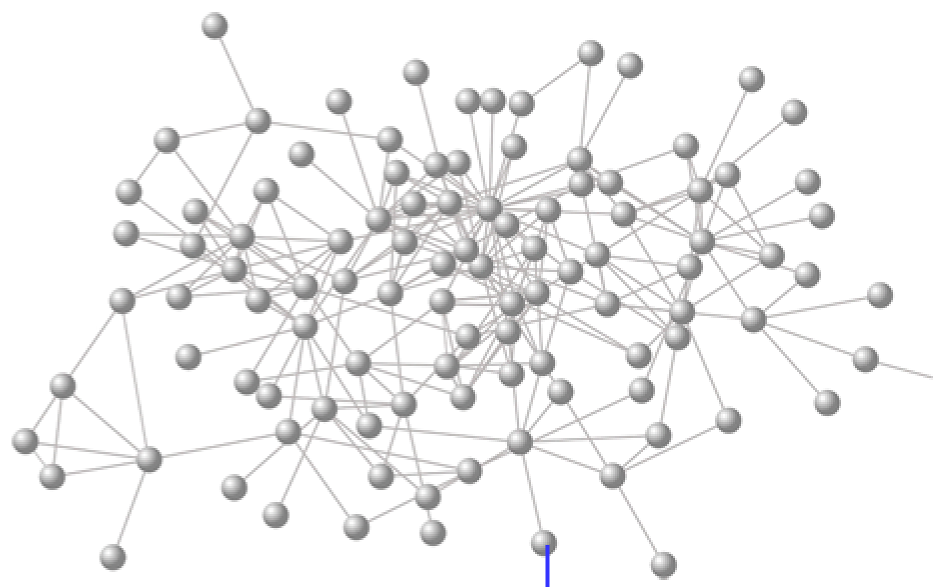
" … very few individuals (approximately **5**%) within honeybee swarms can guide the group to a new nest site."

I.D. Couzin et al., ***Nature***, 3 Feb 2005, vol. 433, p 513

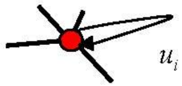
These **5%** of bees can be considered as "**controlling**" or "**controlled**" **agents**

Leader-Followers network

Now … mathematically



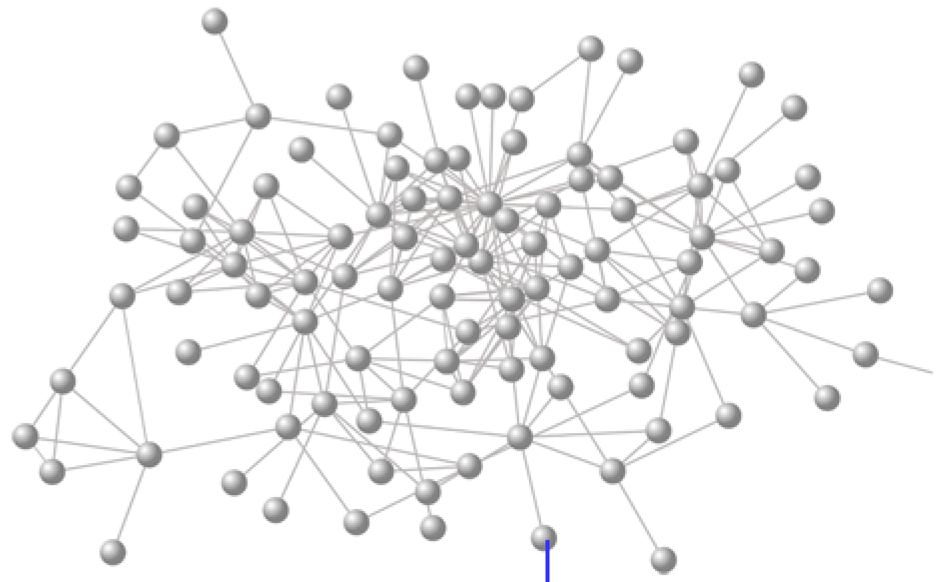
$$\frac{dx_i}{dt} = f(x_i), \qquad x_i \in R^n$$

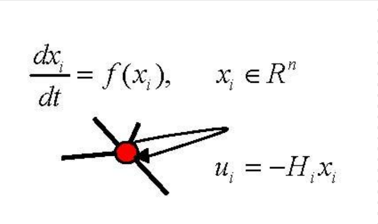


$$u_i = -H_i x_i$$

- Given a network of identical dynamical systems (e.g., ODEs)
- Given a specific control objective (e.g., synchronization)
- Assume: a certain class of controllers (e.g., local linear state-feedback controllers) have been chosen to use

Questions:





Objective: To achieve the control goal with good performance

- How many controllers to use?
- Where to put them? (which nodes to "pin")

--- **"Pinning Control"**

Network Model

Linearly coupled network:

$$\frac{dx_i}{dt} = f(x_i) + c\sum_{j=1}^{N} a_{ij} H x_j \qquad x_i \in R^n \qquad i = 1,2,\ldots,N$$

- a general assumption is that $f(.)$ is Lipschitz
- coupling strength $c > 0$ and H - input matrix
- coupling matrices (undirected):

$$A = [a_{ij}]_{N\times N}$$

A: If node i connects to node j $(j \neq i)$, then $a_{ij} = a_{ji} = 1$; else, $a_{ij} = a_{ji} = 0$;
and $a_{ii} = d_i$ where d_i - degree of node i

Note: For undirected networks, A is symmetrical; for directed networks, it is not so

What kind of controllers? How many? Where?

$$\frac{dx_i}{dt} = f(x_i) + c\sum_{j=1}^{N} a_{ij} H x_j \leftarrow \quad + u_i \qquad i = 1,2,\ldots,N$$

$$(u_i = -\Gamma x_i)$$

$\Longrightarrow$

$$\frac{dx_i}{dt} = f(x_i) + c\sum_{j=1}^{N} a_{ij} H x_j - \delta_i \Gamma x_i \qquad i = 1,2,\ldots,N$$

$$\delta_i = \begin{cases} 1 & if \quad to-control \\ 0 & if \quad not-control \end{cases}$$

Q: How many $\delta_i = 1$? Which i ?

Pinning Control: Our Research Progress

Wang X F, Chen G, Pinning control of scale-free dynamical networks, Physica A, 310: 521-531, 2002.

Li X, Wang X F, Chen G, Pinning a complex dynamical network to its equilibrium, IEEE Trans. Circ. Syst. –I, 51: 2074-2087, 2004.

Sorrentino F, di Bernardo M, Garofalo F, Chen G, Controllability of complex networks via pinning, Phys. Rev. E, 75: 046103, 2007.

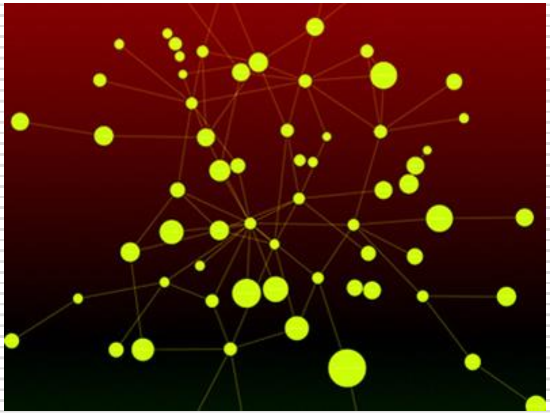
… …

Yu W W, Chen G, Lu J H, Kurths J, Synchronization via pinning control on general complex networks, SIAM J. Contr. Optim., 51: 1395-1416, 2013.

Chen G, Pinning control and synchronization on complex dynamical networks, Int. J. Contr., Auto. Syst., 12: 221-230, 2014.

Xiang L, Chen F, Chen G. Pinning synchronization of networked multi-agent systems: Spectral analysis. Control Theory Tech., 13: 45-54, 2015.

Controllability Theory



In retrospect, …

J.S.I.A.M. Control
Ser. A, Vol. 1, No. 2
Printed in U.S.A., 1963

MATHEMATICAL DESCRIPTION OF LINEAR DYNAMICAL SYSTEMS*

R. E. KALMAN†

Abstract. There are two different ways of describing dynamical systems: (i) by means of state variables and (ii) by input/output relations. The first method may be regarded as an axiomatization of Newton's laws of mechanics and is taken to be the basic definition of a system.

It is then shown (in the linear case) that the input/output relations determine only one part of a system, that which is completely observable and completely controllable. Using the theory of controllability and observability, methods are given for calculating irreducible realizations of a given impulse-response matrix. In particular, an explicit procedure is given to determine the minimal number of state variables necessary to realize a given transfer-function matrix. Difficulties arising from the use of reducible realizations are discussed briefly.

State Controllability

Linear Time-Invariant (LTI) system

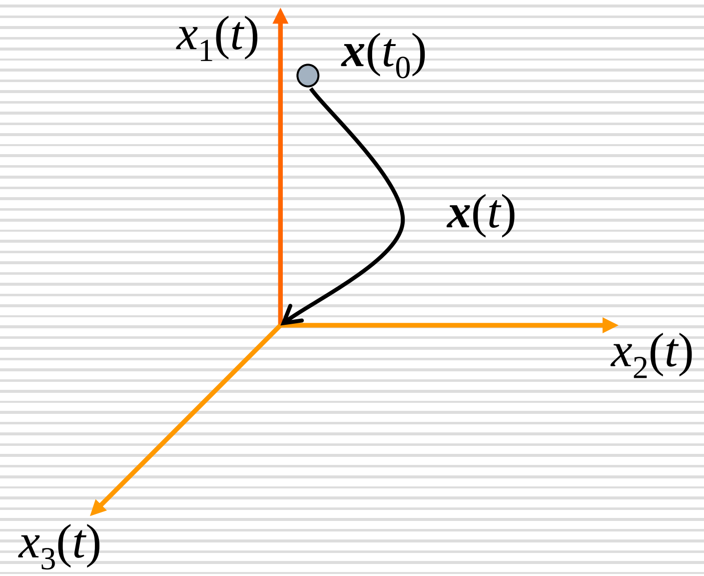
$$\dot{x}(t) = Ax(t) + Bu(t)$$

$x \in R^n$: state vector

$u \in R^p$: control input

$A \in R^{n \times n}$: state matrix

$B \in R^{n \times p}$: control input matrix



[Concept] ***State Controllable***: The system orbit can be driven by an input from any initial state to the origin in finite time

C. K. Chui and G. Chen, **Linear Systems and Optimal Control**, Springer, **1989**

State Controllability Theorems

$$\dot{x}(t) = Ax(t) + Bu(t)$$

(i) Kalman Rank Criterion

The controllability matrix Q has full row rank:

$$Q = [B \;\; AB \;\; \cdots \;\; A^{n-1}B]$$

(ii) Popov-Belevitch-Hautus (PBH) Test

The following relationship holds:

$$v^T A = \lambda v^T, \quad v^T B \neq 0$$

λ : eigenvalue of A

v : nonzero left eigenvactor with λ

What about networks? -- Some earlier attempts

- **Leader-follower multi-agent systems**

  H.G. Tanner, *CDC* , 2004

  …

- **Pinning state-controllability of complex networks**

  F. Sorrentino, M. di Bernardo, F. Garofalo, G. Chen, *Phys. Rev. E*, 2007

  …

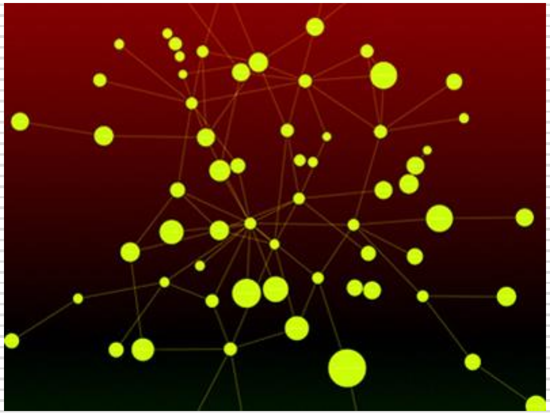
- **Structural controllability of complex networks**

  Y.Y. Liu, J.J. Slotine, A.L. Barabási, *Nature*, 2011

  …

Structural Controllability

A network of single-input/single-output (SISO) node systems, where the node systems can be of higher-dimensional



Structural Controllability

In the controllability matrix Q:

$$Q = [B \;\; AB \;\; \cdots \;\; A^{n-1}B]$$

All 0 are fixed

There is a realization of independent nonzero parameters such that Q has full row-rank

Example 1:

$$Q = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

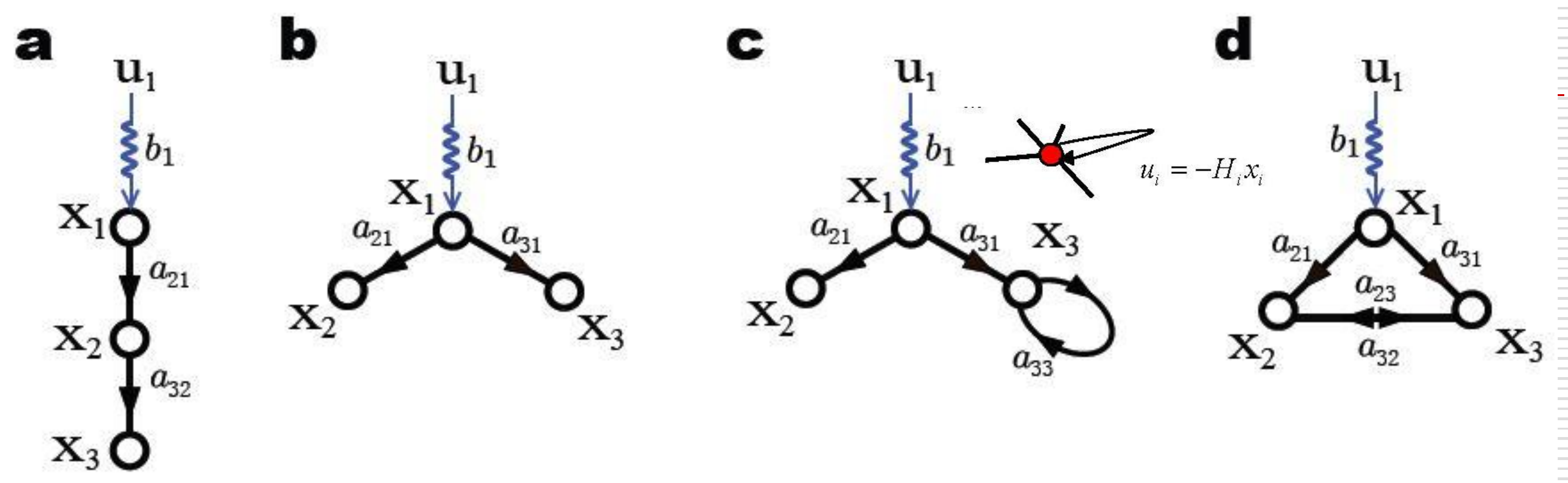
Realization: All admissible parameters

$$a \neq 0, \;\; d \neq 0$$

Example 2: Frobinius Canonical Form

$$Q = \begin{bmatrix} -a_1 & -a_2 & -a_3 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

Examples: Structure matters



$$\mathbf{C} = [\mathbf{B}, \mathbf{A}\cdot\mathbf{B}, \mathbf{A}^2\cdot\mathbf{B}]$$

$$b_1\begin{bmatrix}1 & 0 & 0\\ 0 & a_{21} & 0\\ 0 & 0 & a_{32}a_{21}\end{bmatrix}, \quad b_1\begin{bmatrix}1 & 0 & 0\\ 0 & a_{21} & 0\\ 0 & a_{31} & 0\end{bmatrix}, \quad b_1\begin{bmatrix}1 & 0 & 0\\ 0 & a_{21} & 0\\ 0 & a_{31} & a_{33}a_{31}\end{bmatrix}, \quad b_1\begin{bmatrix}1 & 0 & 0\\ 0 & a_{21} & a_{23}a_{31}\\ 0 & a_{31} & a_{32}a_{21}\end{bmatrix}$$

rank $\mathbf{C} = 3 = n$ — controllable

rank $\mathbf{C} = 2 < n = 3$ — uncontrollable

rank $\mathbf{C} = 3 = n$ — controllable

rank $\mathbf{C} = ?$ — controllable?

Partially controllable

Structurally controllable

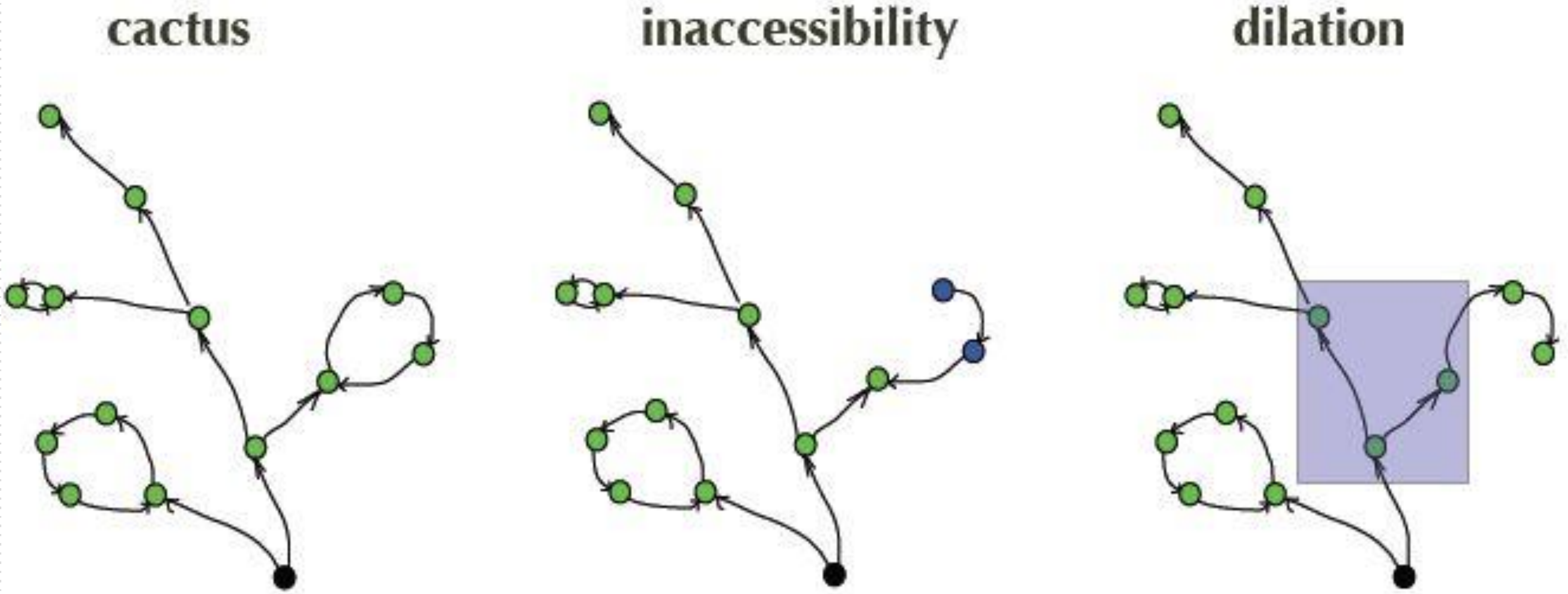
Y.Y. Liu, J.J. Slotine, and A.L. Barabási, ***Nature***, 2011

In retrospect: large-scale systems theory

Structural Controllability (and Structural Observability)

1. C.T. Lin, Structural Controllability, IEEE Trans. Auto Contr., 19(3): 201-208, 1974
2. R.W. Shields, J.B. Pearson, Structural Controllability of Multiinput Linear Systems, IEEE Trans. Auto. Contr., 21(2): 203-212, 1976
3. K. Glover, L.M. Silverman, Characterization of Structural Controllability, IEEE Trans. Auto. Contr., 21(4): 534-537, 1976
4. C.T. Lin, System Structure and Minimal Structure Controllability, IEEE Trans. Auto. Contr., 22(5): 855-862, 1977
5. S. Hosoe, K. Matsumoto, On the Irreducibility Condition in the Structural Controllability Theorem, IEEE Trans. Auto. Contr., 24(6): 963-966, 1979
6. H. Mayeda, On Structural Controllability Theorem, IEEE Trans. Auto. Contr., 26(3): 795-798, 1981
7. A. Linnemann, A Further Simplification in the Proof of the Structural Controllability Theorem, IEEE Trans. Auto. Contr., 31(7): 638-639, 1986
8. J. Willems, Structural Controllability and Observability, Syst. Contr. Lett., 8(1): 5-12, 1986

Building Blocks



Cactus is the minimum structure which contains no inaccessible nodes and no dilations

Structural Controllability Theorem

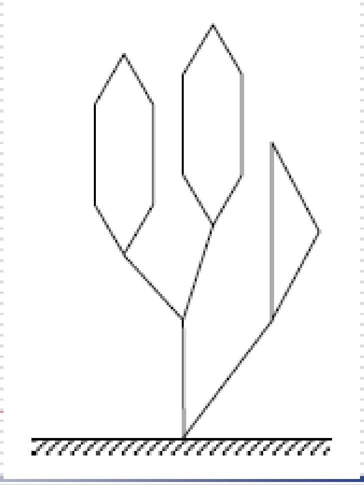
The following two criteria are equivalent:

1. Algebraic:

   The LTI control system (A,B) is structurally controllable

2. Geometric:

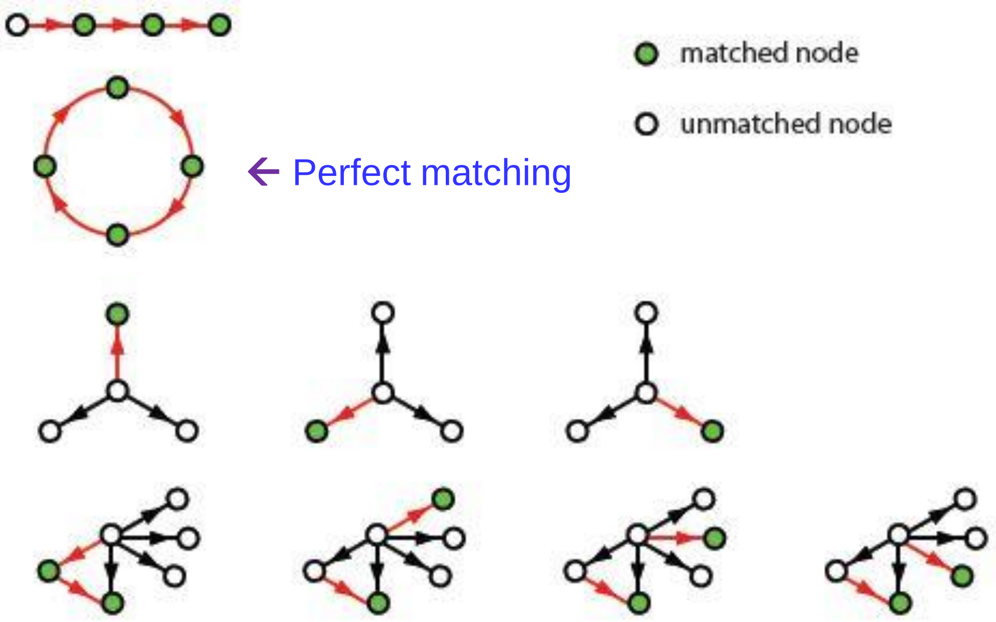
   The digraph $G\ (A,B)$ is spanned by a cactus



C.T. Lin, *IEEE Trans. Auto. Contr.*, 1977

Matching in Directed Networks

- **Matching**: a set of directed edges without common heads and tails
- **Unmatched node**: the tail node of a matching edge



Maximum matching:
Cannot be extended

Perfect matching:
All nodes are matched nodes

← Maximum but not perfect matching

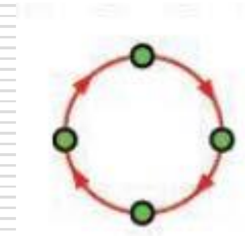
Minimum Inputs Theorem

Q: How many?

A: The minimum number of inputs N_D needed is:

Case 1: If there is a perfect matching, then

$$N_D = 1$$



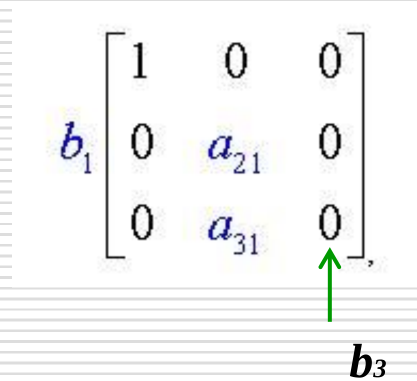
Case 2: If there is no perfect matching, then

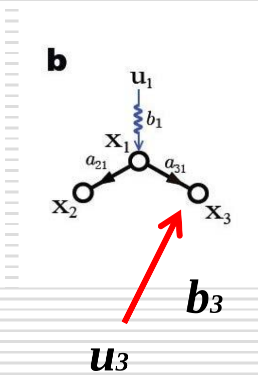
N_D = number of unmatched nodes

Q: Where to put them?

A: Case 1: Anywhere

Case 2: At unmatched nodes

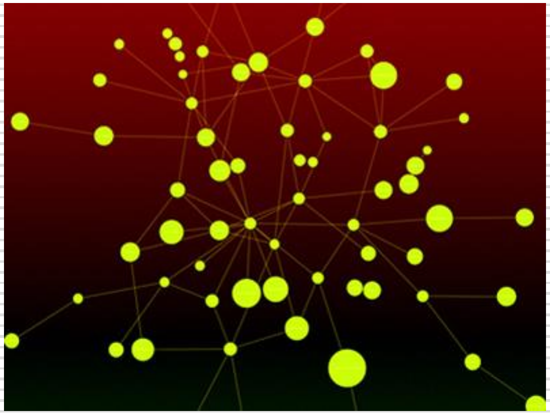




Y. Y. Liu, J. J. Slotine, and A. L. Barabási, ***Nature*, 2011**

State Controllability

A network of multi-input/multi-output (MIMO) node systems, where the node systems are of higher-dimensional



Some Earlier Progress

Consider a network of N identical discrete-time LTI node-systems, with the i th $(i = 1,2,\ldots,N)$ sub-system

$$\begin{bmatrix} x(t+1,i) \\ z(t,i) \\ y(t,i) \end{bmatrix} = \begin{bmatrix} A_{TT}(i) & A_{TS}(i) & B_T(i) & 0 \\ A_{ST}(i) & A_{SS}(i) & B_S(i) & 0 \\ C_T(i) & C_S(i) & D_d(i) & D_w(i) \end{bmatrix} \begin{bmatrix} x(t,i) \\ v(t,i) \\ d(t,i) \\ w(t,i) \end{bmatrix}$$

where $x(t)$ – state; $y(t)$ – observation; $d(t)$ – disturbance; $w(t)$ – noise;

$$A_{*,\#} = col\left\{A_{*,\#}(i)\,\big|_{i=1}^{N}\right\} \quad B_{*} = diag\left\{B_{*}(i)\,\big|_{i=1}^{N}\right\} \quad C_{*} = diag\left\{C_{*}(i)\,\big|_{i=1}^{N}\right\}$$

in which $*,\# \; = T$ or S (**Note:** All nodes are subject to control input)

T. Zhou, *Automatica* (2015)

continued

Result: Assume that all the transfer function matrices $\overline{G}_i^{[1]}|_{i=1}^{N}$ of the network have full column normal rank. Then, the network is controllable **if and only if** for every $k \in \{1,2,\ldots,\overline{m}\}$, where $\overline{m}$ is the number of distinctive transmission zeros of $G^{[1]}(\lambda)$, and for every $\overline{y}^{[k]} \in \mathrm{Y}^{[k]}$, one has $\Phi^T \overline{G}^{[2]}(\overline{\lambda}_0^{[k]})\overline{y}^{[k]} \neq \overline{y}^{[k]}$.

Here, Φ is the transfer matrix in $z(t) = \Phi v(t)$ and, for $i = 1,2,\ldots,N$,

$$G_i^{[1]}(\lambda) = C_S(i) + C_T(i)[\lambda I - A_{TT}(i)]^{-1} A_{TS}(i), \quad G_i^{[2]}(\lambda) = A_{SS}(i) + A_{ST}(i)[\lambda I - A_{TT}(i)]^{-1} A_{TS}(i)$$

$$\bar{\mathbf{Y}}^{[k]} = \left\{ y \;\middle|\; \begin{array}{l} y = \mathbf{col}\left\{ \left(0_{m_{z(\bar{k}(i)+1)}}, \ldots, 0_{m_{z(\bar{k}(i+1)-1)}}, \right.\right. \\ \qquad \left.\bar{y}_{i+1,0}^{[k]}\right)\Big|_{i=0}^{\bar{s}^{[k]}-1}, 0_{m_{z(\bar{k}(\bar{s}^{[k]})+1)}}, \ldots, 0_{m_{zN}} \Big\} \\ \bar{y}_{i,0}^{[k]} \in \bar{\mathbf{Y}}_i^{[k]},\ i = 1, 2, \ldots, \bar{s}^{[k]};\ y \neq 0 \end{array} \right\}$$

T. Zhou, *Automatica* (2015)

A Network of Multi-Input/Multi-Output LTI Systems

Node system $\dot{x}_i = Ax_i + Bu_i \qquad y_i = Cx_i \qquad x_i \in R^n \quad y_i \in R^m \quad u_i \in R^p$

Networked system $\dot{x}_i = Ax_i + \sum_{j=1}^{N} \beta_{ij} H y_j, \quad y_i = Cx_i, \qquad i = 1, 2, \cdots, N$

Networked system with external control $\dot{x}_i = Ax_i + \sum_{j=1}^{N} \beta_{ij} HCx_j + \delta_i Bu_i, \quad i = 1, 2, \cdots, N$

$\delta_i = 1$: **with** external control $\qquad \delta_i = 0$: **without** external control

Some notations

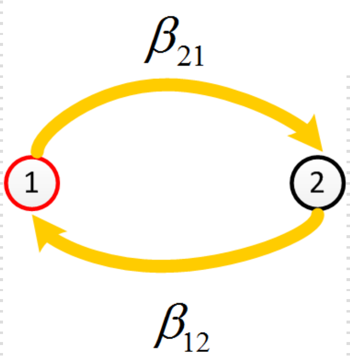
Node system (A,B,C) — Network structure $L = [\beta_{ij}] \in R^{N \times N}$

Coupling matrix H — External control inputs $\Delta = diag(\delta_1, \cdots, \delta_N)$

L. Wang, X.F. Wang, G.R. Chen and W.K.S. Tang, *Automatica* (2016)

Some counter-intuitive examples

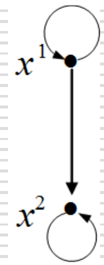
Network structure



$$L=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

structurally controllable

Node system



$$H=\begin{bmatrix}0\\ 1\end{bmatrix}$$

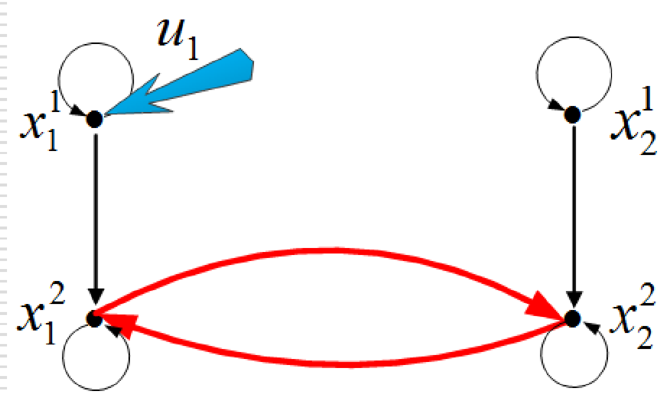
$$A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\quad B=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$C=\begin{bmatrix}0 & 1\end{bmatrix}$$

(A,B) is controllable

(A,C) is observable

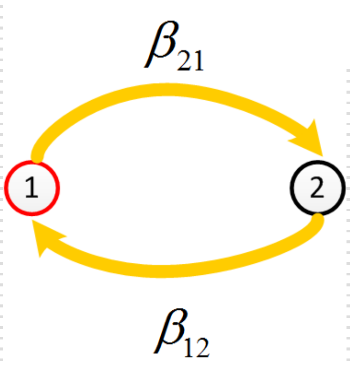
Networked MIMO system



state uncontrollable

Some counter-intuitive examples

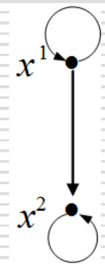
Network structure



$$L = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

structurally controllable

Node system



$$H = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

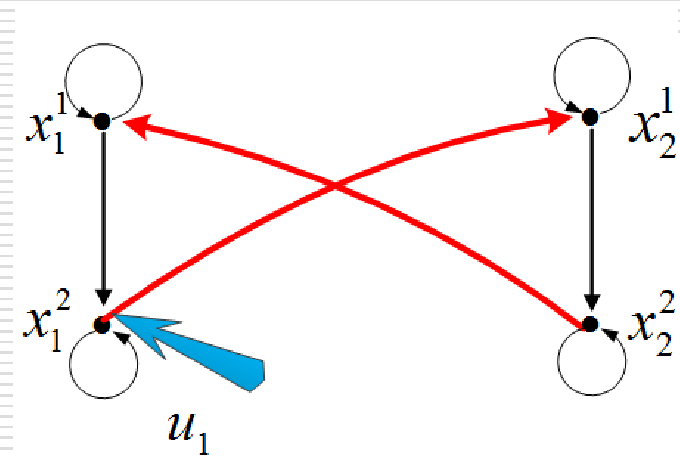
$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

(A,B) is uncontrollable

(A,C) is observable

Networked MIMO system



state controllable

coupling matrix H is important

Hou B Y, Li X, Chen G (2016)

A Network of Multi-Input/Multi-Output LTI Systems

A necessary and sufficient condition

$$\dot{x}_i = Ax_i + \sum_{j=1}^{N} \beta_{ij} HCx_j + \sum_{k=1}^{s} \delta_{ik} Bu_k ,$$

$$y_l = \sum_{j=1}^{N} m_{lj} Dx_j$$

$x_i \in R^n, \quad i = 1, \cdots N$

$u_k \in R^p, \quad k = 1, \cdots s$

$y_l \in R^q, \quad l = 1, \cdots r$

$$L = [\beta_{ij}] \in R^{N \times N} \qquad \Delta = [\delta_{ij}] \in R^{N \times s}$$

State Controllable ⟷ **If and only if**

Matrix equations

$$\Delta^T XB = 0, L^T XHC = X(\lambda I - A)$$

has a unique solution $X = 0$

L. Wang, X.F. Wang, G.R. Chen and W.K.S. Tang, *Automatica* (2016)

General Topology with SISO Nodes

$$\dot{x}_i = Ax_i + \sum_{j=1}^{N} \beta_{ij} HCx_j + \delta_i Bu_i, \quad i = 1, 2, \cdots, N \qquad x_i \in R^n \quad y_i \in R^m \quad u_i \in R^p$$

$$L = [\beta_{ij}] \in R^{N \times N} \quad \Delta = diag(\delta_1, \cdots, \delta_N)$$

A network with SISO nodes is controllable if and only if

(A,H) is controllable,

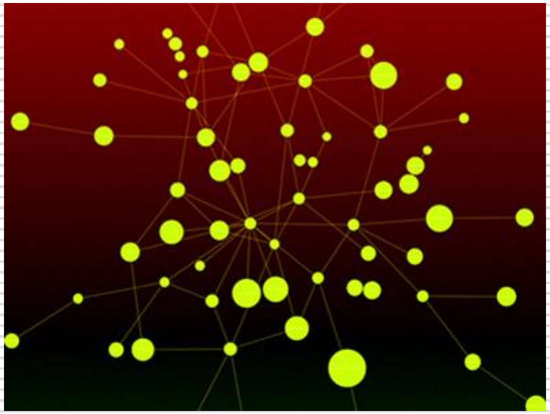
(A,C) is observable,

for any $s \in \sigma(A)$ and $\alpha \in \Gamma(s)$, $\alpha L \neq 0$ if $\alpha \neq 0$,

for any $s \notin \sigma(A)$, $rank(I - L\gamma, \Delta\eta) = N$, with $\gamma = C(sI - A)^{-1} H$, $\eta = C(sI - A)^{-1} B$.

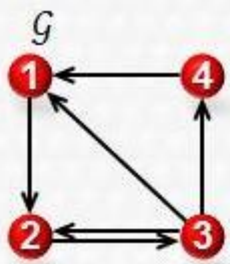
L. Wang, X.F. Wang and G. Chen (2016)

Some most recent progress



Temporally Switching Networks

Edge (i, j, δ_{ij}) from i to j on duration δ_{ij}



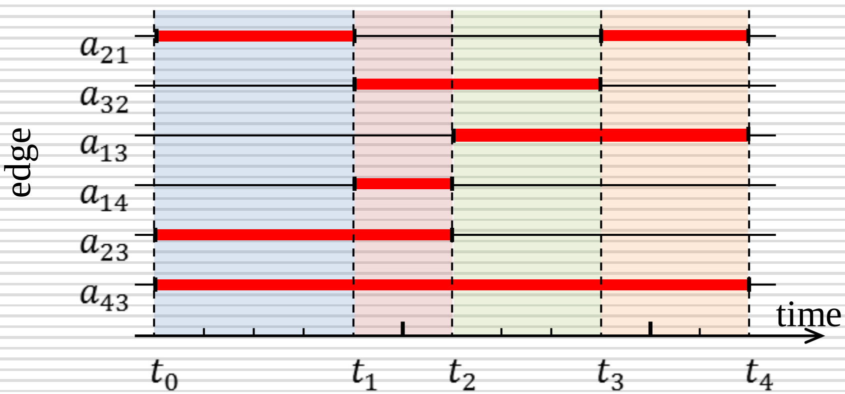
$\delta_{12} = [t_0, t_1) \cup [t_3, t_4]$
$\delta_{23} = [t_1, t_3)$
$\delta_{31} = [t_2, t_4]$
$\delta_{41} = [t_1, t_2)$
$\delta_{32} = [t_0, t_2)$
$\delta_{34} = [t_0, t_4]$

Adjacency matrix:

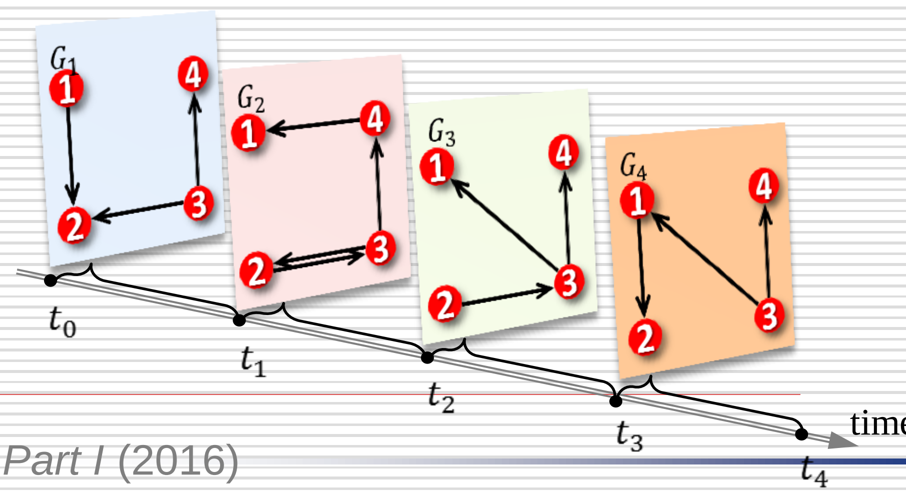
$$[A_k]_{ji} = a_{ji}(k) \begin{cases} \neq 0, \text{ edge}(i, j, [t_{k-1}, t_k)) \neq \emptyset \\ = 0, \text{ otherwise} \end{cases}$$

a_{ji} are constants, but appear and disappear in a temporal manner

Division of time durations



Network topology is temporally switching



B. Y. Hou, X. Li, G. Chen, *IEEE Trans. Circ. Syst. Part I* (2016)

State Controllability of Temporally Switching Systems

Temporally Switching Systems

$\dot{x}(t) = A(t)x(t) + Bu(t), \quad x(t_0) = x_0$

$x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, $B \in \mathbb{R}^{n\times r}$

$A(t) \in \mathbb{R}^{n\times n}$ is piecewise constant

$(A(t), B)$ can be described by matrix pair (A_i, B) when t belongs to $[t_{i-1}, t_i)$

State Controllability:

any given initial state $x(t_0) = x_0$ → input signal $u(\cdot)$ defined on $[t_0, t_1]$ → final state $x(t_m) = 0$

Necessary and Sufficient Condition

State Controllable *if and only if* ⟺ **Controllability matrix**

$$\mathcal{C} = \left(e^{A_m(t-t_{m-1})} \cdots e^{A_2(t_2-t_1)} C_1, \cdots, e^{A_m(t-t_{m-1})} C_{m-1}, C_m\right)$$

has full rank, where $C_i = \left(A_i^{n-1}B, \cdots, A_iB, B\right)$

Structural Controllability of Temporally Switching Networks

Temporally Switching Systems

$$\dot{x}(t) = A(t)x(t) + Bu(t), \quad x(t_0) = x_0$$

$x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, $B \in \mathbb{R}^{n\times r}$

$A(t) \in \mathbb{R}^{n\times n}$ is piecewise constant

$(A(t), B)$ can be described by matrix pair (A_i, B) when t belongs to $[t_{i-1}, t_i)$

Structural Controllability: There exist a set of parameter values such that

any given initial state $x(t_0) = x_0$ → input signal $u(\cdot)$ defined on $[t_0, t_1]$ → final state $x(t_m) = 0$

Necessary and Sufficient Condition

Structural Controllability if and only if $\Longleftrightarrow$ **Controllability matrix**

$$\mathcal{C} = \left(e^{A_m(t-t_{m-1})} \cdots e^{A_2(t_2-t_1)} C_1, \cdots, e^{A_m(t-t_{m-1})} C_{m-1}, C_m\right)$$

has full rank for some set of parameter values

Research Outlook

General networks of linear time-varying (LTV) node-systems

General networks of non-identical node-systems

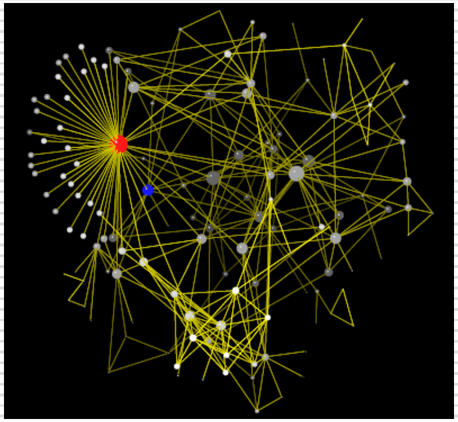
General temporal networks of LTI or LTV node-systems

Some special types of networks of nonlinear node-systems

……

There are more, of course

Thanks



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