普遍法则与架构

道耀

Ca#1tech

普遍法则与架构

Universal laws and architectures:

Theory and lessons from

brains, hearts, cells, grids, nets, bugs,
fluids, bodies, planes, docs, fire, fashion,
earthquakes, music, buildings, cities, art, running,
cycling, throwing, Synesthesia, spacecraft, statistical mechanics

and zombies

John Doyle 道耀
Jean-Lou Chameau Professor
Control and Dynamical Systems, EE, & BioE
Ca#1tech
https://www.cds.caltech.edu/~doyle

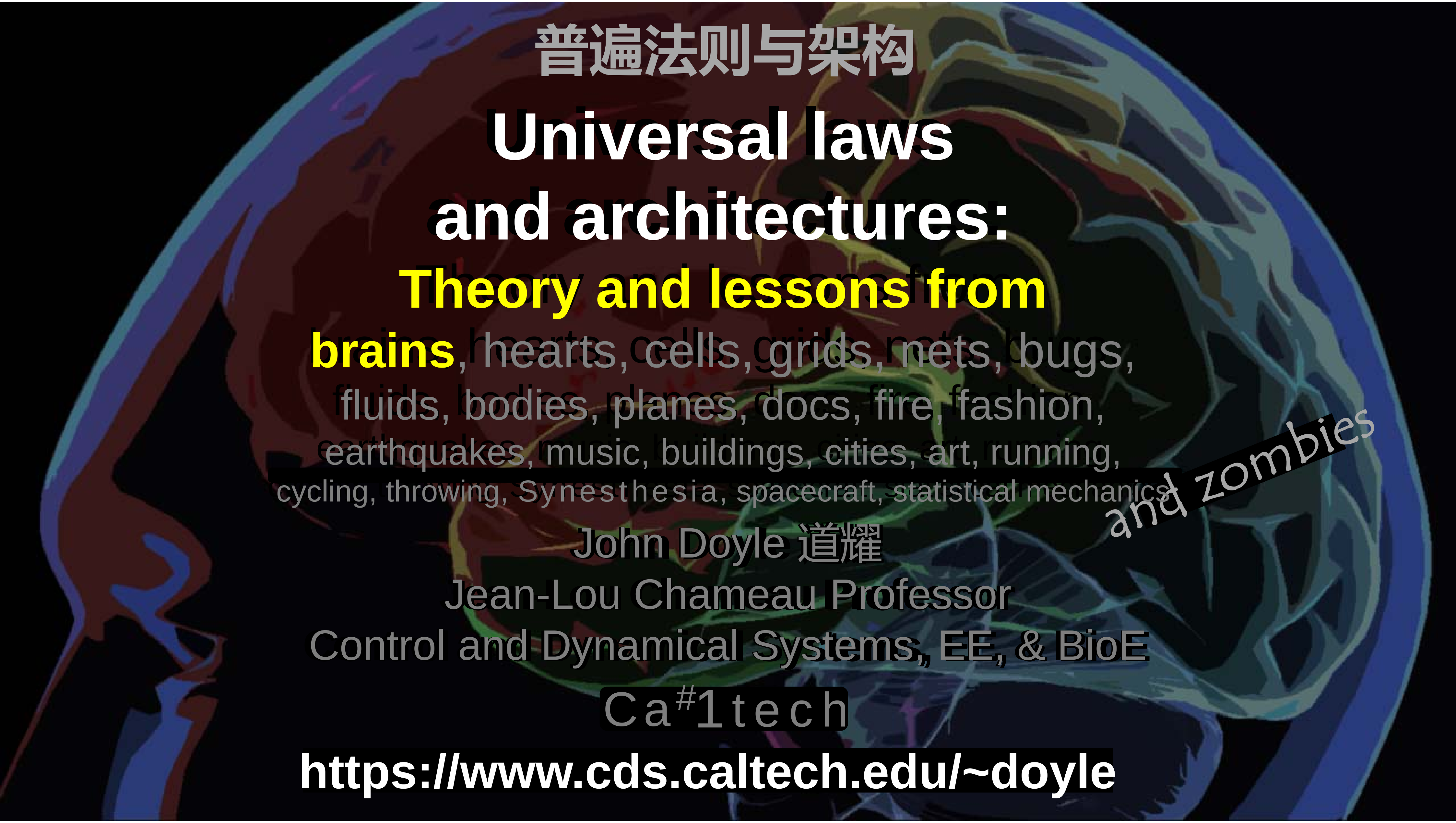
Universal laws
and architectures:

The videos

The following preview is
approved for all audiences. ~~and zombies~~

https://rigorandrelevance.wordpress.com/author/doyleatcaltech/

https://www.cds.caltech.edu/~doyle



普遍法则与架构

Universal laws and architectures:

Theory and lessons from

brains, hearts, cells, grids, nets, bugs, fluids, bodies, planes, docs, fire, fashion, earthquakes, music, buildings, cities, art, running, cycling, throwing, Synesthesia, spacecraft, statistical mechanics

and zombies

John Doyle 道耀

Jean-Lou Chameau Professor

Control and Dynamical Systems, EE, & BioE

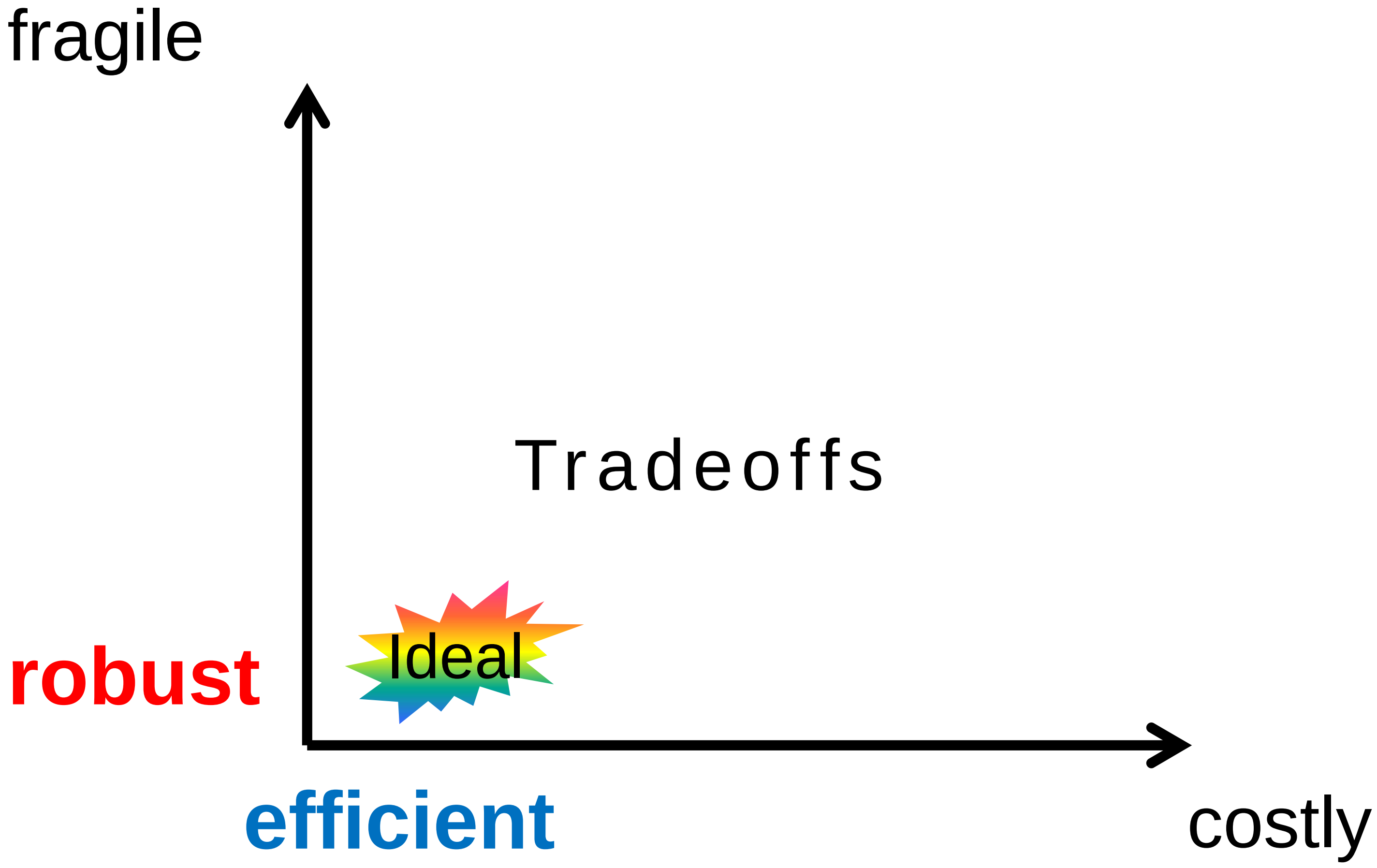
Ca#1tech

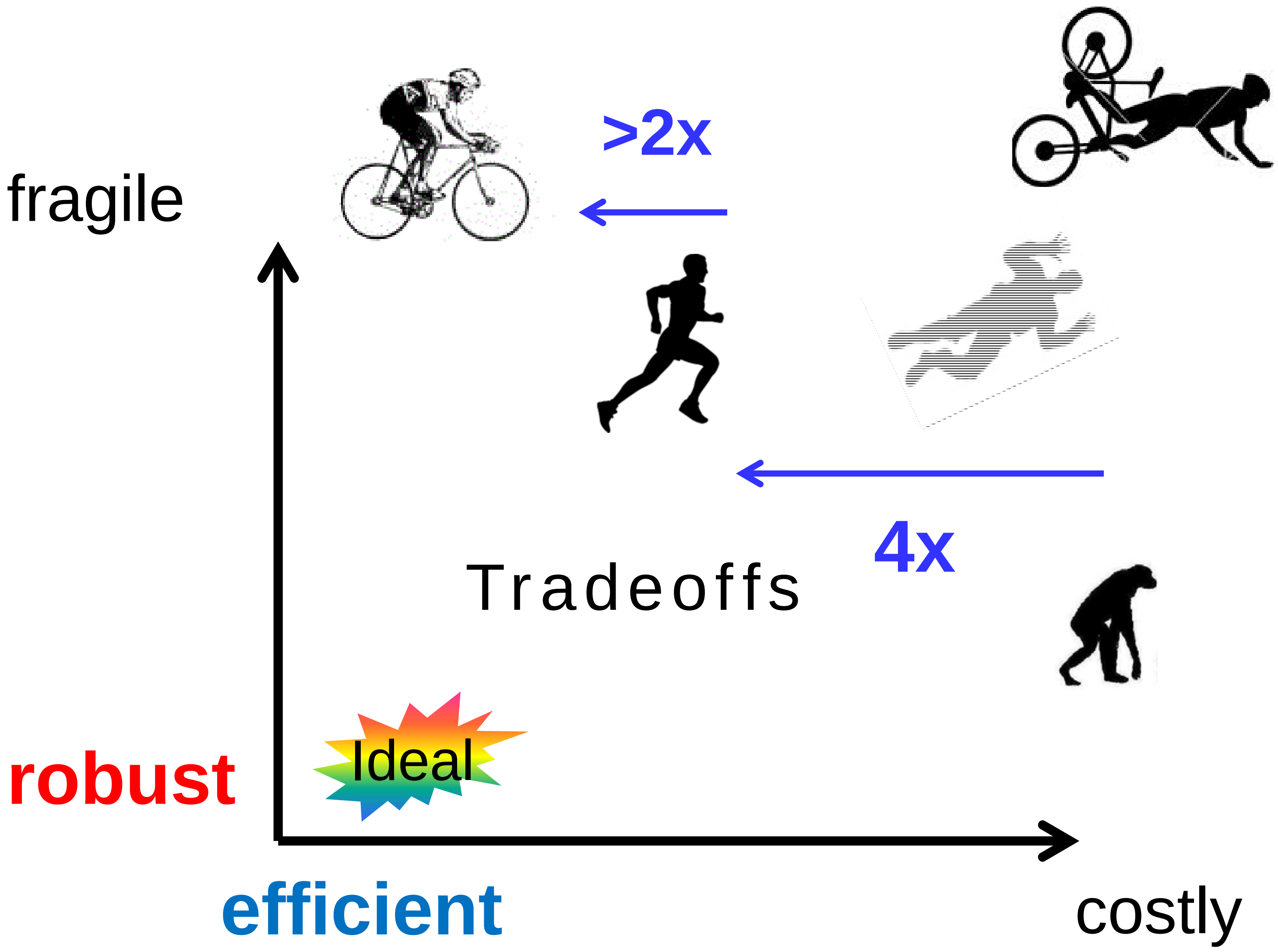
https://www.cds.caltech.edu/~doyle

High
vision

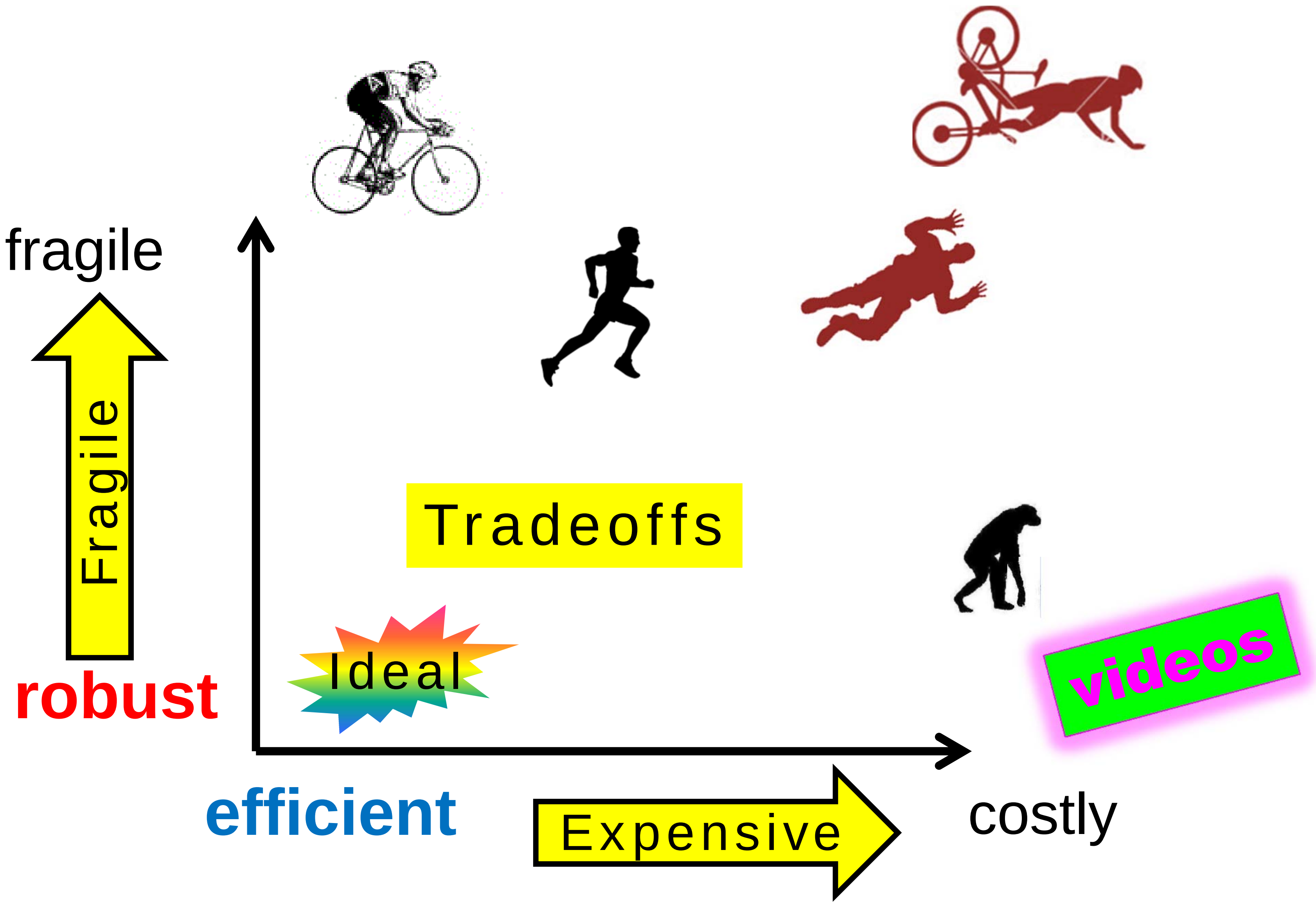
How can humans do this?

Lower
reflex


fragile
Tradeoffs
robust
Ideal
efficient
costly



fragile
>2x
4x
Tradeoffs
robust
Ideal
efficient
costly


fragile
Fragile
robust
Tradeoffs
Ideal
videos
efficient
Expensive
costly

fragile

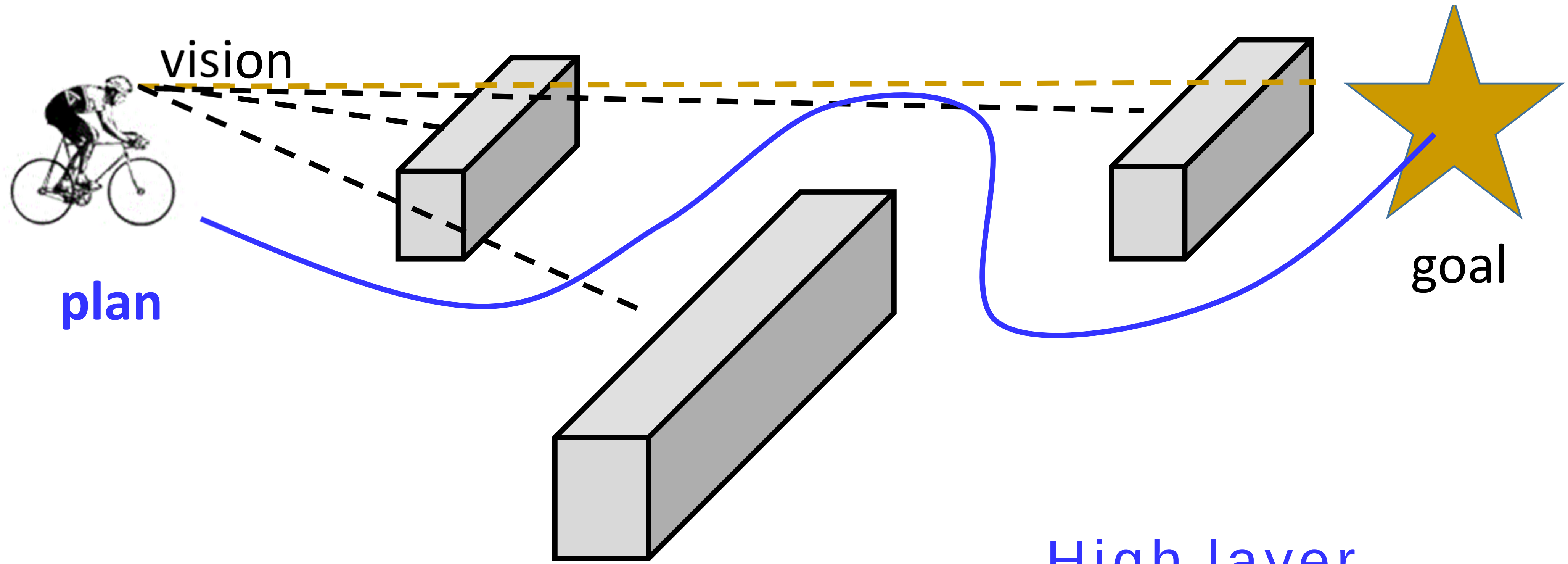
Fragile

Robustness Tradeoffs

robust

~~**efficient**~~

fixed resources



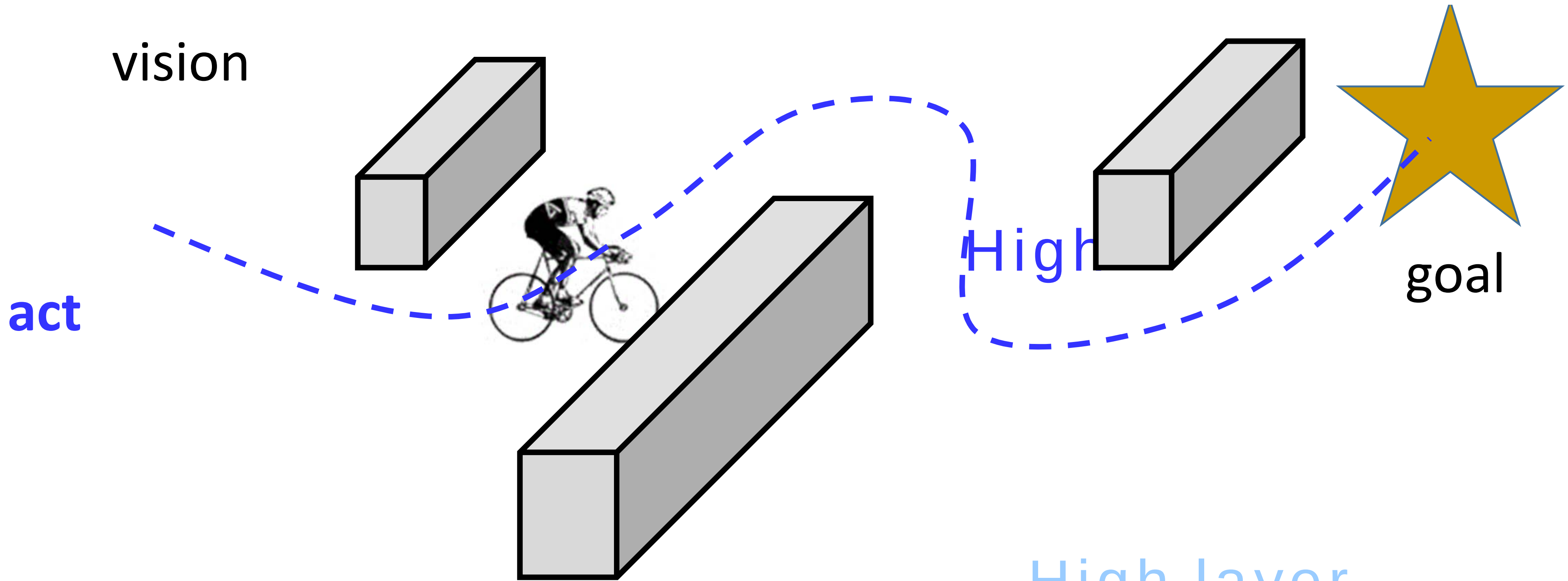
High layer

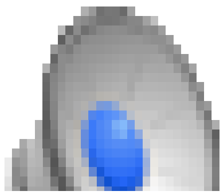
advanced warning

planning

large disturbance

Also tool use





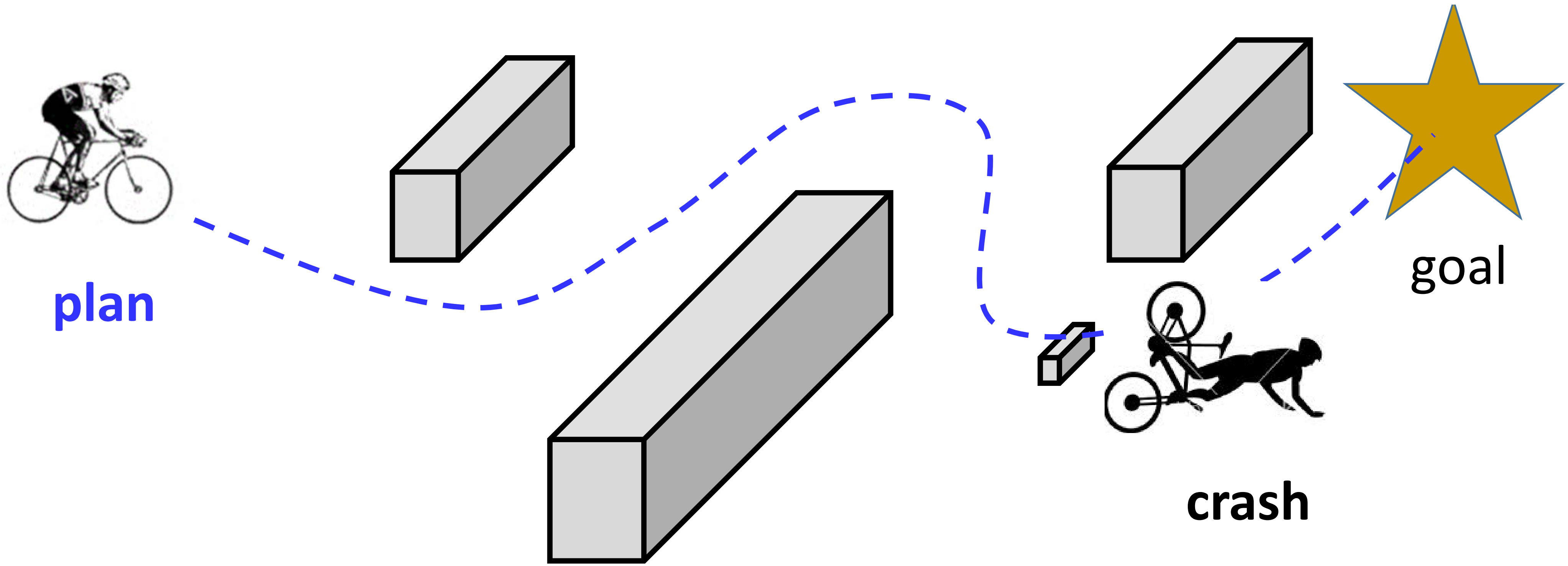
High layer

advanced warning

planning

large disturbance

small error



Lower

~~vision~~

~~plan~~

avoiding a crash

Lower layer

delayed

reflexes

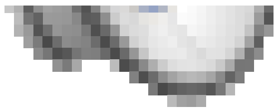
small disturbance

large error

vision

plan

avoiding a crash



Lower layer

delayed

reflexes

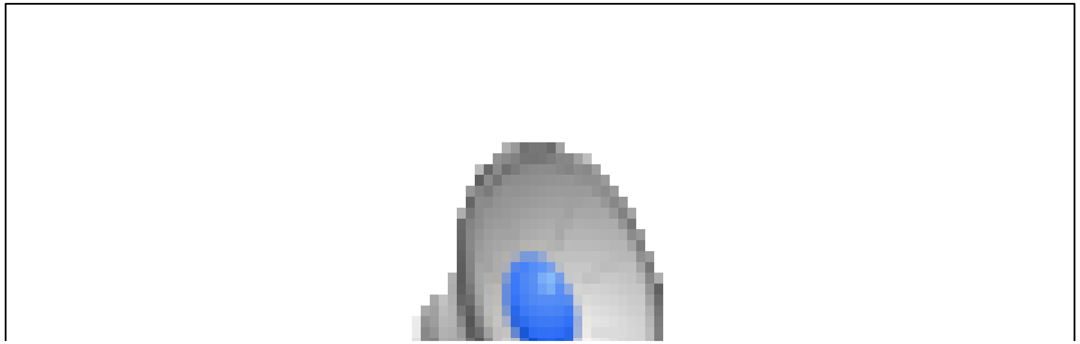
small disturbance

large error

Speed vs Accuracy

robust

High
vision



advanced
planning
large disturbance
small error
need accuracy

Speed vs Accuracy

delayed
reflexes
small disturbance
large error
need speed

Lower
reflex

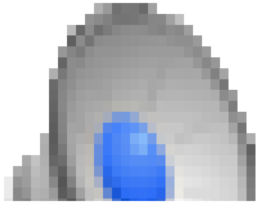
fragile

Speed vs Accuracy

robust

High
vision

advanced
planning
large disturbance
small error
need accuracy



Extreme diversity

delayed
reflexes
small disturbance
large error
need speed

Lower
reflex

fragile

Speed vs Accuracy

robust

High
vision

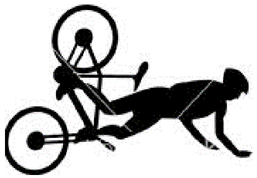
advanced
planning
large disturbance
small error
need accuracy

Extreme *opposites*

delayed
reflexes
small disturbance
large error
need speed

Lower
reflex

fragile



fragile

$$\log(gain) = \log\left(\frac{error}{dist}\right)$$

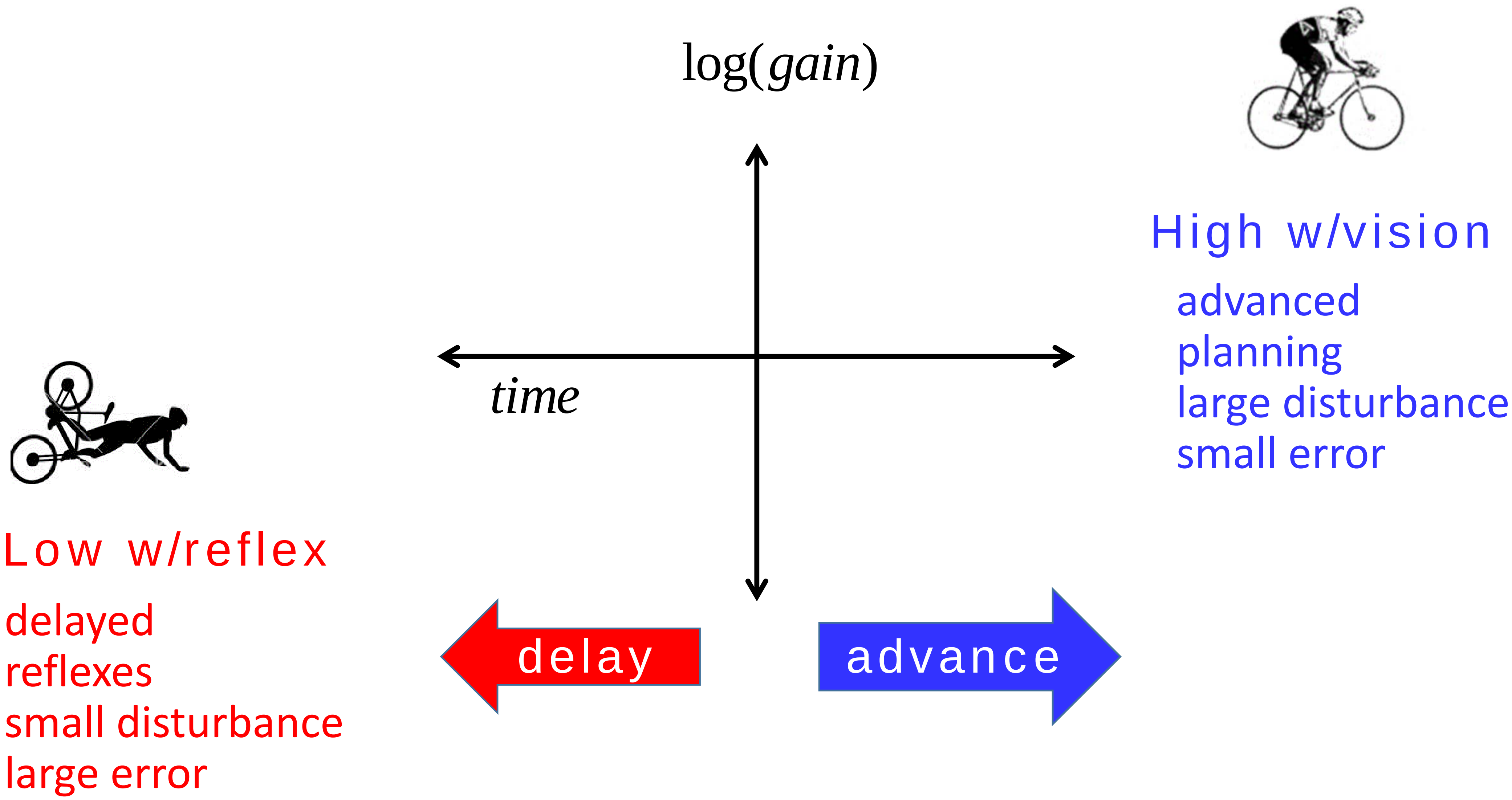
robust

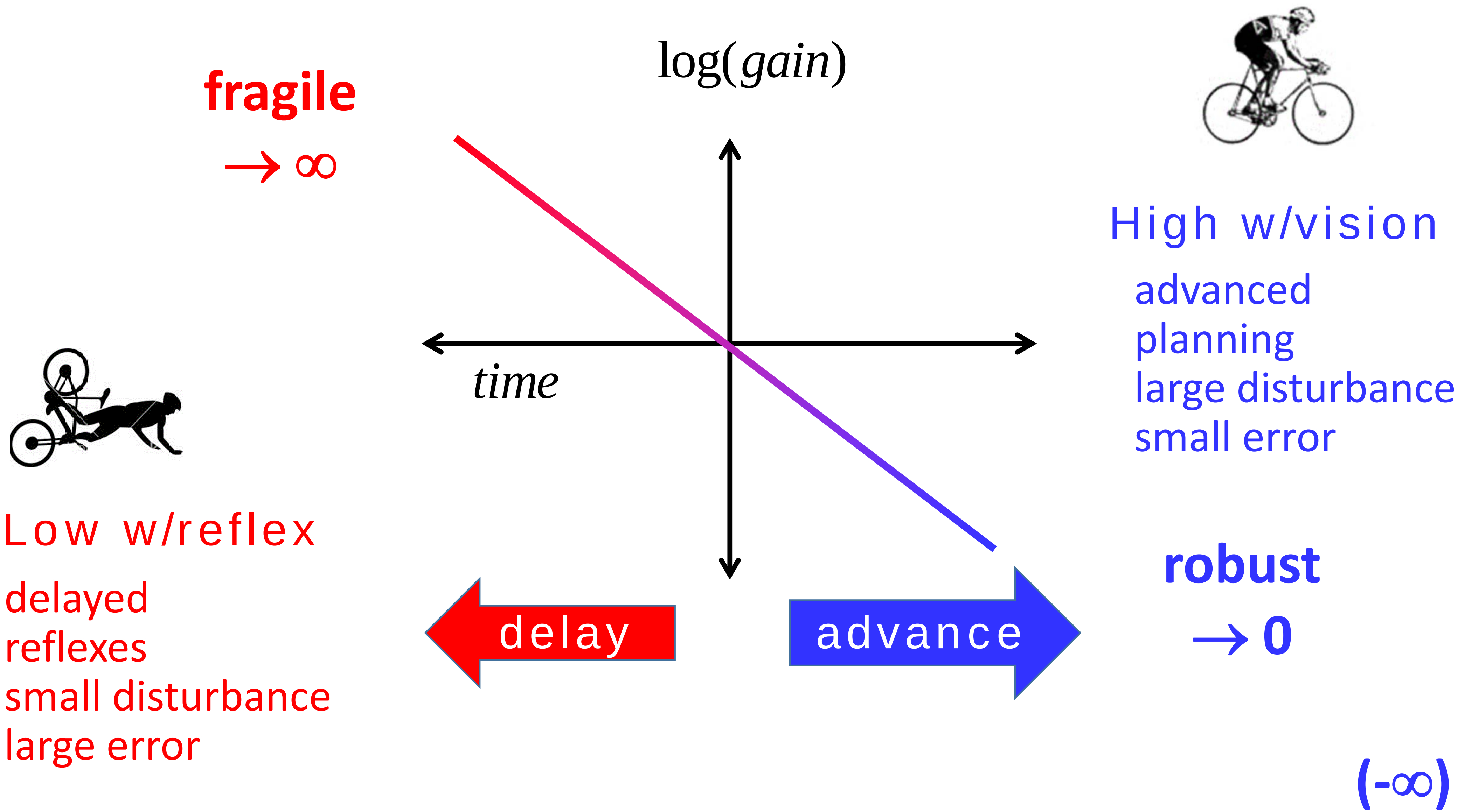
Low w/reflex

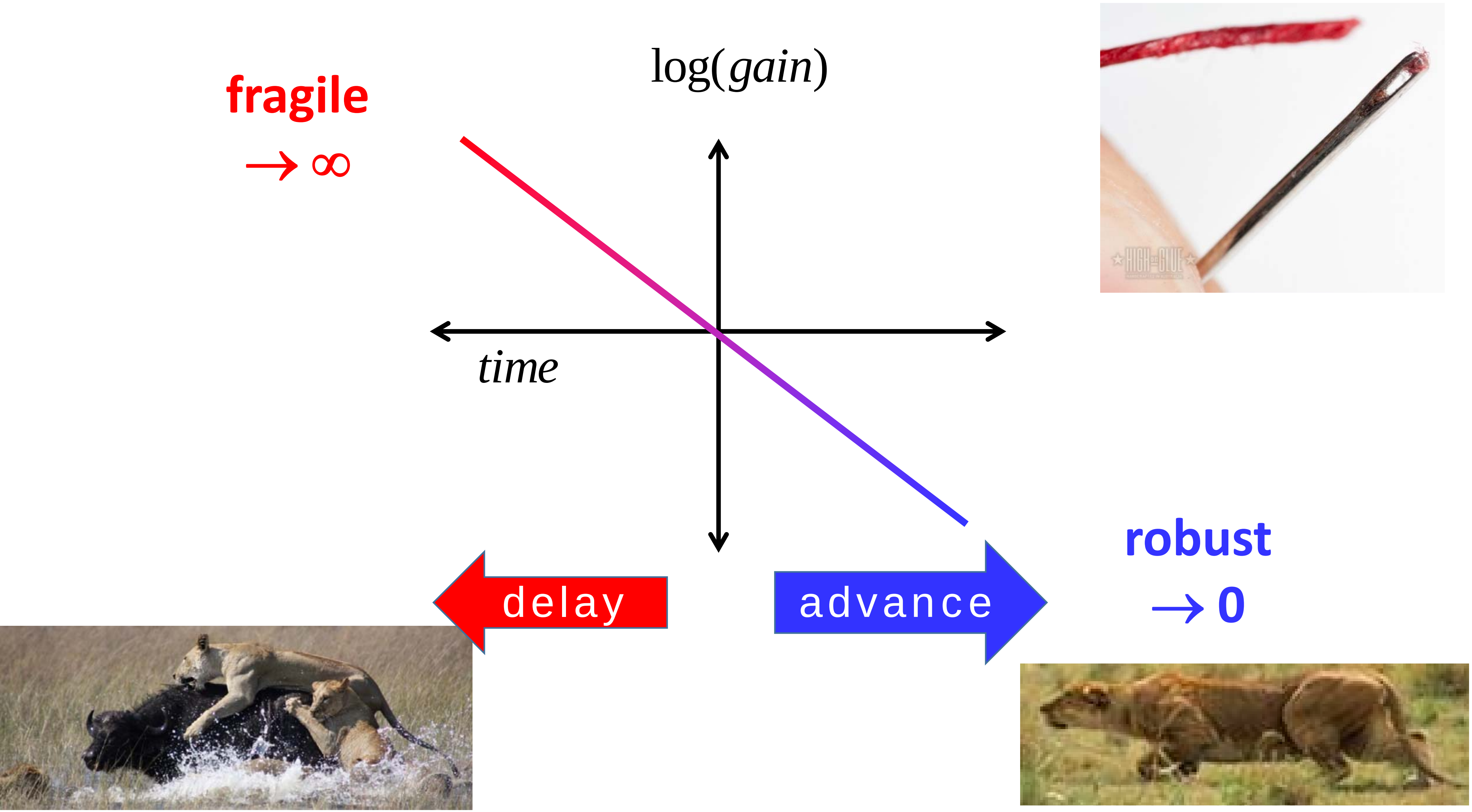
delayed
reflexes
small disturbance
large error

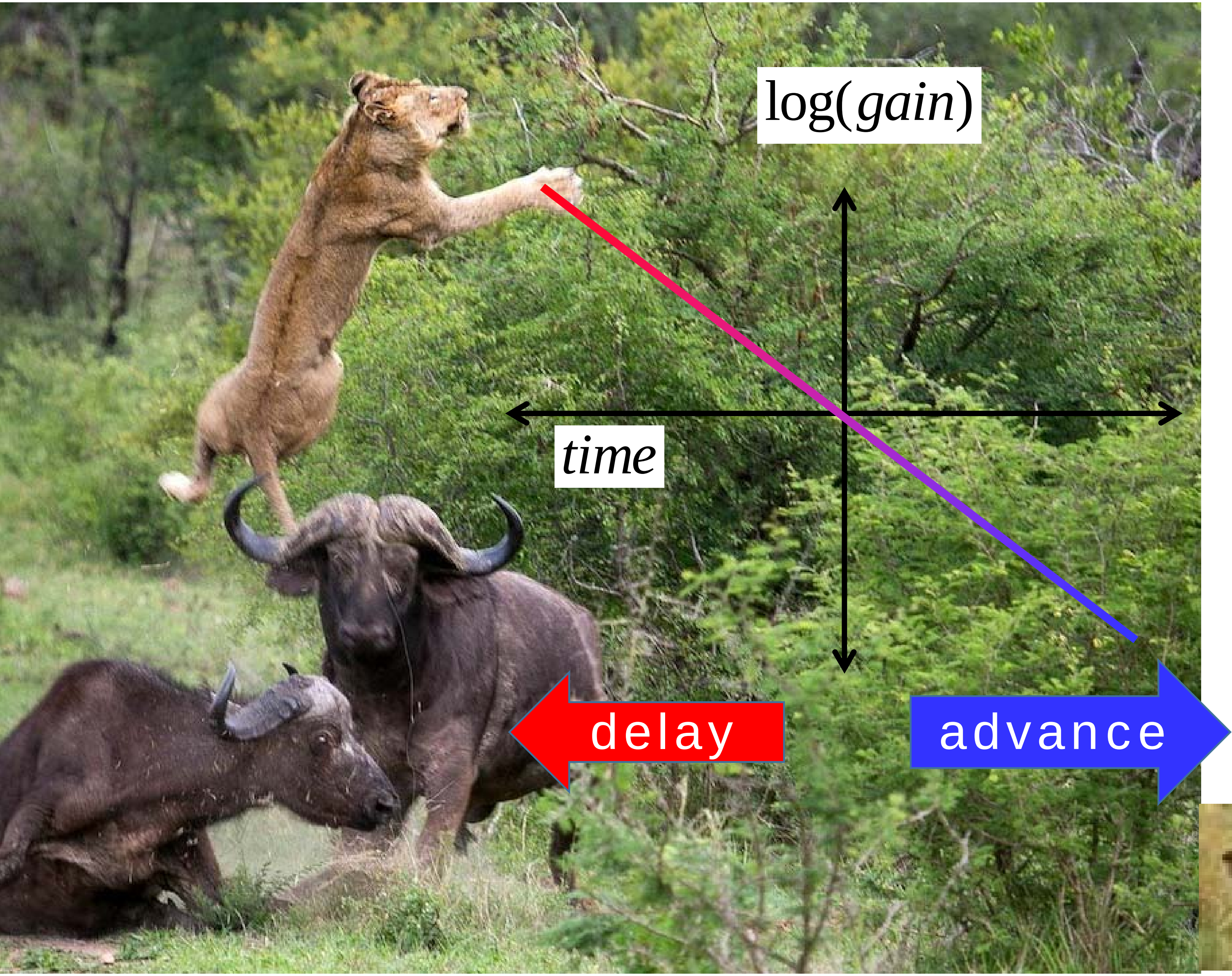
High w/vision

advanced
planning
large disturbance
small error


log(*gain*)
time
High w/vision
advanced
planning
large disturbance
small error
Low w/reflex
delayed
reflexes
small disturbance
large error
delay
advance


fragile
→ ∞
log(gain)
High w/vision
advanced
planning
large disturbance
small error
time
Low w/reflex
delayed
reflexes
small disturbance
large error
delay
advance
robust
→ 0
(-∞)


fragile
→ ∞
log(gain)
time
delay
advance
robust
→ 0



robust
$\rightarrow 0$

Low delay

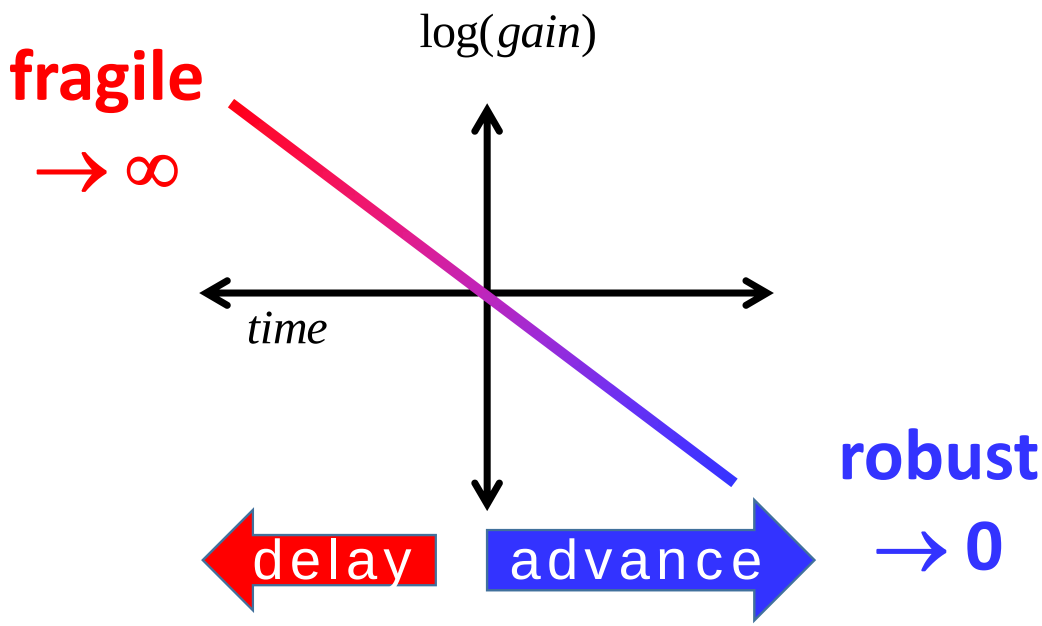
Needs

What the network must provide.

High bandwidth

High w/vision

advanced
planning
large disturbance
small error



Low w/reflex

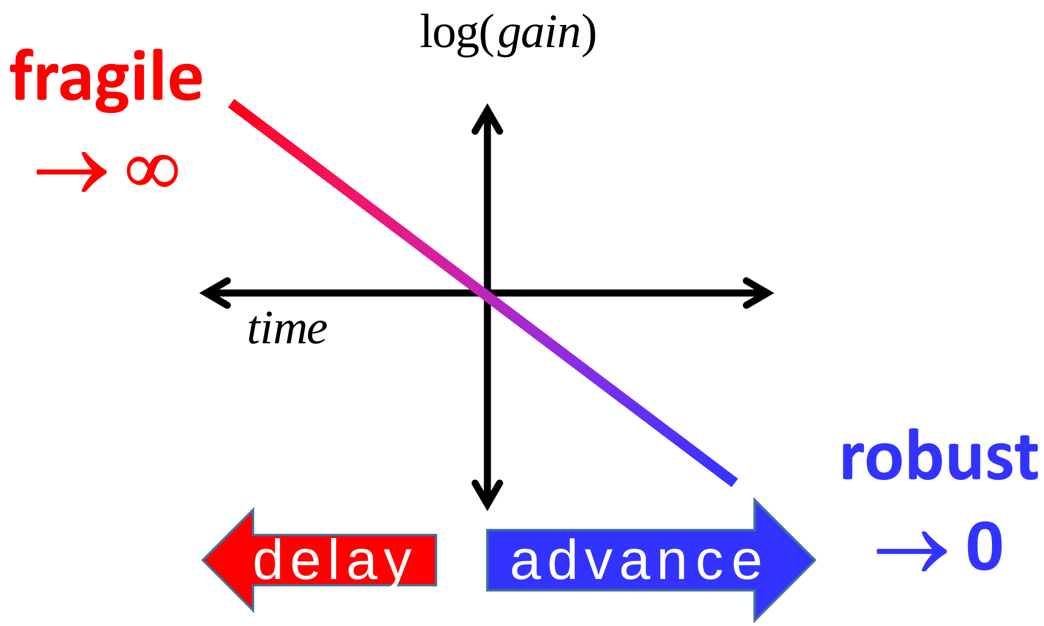
delayed
reflexes
small disturbance
large error

Low delay

Needs

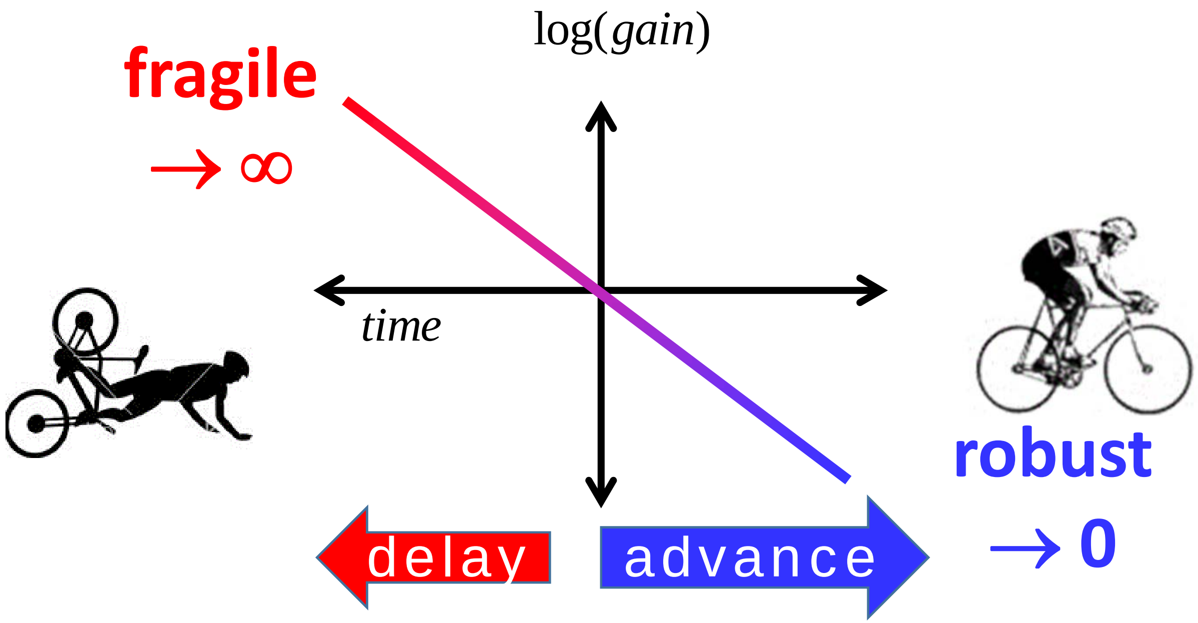
What the network must provide.

High bandwidth



Low w/reflex

delayed
reflexes
small disturbance
large error

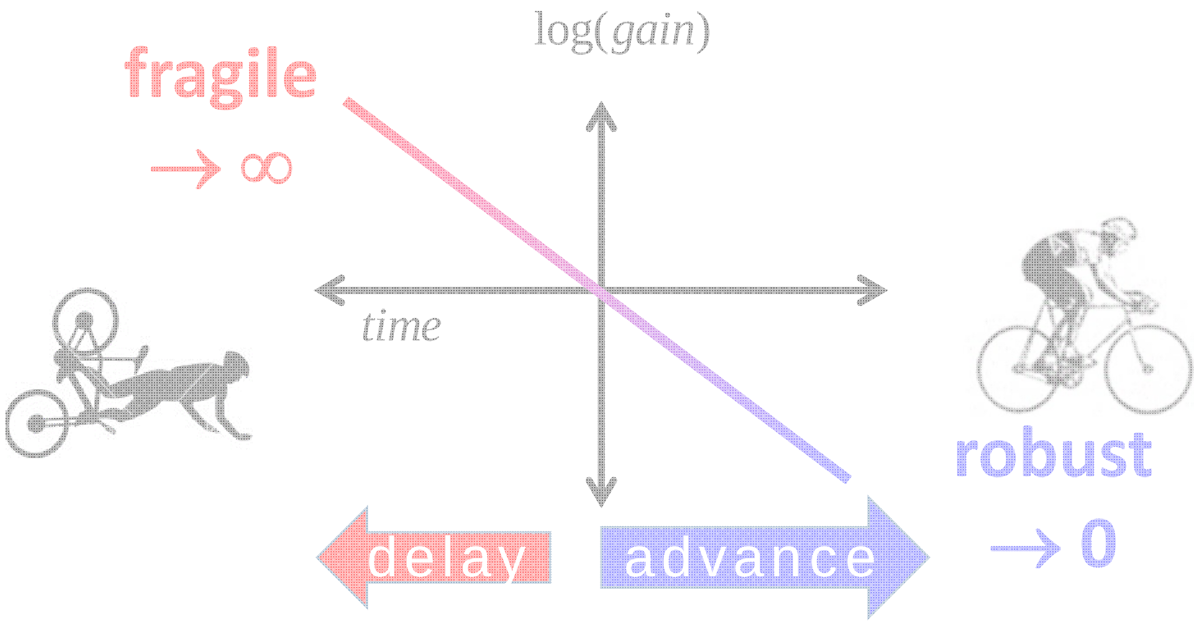


High w/vision

advanced
planning
large disturbance
small error

- model extreme behavior
- formalize tradeoffs
- hard constraints (laws)
- optimal design (architectures)
- mechanistic physiology

Simple and Familiar models, Easy proofs, and Modular (scalable, extendable)



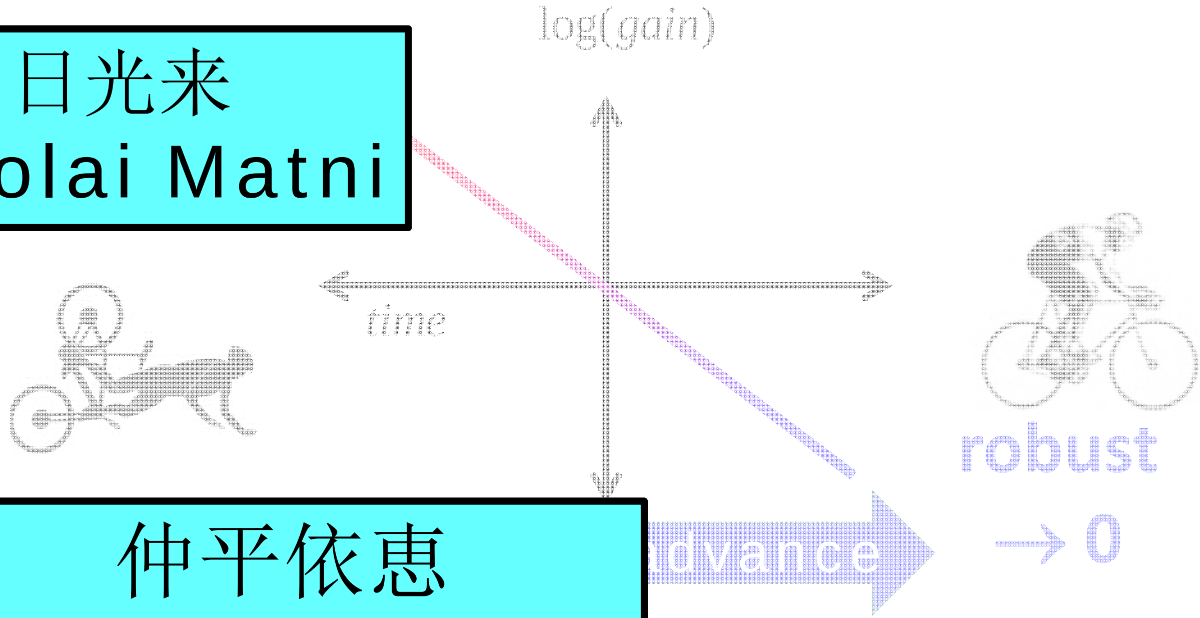
- model extreme behavior
- formalize tradeoffs
- hard constraints (laws)
- optimal design (architectures)
- mechanistic physiology

High w/vision

advanced
planning
large disturbance
small error

日光来
Nikolai Matni

Simple
Familiar
Easy proofs
Modular



仲平依惠
Yorie Nakahira

High w/vision

advanced
planning

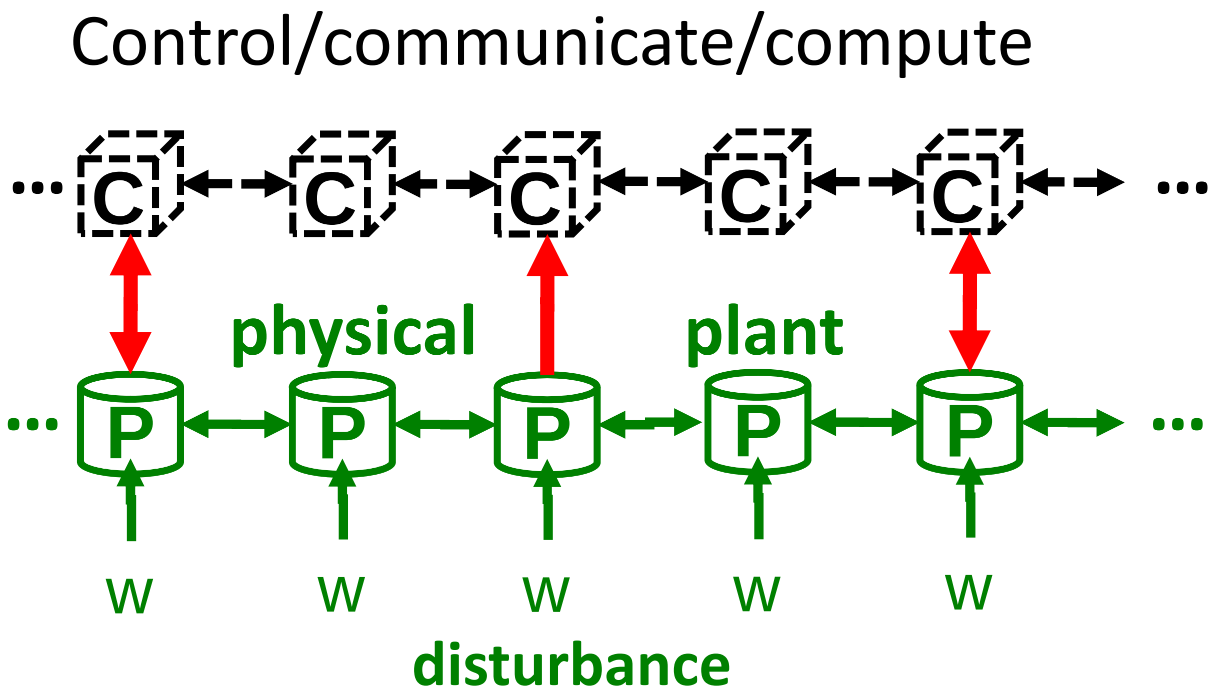
- model extreme behavior
- formalize tradeoffs

王郁翔
Yuh Shyang Wang
(Mickey)

梁玉萍
Yoke Peng Leong

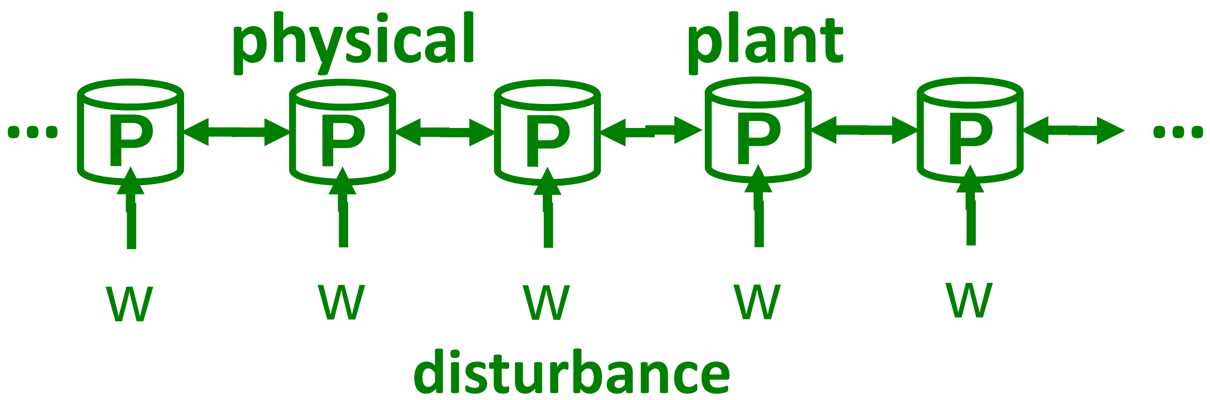
Challenges for control

- Communications
  - delay, sparse
  - quantized
- Actuation and sensing
  - delay, sparse
  - saturates
  - quantized
- Dynamic plant
  - distributed, sparse
  - unstable
- Disturbance (w)
  - worst case
  - measured (or not)
  - advanced warning T



Challenges for control

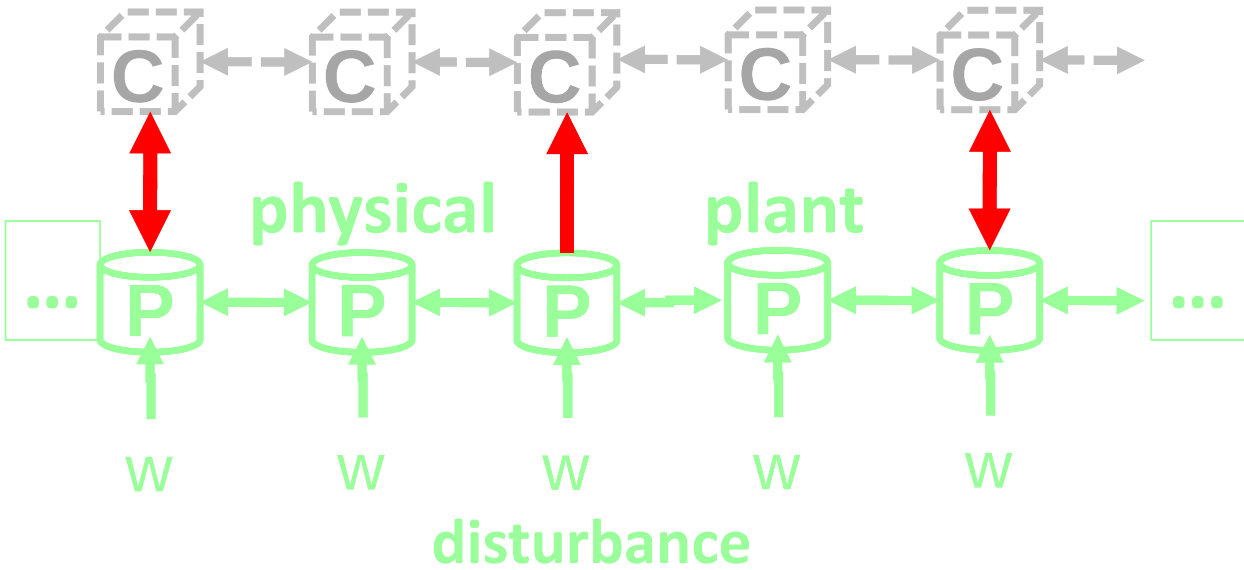
- Communications
  - delay, sparse
  - quantized
- Actuation and sensing
  - delay, sparse
  - saturates
  - quantized
- **Dynamic plant**
  - **distributed, sparse**
  - **unstable**
- **Disturbance (w)**
  - **worst case**
  - **measured (or not)**



Challenges for control

- Communications
  - delay, sparse
  - quantized
- **Actuation and sensing**
  - **delay, sparse**
  - **saturates**
  - **quantized**
- Dynamic plant
  - distributed, sparse
  - unstable
- Disturbance (w)
  - worst case
  - measured (or not)
  - advanced warning T

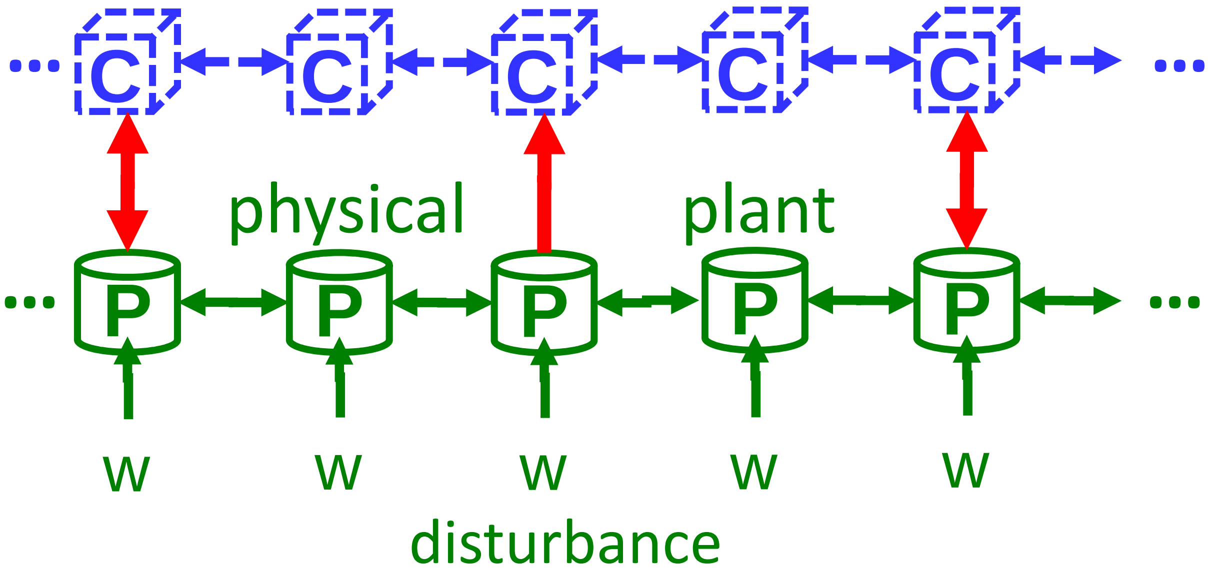
Control/communicate/compute



Challenges for control

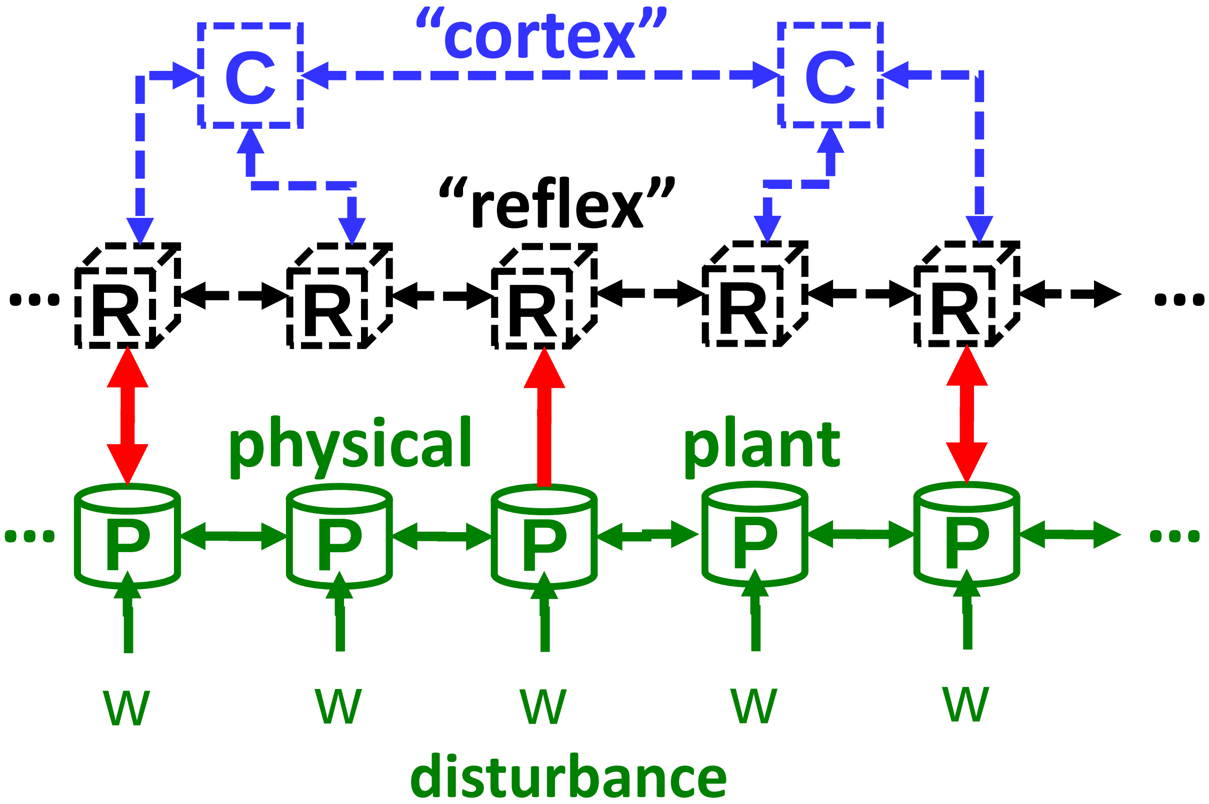
- **Communications**
  - **delay, sparse**
  - **quantized**
- Actuation and sensing
  - delay, sparse
  - saturates
  - quantized
- Dynamic plant
  - distributed, sparse
  - unstable
- Disturbance (w)
  - worst case
  - measured (or not)
  - advanced warning T

Control/communicate/compute



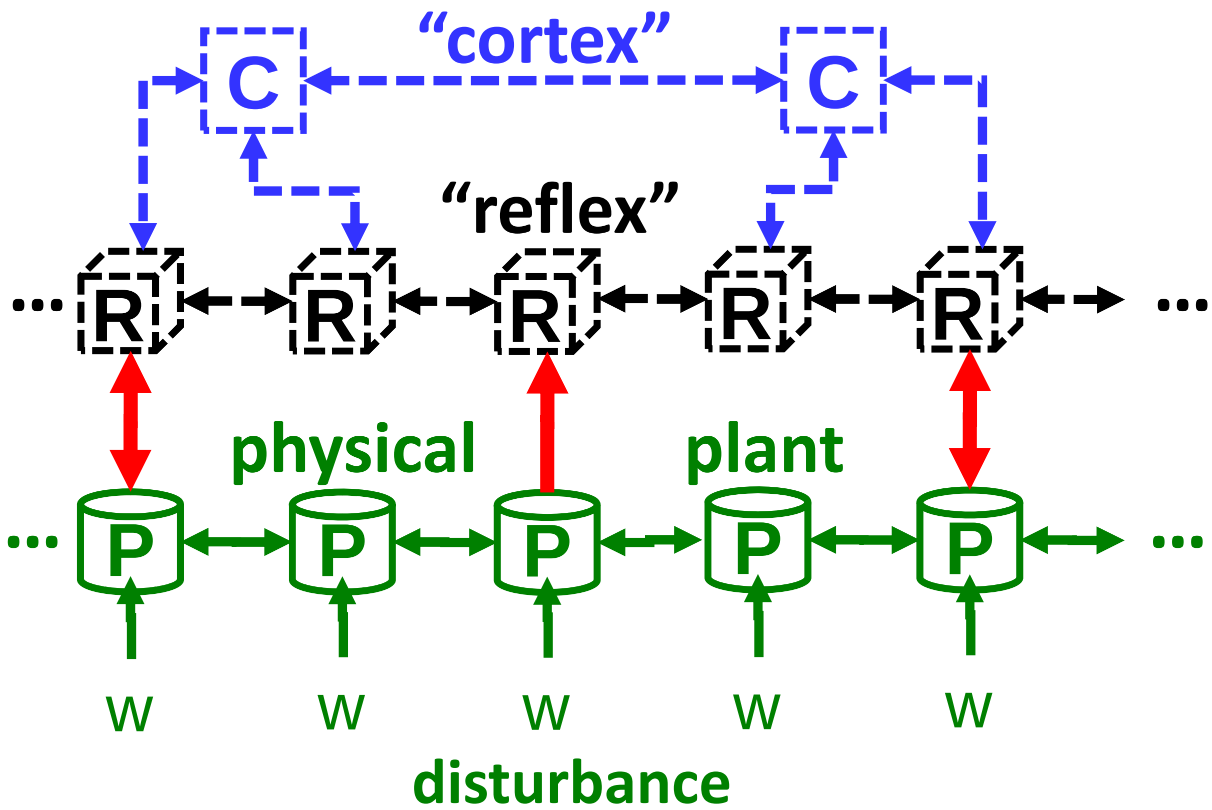
Challenges for control

- Localized
  - layered
  - scalable→local
- Communications
  - delay, sparse
  - quantized
- Actuation and sensing
  - delay, sparse
  - saturates
  - quantized
- Dynamic plant
  - distributed, sparse
  - unstable
- Disturbance (w)
  - worst case
  - measured (or not)
  - advanced warning T



- Localized
  - layered
  - scalable→local
- Communications
  - delay, sparse
  - quantized
- Actuation and sensing
  - delay, sparse
  - saturates
  - quantized
- Dynamic plant
  - distributed, sparse
  - unstable
- Disturbance (w)
  - worst case
  - measured (or not)
  - advanced warning T

Hurts or helps



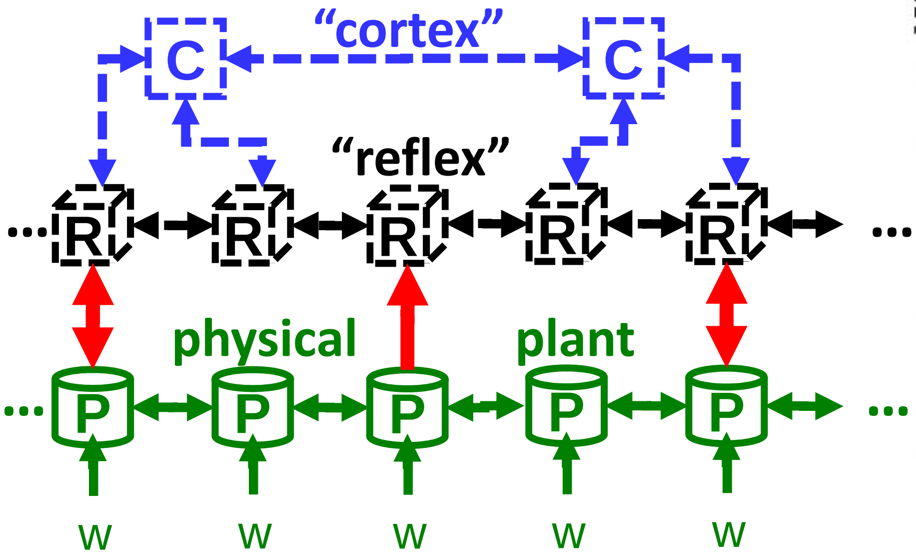
- Localized
  - layered
  - scalable→local
- Communications
  - delay, sparse
  - quantized
- Actuation and sensing
  - delay, sparse
  - saturates
  - quantized
- Dynamic plant
  - distributed, sparse
  - unstable
- Disturbance (w)
  - worst case
  - measured (or not)
  - advanced warning T

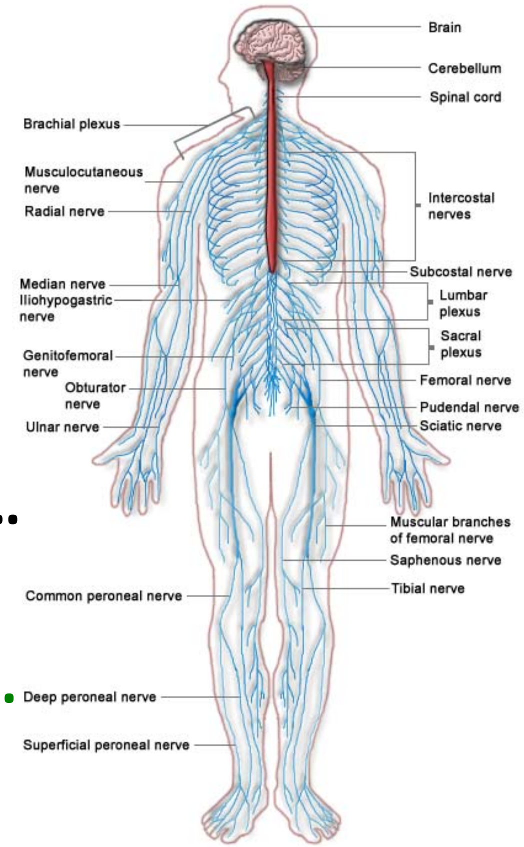
Brains

- sensorimotor control
- maintain homeostasis

Case study

- complex
- informative
- live demos
- wide interest





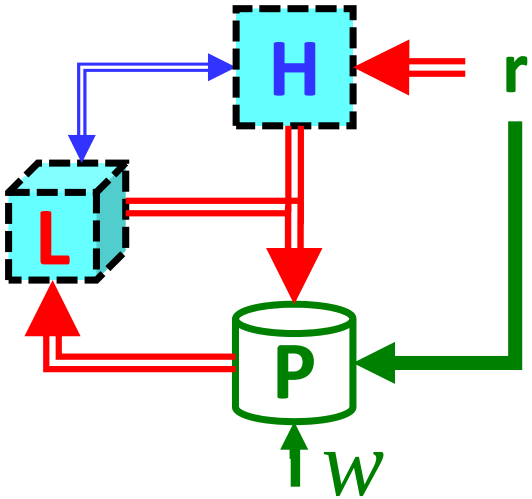
- Localized
  - layered
  - scalable→local
- Communications
  - delay, ~~sparse~~
  - quantized
- Actuation and sensing
  - delay, ~~sparse~~
  - saturates
  - quantized
- Dynamic plant
  - ~~distributed, sparse~~
  - unstable
- Disturbance (w)
  - worst case
  - measured (or not)
  - advanced warning T

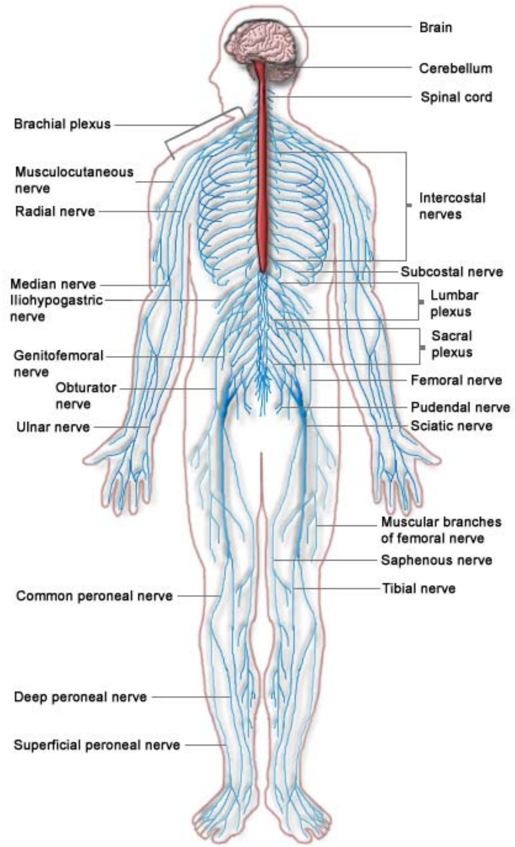
Brains

- sensorimotor control
- ~~maintain homeostasis~~

Case study

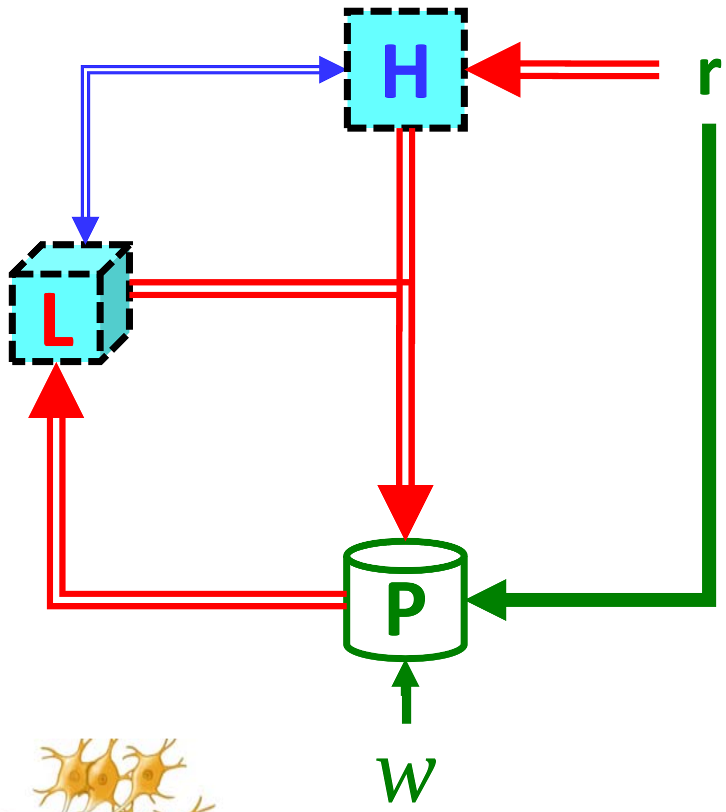
- complex
- informative
- live demos
- wide interest





- Localized
  - layered
  - scalable→local
- Communications
  - delay, ~~sparse~~
  - quantized
- Actuation and sensing
  - delay, ~~sparse~~
  - saturates
  - quantized
- Dynamic plant
  - ~~distributed, sparse~~
  - unstable
- Disturbance (w)
  - worst case
  - measured (or not)
  - advanced warning T

Simple starting point



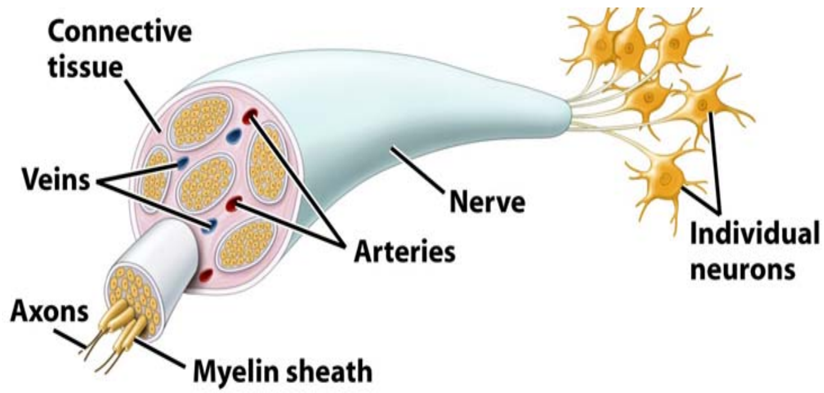
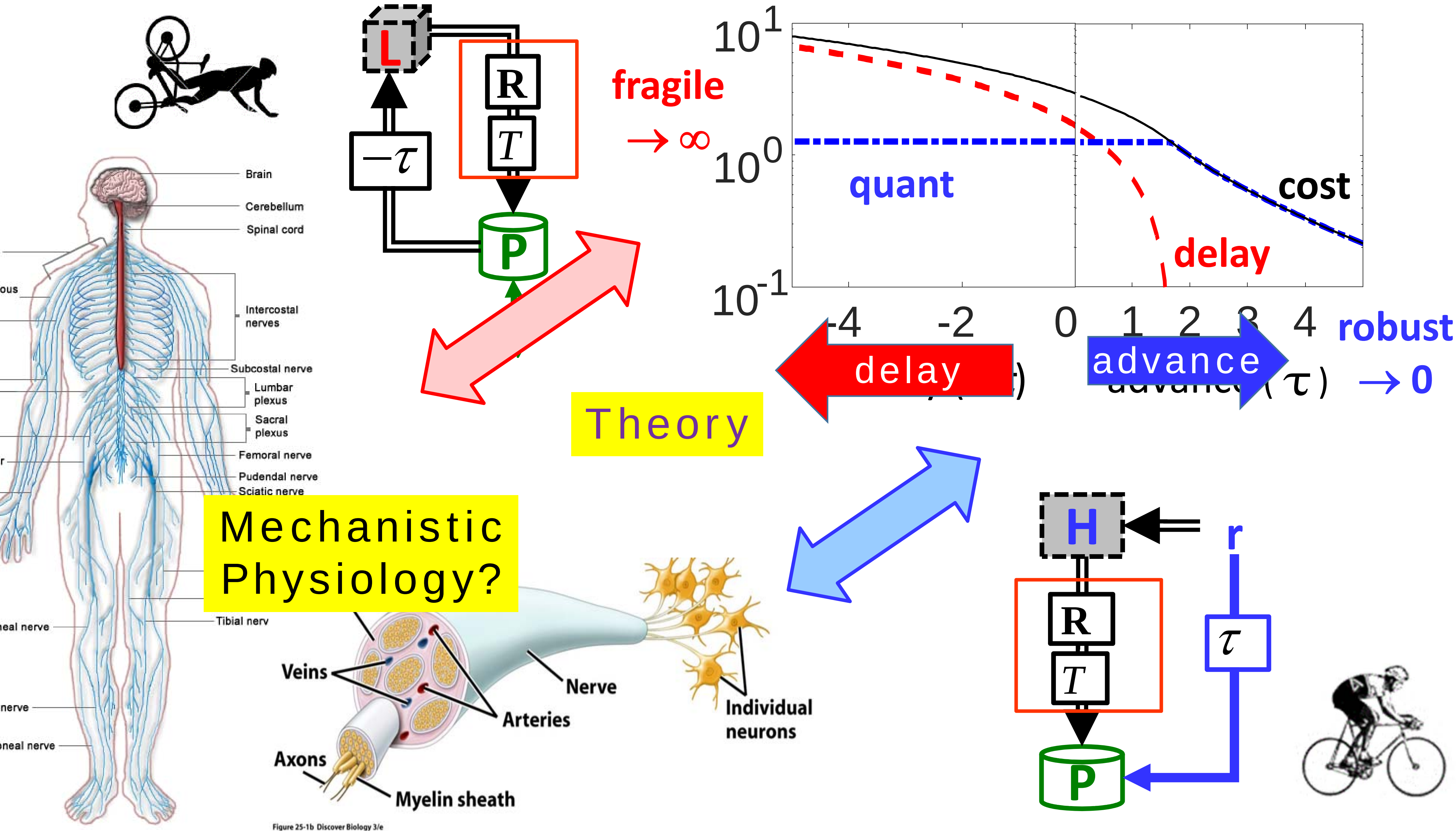
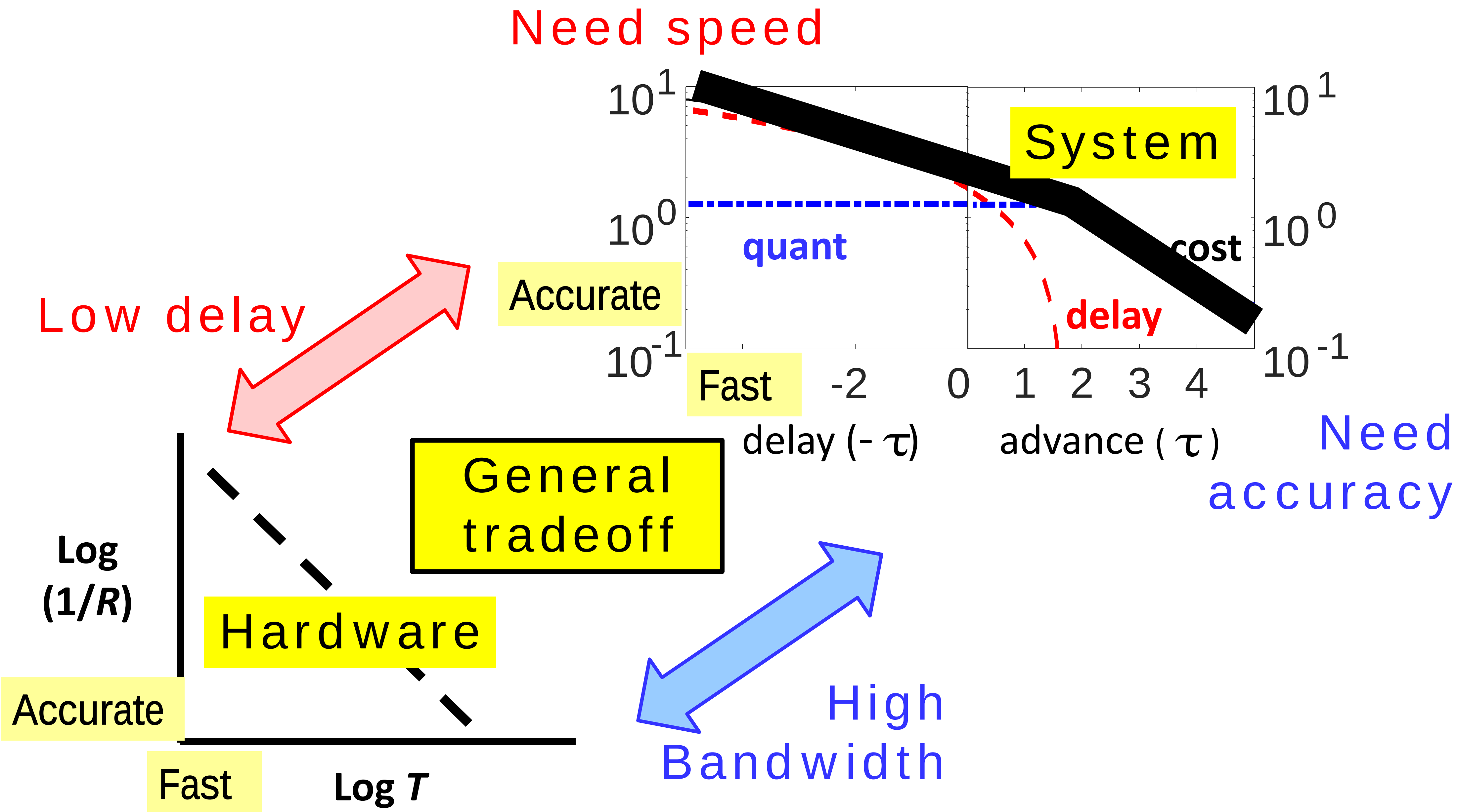


Figure 25-1b Discover Biology 3/e
© 2006 W. W. Norton & Company, Inc.



Brain
Cerebellum
Spinal cord
Intercostal nerves
Subcostal nerve
Lumbar plexus
Sacral plexus
Femoral nerve
Pudendal nerve
Sciatic nerve
Tibial nerv
L
R
T
−τ
P
fragile
→ ∞
10^1
10^0
10^-1
quant
cost
delay
-4
-2
0
1
2
3
4
robust
delay
advance
→ 0
Theory
Mechanistic Physiology?
Veins
Arteries
Nerve
Individual neurons
Axons
Myelin sheath
Figure 25-1b Discover Biology 3/e
H
r
τ
R
T
P



Need speed
10^1
10^0
10^{-1}
System
quant
cost
delay
Accurate
Fast
-2
0
1
2
3
4
delay (- τ)
advance (τ)
Low delay
Need accuracy
General tradeoff
Log (1/R)
Hardware
Accurate
Fast
Log T
High Bandwidth

Lower

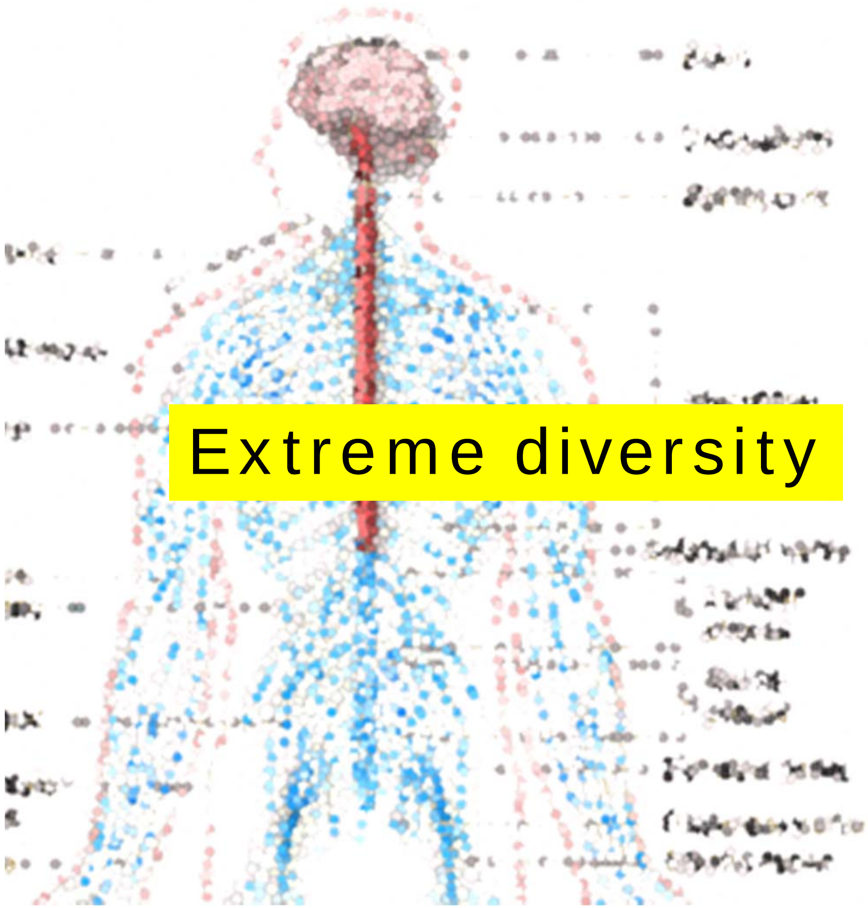
delayed
reflexes
small disturbance
large error
need speed

unstable(real)
distributed
local

unconscious
automatic

Huge range of nerve and axon

- lengths
- diameters



High

advanced
planning
large disturbance
small error
need accuracy

stable(virtual)
centralized
global

conscious
deliberate

Lower

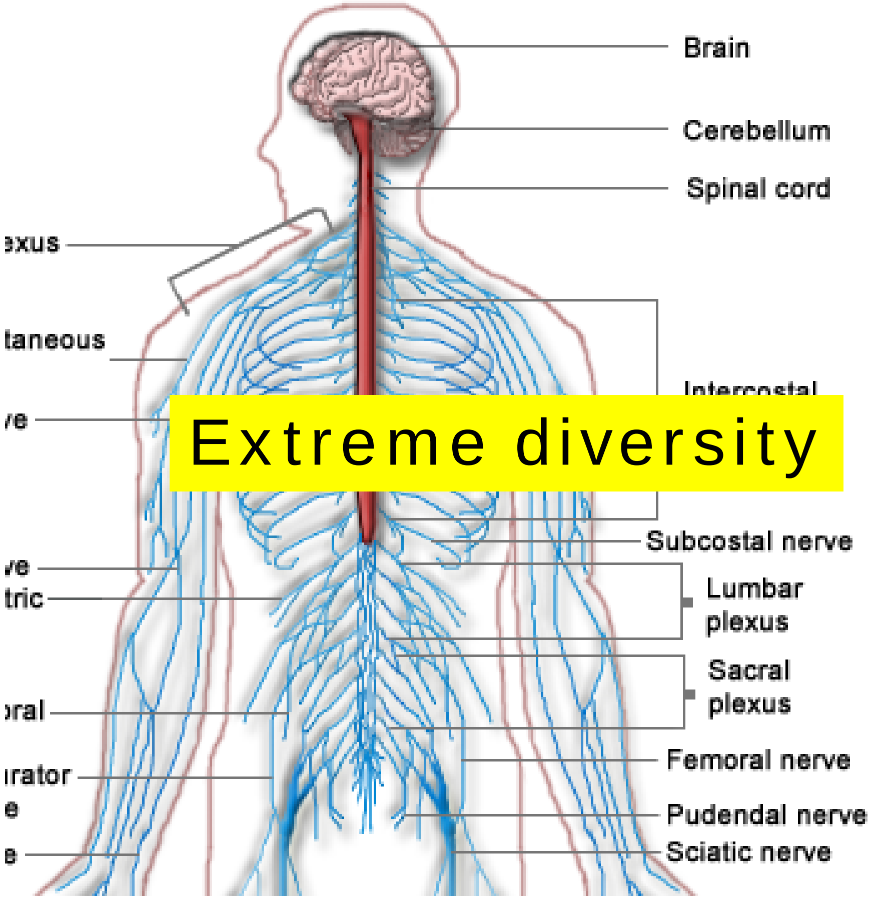
delayed
reflexes
small disturbance
large error
need speed

unstable(real)
distributed
local

unconscious
automatic

Huge range of nerve and axon

- lengths
- diameters



Extreme diversity

High

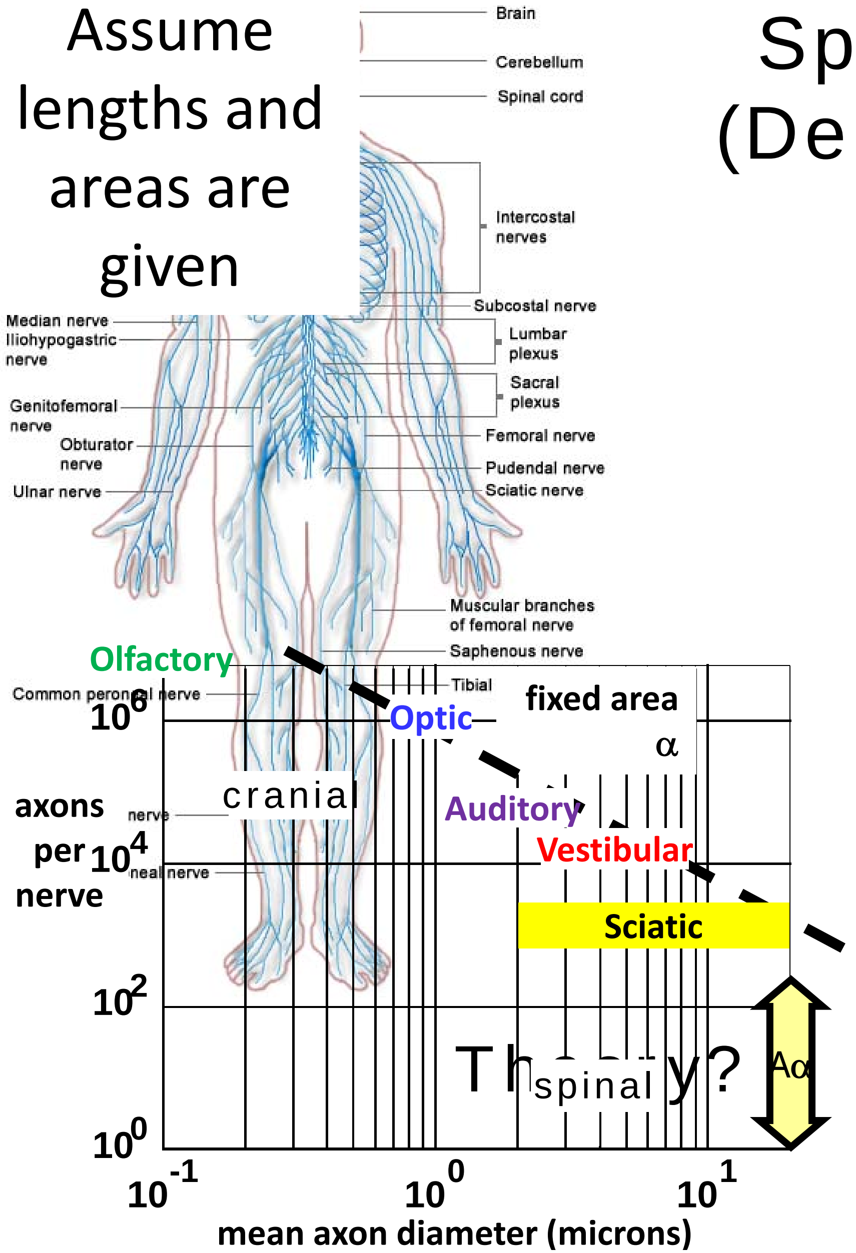
advanced
planning
large disturbance
small error
need accuracy

stable(virtual)
centralized
global

conscious
deliberate

Speed vs Accuracy
(Delay vs Bandwidth)

Assume lengths and areas are given



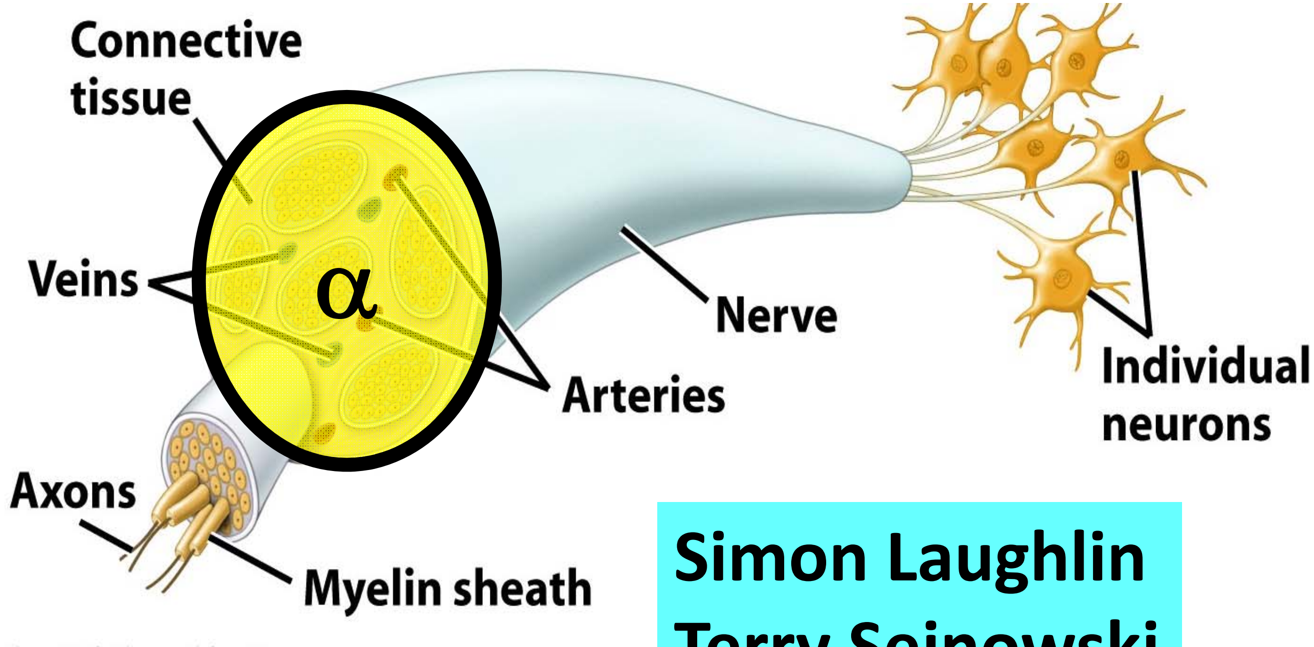
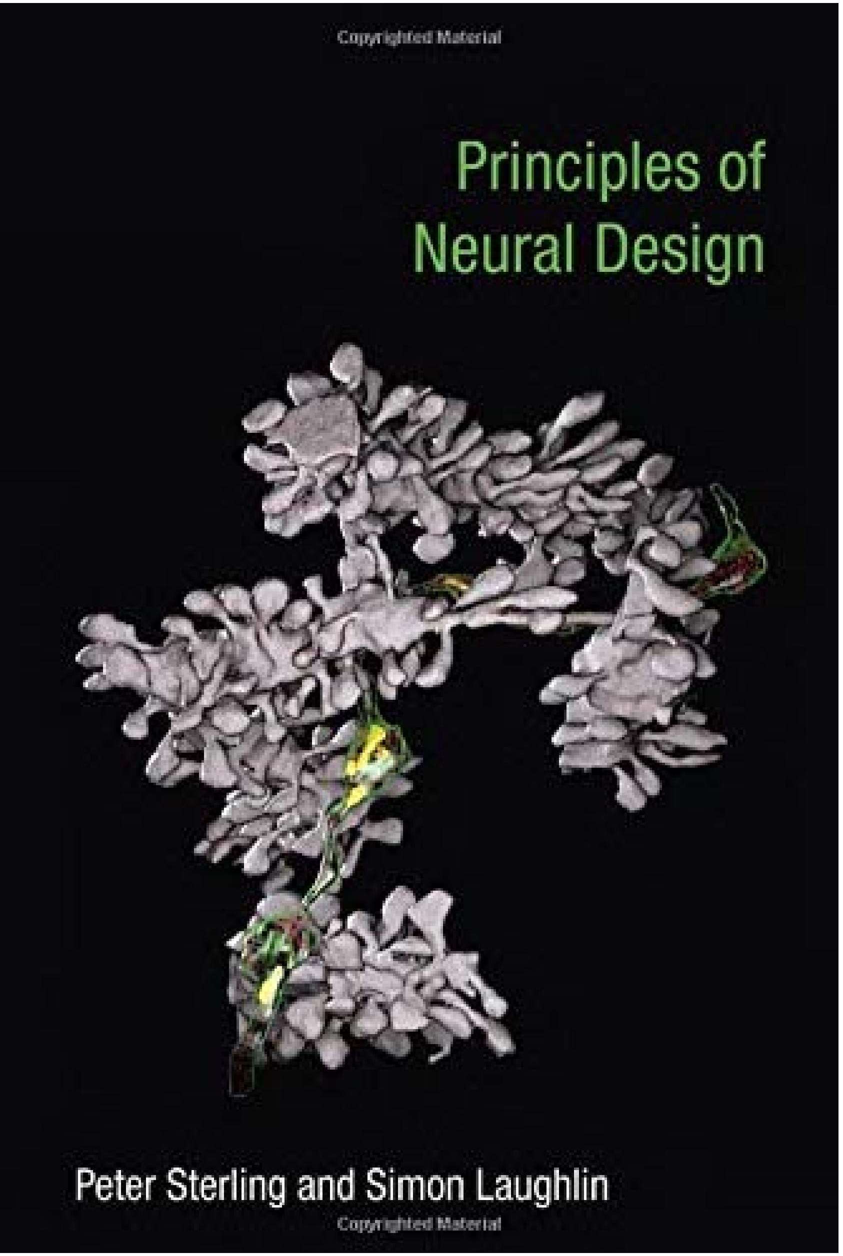
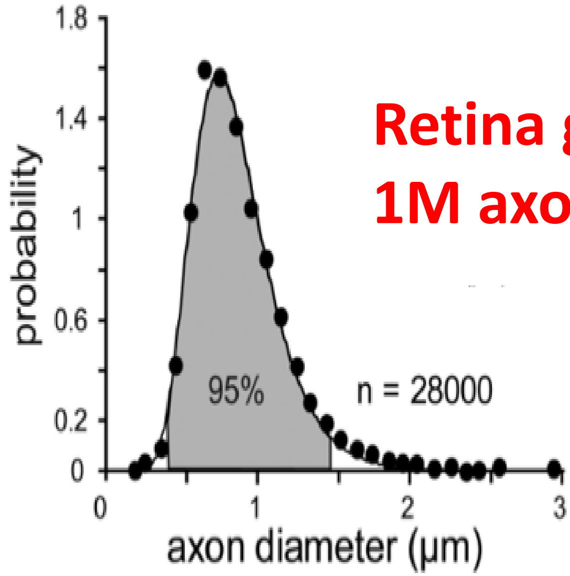


Figure 25-1b Discover Biology 3/e
© 2006 W. W. Norton & Company, Inc.

Simon Laughlin
Terry Sejnowski

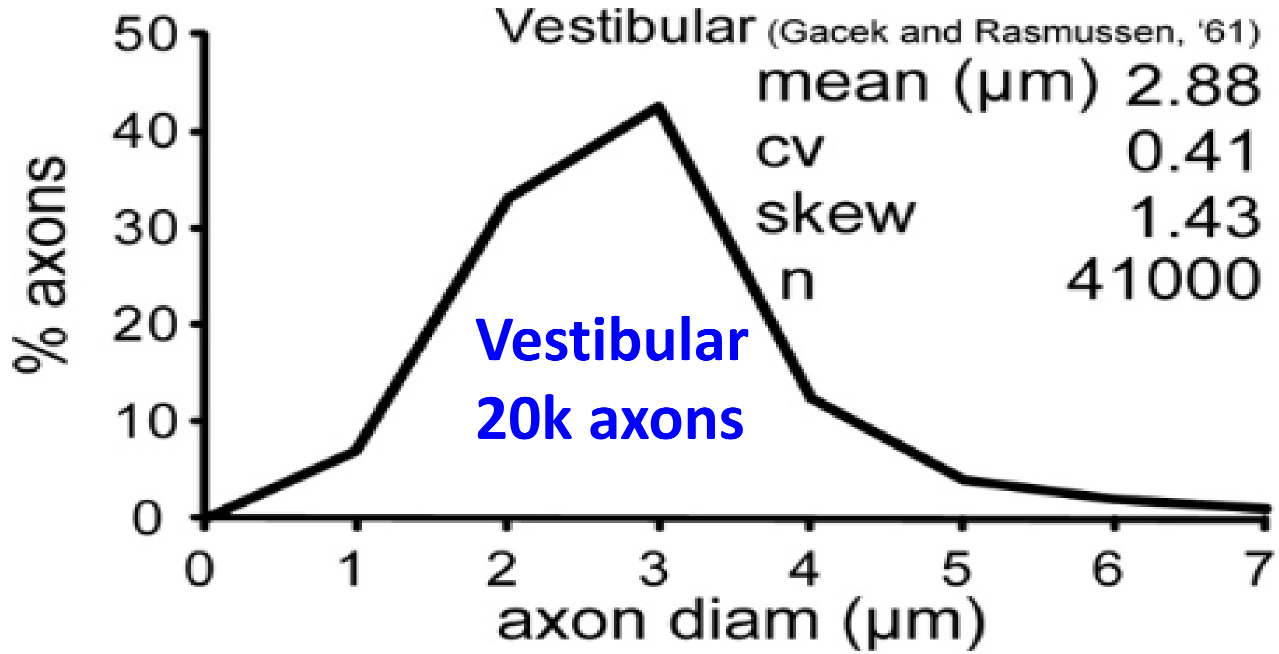
cost ∝ area α
cost = resources to build and maintain



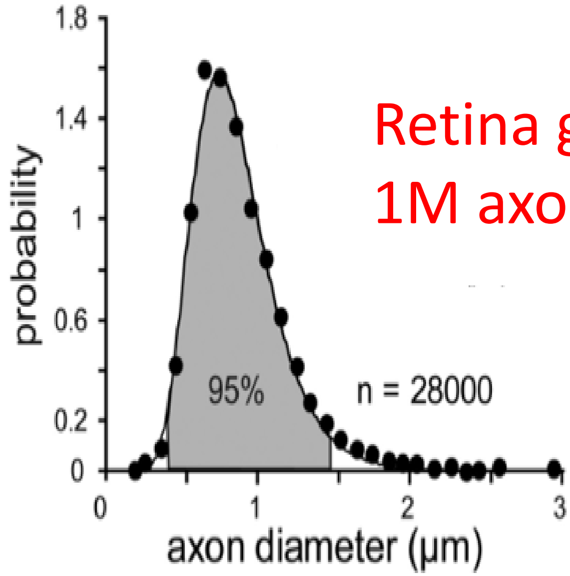


Retina ganglion
1M axons

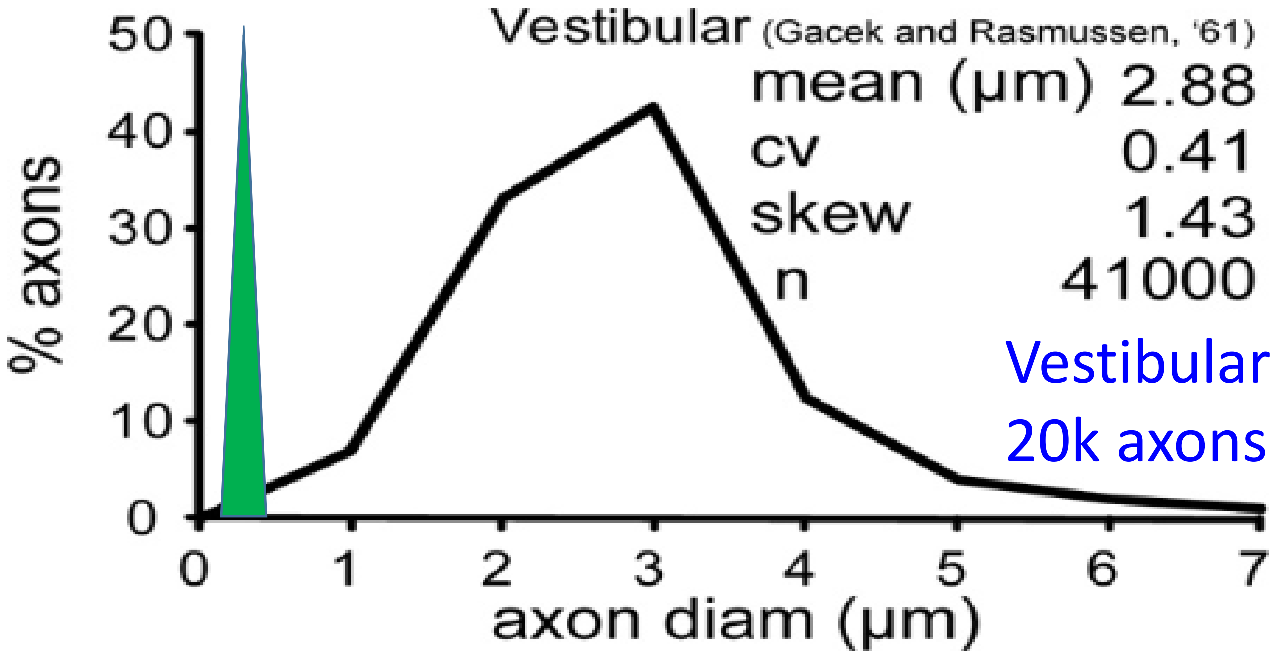
Sterling and Laughlin

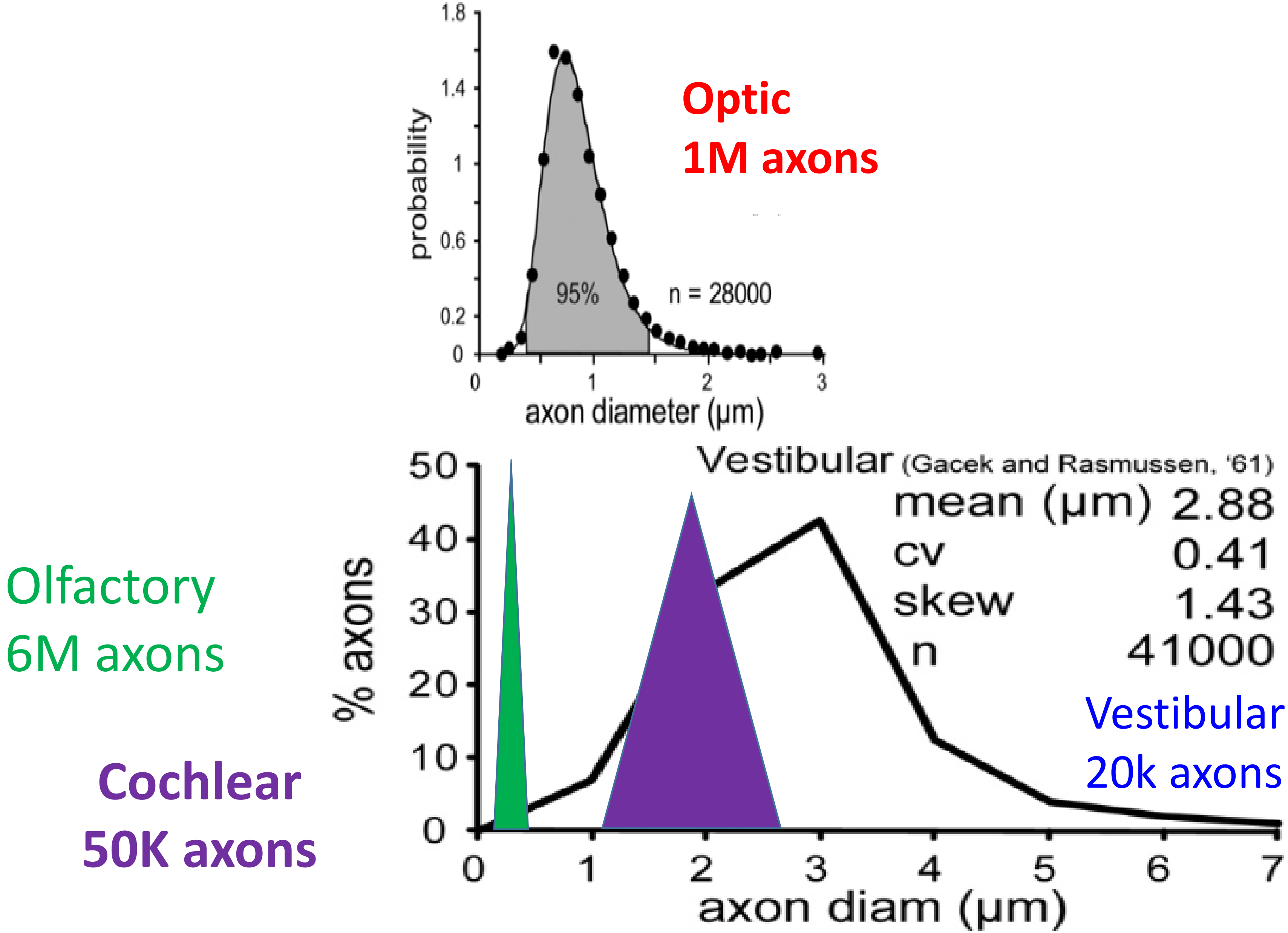


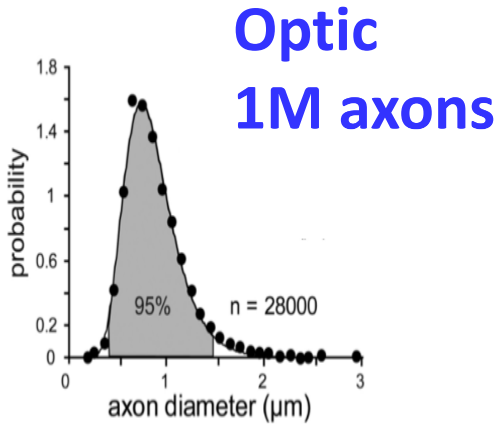
Vestibular
20k axons

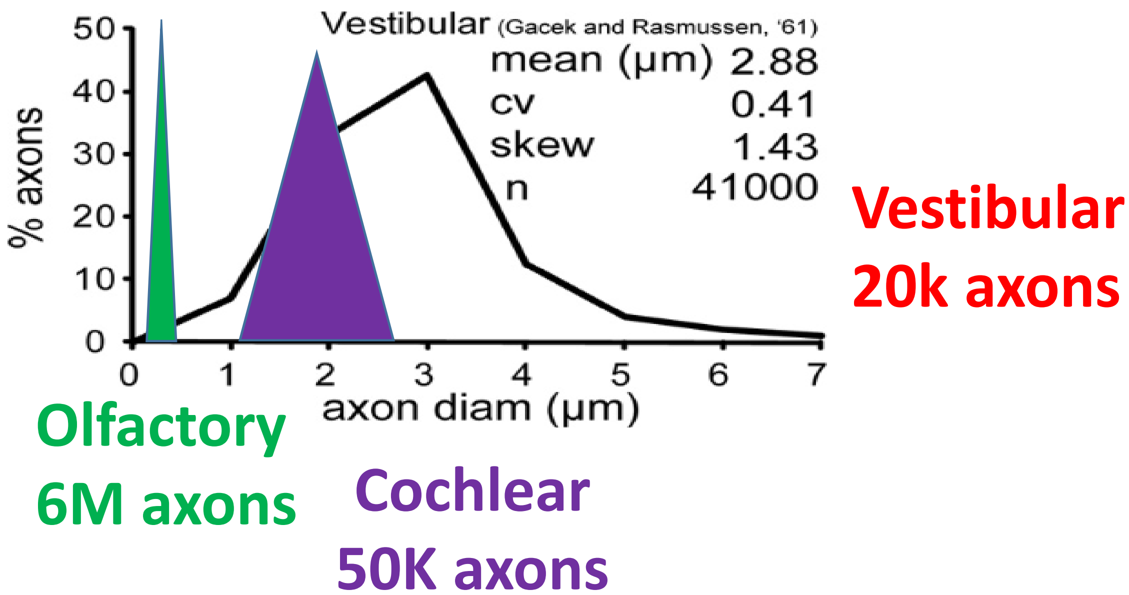


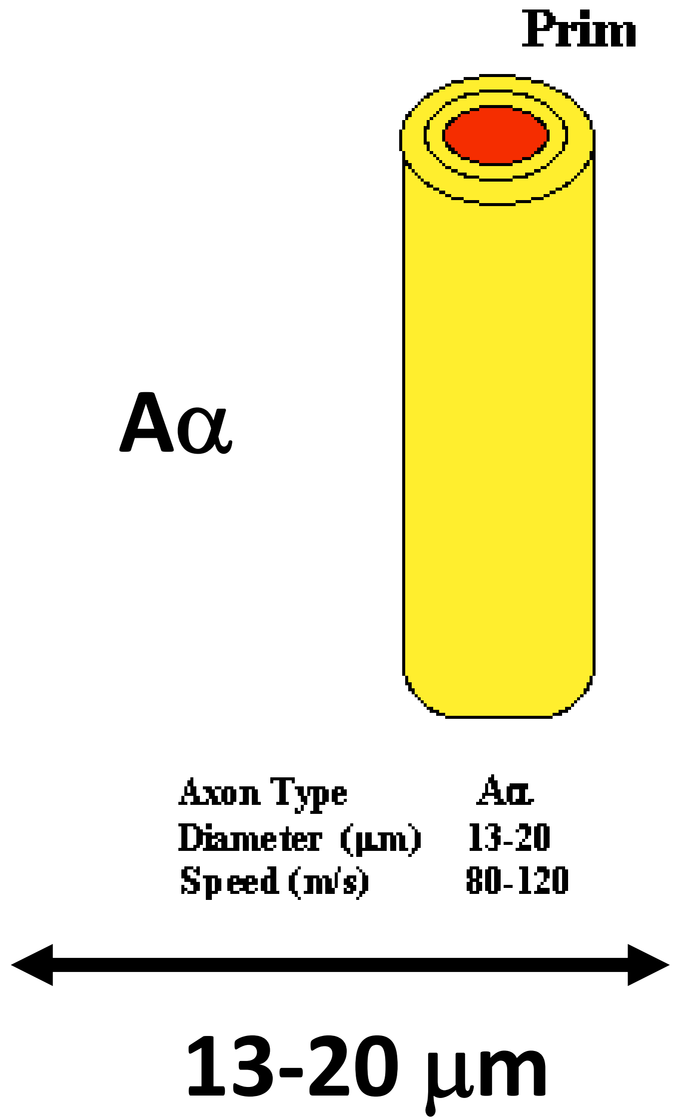
Olfactory
6M axons

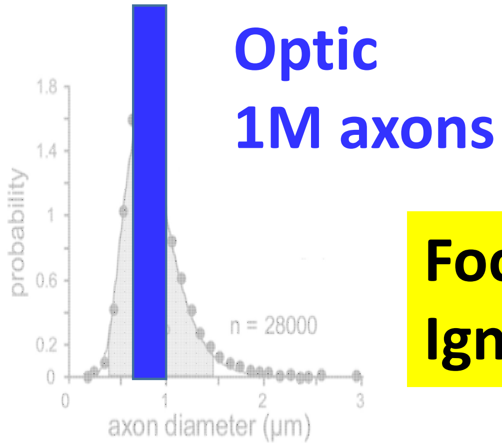



Optic
1M axons
probability
1.8
1.4
1
0.6
0.2
0
95%
n = 28000
0
1
2
3
axon diameter (μm)
Vestibular (Gacek and Rasmussen, '61)
mean (μm) 2.88
cv 0.41
skew 1.43
n 41000
% axons
50
40
30
20
10
0
0
1
2
3
4
5
6
7
axon diam (μm)
Olfactory
6M axons
Cochlear
50K axons
Vestibular
20k axons

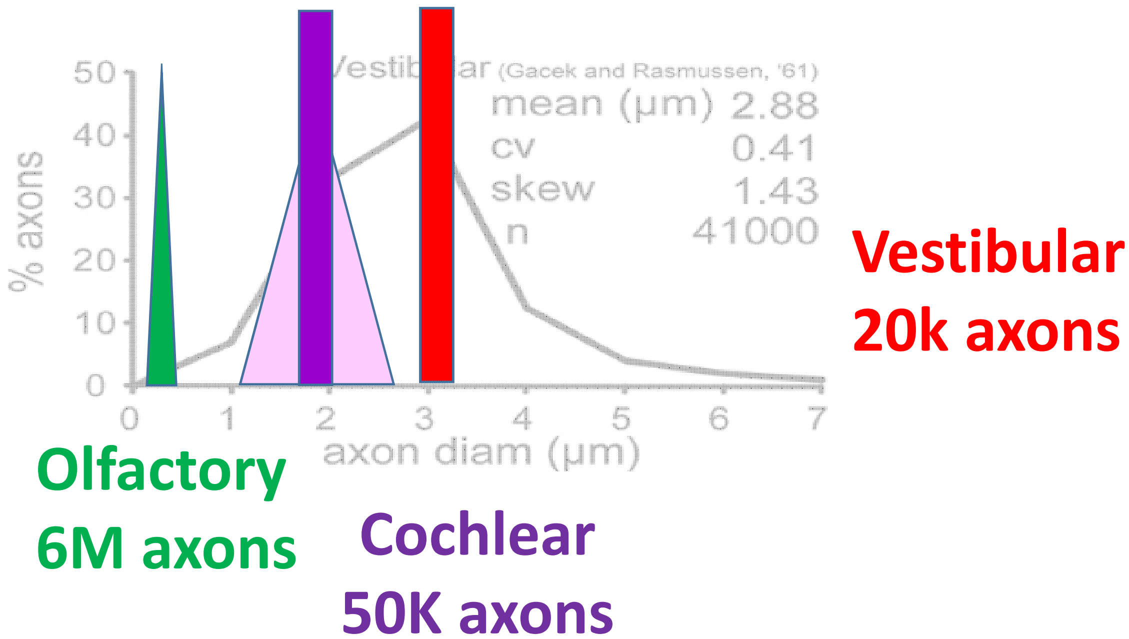

Optic
1M axons
probability
1.8
1.4
1
0.6
0.2
0
95%
n = 28000
0
1
2
3
axon diameter (µm)

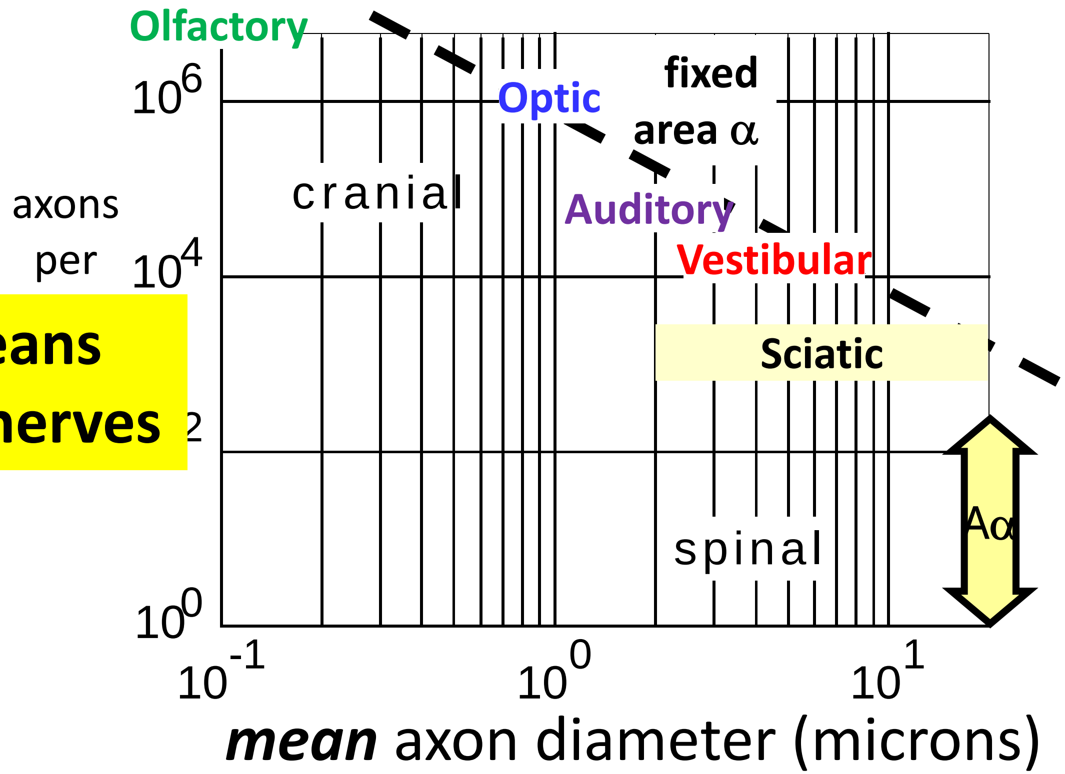

Vestibular (Gacek and Rasmussen, '61)
mean (µm) 2.88
cv 0.41
skew 1.43
n 41000
% axons
50
40
30
20
10
0
0 1 2 3 4 5 6 7
axon diam (µm)
Vestibular
20k axons
Olfactory
6M axons
Cochlear
50K axons

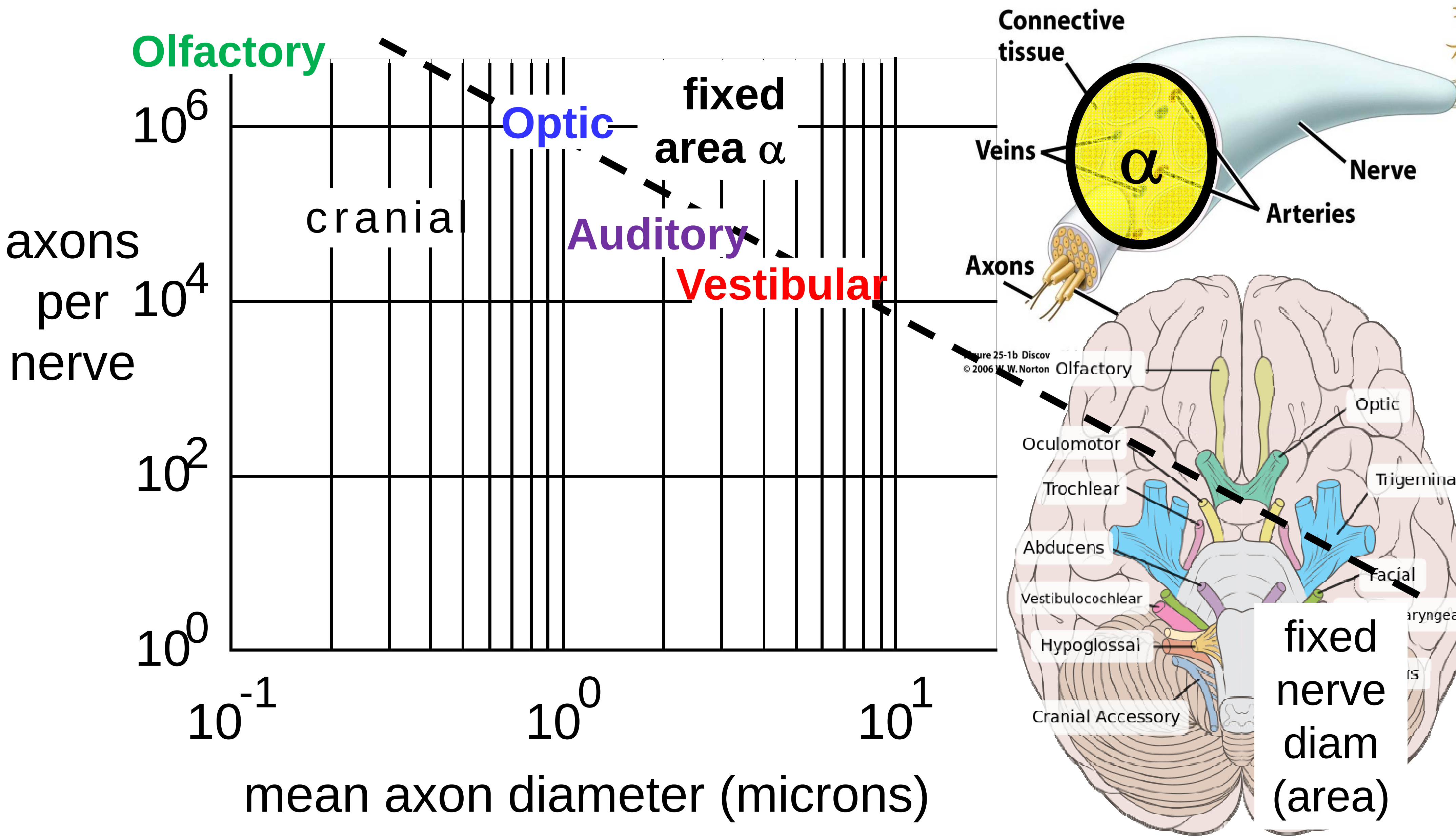

Prim
Aα
Axon Type Aα
Diameter (µm) 13-20
Speed (m/s) 80-120
13-20 µm



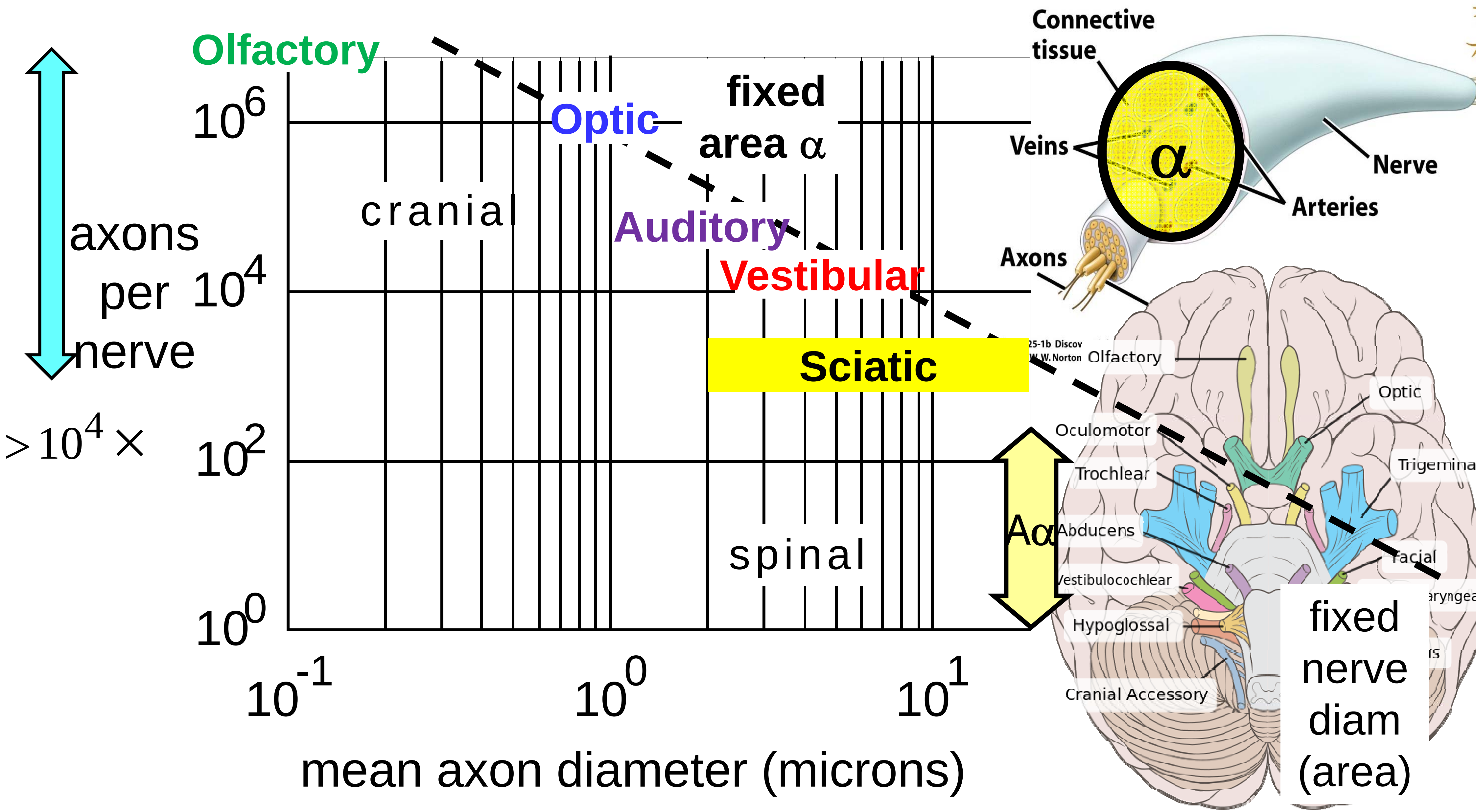
Focus on diversity in means
Ignore diversity *within* nerves







Olfactory
Optic
Auditory
Vestibular
fixed area α
cranial
axons per nerve
10^6
10^4
10^2
10^0
10^{-1}
10^0
10^1
mean axon diameter (microns)
Connective tissue
Veins
α
Nerve
Arteries
Axons
Olfactory
Optic
Oculomotor
Trochlear
Trigemina
Abducens
Facial
Vestibulocochlear
Hypoglossal
Cranial Accessory
fixed nerve diam (area)

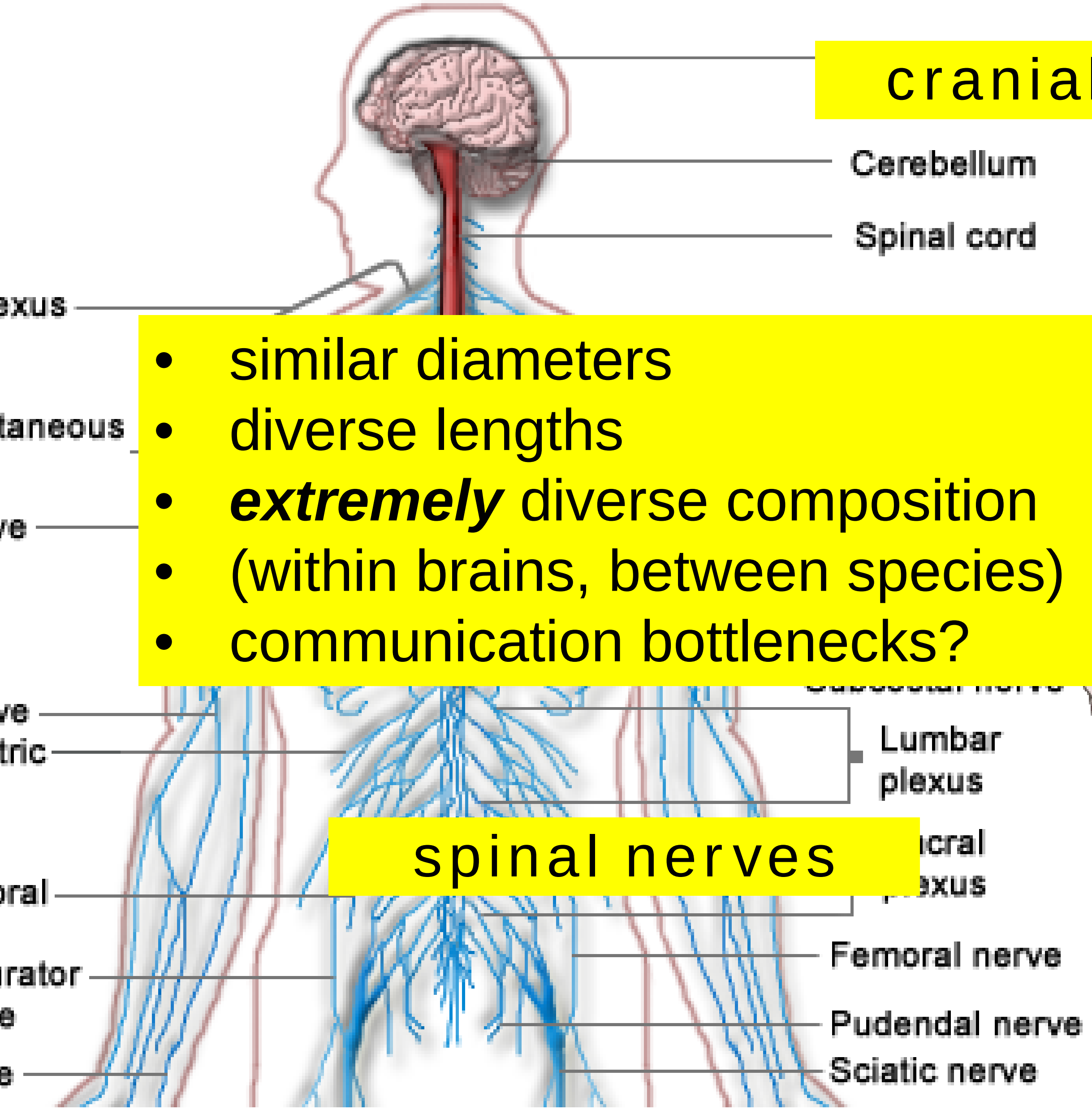


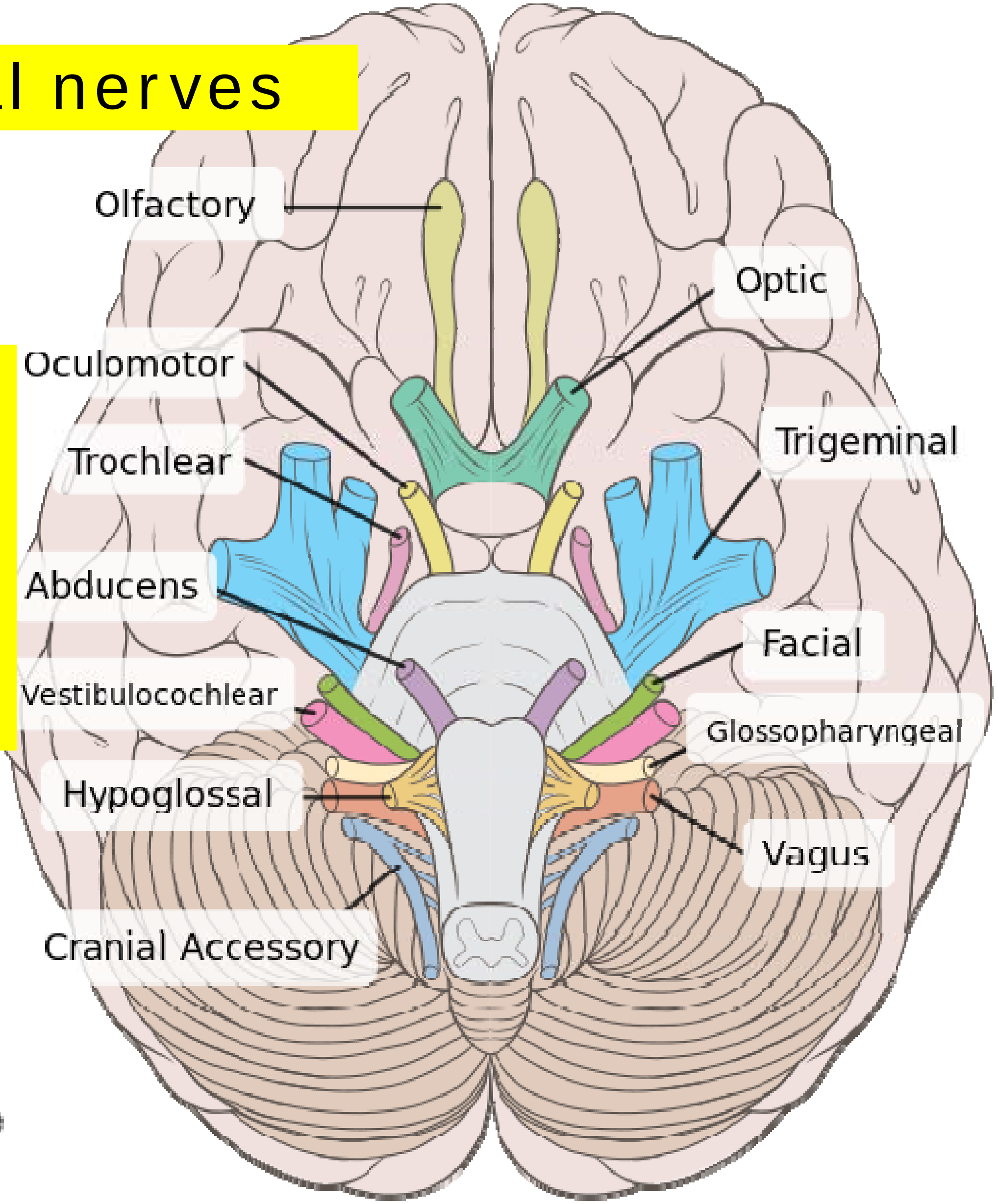
Olfactory
Optic
Auditory
Vestibular
Sciatic
fixed area α
cranial
spinal
axons per nerve
> 10^4 ×
10^6
10^4
10^2
10^0
10^-1
10^0
10^1
mean axon diameter (microns)
Aα
Connective tissue
Veins
Nerve
Arteries
Axons
α
Olfactory
Optic
Oculomotor
Trochlear
Abducens
Facial
Hypoglossal
Cranial Accessory
fixed nerve diam (area)

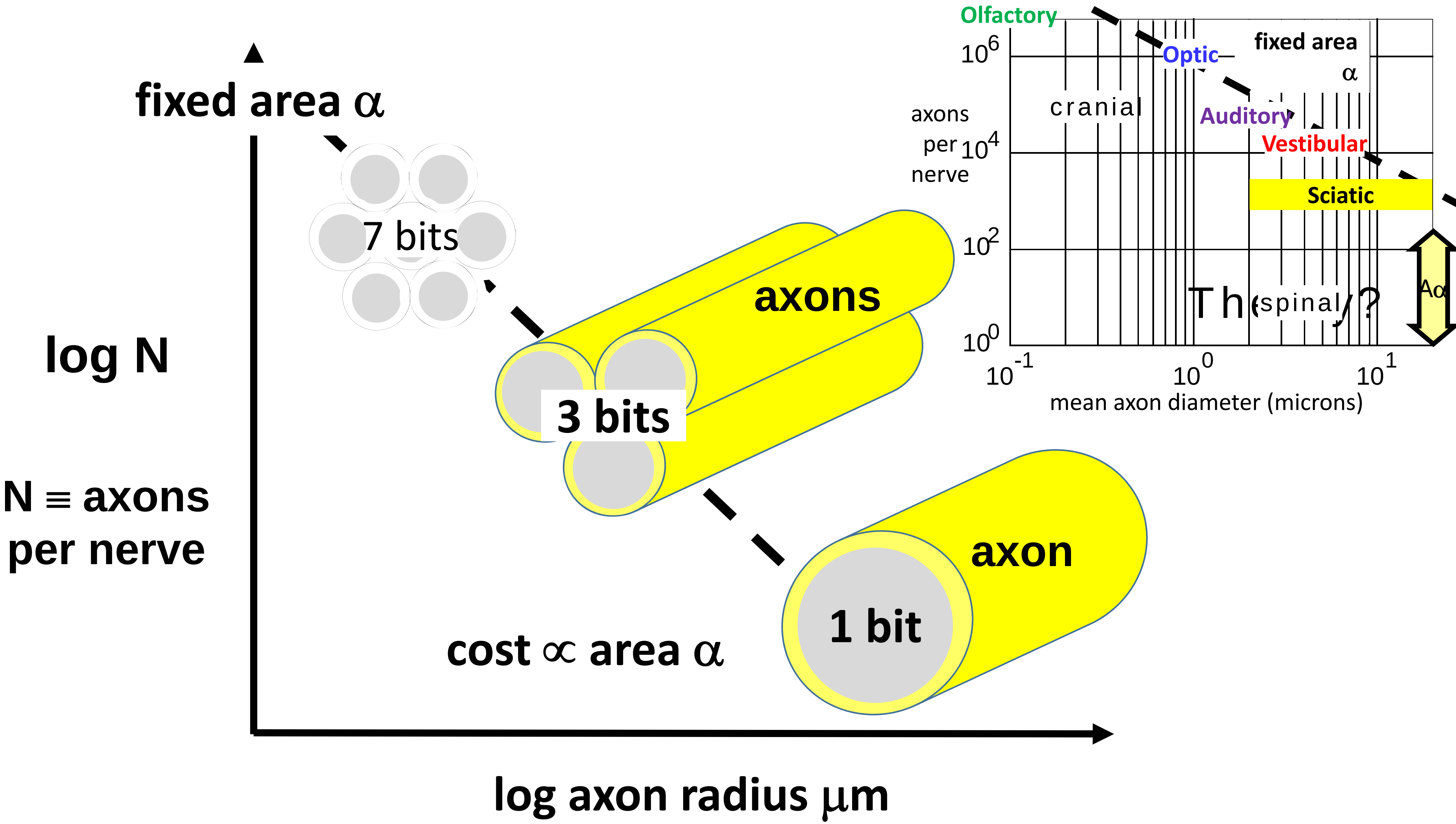
cranial nerves

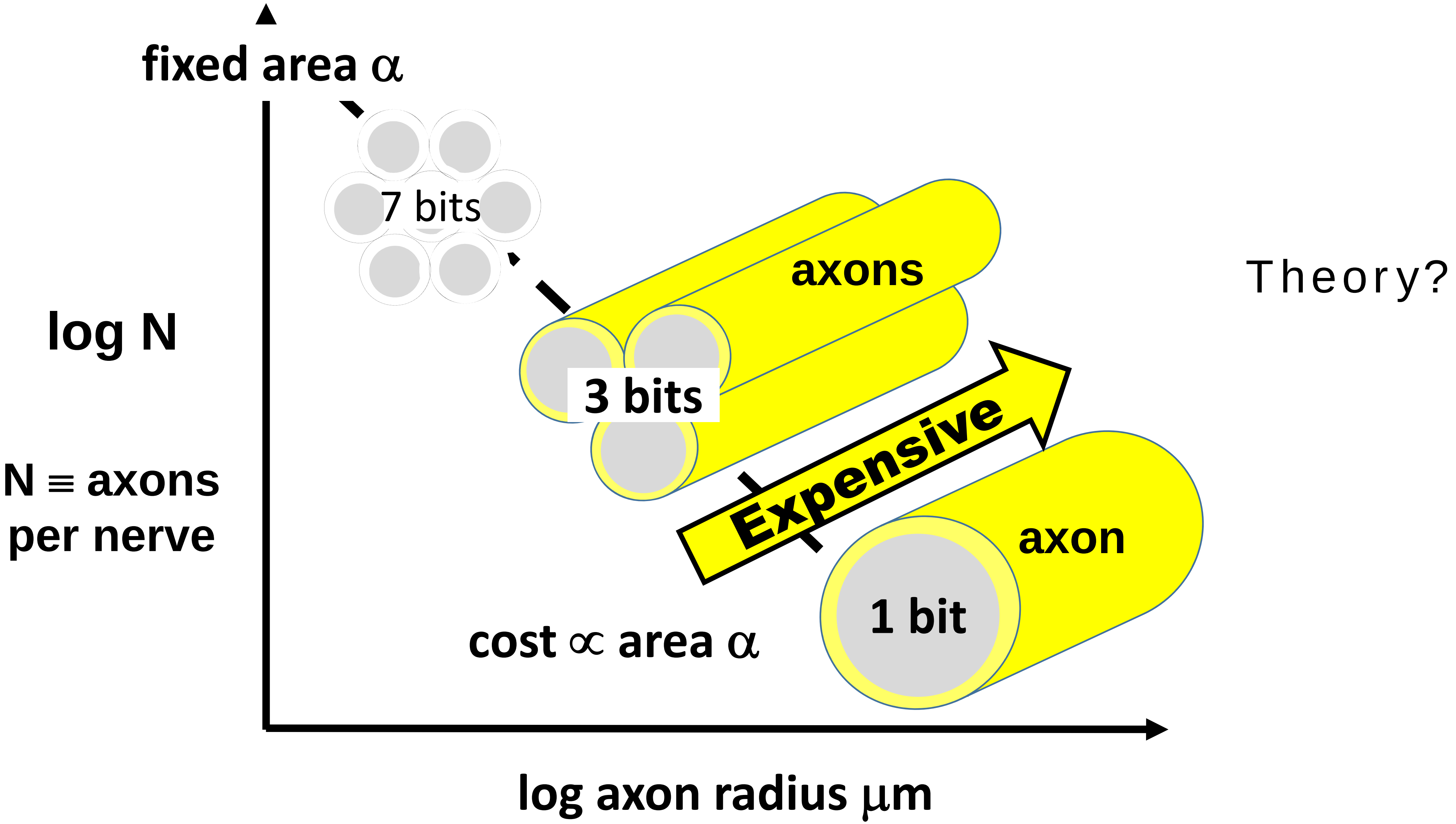
- similar diameters
- diverse lengths
- ***extremely*** diverse composition
- (within brains, between species)
- communication bottlenecks?

spinal nerves



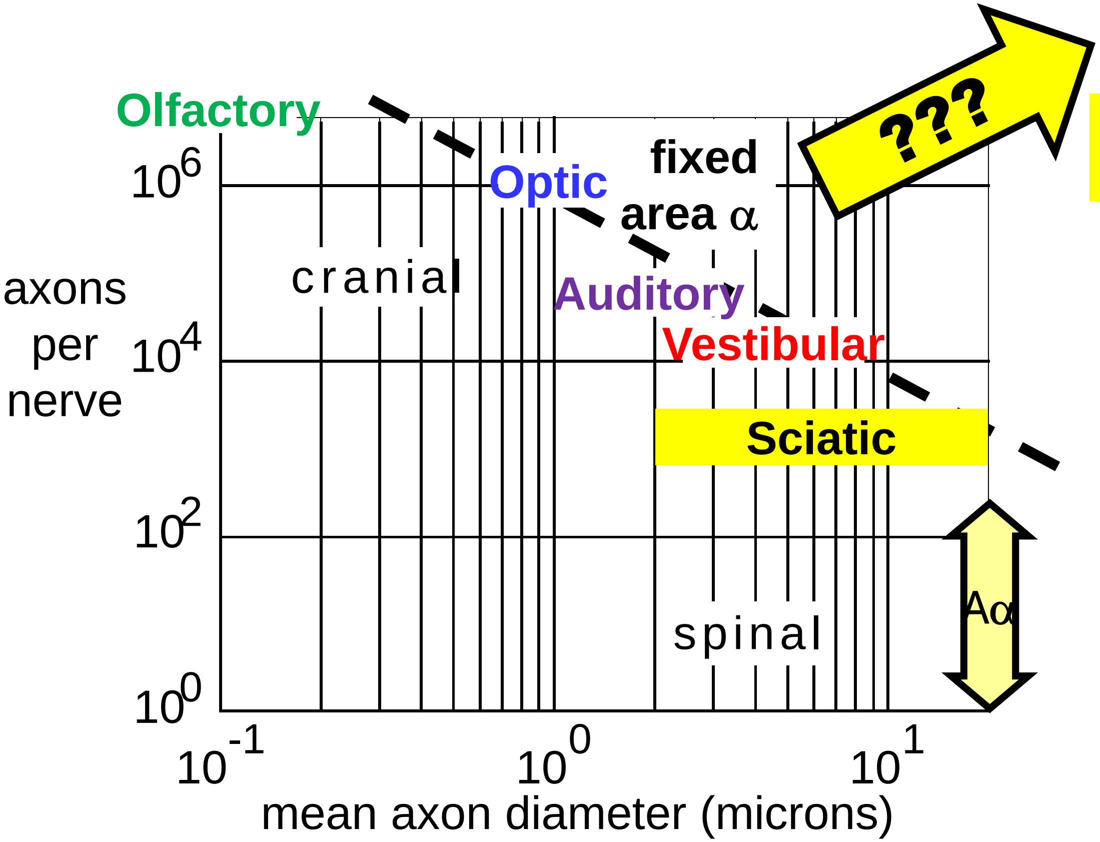


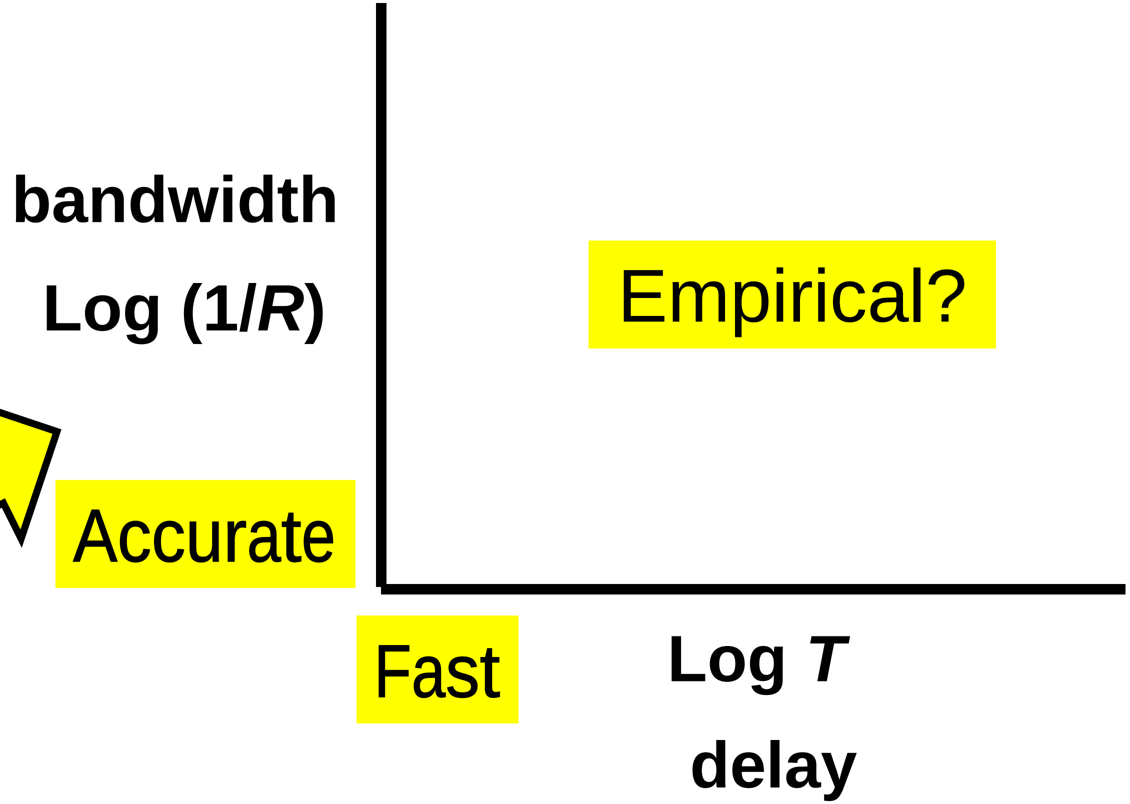

fixed area α
log N
N ≡ axons
per nerve
7 bits
axons
3 bits
axon
1 bit
cost ∝ area α
log axon radius μm
Olfactory
Optic
Auditory
Vestibular
Sciatic
fixed area
α
cranial
spinal
axons
per
nerve
10^6
10^4
10^2
10^0
10^{-1}
10^0
10^1
mean axon diameter (microns)


fixed area α
7 bits
axons
3 bits
Theory?
log N
N ≡ axons
per nerve
Expensive
axon
1 bit
cost ∝ area α
log axon radius μm

Speed vs Accuracy?
(Delay *T* vs Bandwidth *R*?)

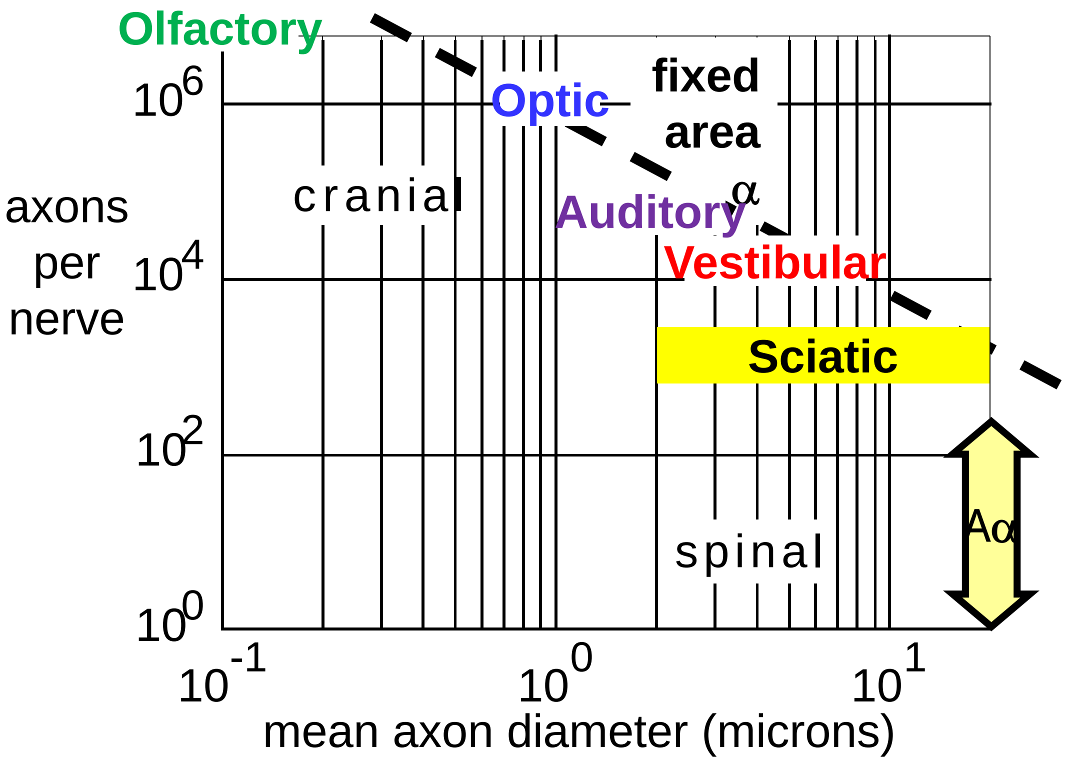
***R*=bits/time**
1/*R* ∝ accuracy

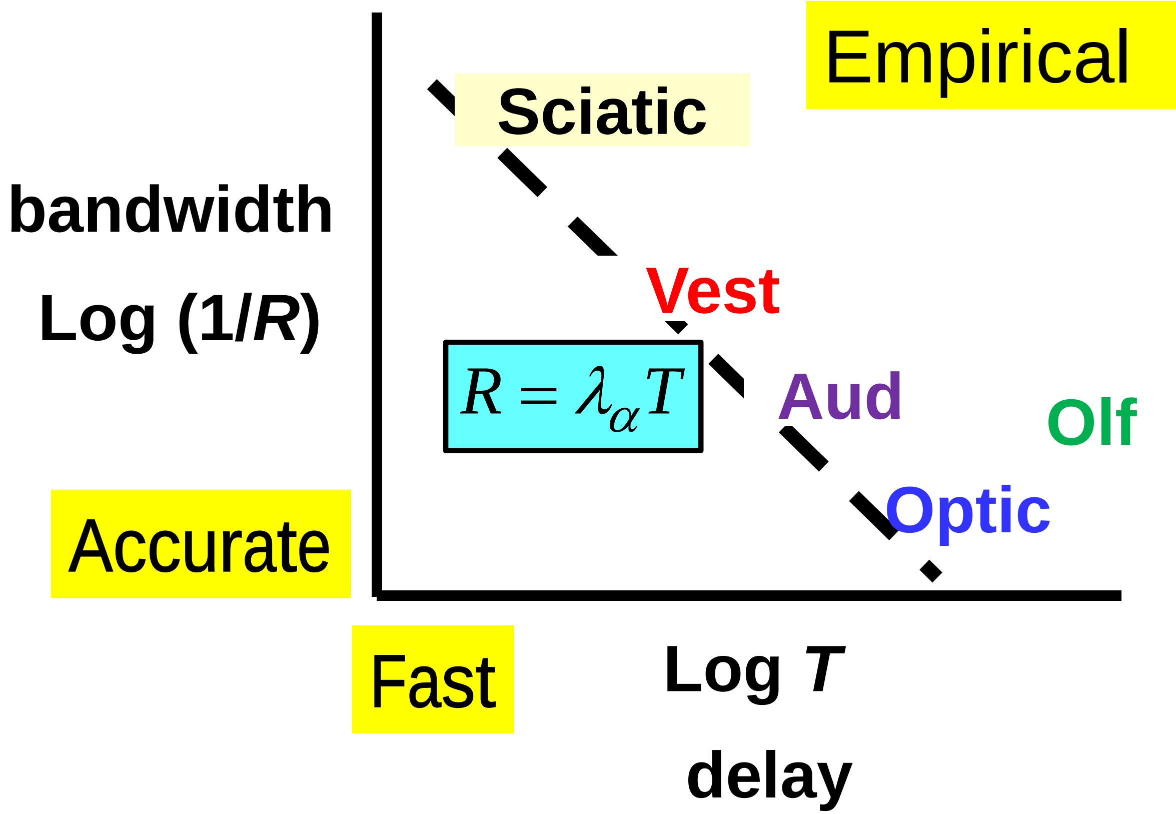




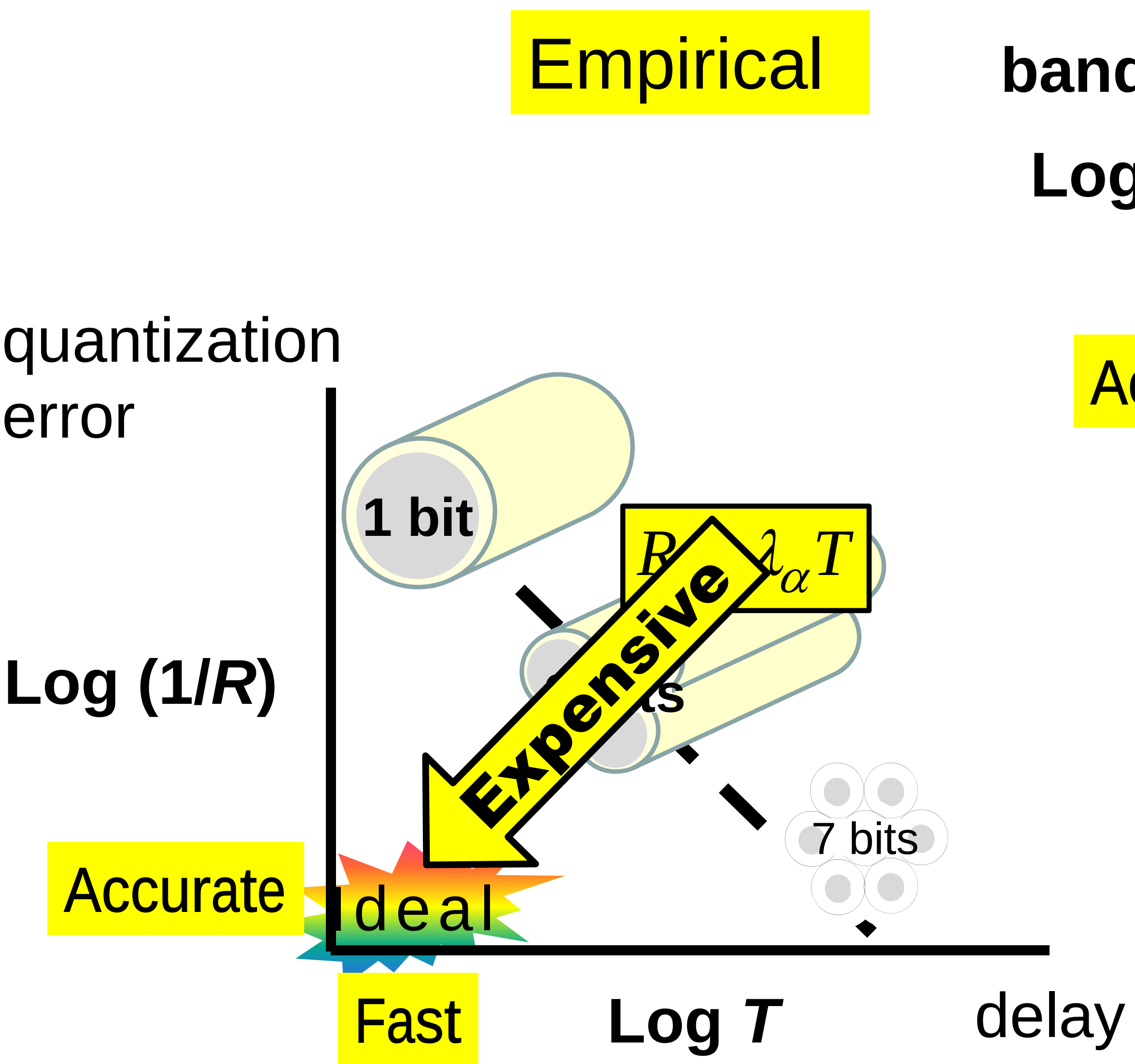
Speed vs Accuracy?
(Delay T vs Bandwidth R?)

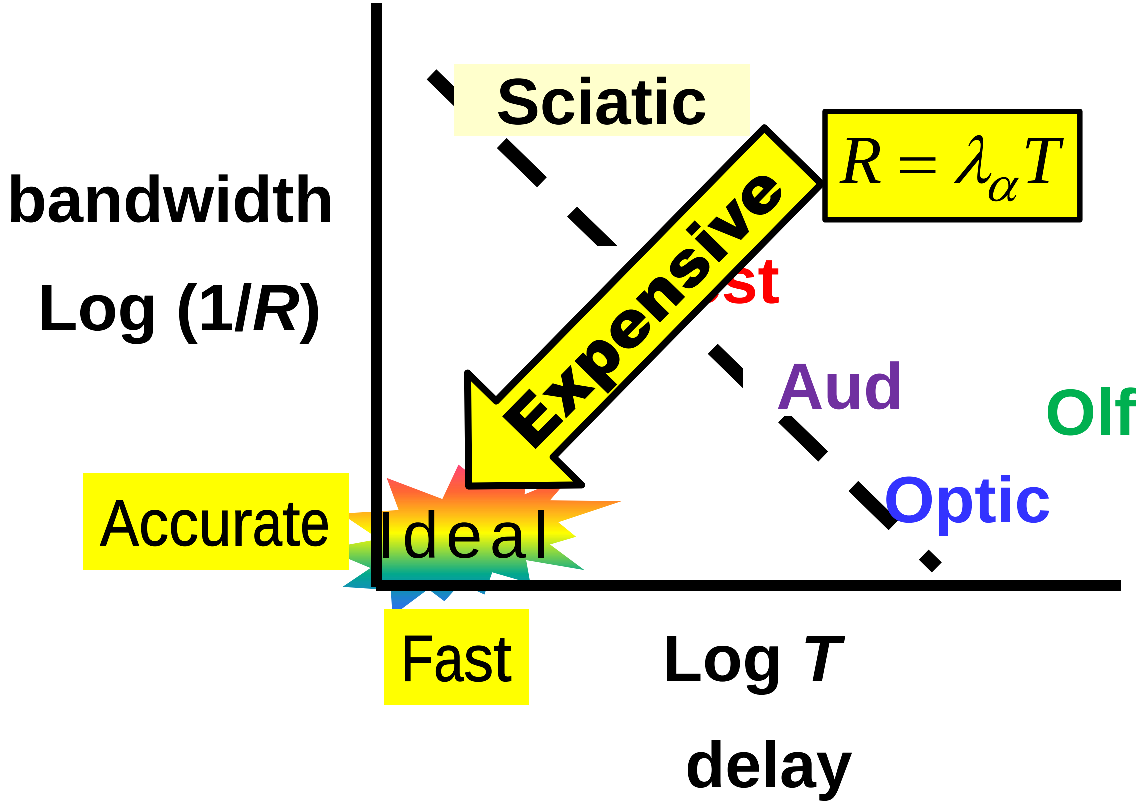
***R*=bits/time**
1/*R* ∝ accuracy



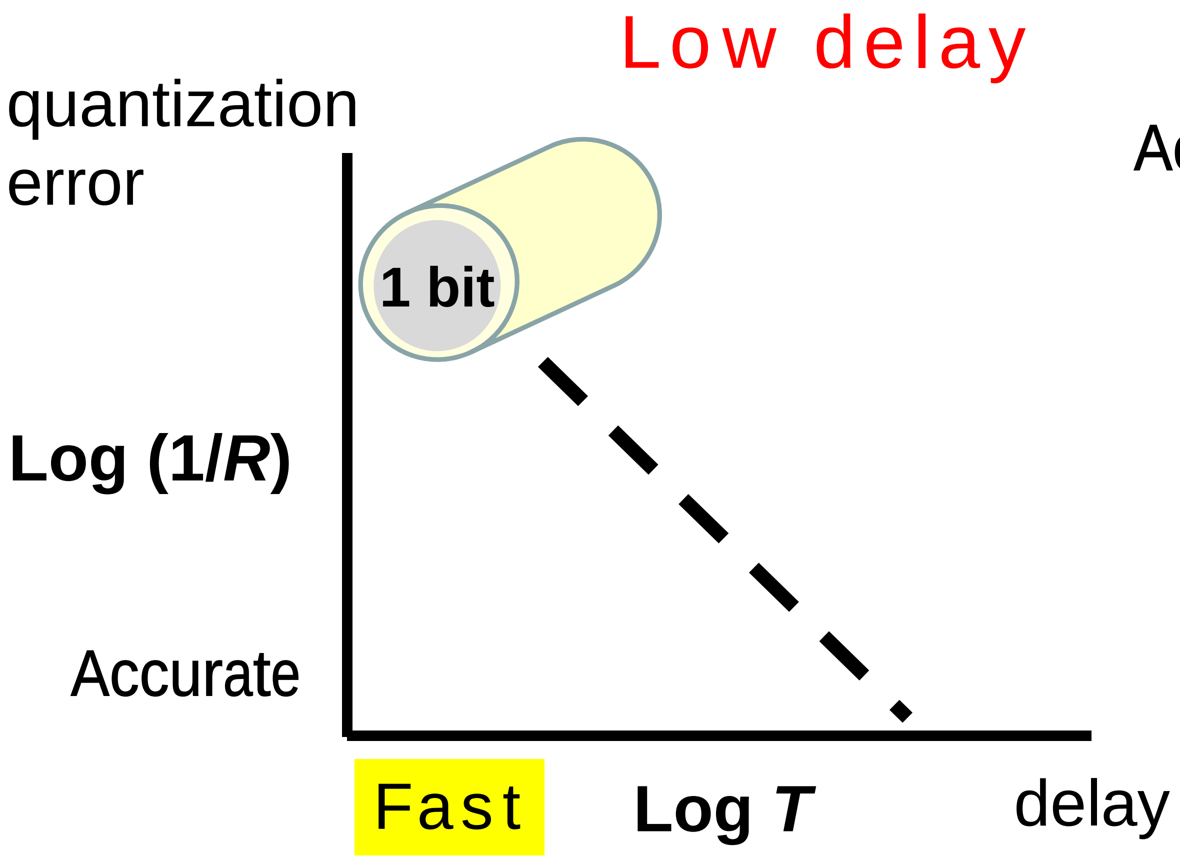


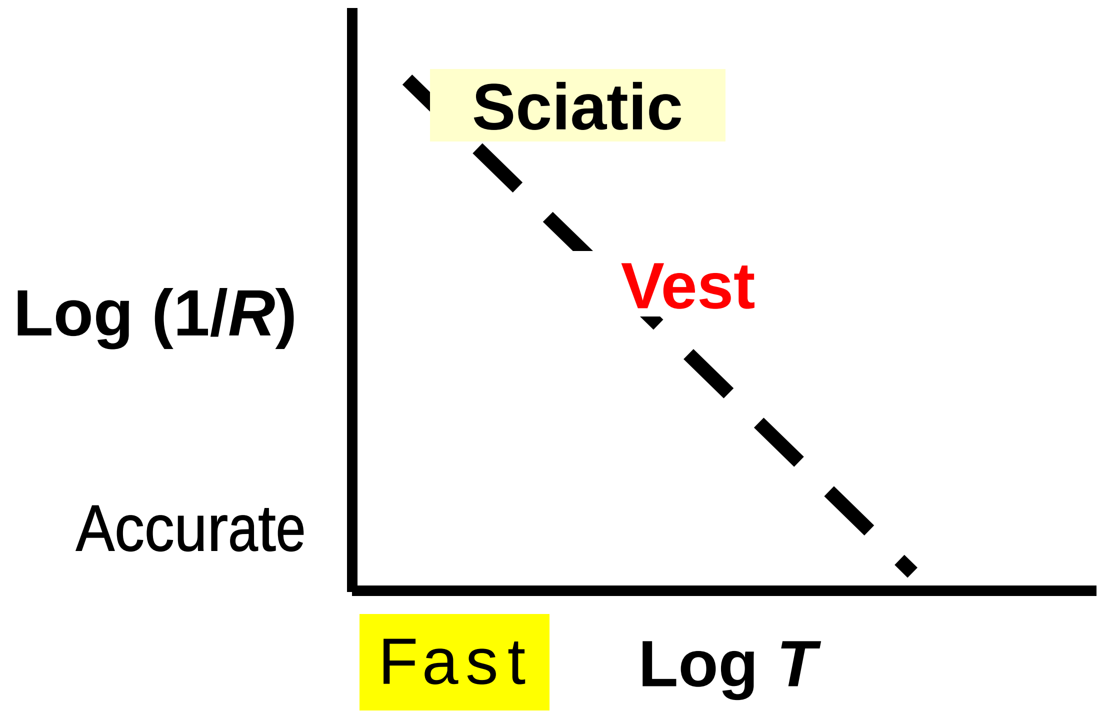
Simon Laughlin
Terry Sejnowski

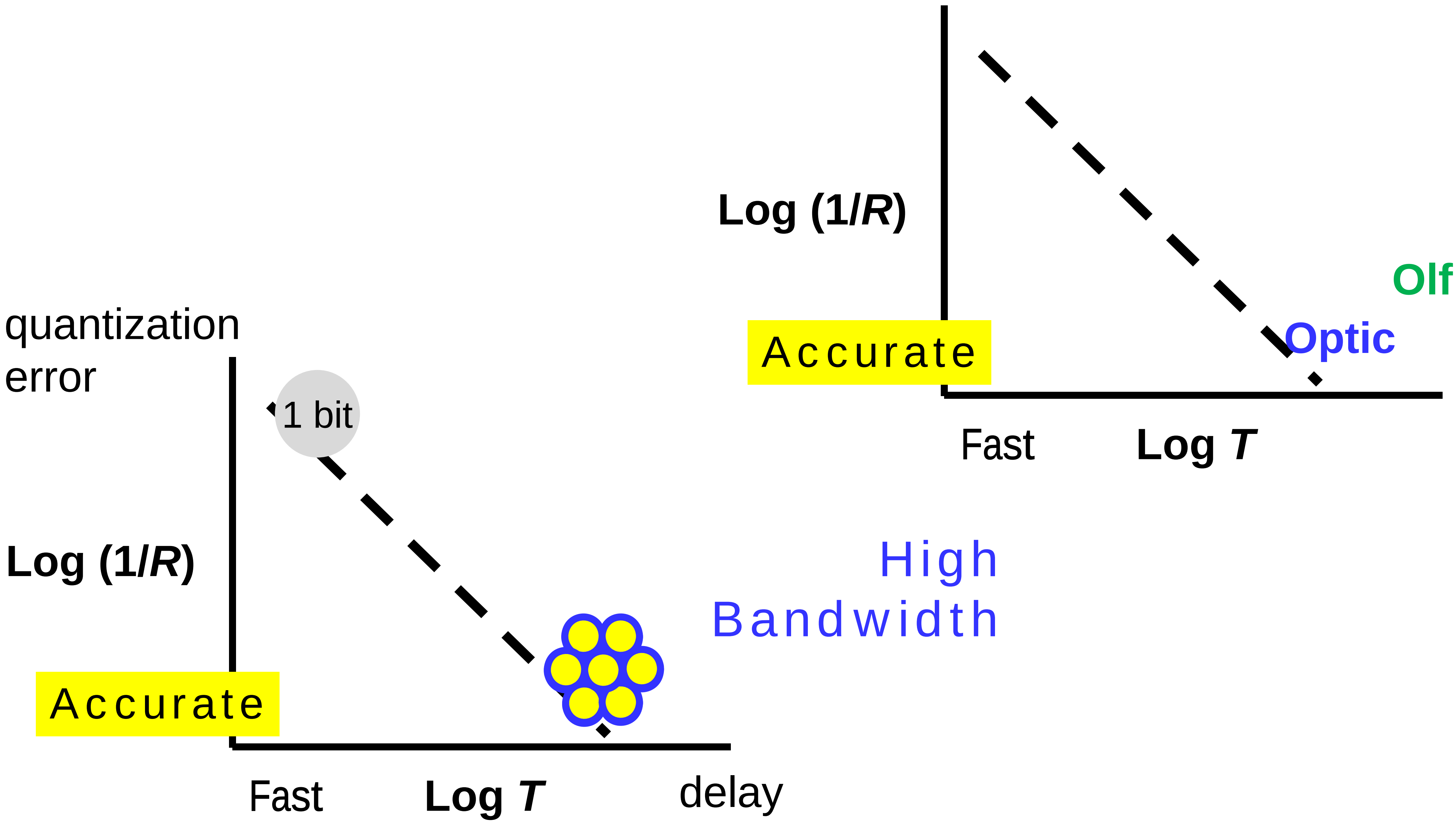

Empirical
quantization error
Log (1/R)
1 bit
Expensive
7 bits
Accurate
Ideal
Fast
Log T
delay

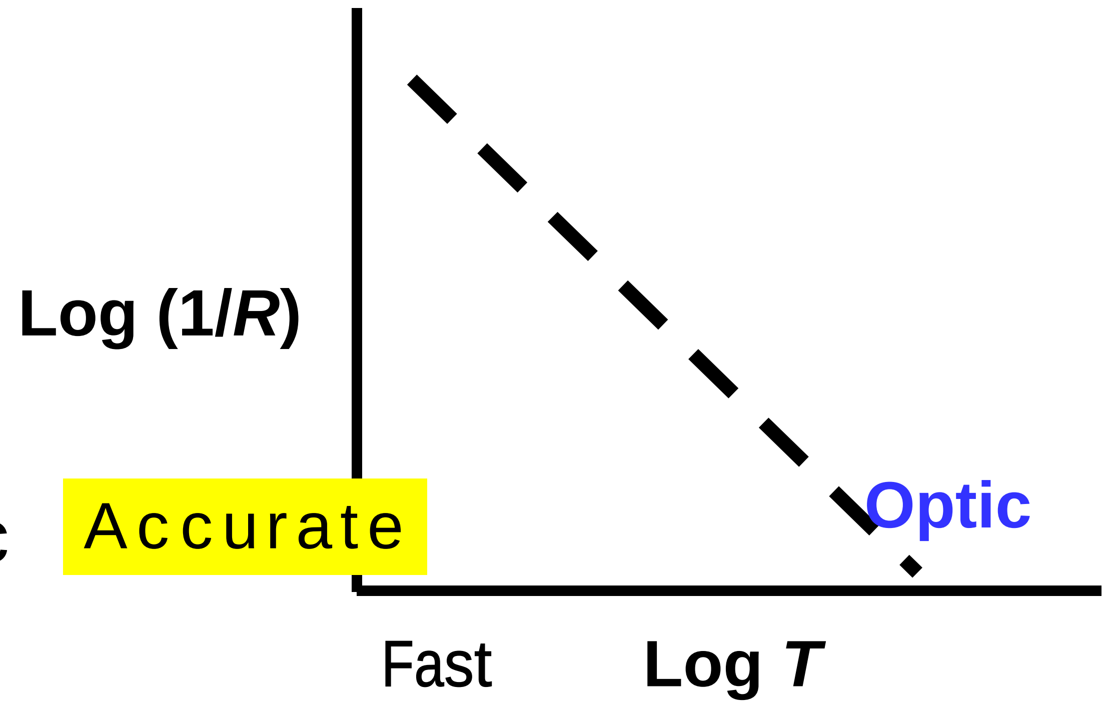

bandwidth
Log (1/R)
Sciatic
$R = \lambda_\alpha T$
Expensive
Aud
Olf
Optic
Accurate
Ideal
Fast
Log T
delay

R=bits/time


Low delay
quantization
error
1 bit
Log (1/R)
Accurate
Fast
Log T
delay


Sciatic
Vest
Log (1/R)
Accurate
Fast
Log T


Log (1/R)
Olf
Optic
Accurate
Fast
Log T
quantization
error
1 bit
Log (1/R)
High
Bandwidth
Accurate
Fast
Log T
delay



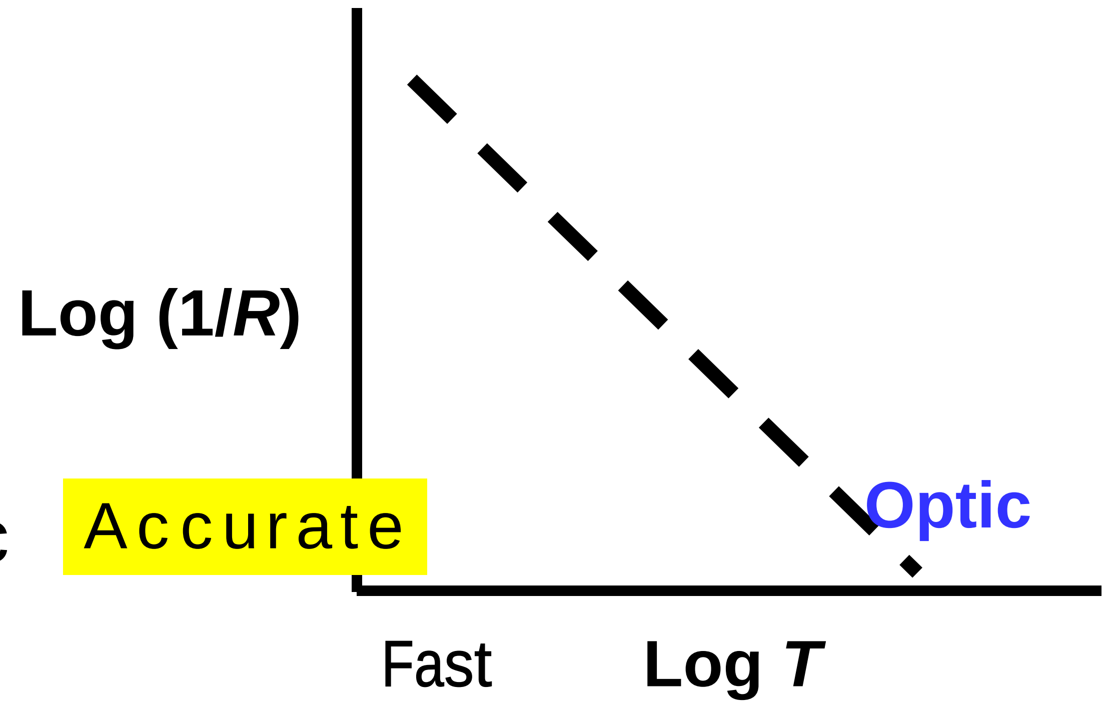
R=10Mb/sec

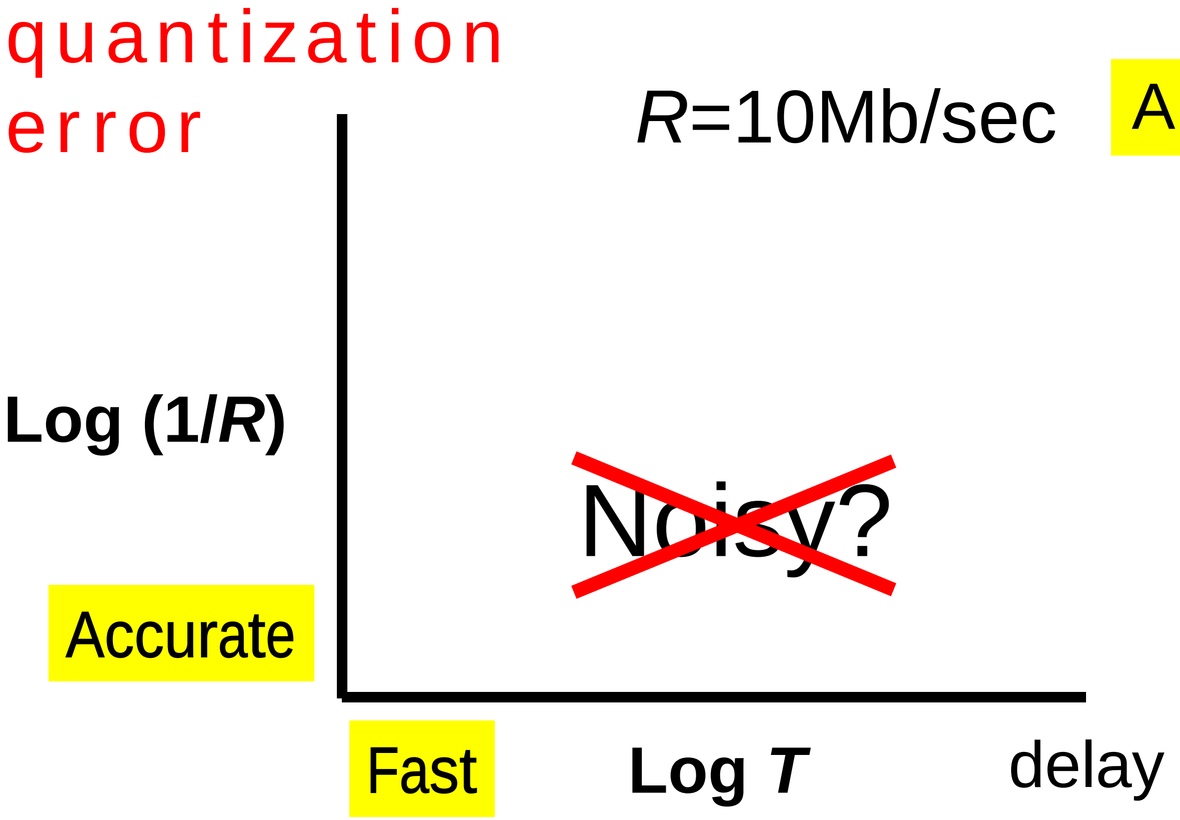
Accurate

50ms<*T*<300ms

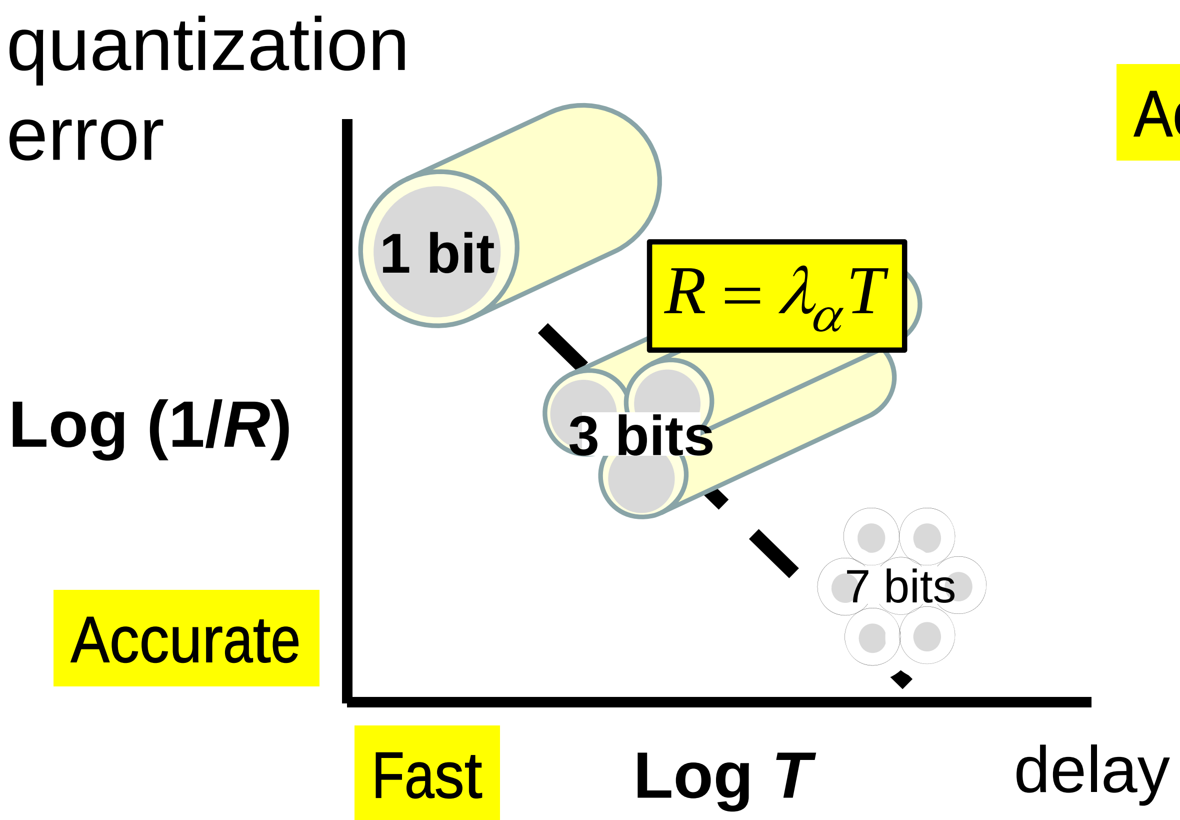
~~Noisy?~~

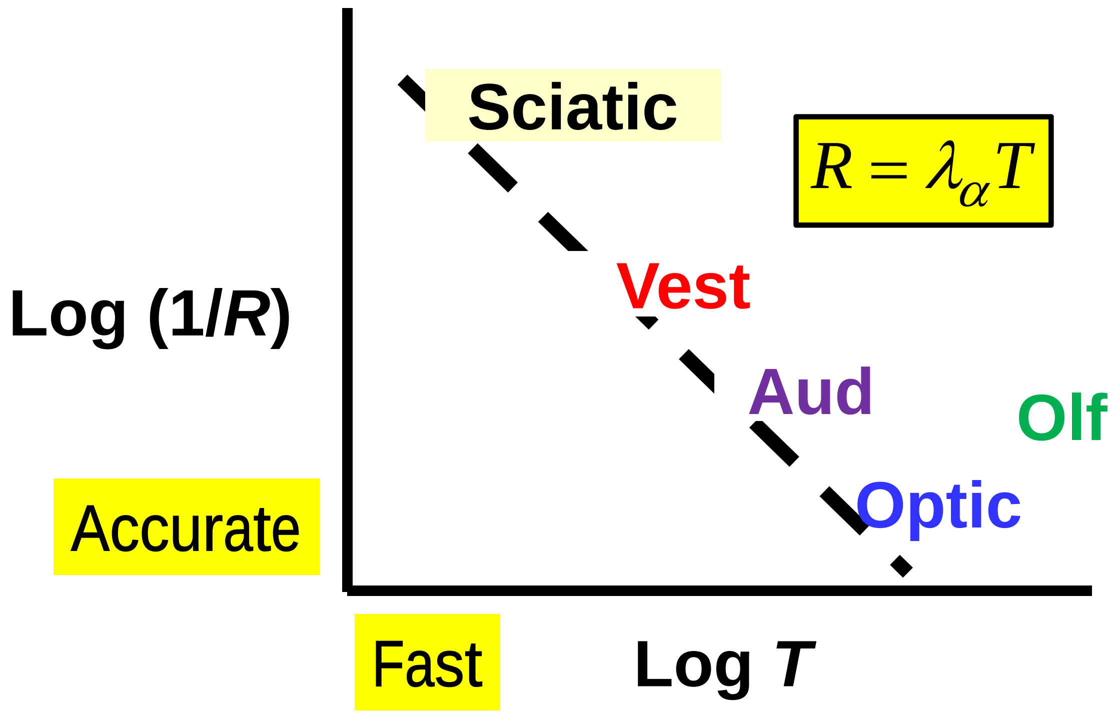
By Jeff Dahl - Own work by uploader, Based on the public domain document: [1], CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=4262200



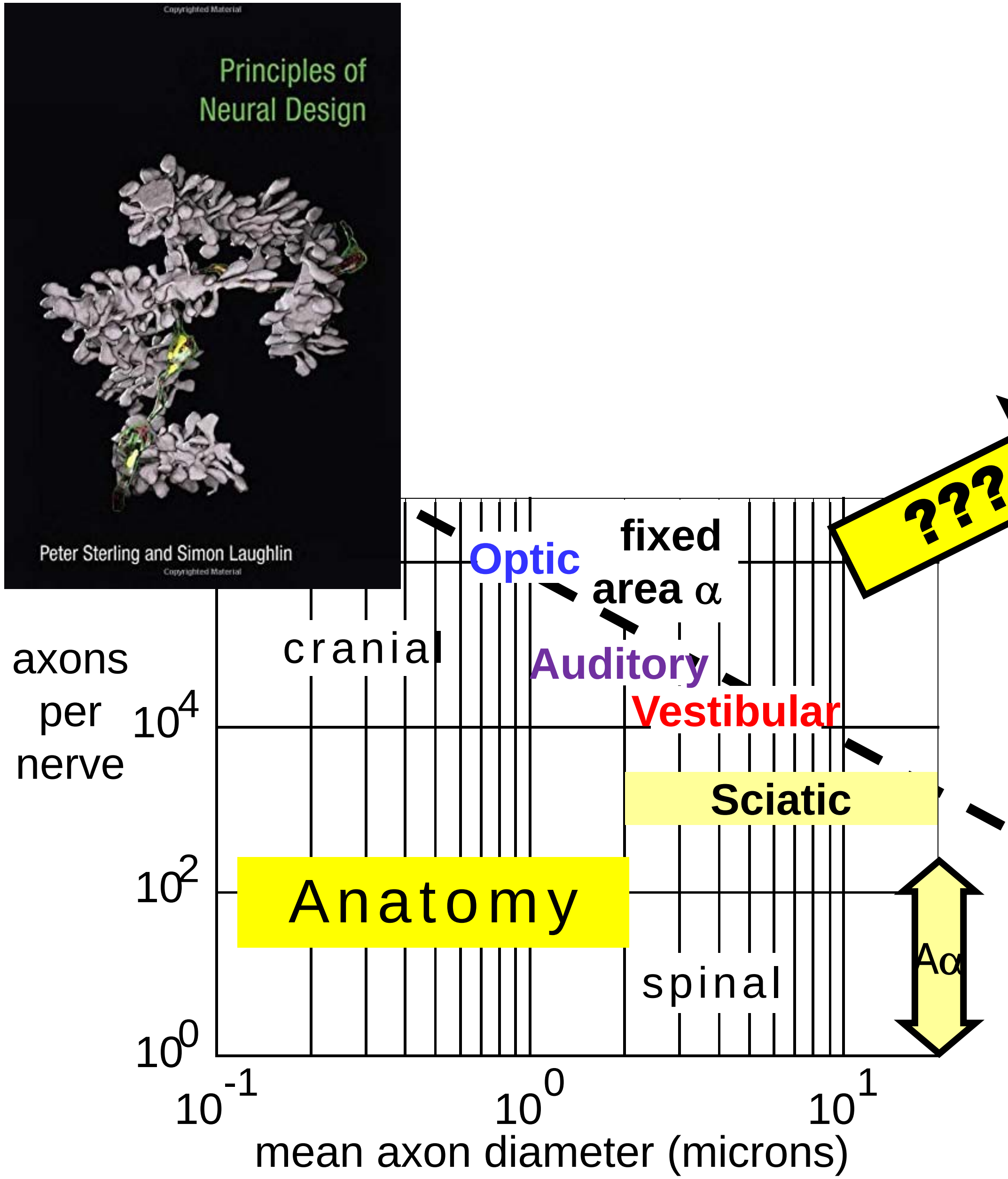


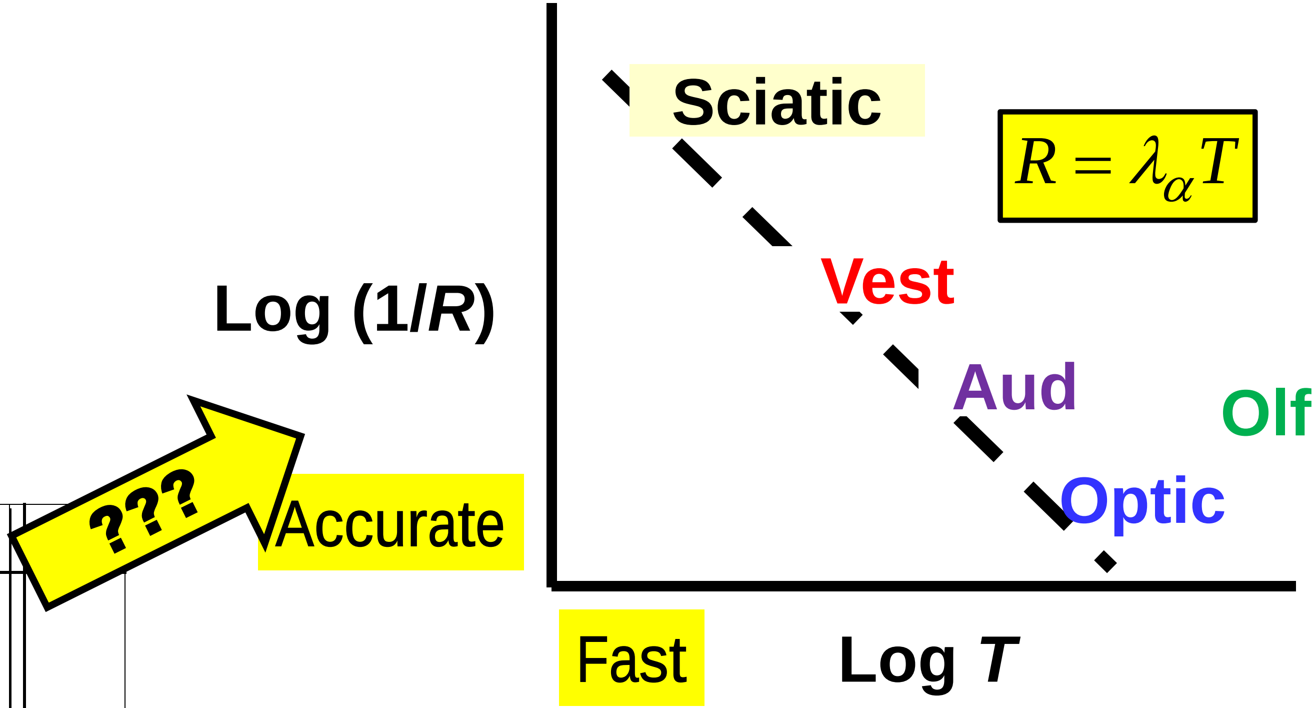
Focus on
nerve diversity
and R vs T





Focus on
nerve diversity
and R vs T





How would Laughlin "explain" this?

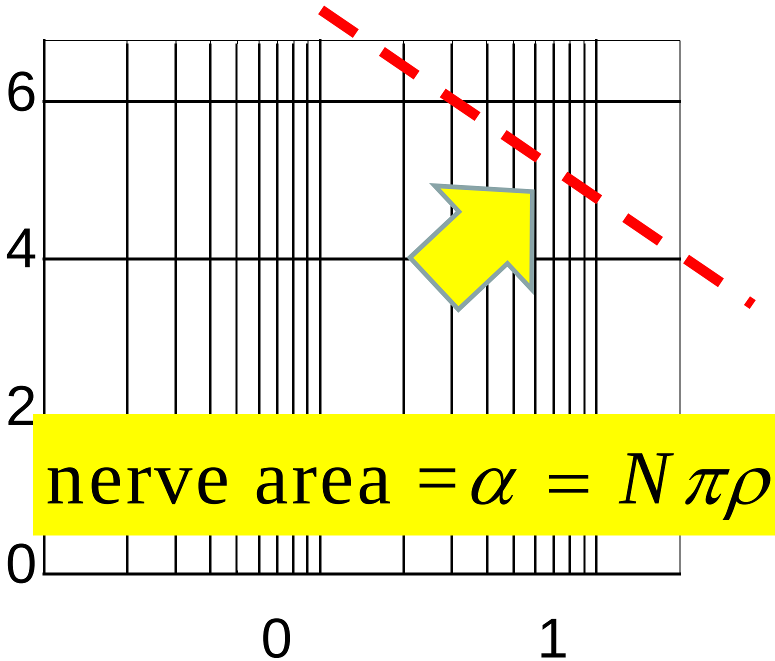
spike speed $\propto \rho$

(propagation)

$$\frac{1}{T} = \frac{1}{delay} \propto \rho$$

resource (cost) ∝ area α

N= axons per nerve



$$\text{nerve area} = \alpha = N \pi \rho^2$$

ρ= axon radius (μm)

Tradeoffs in spiking neurons

$$\text{spike rate} \propto \rho$$

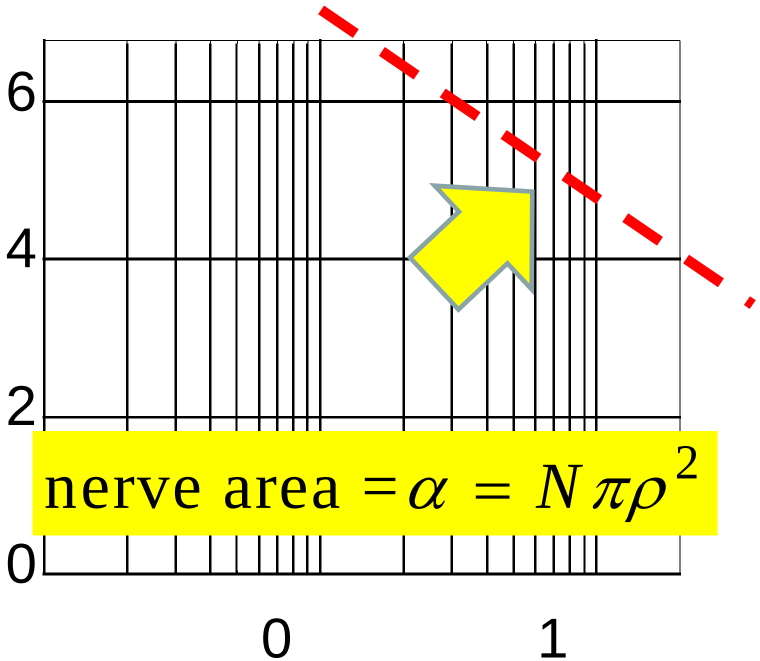
$$\text{spike speed} \propto \rho$$

(propagation)

$$\frac{1}{T} = \frac{1}{delay} \propto \rho$$

resource (cost) ∝ area α

N= axons per nerve



$$\text{nerve area} = \alpha = N\pi\rho^2$$

ρ= axon radius (μm)

Tradeoffs in spiking neurons

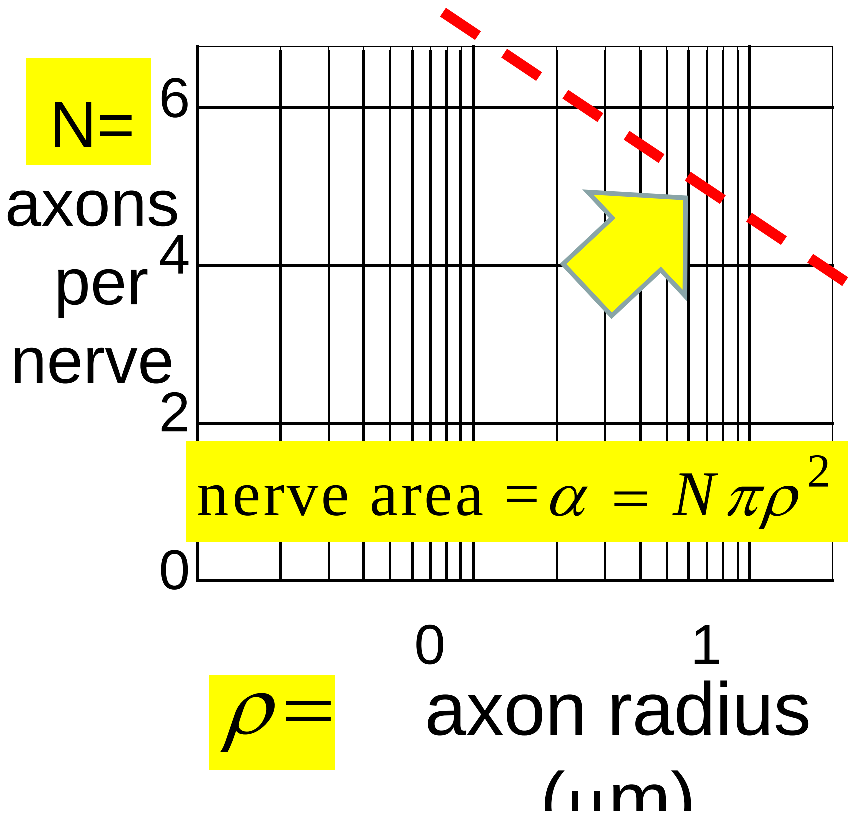
spike rate $\propto \rho$

spike speed $\propto \rho$

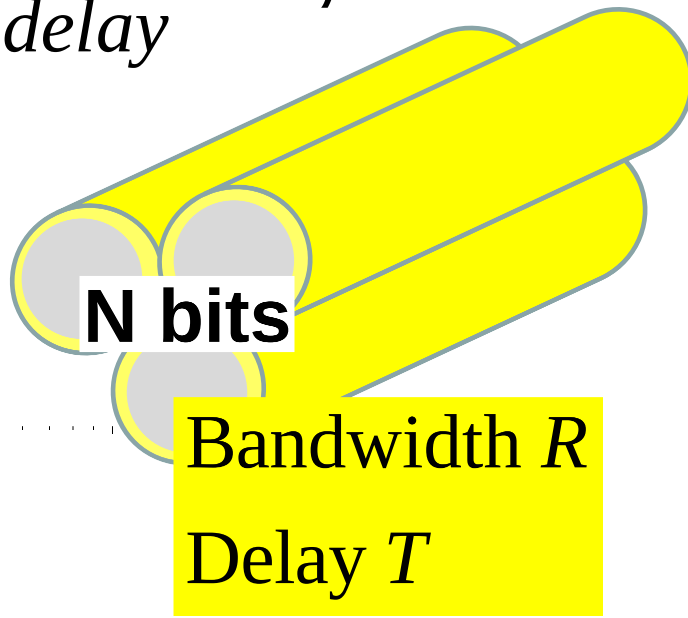
bandwidth R (bits/time) $\propto N\rho$

(propagation)

$$\frac{1}{T} = \frac{1}{delay} \propto \rho$$



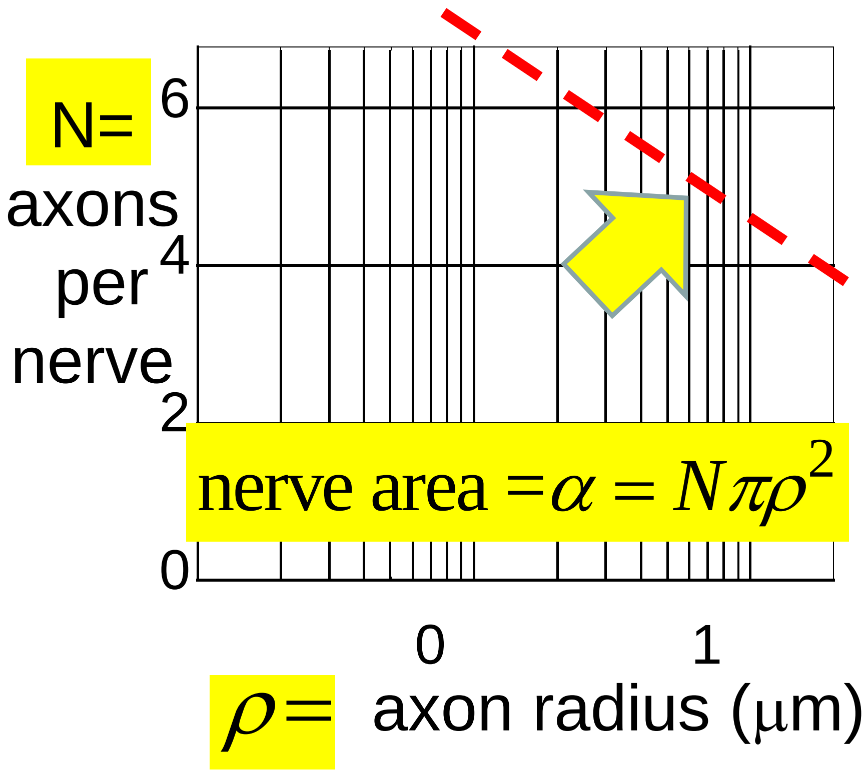
Digital
tradeoffs
in
spiking
neurons



Terry Sejnowski
Simon Laughlin

$$\text{spike rate} \propto \rho$$

$$\text{bandwidth } R \text{ (bits/time)} \propto N\rho$$



$$N \propto \frac{\alpha}{\rho^2}$$

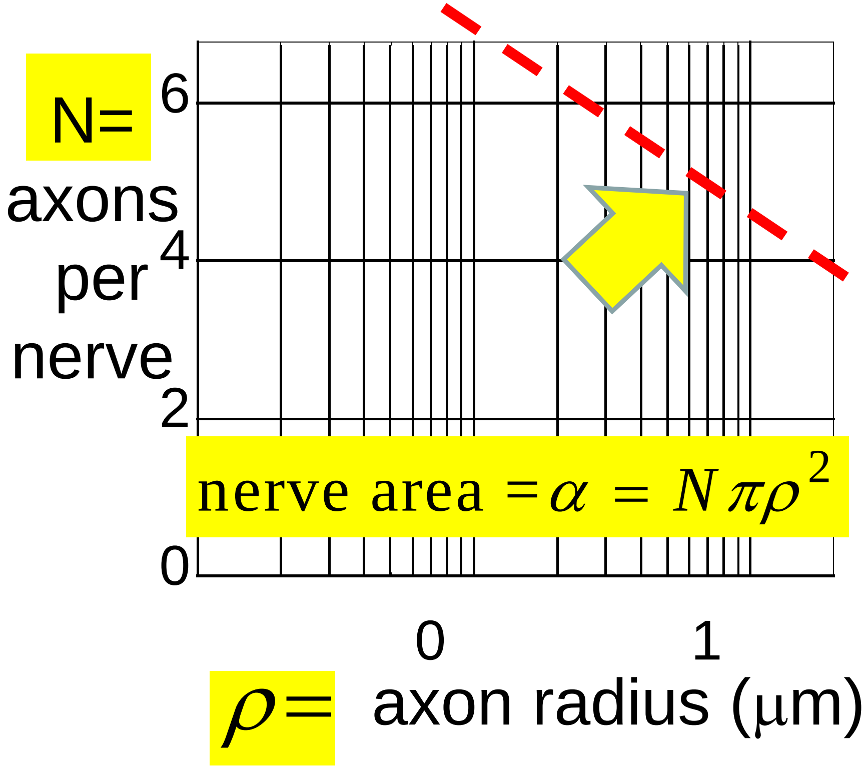
$$R \propto \left(\frac{\alpha}{\rho^2}\right)\rho \propto \left(\frac{\alpha}{\rho}\right)$$

spike rate $\propto \rho$

R (bits/time) $\propto N\rho$

spike speed $\propto \rho$

$$\therefore \text{delay } T_s \propto \frac{1}{\rho}$$



$$R \propto \left(\frac{\alpha}{\rho^2}\right)\rho \propto \left(\frac{\alpha}{\rho}\right) \propto \alpha T$$

$$\therefore R = \lambda_\alpha T$$

$$\lambda_\alpha \propto \alpha$$

spike rate $\propto \rho$

R (bits/time) $\propto N\rho$

spike speed $\propto \rho$

$$\therefore \text{delay } T_s \propto \frac{1}{\rho}$$

$T = 1$

$R = 1$

$T = \sqrt{3}$

$R = \sqrt{3}$

$$\therefore R = \lambda_\alpha T$$

$$\lambda_\alpha \propto \alpha$$

spike rate $\propto \rho$

R (bits/time) $\propto N\rho$

spike speed $\propto \rho$

$\therefore$ delay $T_s \propto \dfrac{1}{\rho}$

$T = 1$

$R = 1$

$T = \sqrt{3}$

$R = \sqrt{3}$

$$\therefore R = \lambda_\alpha T$$

$$\lambda_\alpha \propto \alpha$$

spike rate $\propto \rho$

R (bits/time) $\propto N\rho$

spike speed $\propto \rho$

$\therefore$ delay $T_s \propto \dfrac{1}{\rho}$

“packet switching”
by
“population codes”

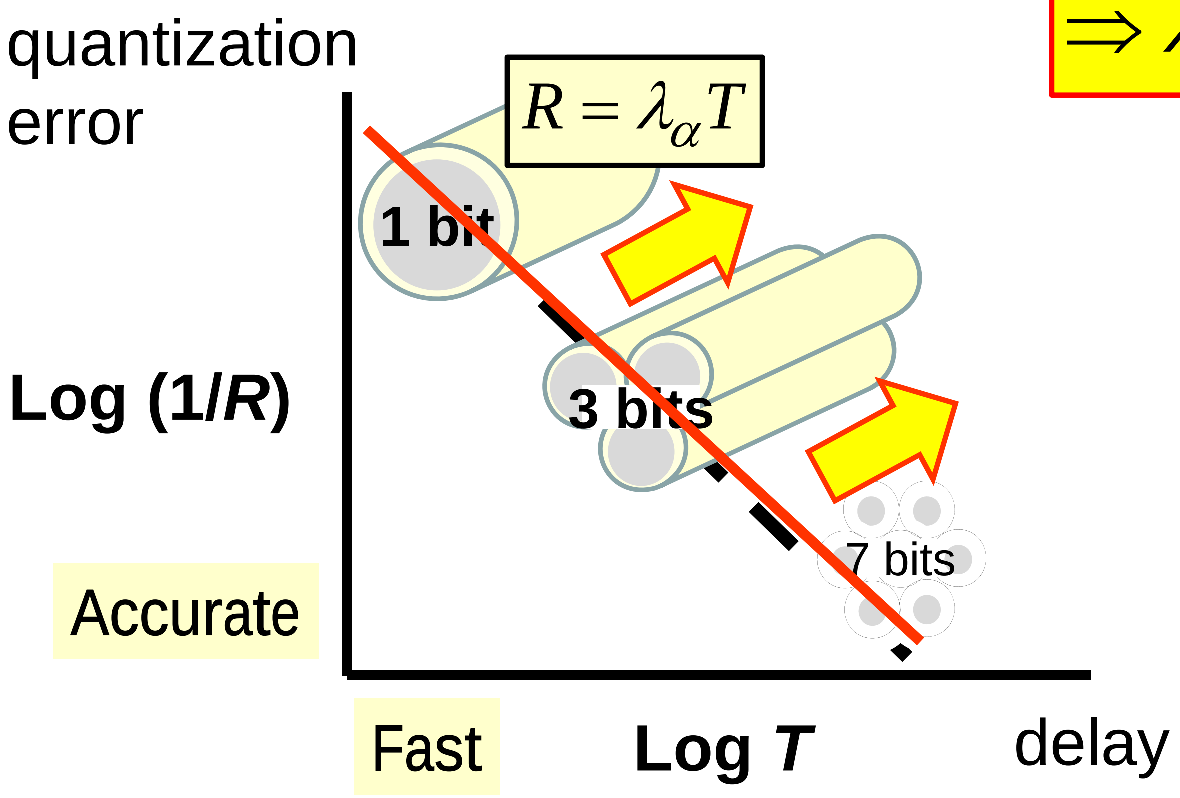
$T = \sqrt{3}$

$R = \sqrt{3}$

$$\therefore R = \lambda_\alpha T$$

$$\lambda_\alpha \propto \alpha$$

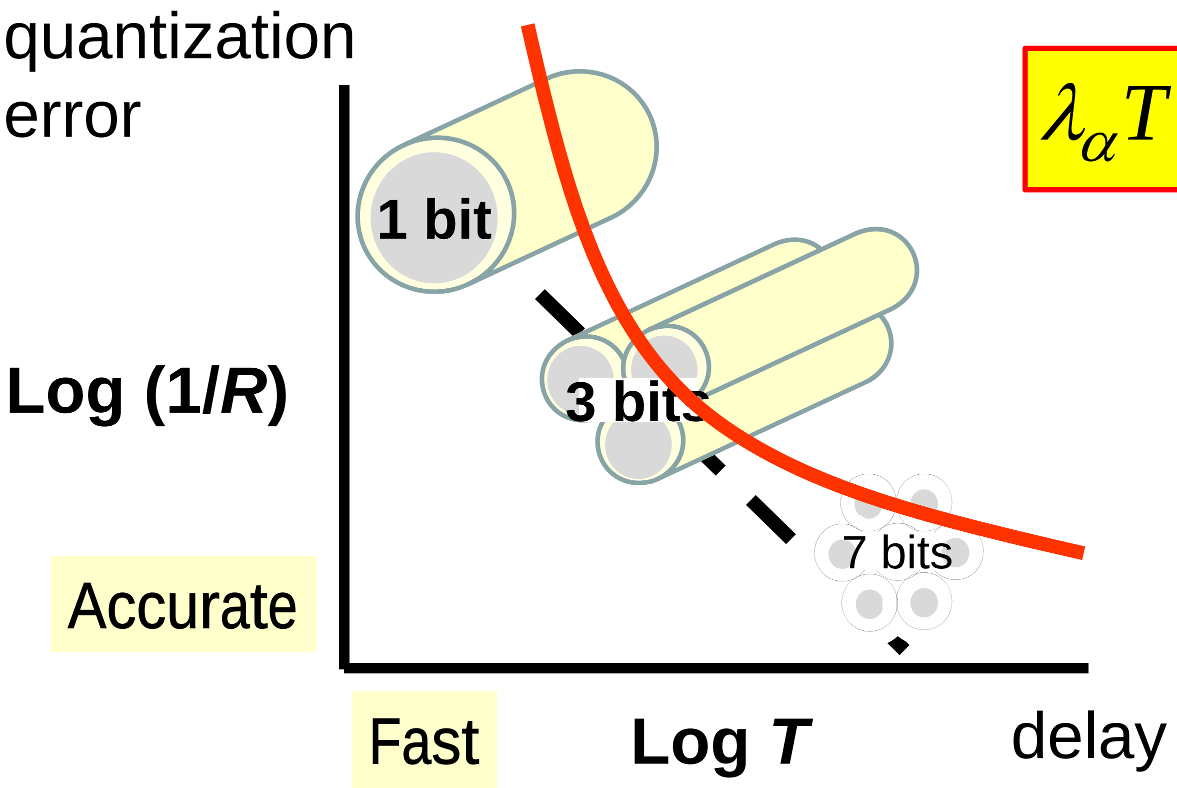
Easy to add bit errors and
error correcting codes



$\Rightarrow \lambda_\alpha$ reduced

≈ reducing
area

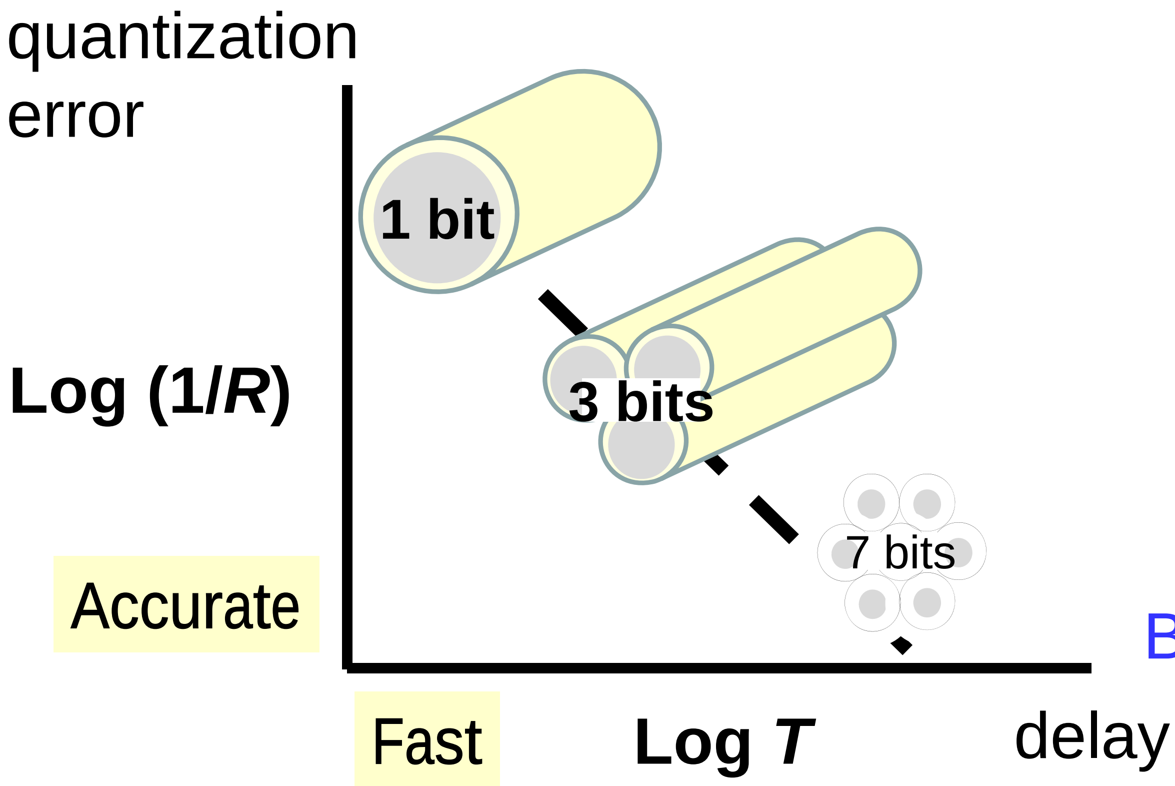
Easy to use other tradeoffs



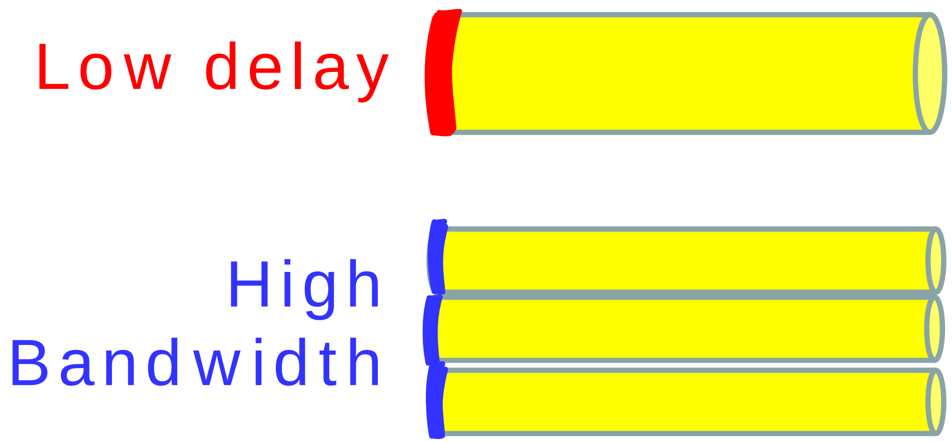
$$\lambda_\alpha T \Rightarrow \lambda_\alpha(T)$$

Start with this for simplicity

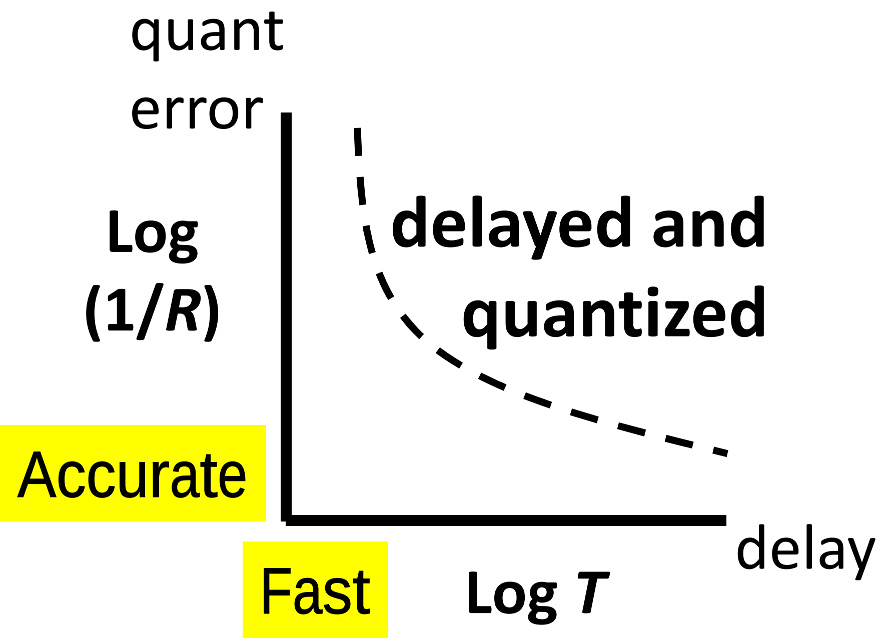
$$R = \lambda_\alpha T$$



"delayed and quantized"

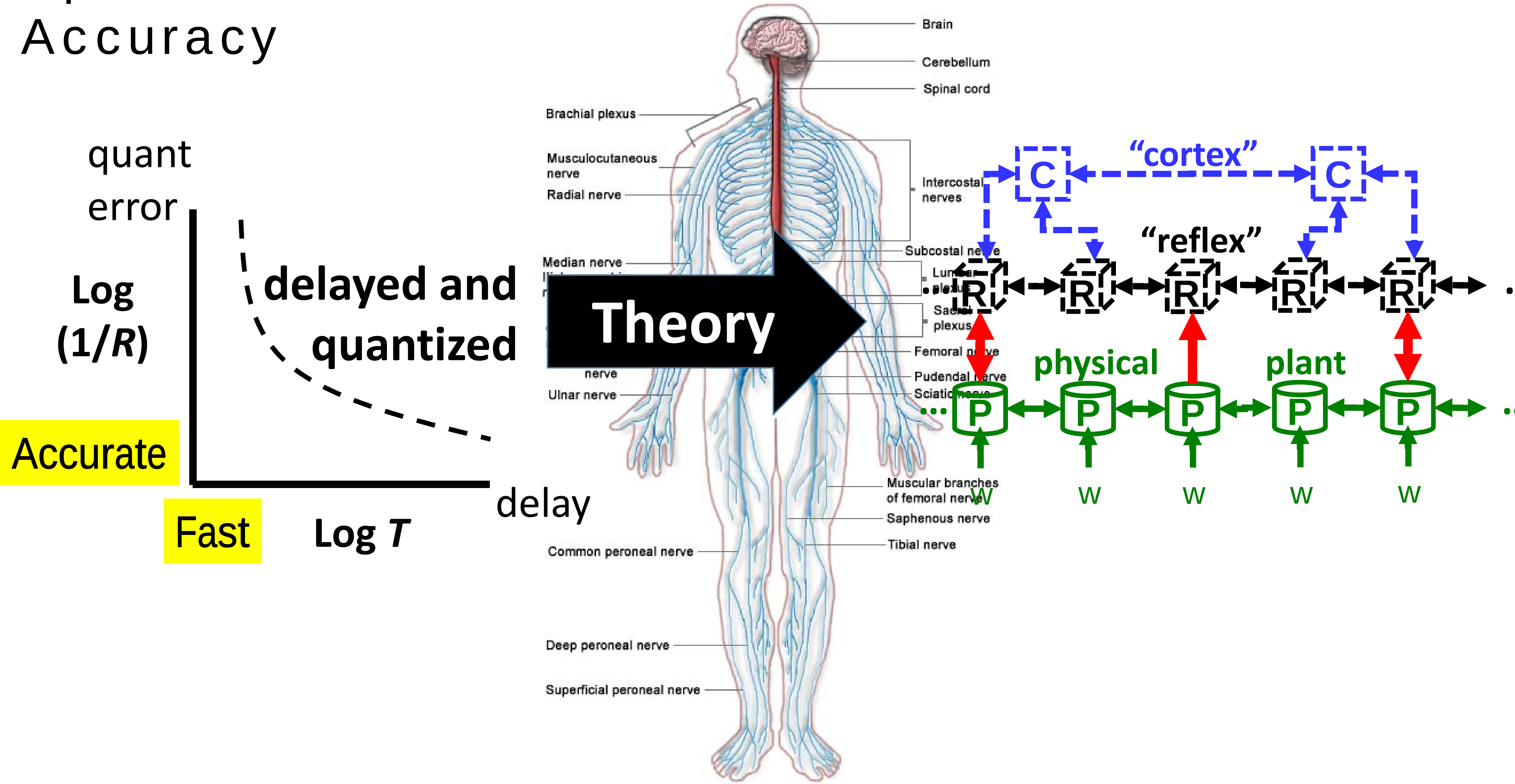


Speed vs Accuracy



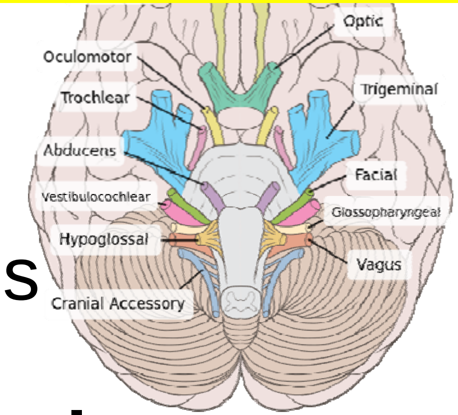
Different technologies will have different tradeoffs

Speed vs Accuracy



Extremely diverse/different (& complementary?)

Nerves



quant error

Log (1/*R*)

$$R = \lambda_\alpha T$$

Hardware

Component

Accurate

Fast

Log *T*

delay

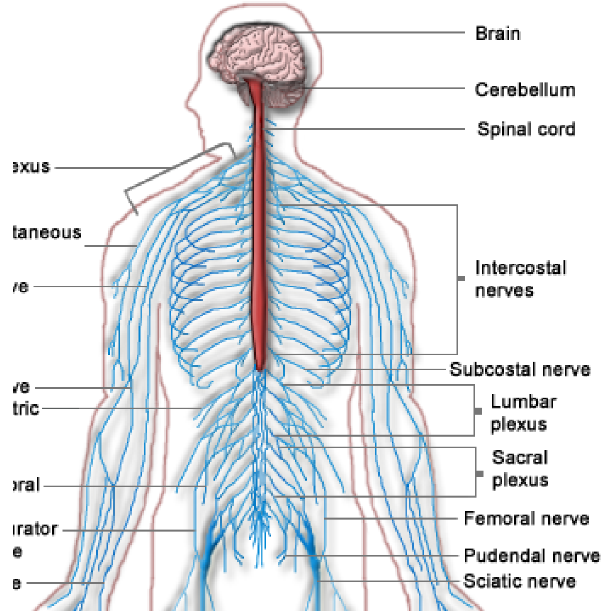
High

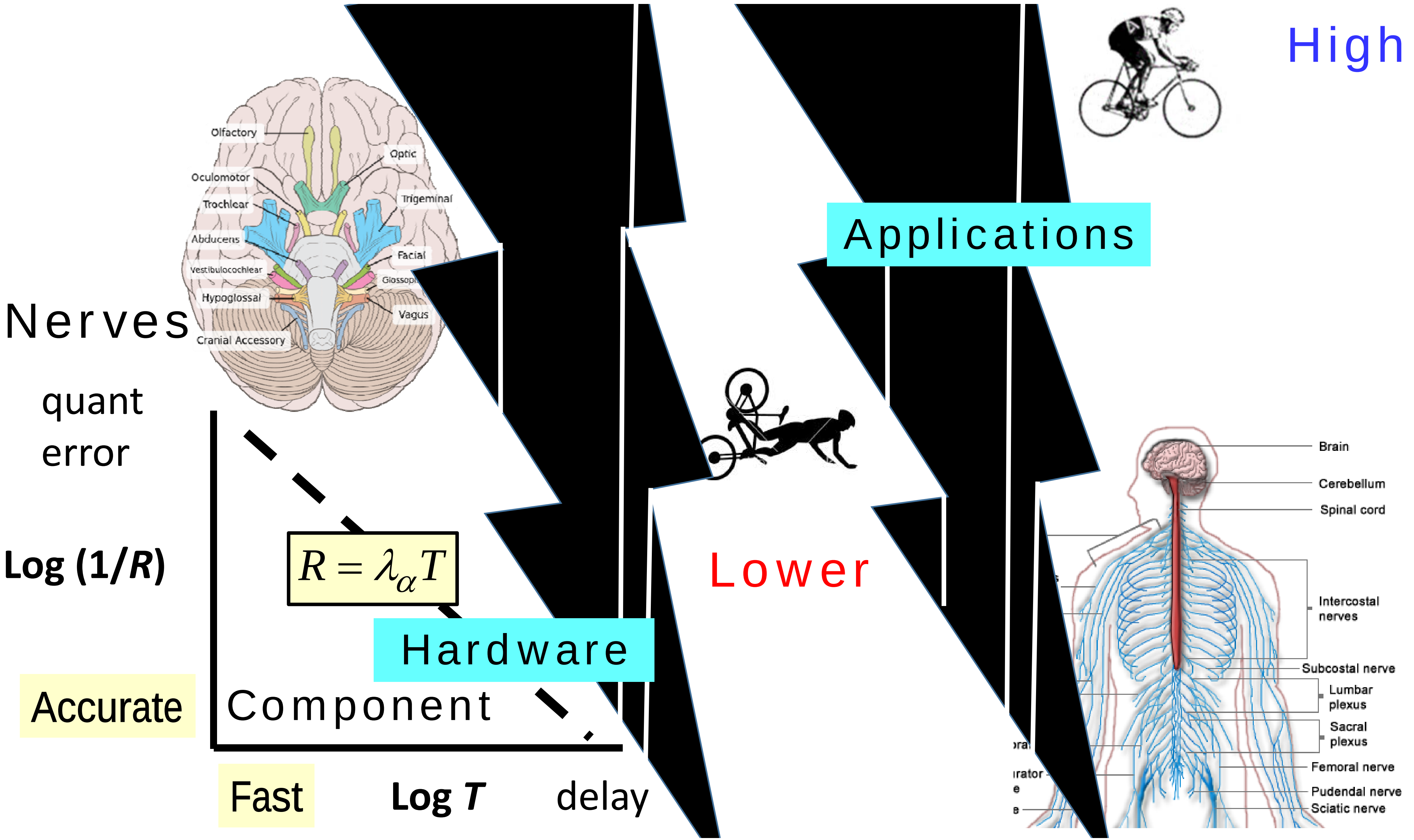
advanced planning
large disturbance
small error
need accuracy

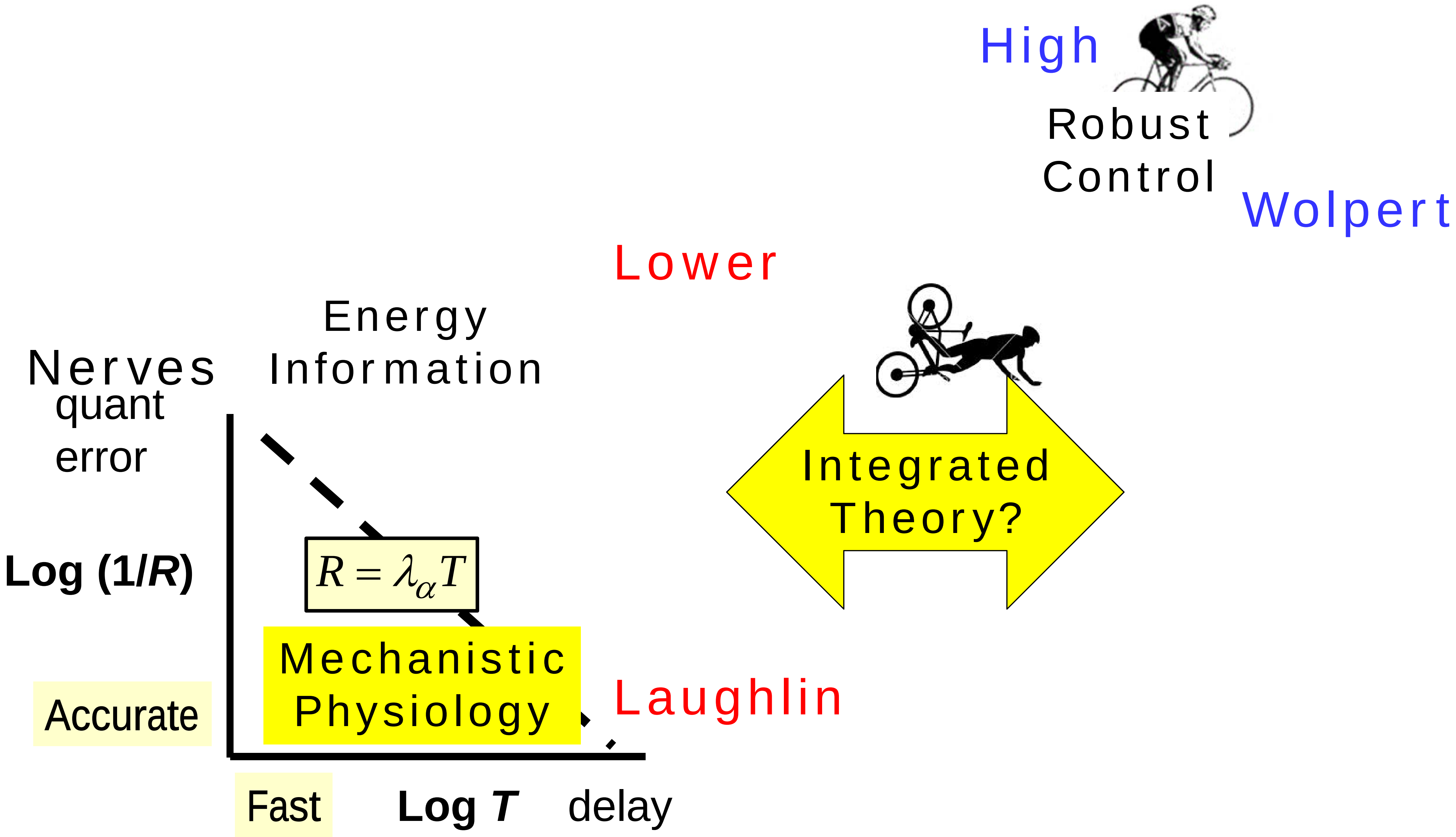
Applications

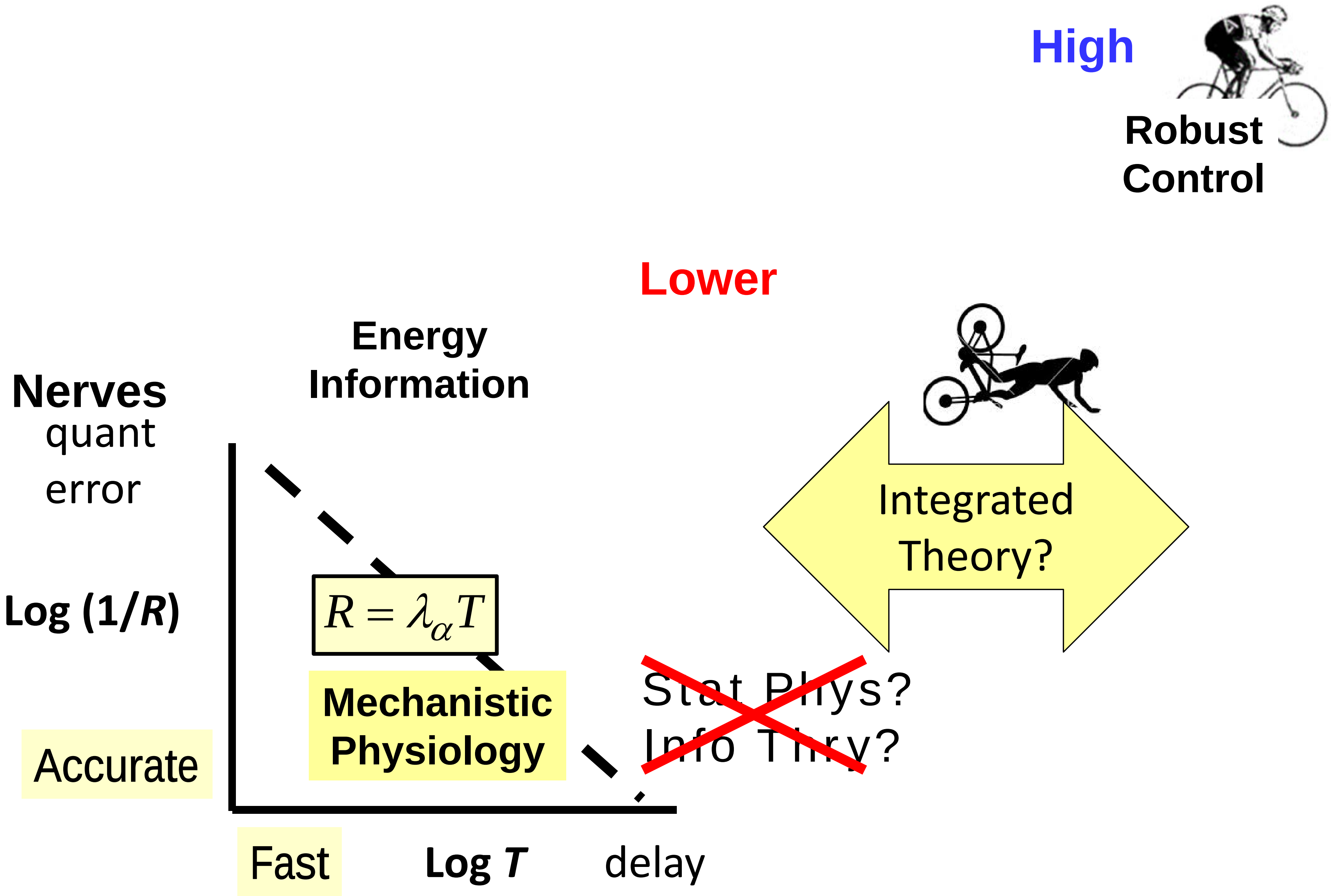
Lower

delayed reflexes
small disturbance
large error
need speed

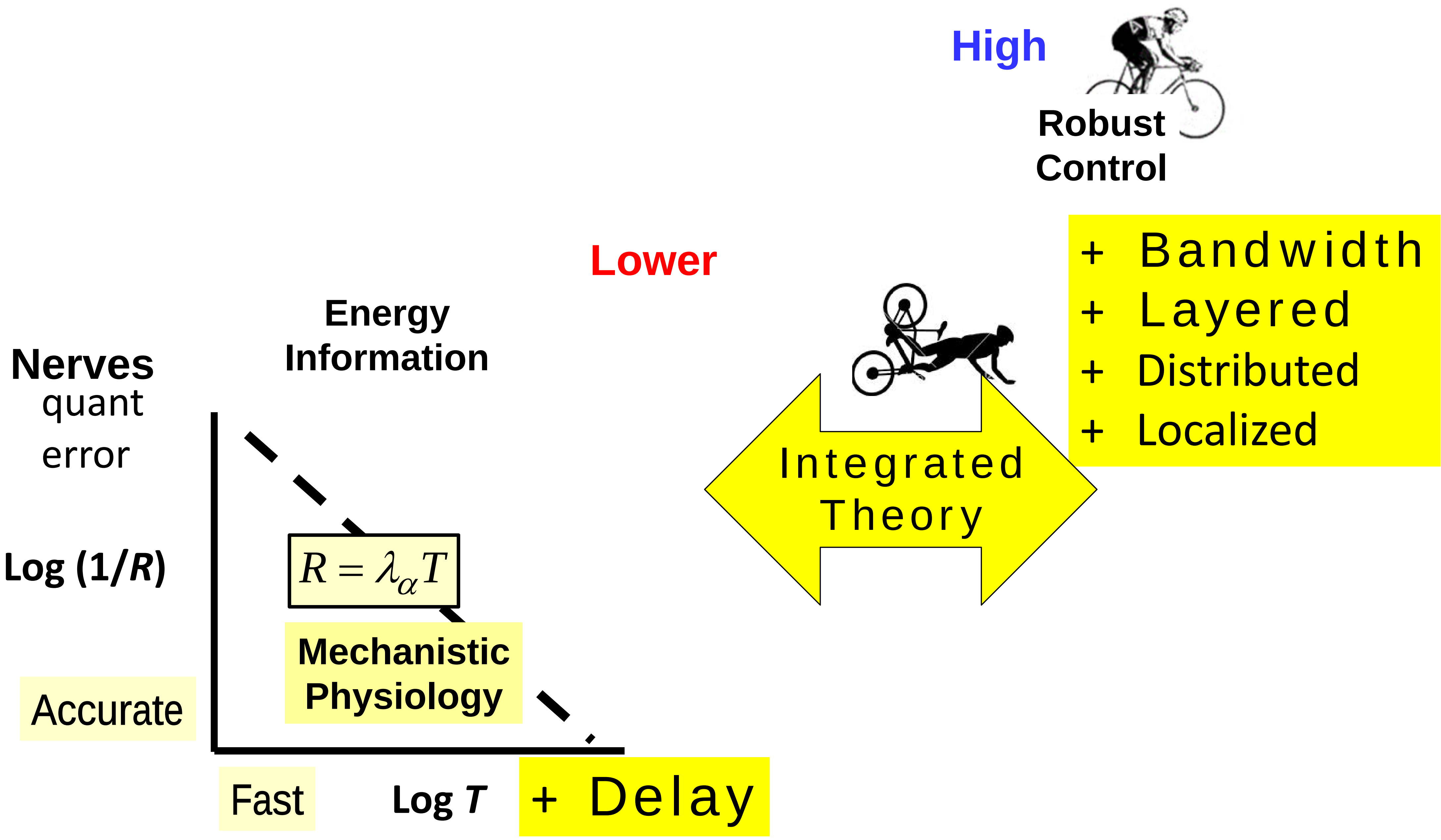



High
Applications
Lower
Nerves
quant
error
Log (1/R)
Accurate
$R = \lambda_\alpha T$
Hardware
Component
Fast
Log T
delay
Olfactory
Optic
Oculomotor
Trochlear
Trigeminal
Abducens
Facial
Vestibulocochlear
Hypoglossal
Vagus
Cranial Accessory
Brain
Cerebellum
Spinal cord
Intercostal nerves
Subcostal nerve
Lumbar plexus
Sacral plexus
Femoral nerve
Pudendal nerve
Sciatic nerve


High
Robust
Control
Wolpert
Lower
Energy
Information
Nerves
quant
error
Log (1/R)
$R = \lambda_\alpha T$
Mechanistic
Physiology
Accurate
Laughlin
Fast
Log T
delay
Integrated
Theory?



High
Robust
Control
Lower
Energy
Information
Nerves
quant
error
Log (1/*R*)
$R = \lambda_\alpha T$
Mechanistic
Physiology
Accurate
Integrated
Theory?
Stat Phys?
Info Thry?
Fast
Log *T*
delay



High
Robust Control
Lower
Energy Information
Nerves
quant error
Log (1/R)
$R = \lambda_\alpha T$
Mechanistic Physiology
Accurate
Fast
Log T
+ Delay
Integrated Theory
+ Bandwidth
+ Layered
+ Distributed
+ Localized

Inspirations from neuroscience

Gazzaniga

High

Robust Control

Wolpert

Sejnowski

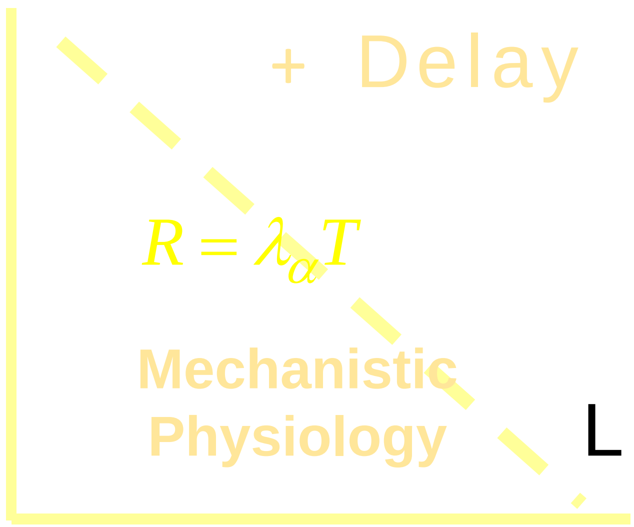
Lower

Energy
Information

+ Bandwidth
+ Layered
+ Distributed
+ Localized

Nerves
quant
error

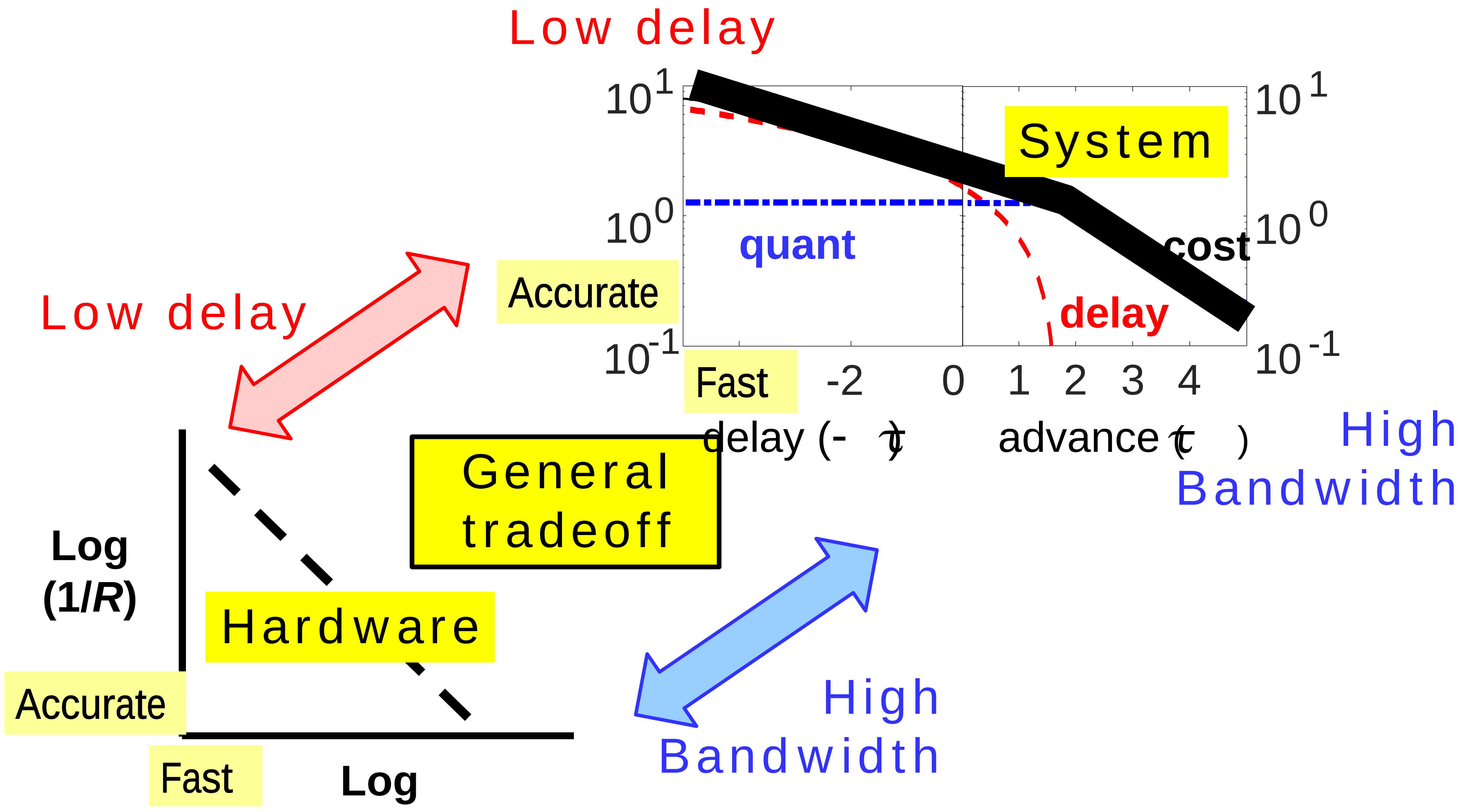
+ Delay



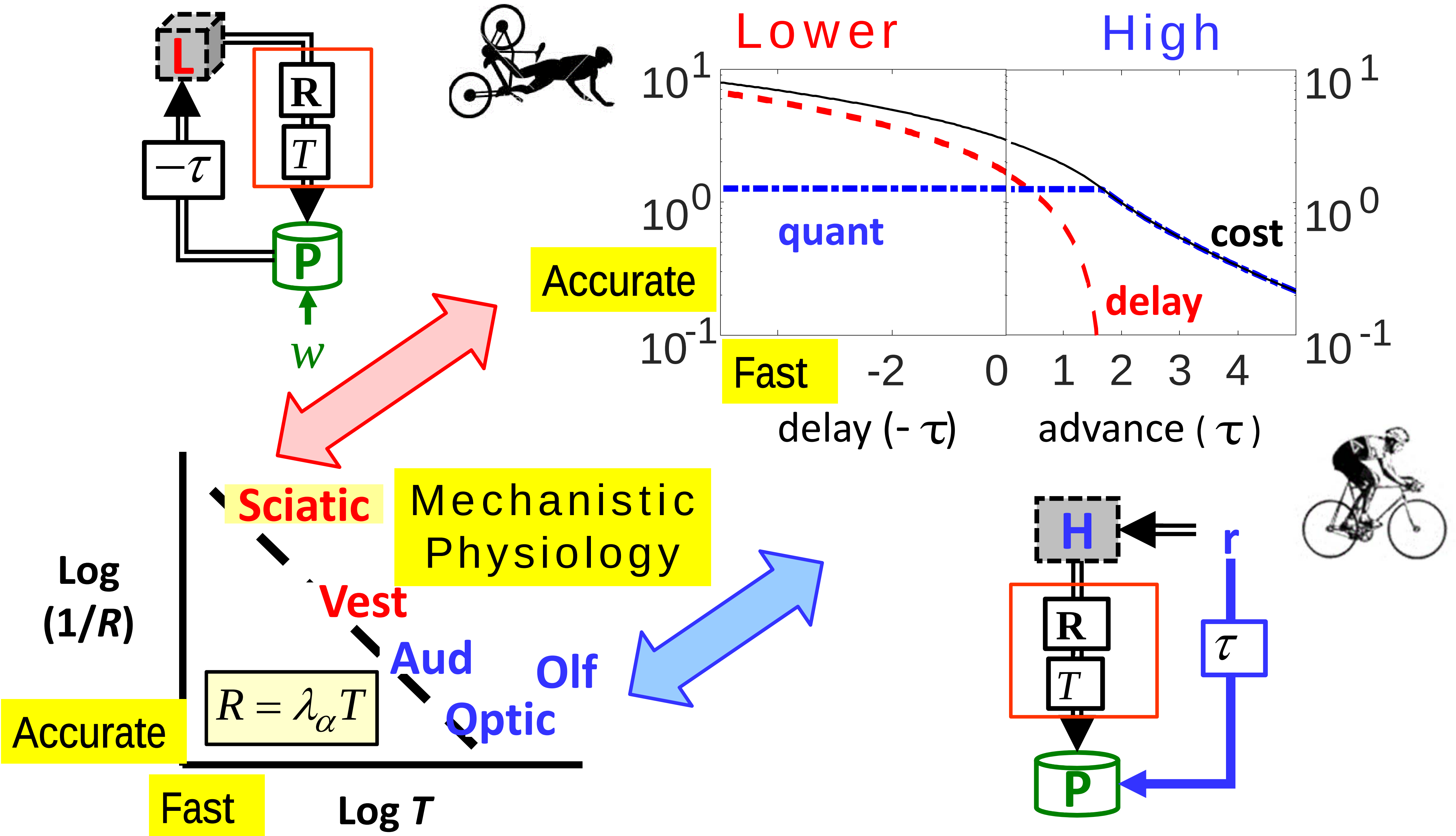
Integrated
Theory

Marder

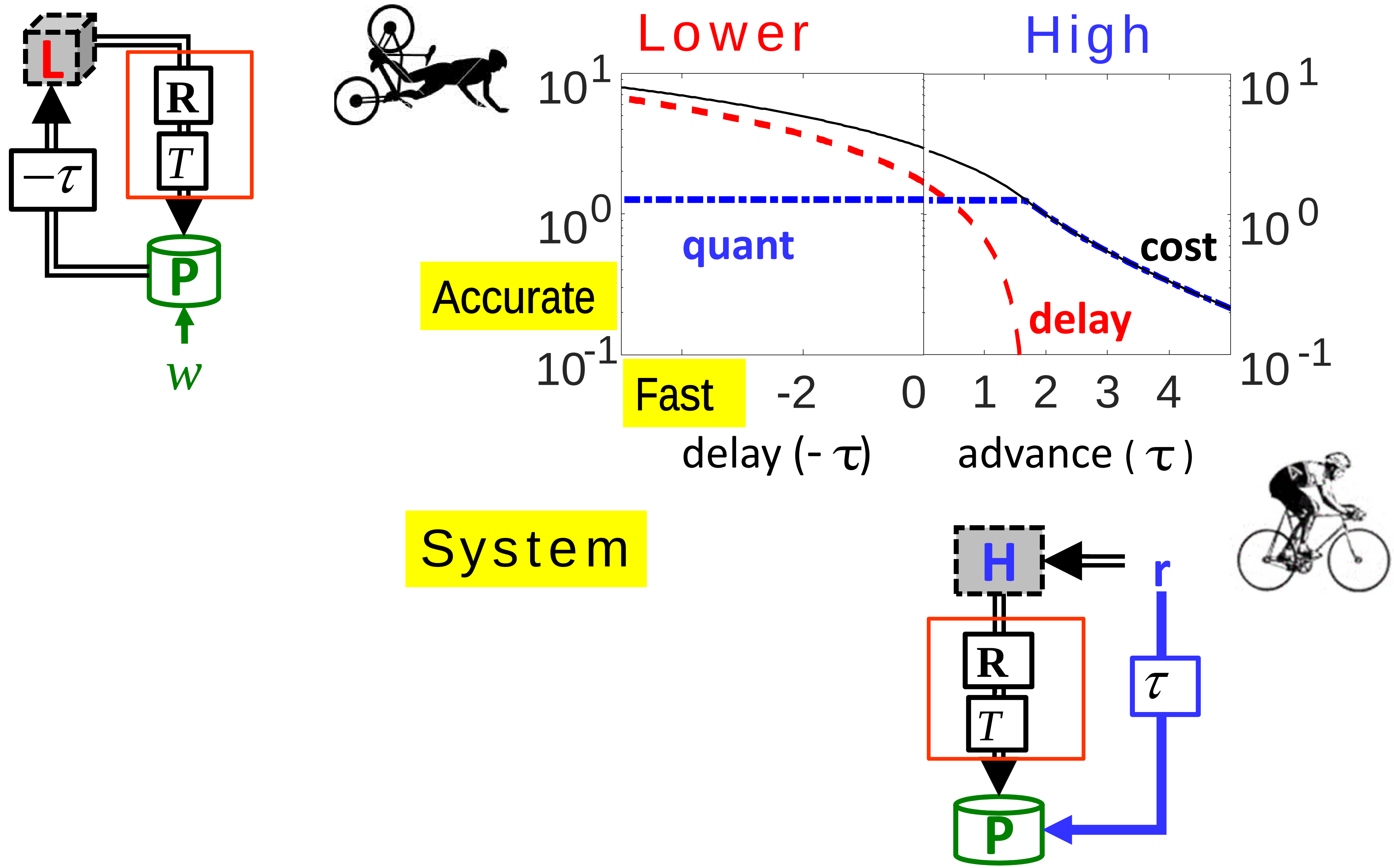
Laughlin



Low delay
System
quant
cost
delay
Accurate
Fast
-2
0
1
2
3
4
delay (-τ)
advance (τ)
High Bandwidth
Low delay
General tradeoff
Log (1/R)
Hardware
Accurate
Fast
Log
High Bandwidth



L
R
T
−τ
P
w
Lower
High
10^1
10^0
10^-1
quant
cost
delay
Accurate
Fast
-2
0
1
2
3
4
delay (- τ)
advance (τ)
Sciatic
Mechanistic Physiology
Log (1/R)
Vest
Aud
Olf
Optic
R = λ_α T
Accurate
Fast
Log T
H
r
τ
R
T
P


L
R
T
−τ
P
w
Lower
High
quant
cost
delay
Accurate
Fast
10^1
10^0
10^-1
-2
0
1
2
3
4
delay (- τ)
advance (τ)
System
H
r
R
T
τ
P

Caveats and Issues (talks and videos)

- **Bad** scholarship, cinematography, diplomacy, organization
- **Badly organized** (Videos, slides, papers in Dropbox)
- More ***breadth*** than depths, more **questions** than answers
- Trailer for videos and papers with ***lots*** of details

But

- **Widely applicable**
- **Accessible** (even latest theory research is relatively…)
- (Almost) **undergrad** for (almost) everything, lots just ***high school***
- “**Obvious**” (if in retrospect) and with familiar components
- Eager for **feedback** (and also new material)
- All **answers** due to students

Caveats and Issues (talks and videos)

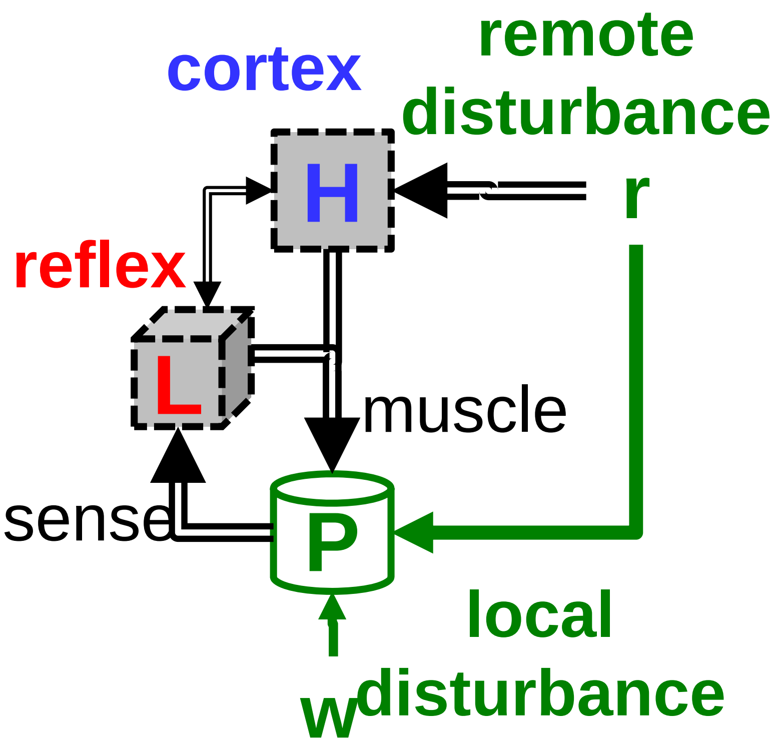
- **Bad** scholarship, cinematography, diplomacy, organization
- **Badly organized** (Videos, slides, papers in Dropbox)
- More ***breadth*** than depths, more **questions** than answers
- Trailer for videos and papers with ***lots*** of details

But

- **Widely applicable**
- **Accessible** (even latest theory research is relatively…)
- (Almost) **undergrad** for (almost) everything, lots just ***high school***
- "**Obvious**" (if in retrospect) and with familiar components
- Eager for **feedback** (and also new material)
- All **answers** due to students

Low layers
past
reflex
delayed
unstable (real)
small ||w|| *but*
large ||CL|| >1
need speed
unconscious
>>10Mb/sec
distributed
local

dynamic/feedback
digital/quantized
worst case $||\bullet||_\infty$
saturation
layered/recursive

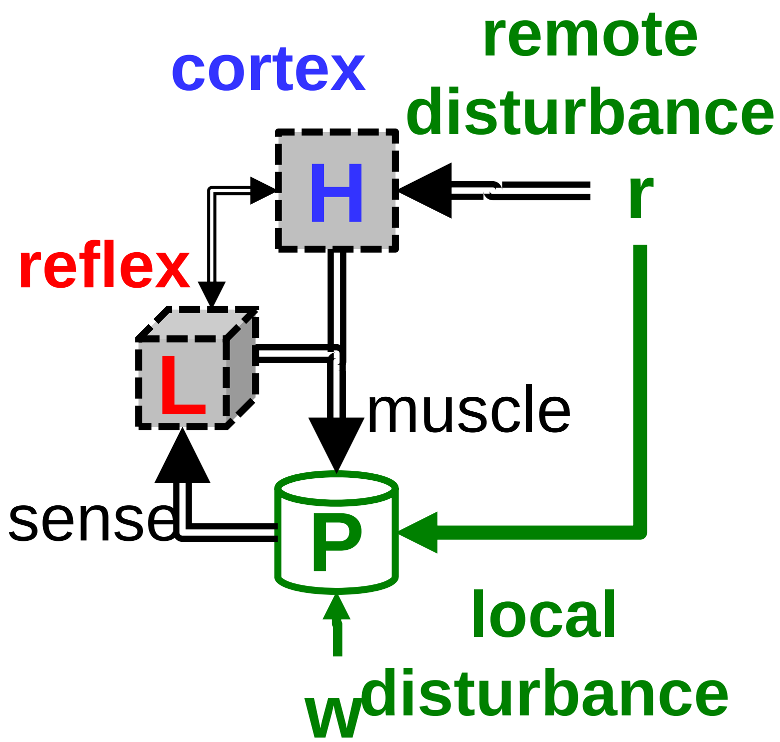


High layers
future
planning
advance warning
stable (virtual)
large ||r|| *but*
small ||CL|| <<1
need accuracy
conscious
<100 bits/sec
centralized
global

Low layers

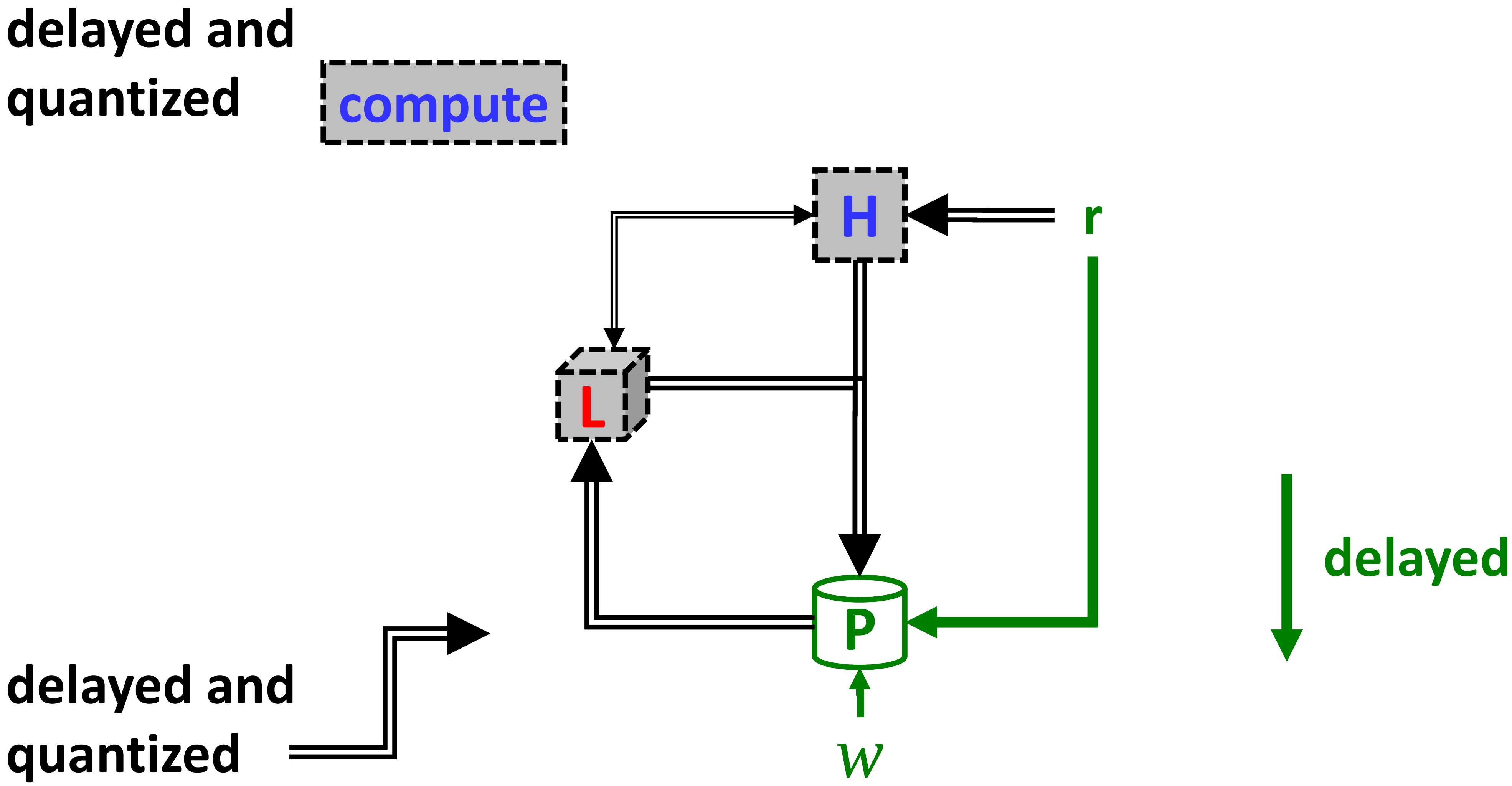
past
reflex
delayed
unstable (real)
small ||w|| *but*
large ||CL|| >1
need speed
unconscious
>>10Mb/sec
distributed
local

dynamic/feedback
digital/quantized
worst case $||\bullet||_\infty$
saturation
layered/recursive

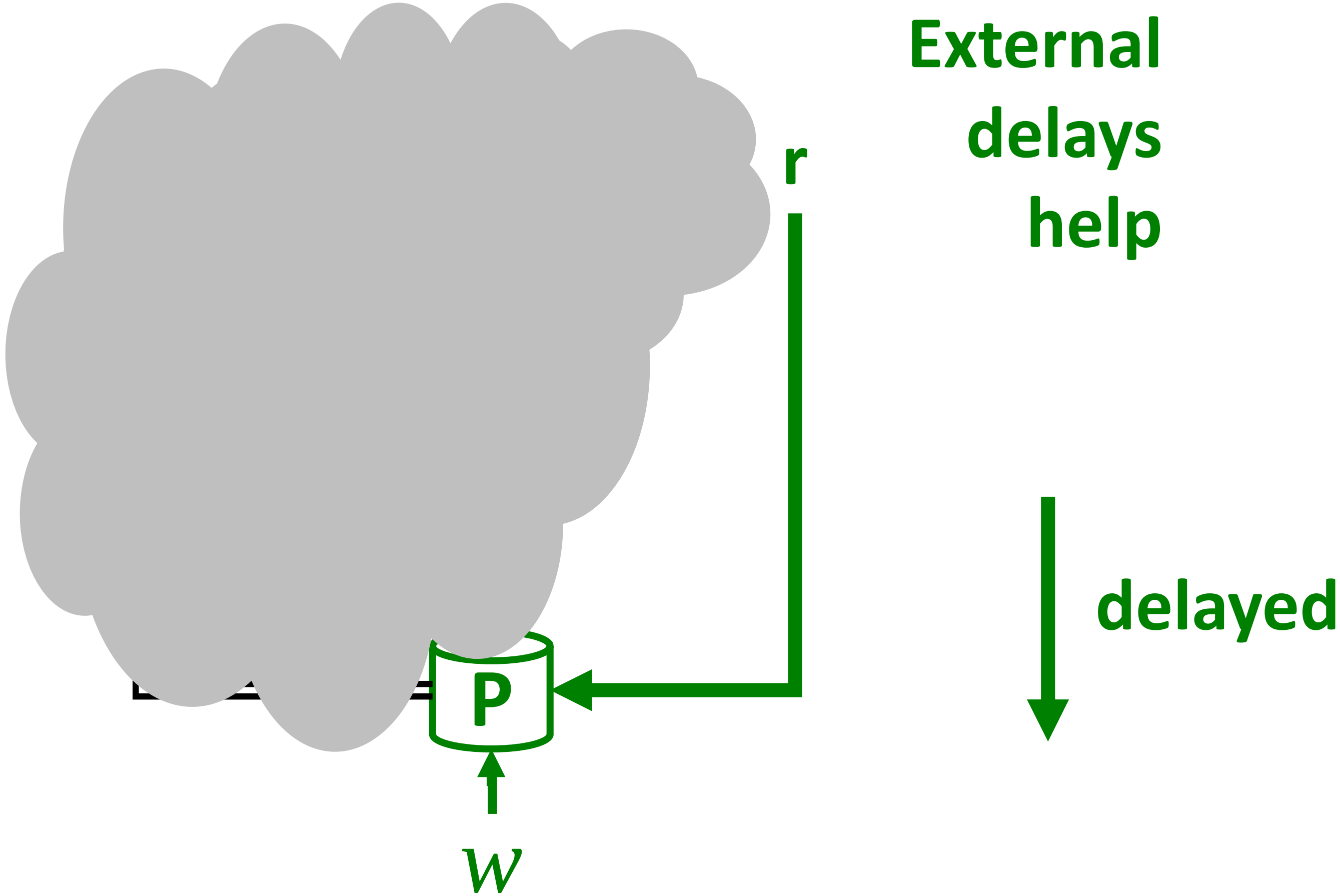


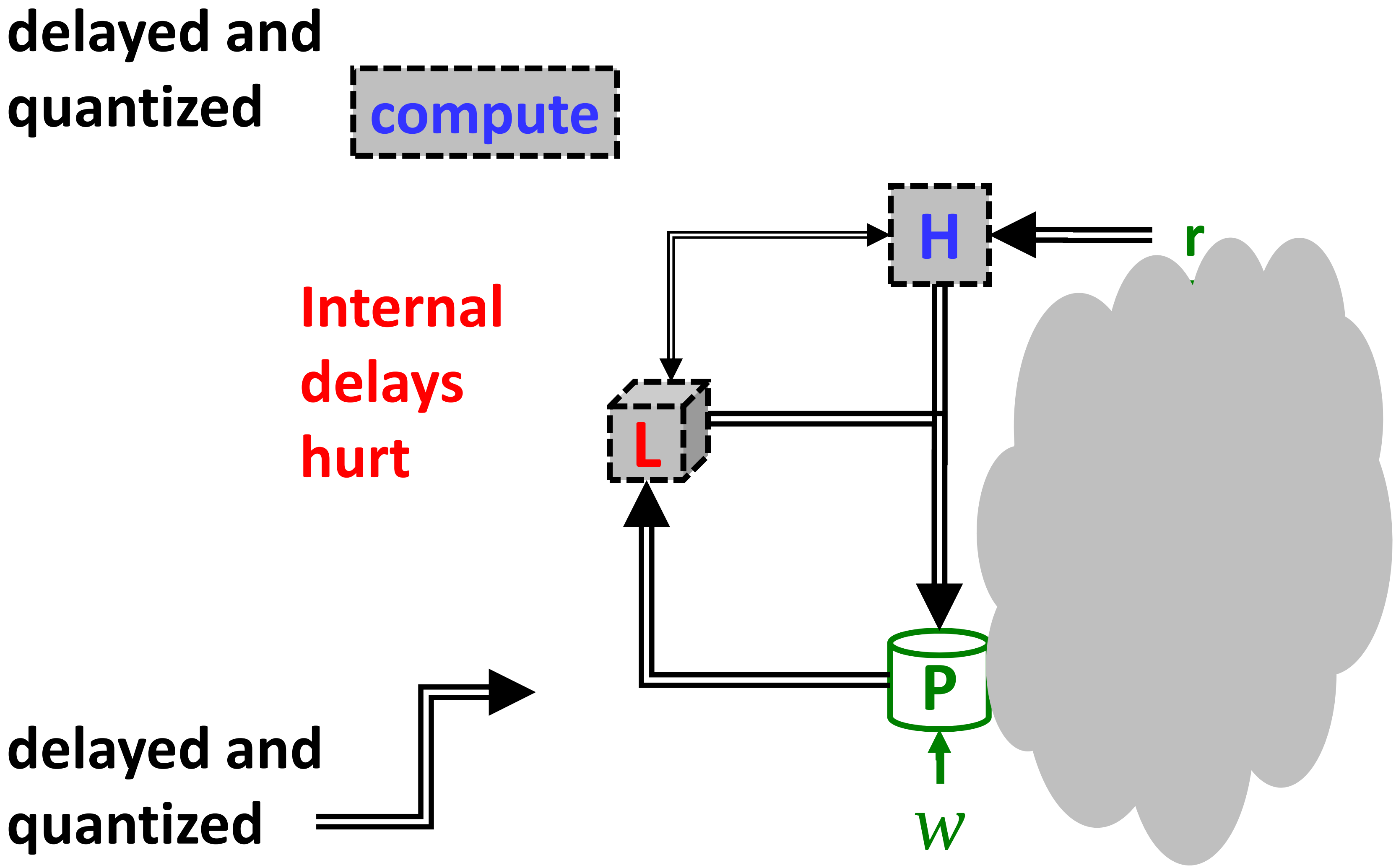
High layers

future
planning
advance warning
stable (virtual)
large ||r|| *but*
small ||CL|| <<1
need accuracy
conscious
<100 bits/sec
centralized
global



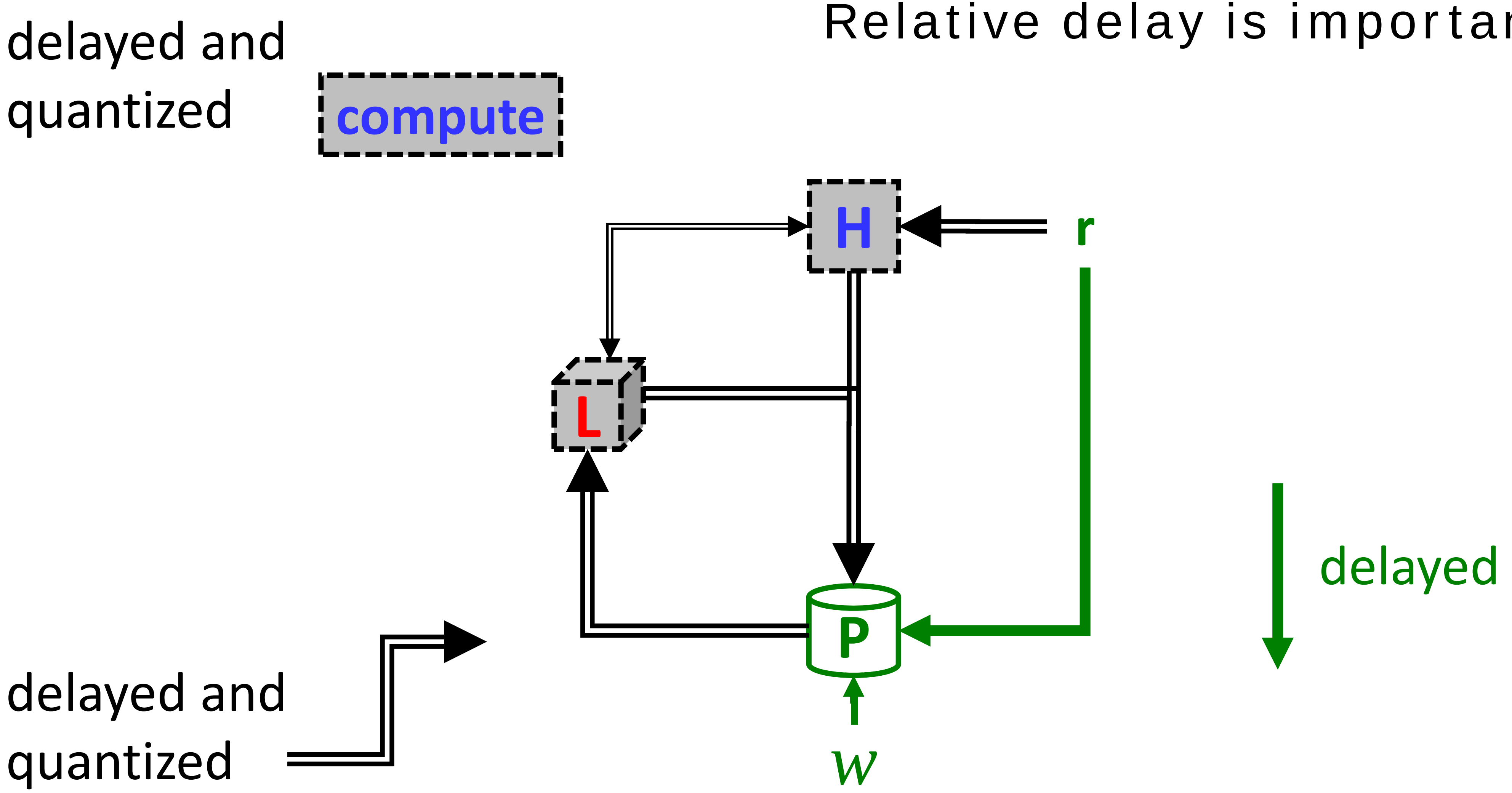
delayed and quantized
compute
H
r
L
delayed
P
delayed and quantized
w


External
delays
help
r
P
w
delayed

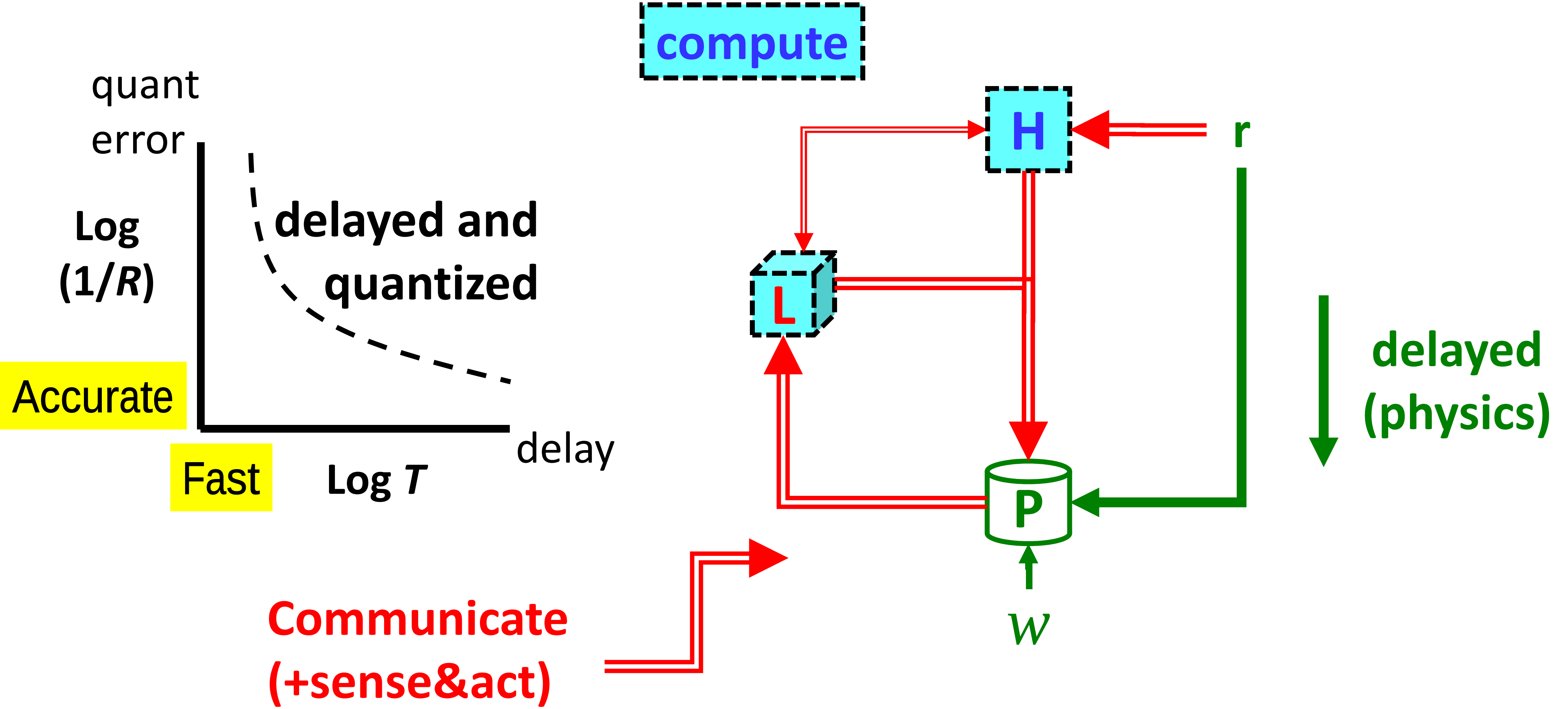


delayed and
quantized
compute
Internal
delays
hurt
H
r
L
P
w
delayed and
quantized

Relative delay is important

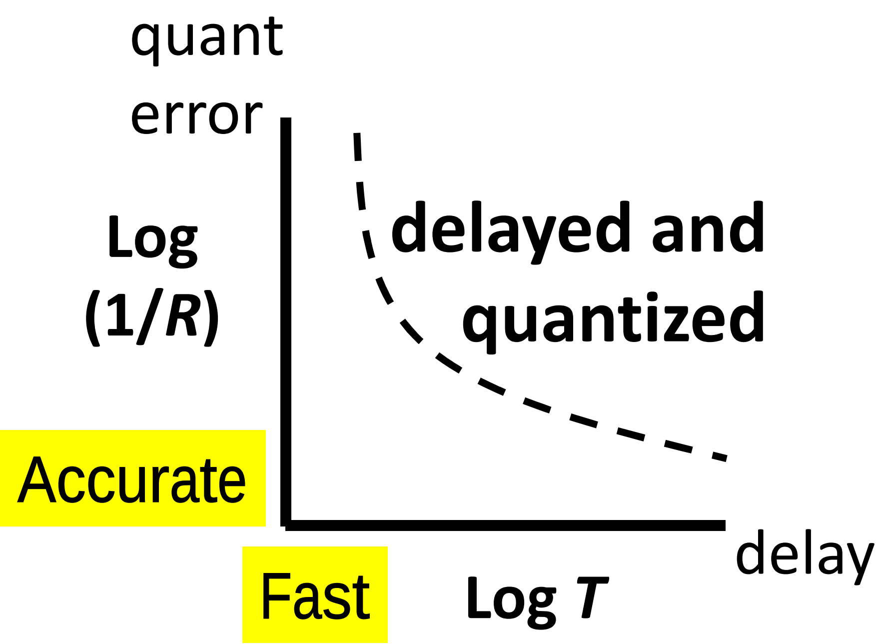


Speed vs Accuracy



Speed vs Accuracy

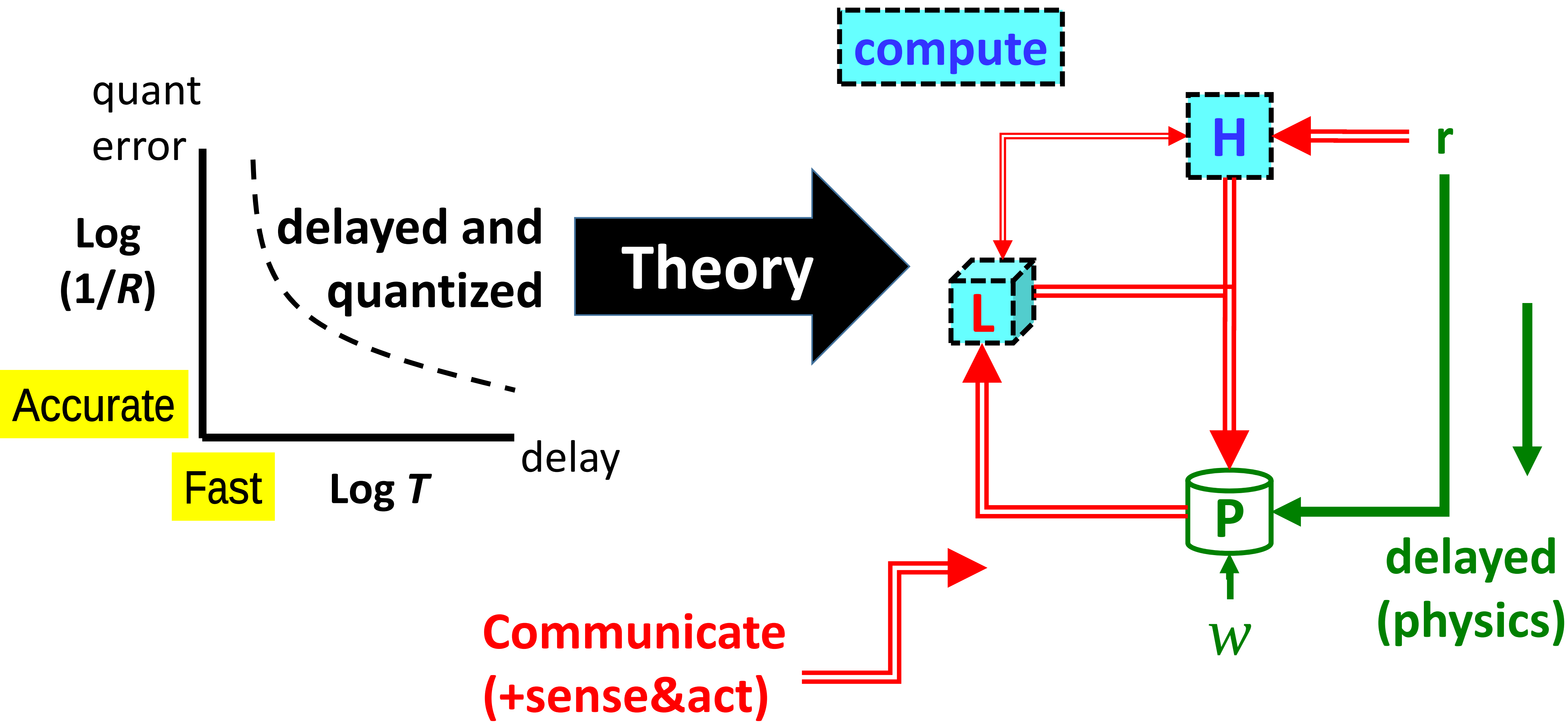
compute

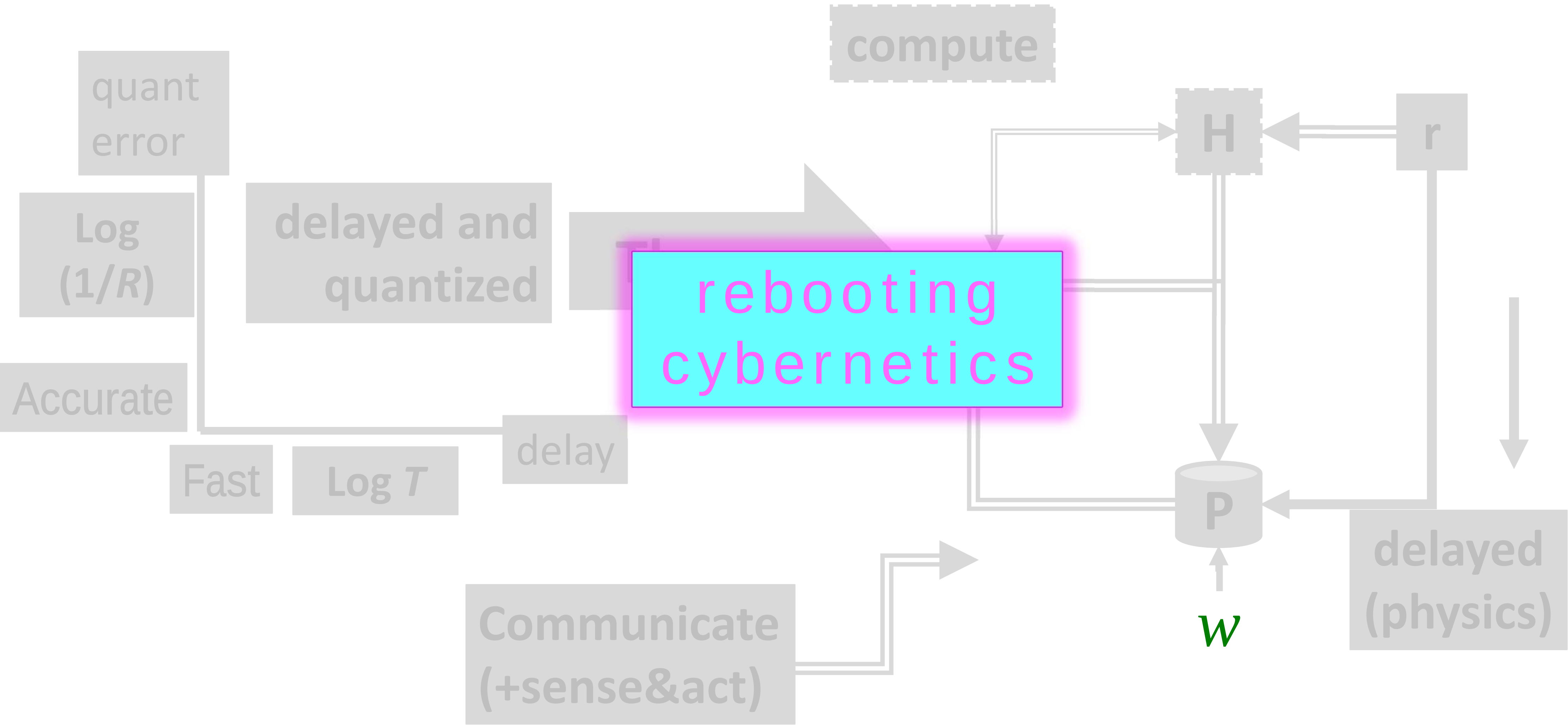


Different technologies will have different tradeoffs

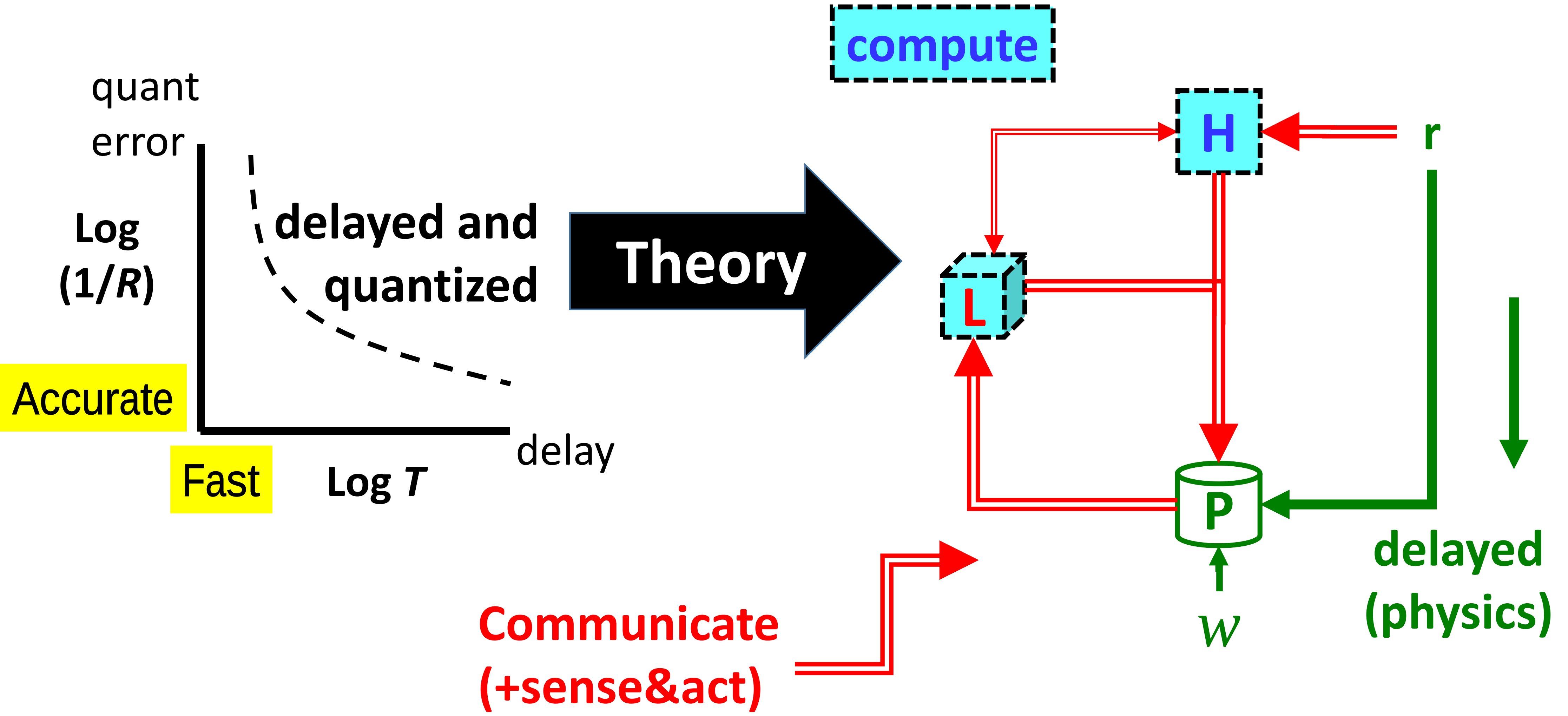
Communicate (+sense&act)

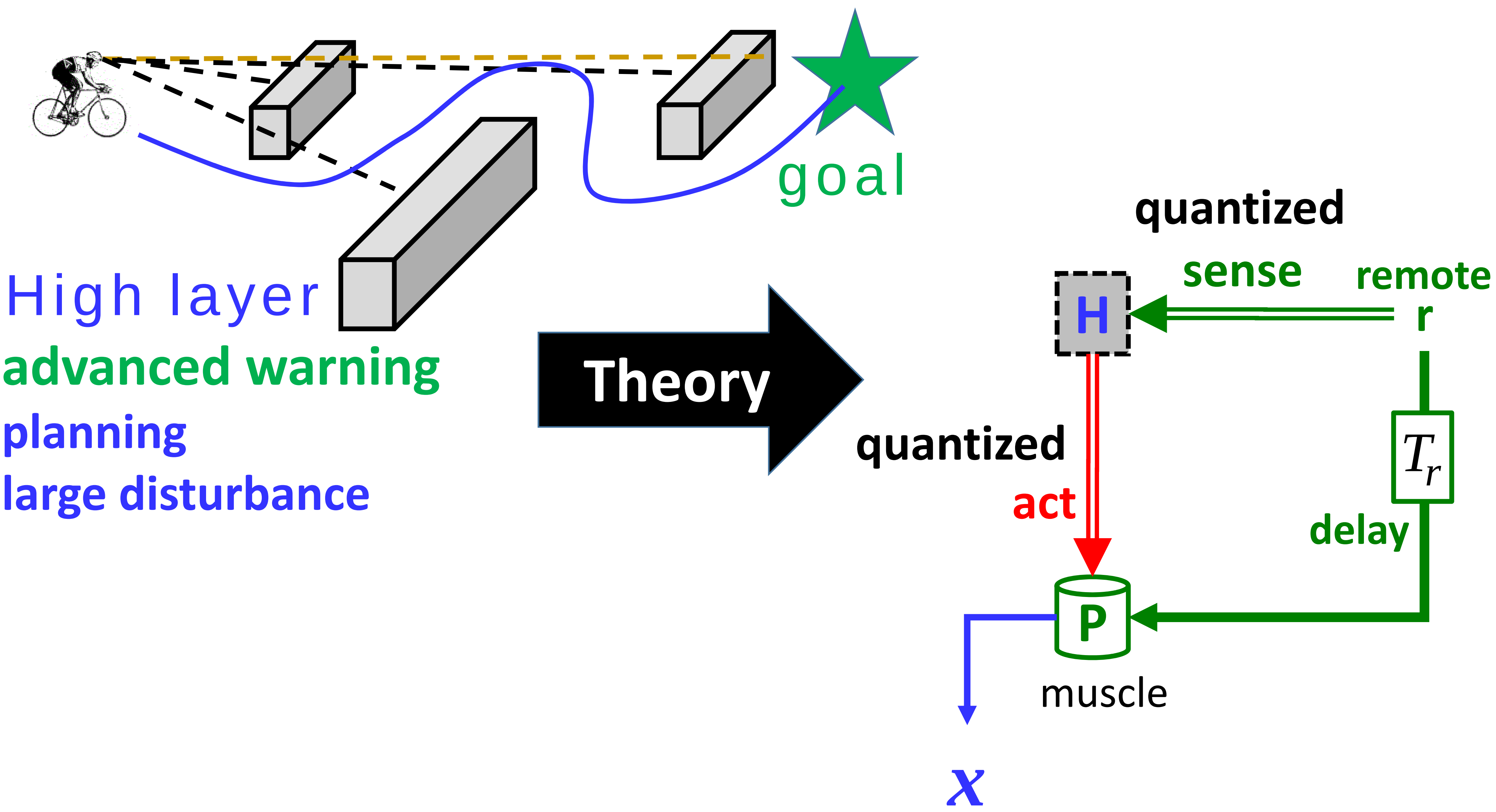
Speed vs Accuracy

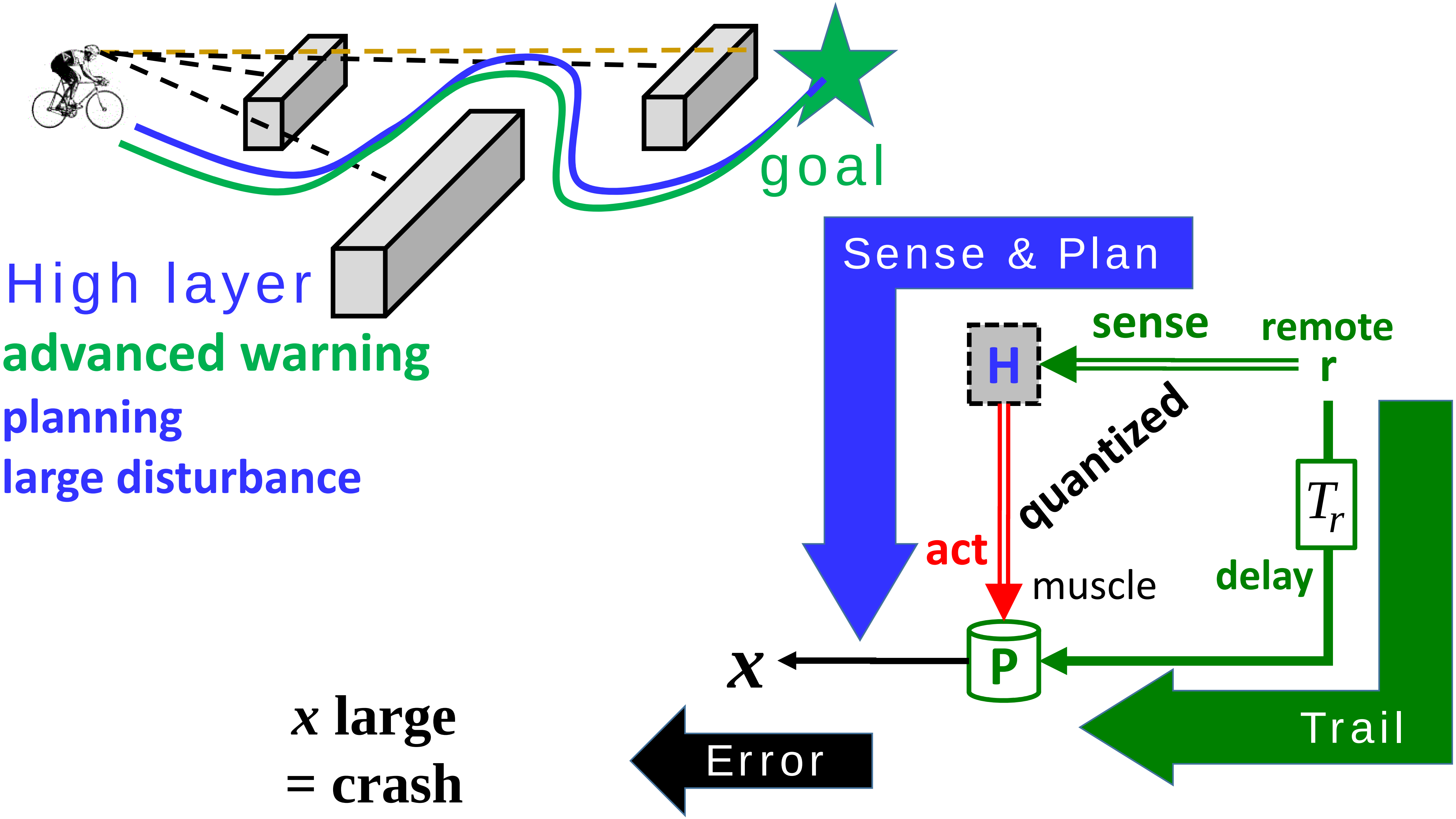


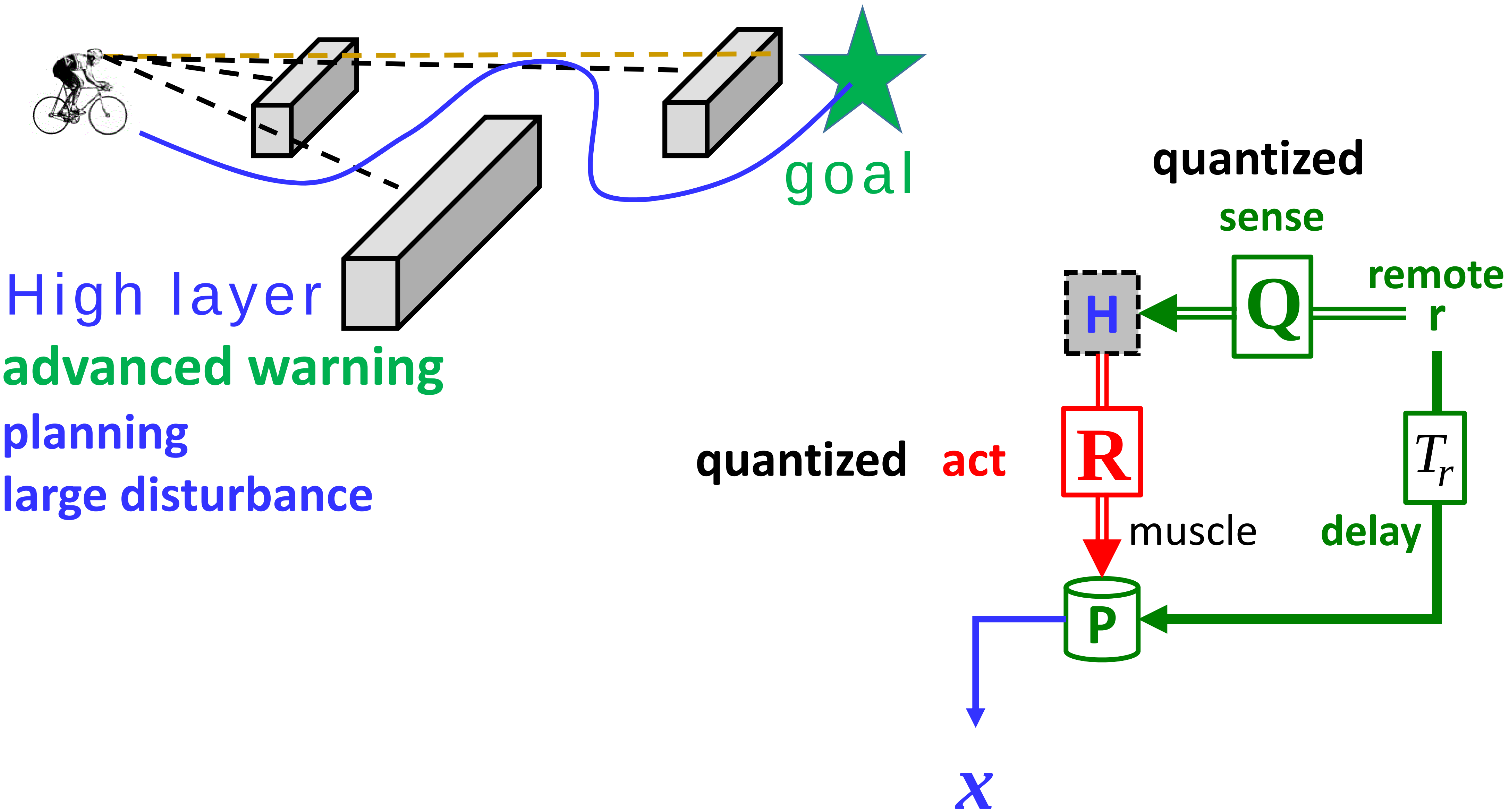

compute
quant error
Log (1/R)
delayed and quantized
rebooting cybernetics
H
r
Accurate
delay
Fast
Log T
P
Communicate (+sense&act)
w
delayed (physics)

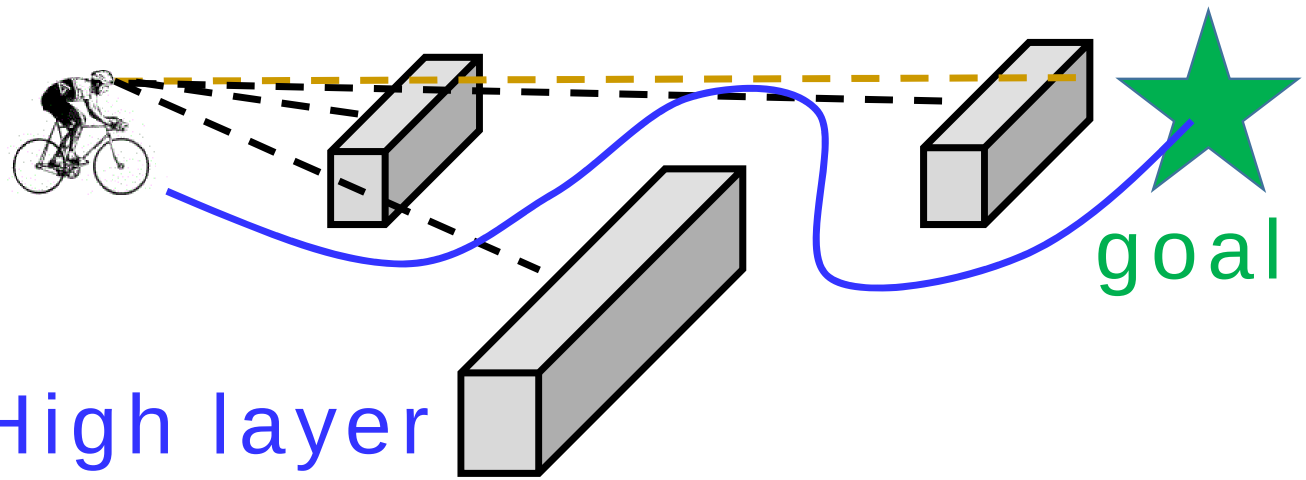
Speed vs Accuracy




goal
High layer
advanced warning
planning
large disturbance
Theory
quantized
sense
remote
r
H
quantized
act
T_r
delay
P
muscle
x


goal
Sense & Plan
High layer
advanced warning
planning
large disturbance
H
sense
remote
r
quantized
act
muscle
T_r
delay
P
x
Trail
x large
= crash
Error


goal
High layer
advanced warning
planning
large disturbance
quantized
sense
remote
r
H
Q
quantized
act
R
T_r
muscle
delay
P
x

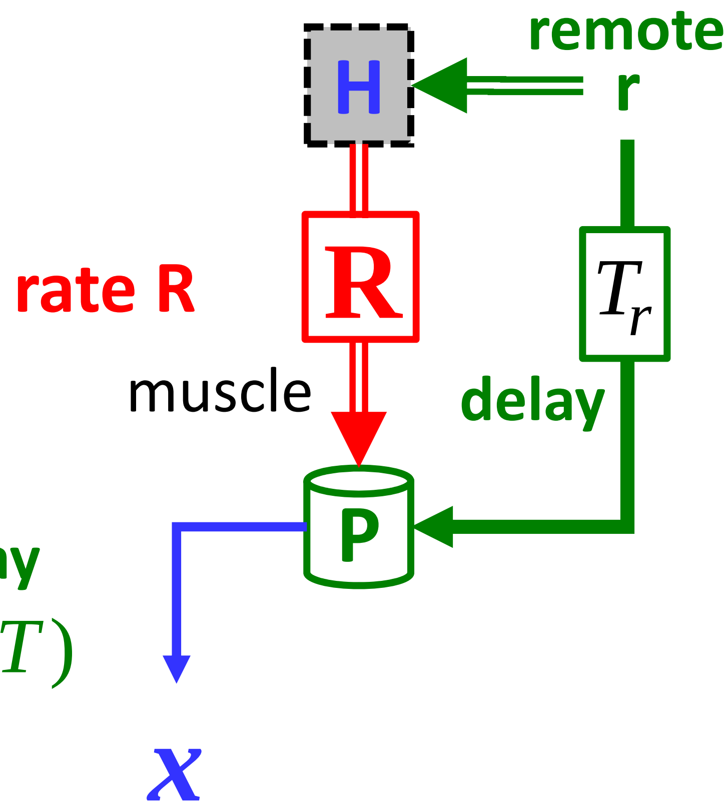


High layer

advanced warning

planning

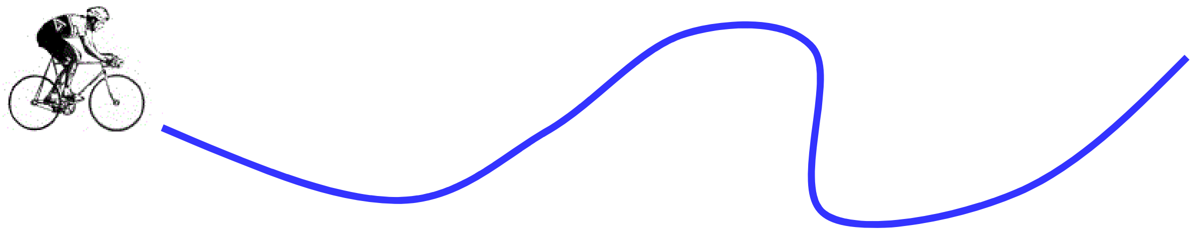
large disturbance



quant **delay**

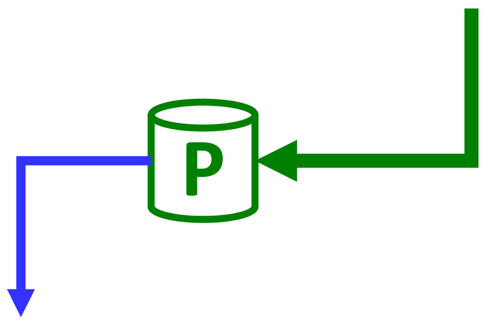
$$x(t+1) = ax(t) - \mathbf{R}\left[u(t)\right] + r(t-T)$$

dynamics



impulse response

remote r

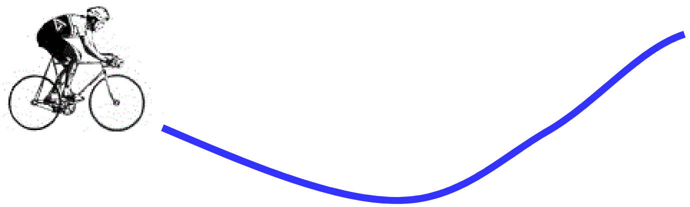


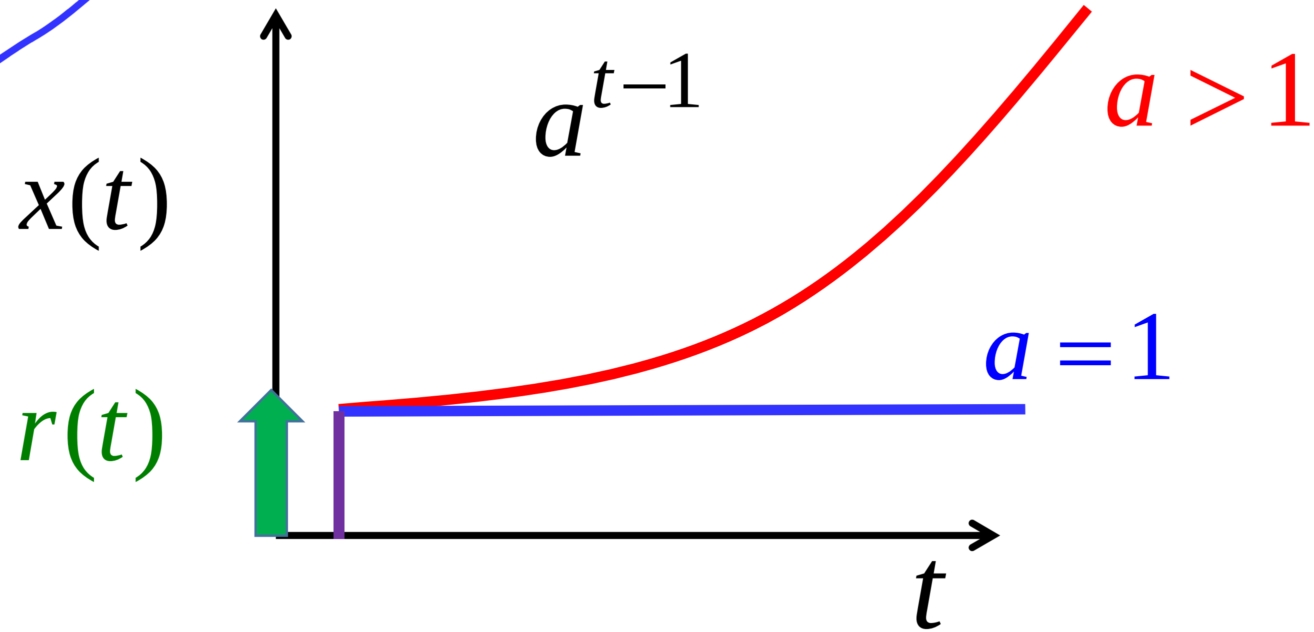
$$x(t+1) = ax(t) \quad + r(t)$$

dynamics

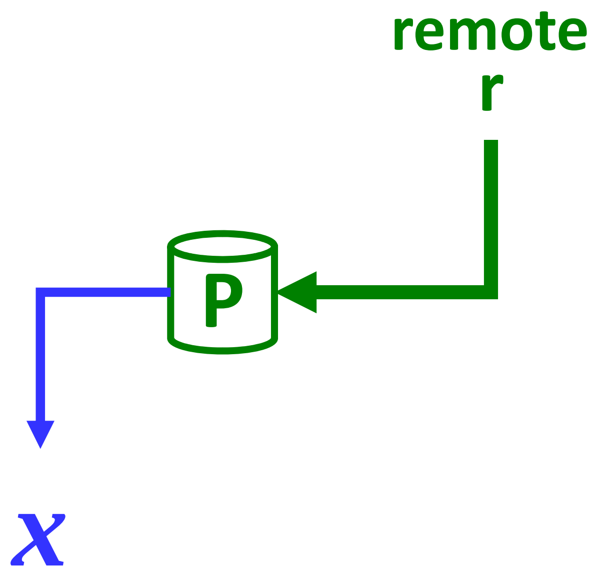
Error

x



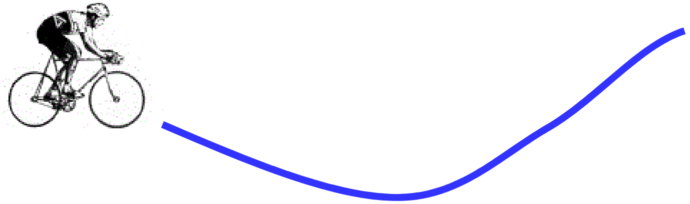


impulse response

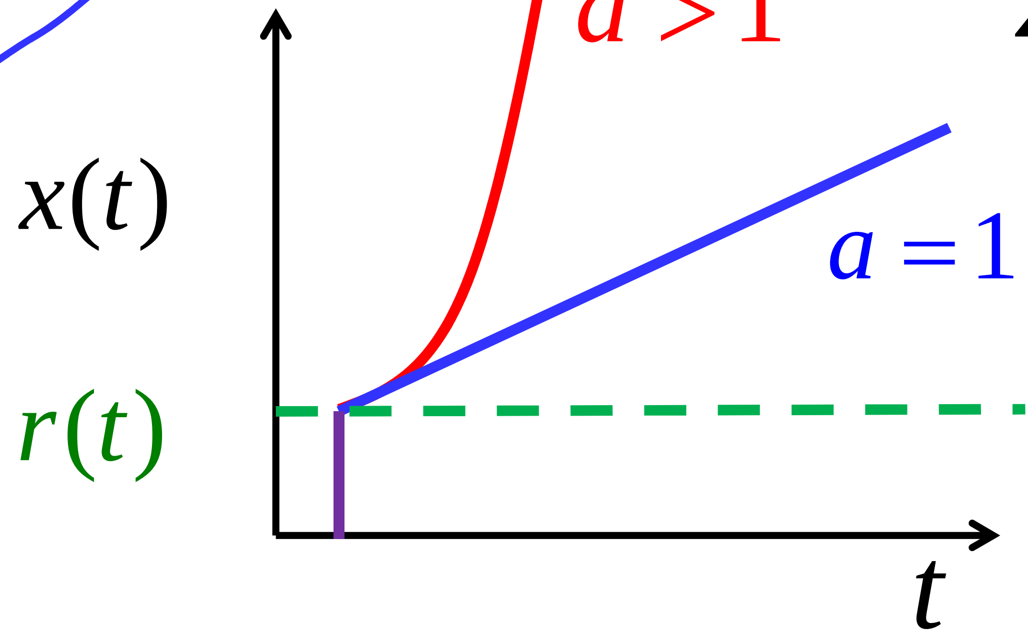


$$x(t+1) = ax(t) \quad + r(t)$$

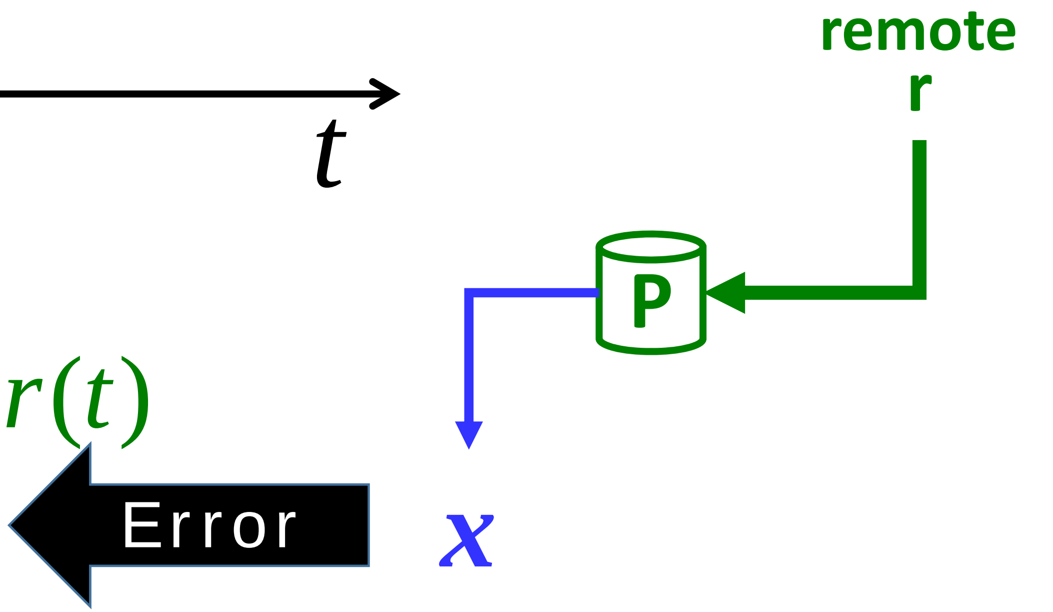
dynamics



$$\sum_{i=1}^{T} \left| a^{i-1} \right|$$

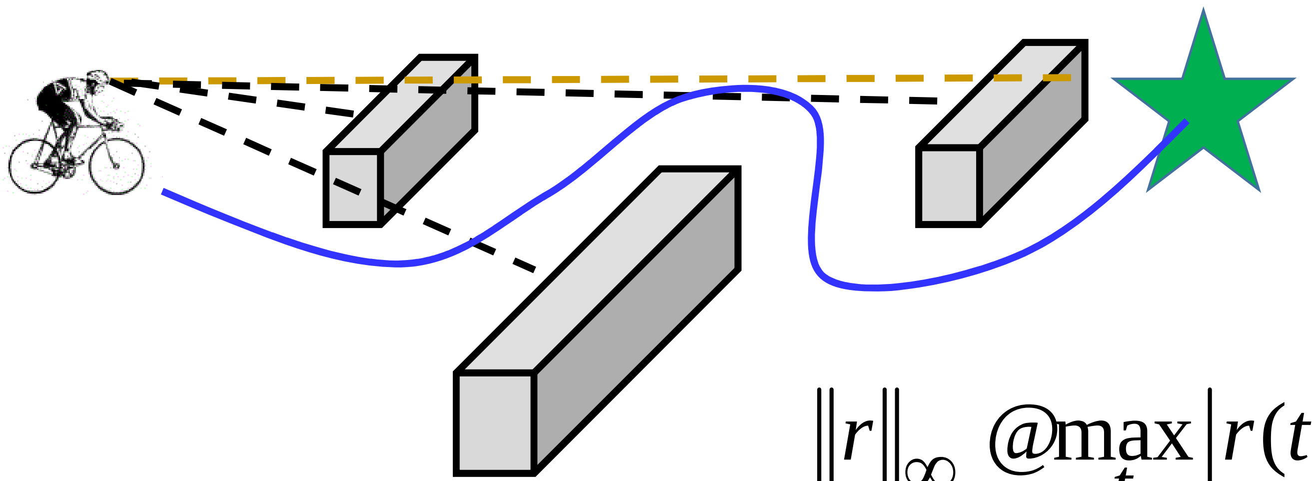


step response



$$x(t+1) = ax(t) \quad + r(t)$$

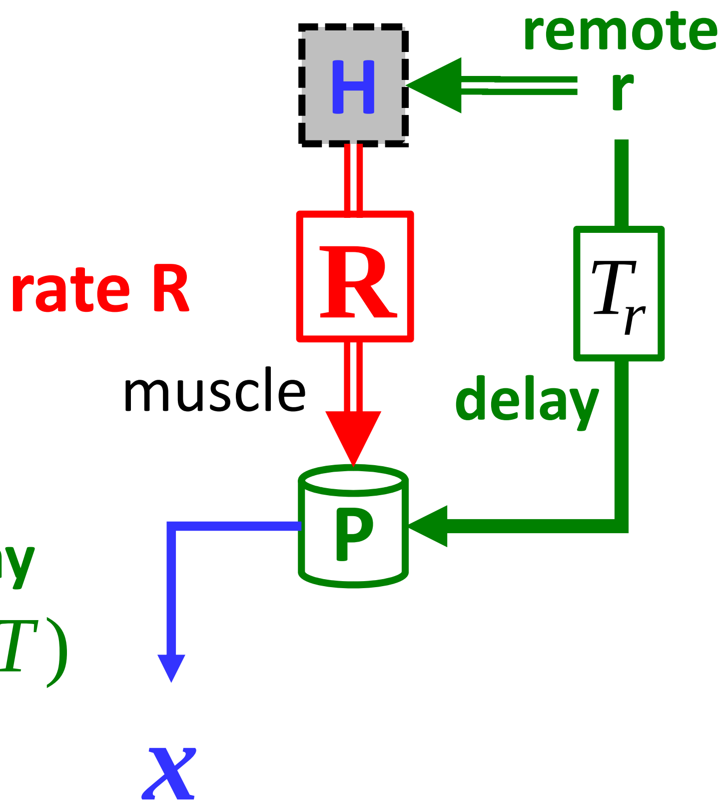
dynamics



$$\|r\|_\infty \le 1$$

$$\|r\|_\infty \triangleq \max_t |r(t)|$$

$$\|x\|_\infty = ?$$



quant delay

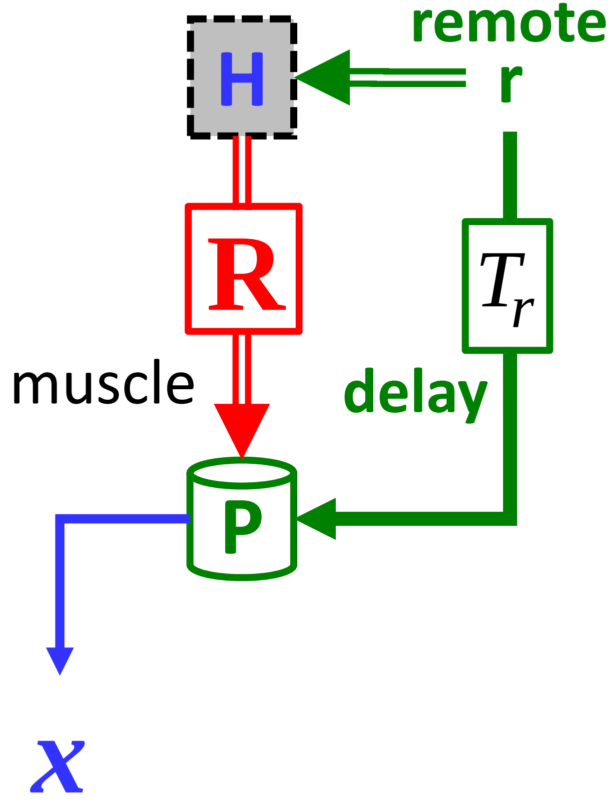
$$x(t+1) = ax(t) - \mathbf{R}\left[u(t)\right] + r(t-T)$$

dynamics

Static rate distortion + dynamics

$$\min_{H,\ R} \max_{\|r\|_\infty \le 1} \|x\|_\infty = \left(2^R - |a|\right)^{-1}$$

$$\|r\|_\infty \le 1$$



quant / delay

$$x(t+1) = ax(t) - \mathbf{R}\left[u(t)\right] + r(t - T)$$

dynamics

Static rate distortion

(Information theory)

$$\min_{\boxed{H}\ \boxed{\mathbf{R}}} \max_{\|r\|_\infty \le 1} \| x \|_\infty = \left(2^R - |a|\right)^{-1} = \left(2^R\right)^{-1}$$

R bits $= 2^R$ levels

$$\mathbf{R}\left[u(t)\right] \approx r(t - T)$$

$$a = 0$$

quant delay

$$x(t+1) = ax(t) - \mathbf{R}\left[u(t)\right] + r(t - T)$$

dynamics

$$\min \max \| x \|_{\infty} = \left(2^{R}\right)^{-1}$$

$$R \text{ bits} = 2^{R} \text{ levels}$$

$$\mathbf{R}\left[u(t)\right] \approx r(t - T)$$

quant delay

$$x(t+1) = -\mathbf{R}\left[u(t)\right] + r(t - T)$$

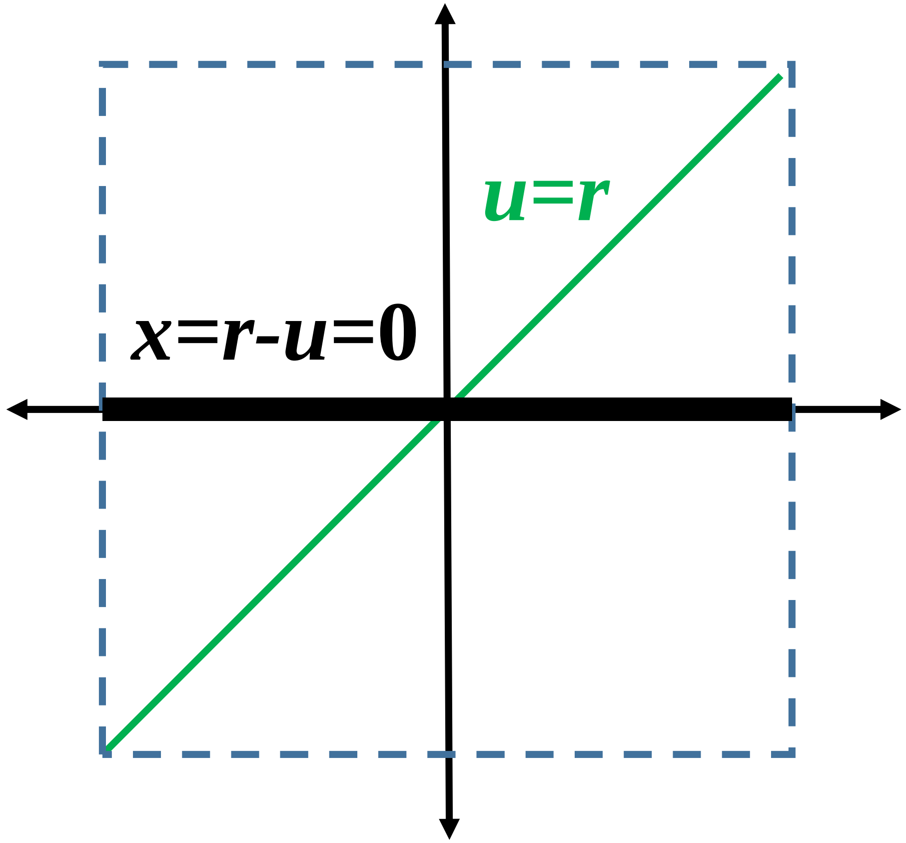
unquantized ($R \to \infty$)

$$\min \max \| x \|_\infty = \left(2^R\right)^{-1} \to 0$$

$$u = r(t - T)$$

quant **delay**

$$x(t+1) = -\mathbf{R}\left[u(t)\right] + r(t - T)$$



unquantized (R=∞)

Ideally u=r

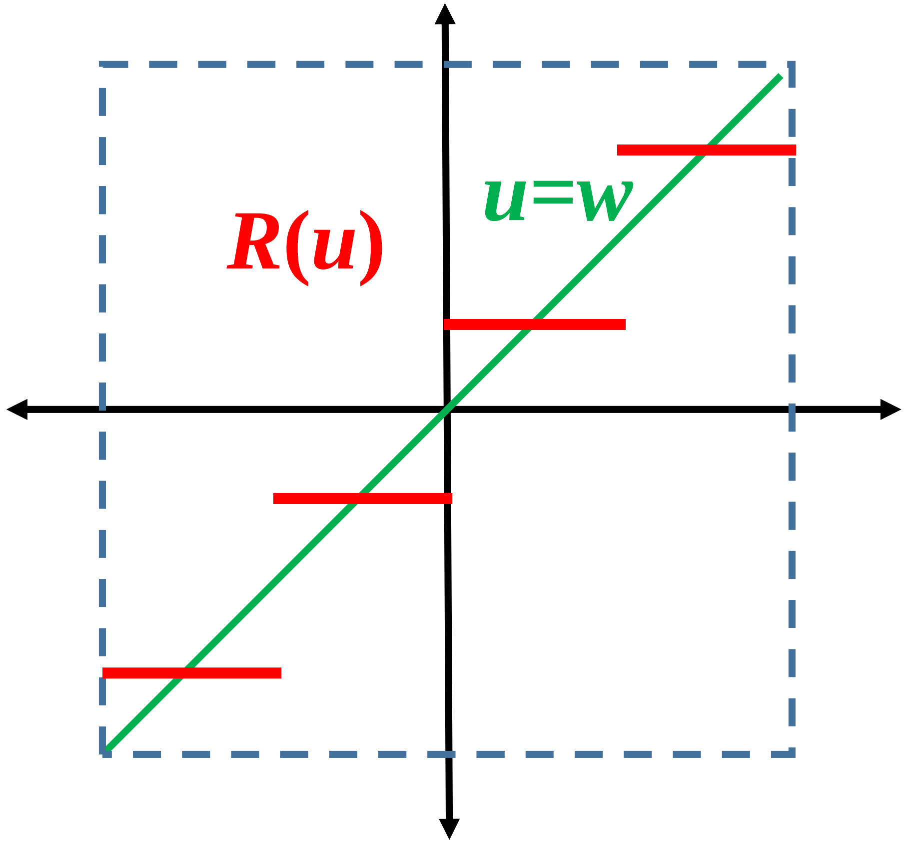
$$\min \max \| x \|_\infty = \left(2^R\right)^{-1}$$

$$R \text{ bits} = 2^R \text{ levels}$$

$$\mathbf{R}\left[u(t)\right] \approx r(t - T)$$

quant delay

$$x(t+1) = -\mathbf{R}\left[u(t)\right] + r(t - T)$$



quantized *R*=2

2^2 = 4 levels

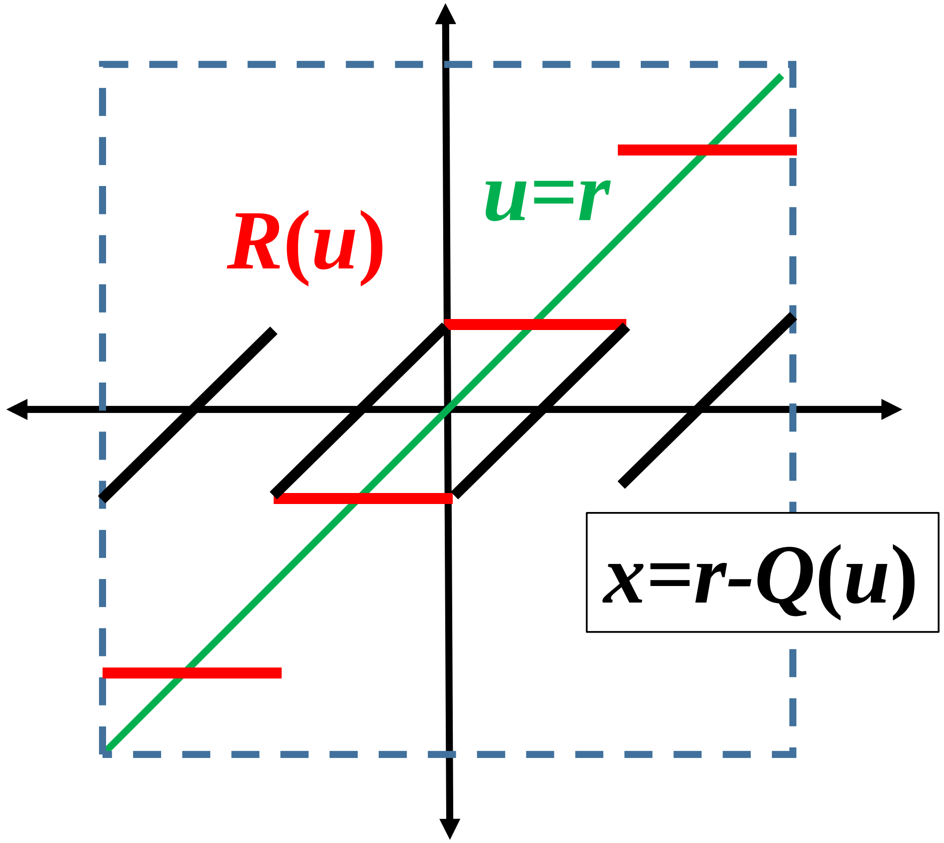
$$\min \max \| x \|_{\infty} = \left(2^{R}\right)^{-1}$$

$$R \text{ bits} = 2^{R} \text{ levels}$$

$$\mathbf{R}\left[u(t)\right] \approx r(t - T)$$

quant delay

$$x(t+1) = -\mathbf{R}\left[u(t)\right] + r(t - T)$$

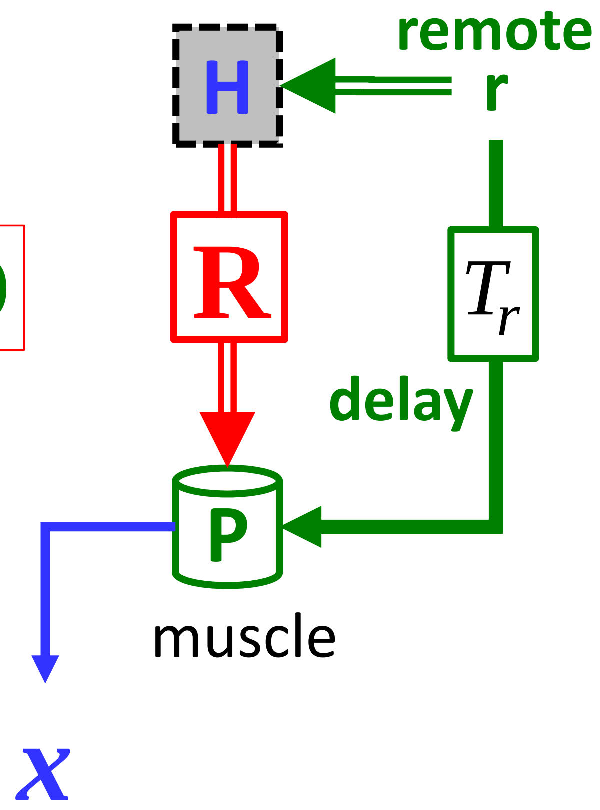


$|r\text{-}R(u)| = 2^{-R} = 2^{-2} = 1/4$

Static rate distortion + dynamics

$$\min_{\mathbf{H}\ \mathbf{R}} \max_{\|r\|_\infty \le 1} \| x \|_\infty = \left(2^R - |a|\right)^{-1}$$

$$\|r\|_\infty \le 1$$



$$\mathbf{R}\left[u(t)\right] \approx ax(t) + r(t - T)$$

quant delay

$$x(t+1) = ax(t) - \mathbf{R}\left[u(t)\right] + r(t - T)$$

dynamics

Summary so far

H $u(t) = f(x(0:t), r(0:t), u(0:t-1))$ Full information

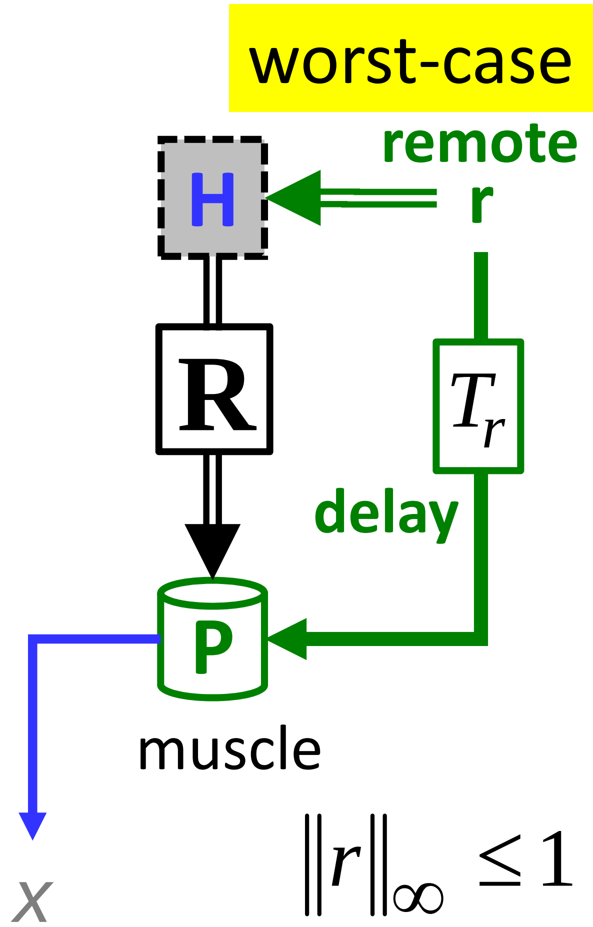
R **Quantizer (*R* bits/time)**

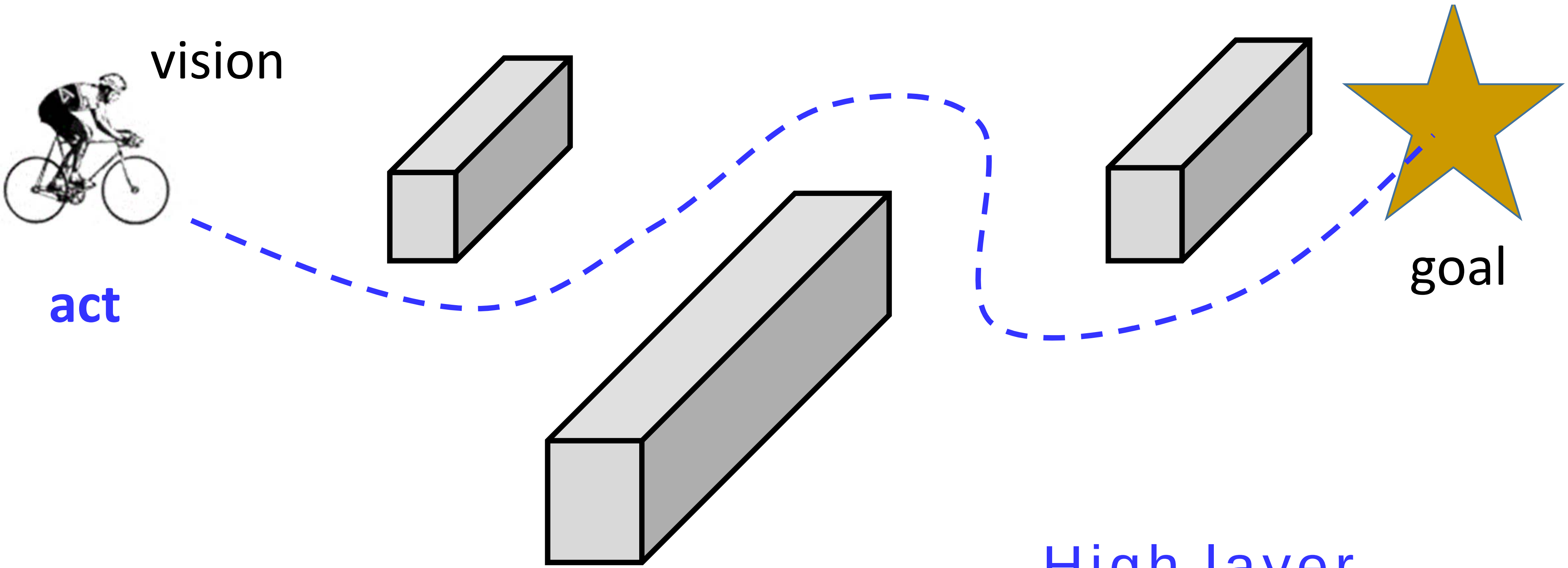
$$x(t+1) = ax(t) - \mathbf{R}\left[u(t)\right] + r(t - T)$$

dynamics quant advance

optimal $\mathbf{R}\left[u(t)\right] = \mathbf{R}\left[ax(t) + r(t - T)\right]$

$$\min_{\mathbf{H}\ \mathbf{R}} \max_{\|r\|_\infty \le 1} \|x\|_\infty = \left(2^R - |a|\right)^{-1}$$





High layer

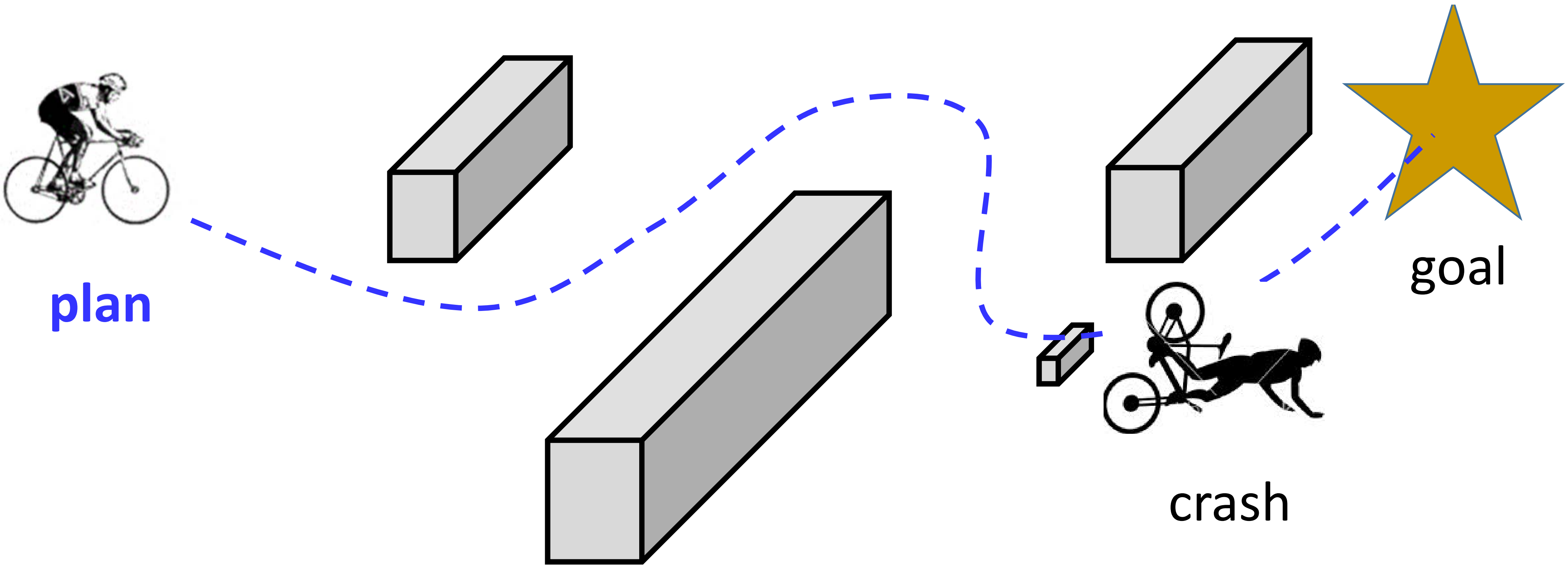
advanced warning

planning

large disturbance

small error

$$\min_{H,\ R} \max_{\|r\|_\infty \le 1} \| x \|_\infty = \left(2^R - |a|\right)^{-1}$$


plan
goal
crash

avoiding a crash

Lower layer

delayed

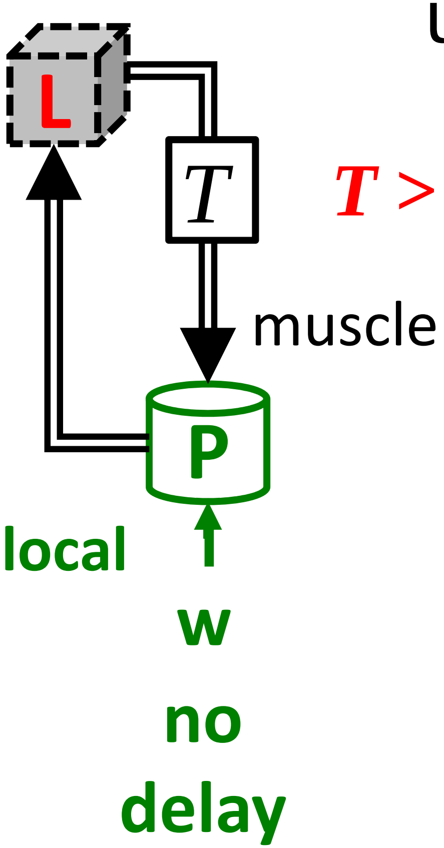
reflexes

small disturbance

large error

crash

unstable+delay



$T > 0$

no delay

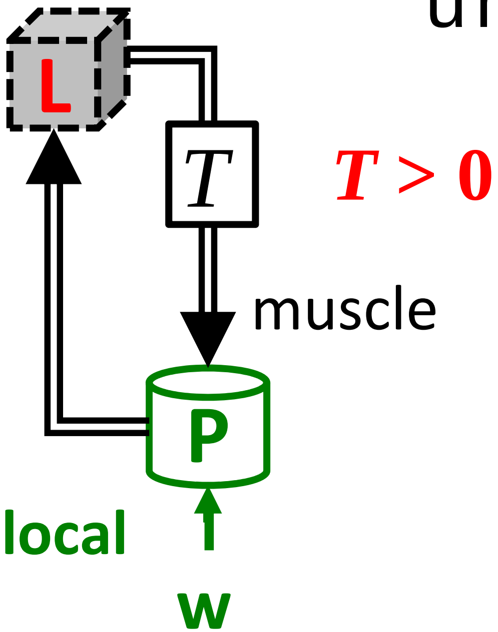
$$x(t+1) = ax(t) - u(t-T) + w(t)$$

delay

$$\| w \|_\infty = \sup_t |w(t)| \le \delta << 1$$

But temporarily $\delta = 1$

unstable+delay



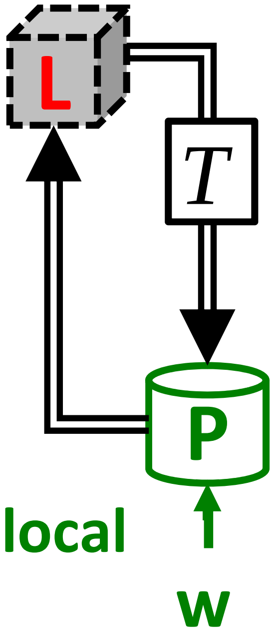
no delay

$$x(t+1) = ax(t) - u(t-T) + w(t)$$

delay

$$\min_{L} \sup_{\|w\|_\infty \le 1} \| x \|_\infty \ ?$$

unstable+delay



$T > 0$

Full information

$$u(t) = f(x(0:t), w(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - u(t-T) + w(t)$$

delay

$$\min_{L} \sup_{\|w\|_\infty \le 1} \| x \|_\infty = \sum_{i=1}^{T} \left| a^{i-1} \right|$$

unstable+delay

Full information

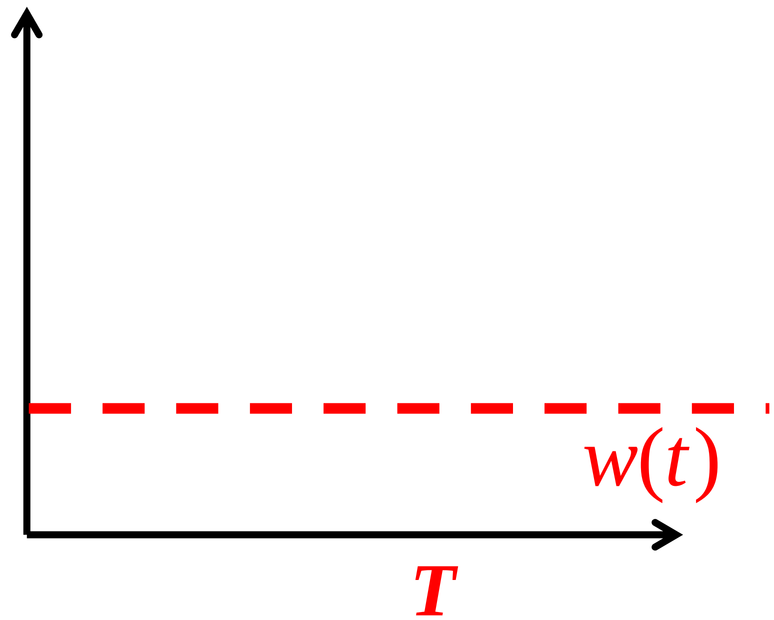
$$u(t) = f(x(0:t), w(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - u(t-T) + w(t)$$

delay

- **Worst case $w(t)$ = unit step.**
- **$u(t)$=0 for $t<T$**

$$\min_{L} \sup_{\|w\|_\infty \le 1} \|x\|_\infty = \sum_{i=1}^{T} \left|a^{i-1}\right|$$



unstable+delay

Full information

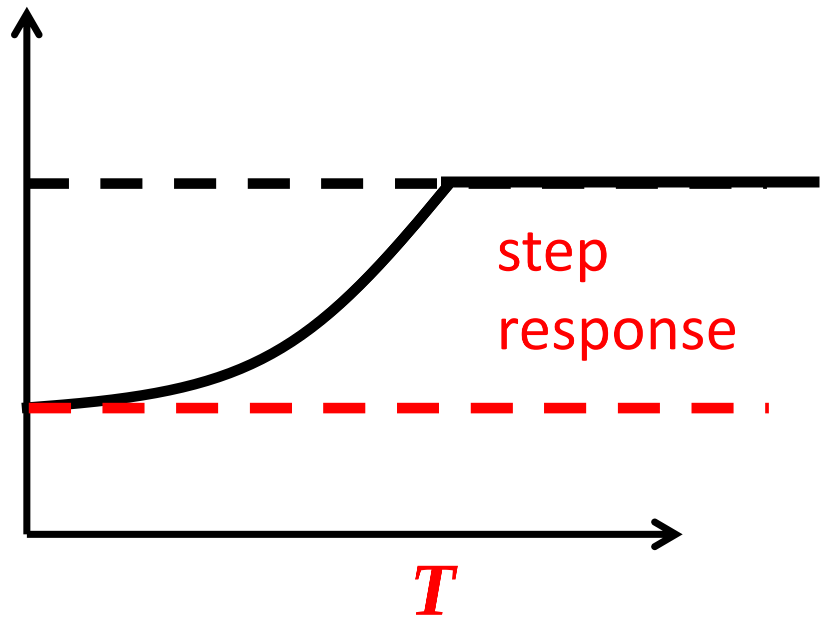
$$u(t) = f(x(0:t), w(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - u(t-T) + w(t)$$

delay

- **Worst case $w(t)$ = unit step.**
- **$u(t)$=0 for $t<T$**

$$\min_{L} \sup_{\|w\|_\infty \le 1} \| x \|_\infty = \sum_{i=1}^{T} \left| a^{i-1} \right|$$



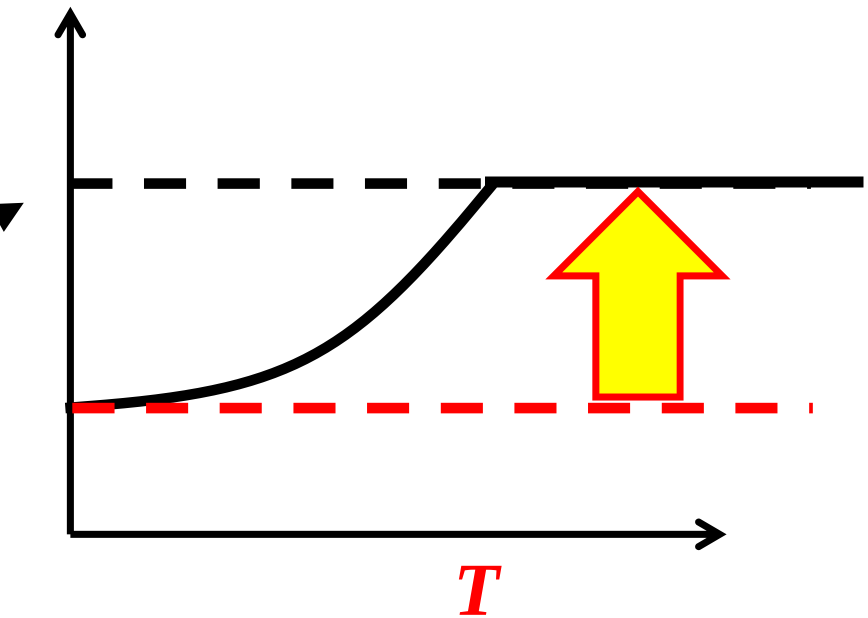
unstable+delay = amplify disturbance

$$x(t+1) = ax(t) - u(t-T) + w(t)$$

delay

- **Worst case $w(t)$ = unit step.**
- **$u(t)=0$ for $t<T$**

$$\min_{L} \sup_{\|w\|_\infty \le 1} \|x\|_\infty = \sum_{i=1}^{T} \left| a^{i-1} \right|$$



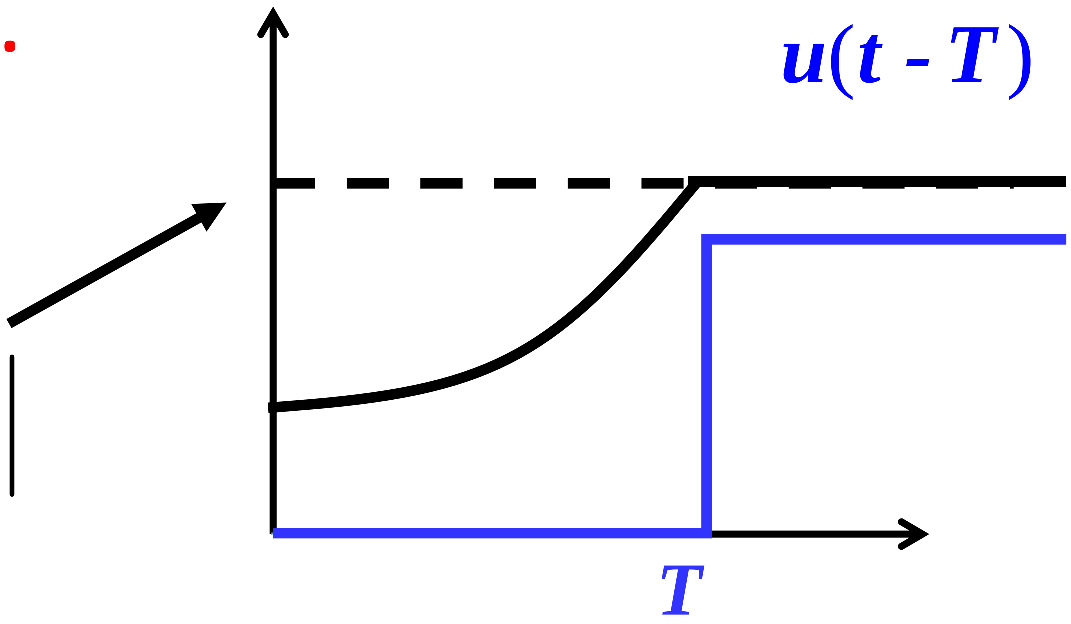
Intuition (almost a proof)

$$x(t+1) = ax(t) - \mathbf{u(t - T)} + w(t)$$

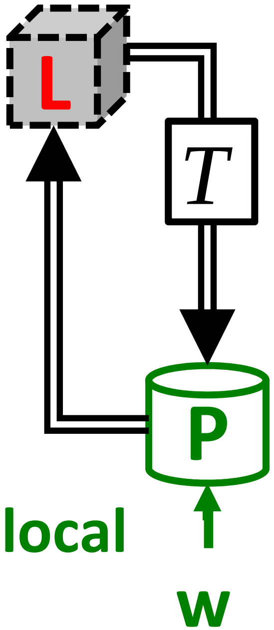
delay

- **Worst case $w(t)$ = unit step.**
- **$u(t)=0$ for $t<T$**

$$\min_{L} \sup_{\|w\|_\infty \le 1} \|x\|_\infty = \sum_{i=1}^{T} \left|a^{i-1}\right|$$



unstable+delay



$T > 0$

Full information

$$u(t) = f(x(0:t), w(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - u(t-T) + w(t)$$

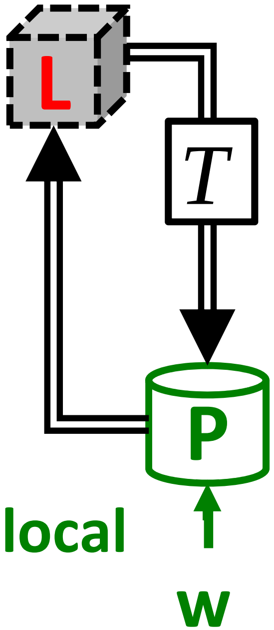
delay

$$l_\infty \Rightarrow l_1$$

$$\min_{L} \sup_{\|w\|_\infty \le 1} \|x\|_\infty = \sum_{i=1}^{T} \left|a^{i-1}\right| = \|h\|_1$$

$h(t)$ = closed loop impulse response

control saturation



$T > 0$

Full information

$$u(t) = f(x(0:t), w(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - u(t-T) + w(t)$$

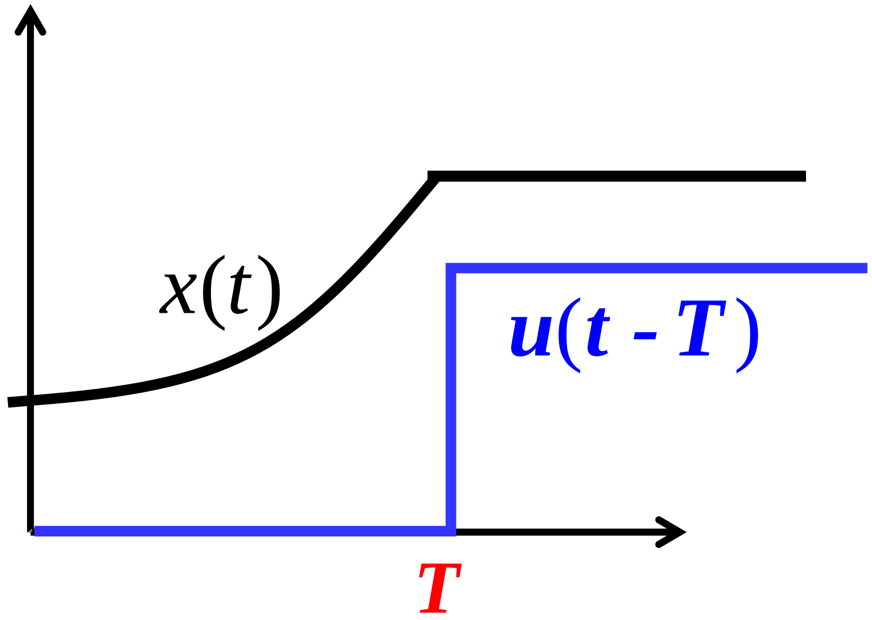
delay

Minimum control

$$\min_{\text{L}} \sup_{\|w\|_\infty \le 1} \|x\|_\infty = \sum_{i=1}^{T} \left| a^{i-1} \right|$$

$$\sup_{\|w\|_\infty \le 1} \|u\|_\infty = |a^T|$$

control saturation



Minimum saturation level that stabilizes

$$\min_{\text{L stabilizing}} \sup_{\|w\|_\infty \le 1} \|u\|_\infty = \begin{cases} |a^T| & |a| \ge 1 \\ 0 & |a| < 1 \end{cases}$$

unstable → ≈ same

$$\min_{\text{L}} \sup_{\|w\|_\infty \le 1} \|x\|_\infty = \sum_{i=1}^{T_S} \left|a^{i-1}\right|$$

Minimum saturation level that achieves optimal performance

$$\sup_{\|w\|_\infty \le 1} \|u\|_\infty = |a^T|$$

$$x(t+1) = ax(t) - \boldsymbol{u(t - T)} + w(t)$$

$$x = ax - \boldsymbol{u} + 1$$

$$u = (a-1)x + 1 = (a-1)\left(\frac{a^T - 1}{a-1}\right) + 1 = a^T$$

$$\frac{a^T - 1}{a-1} ? a^T$$

$$a^T - 1 ? a^{T+1} - a^T$$

$$0 ? a^{T+1} - 2a^T + 1$$

$$0 ? a^T(a-2) + 1$$

$$0 >$$

Lower

delayed

reflexes

small disturbance

large error

need speed

unstable

distributed

local

$$\min_{\boxed{L}} \sup_{\|w\|_\infty \le 1} \| x \|_\infty = \sum_{i=1}^{T_S} \left| a^{i-1} \right|$$

Lower

delayed

reflexes

small disturbance

large error

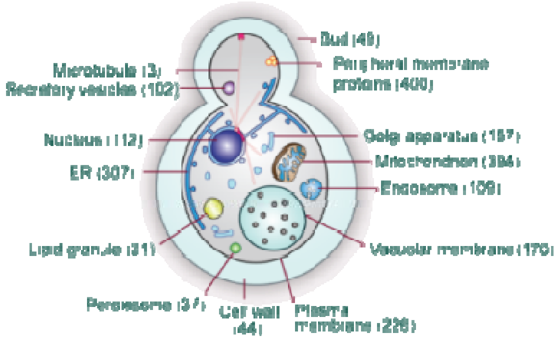
need speed

unstable

distributed

local

$$\min \sup \| x \|_\infty = \sum_{i=1}^{T_S} \left| a^{i-1} \right|$$



metabolism

biosynthesis

...

Unstable due to Positive feedback

- autocatalysis
- gravity

balance

posture

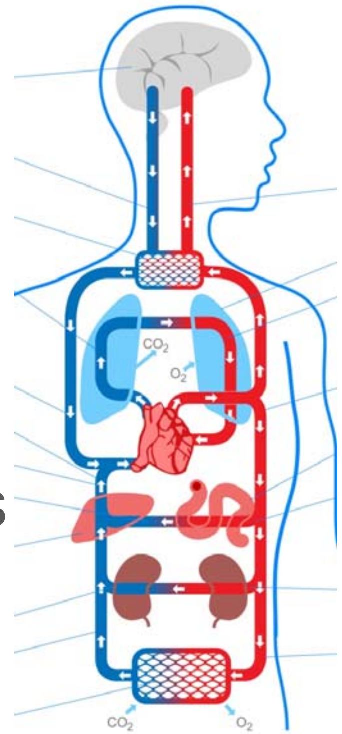
SaO2

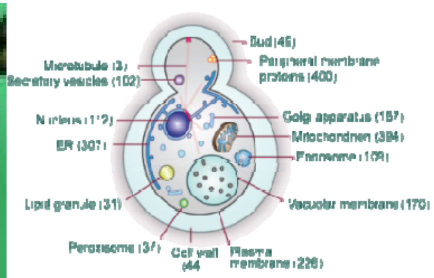
BP

pH

infections

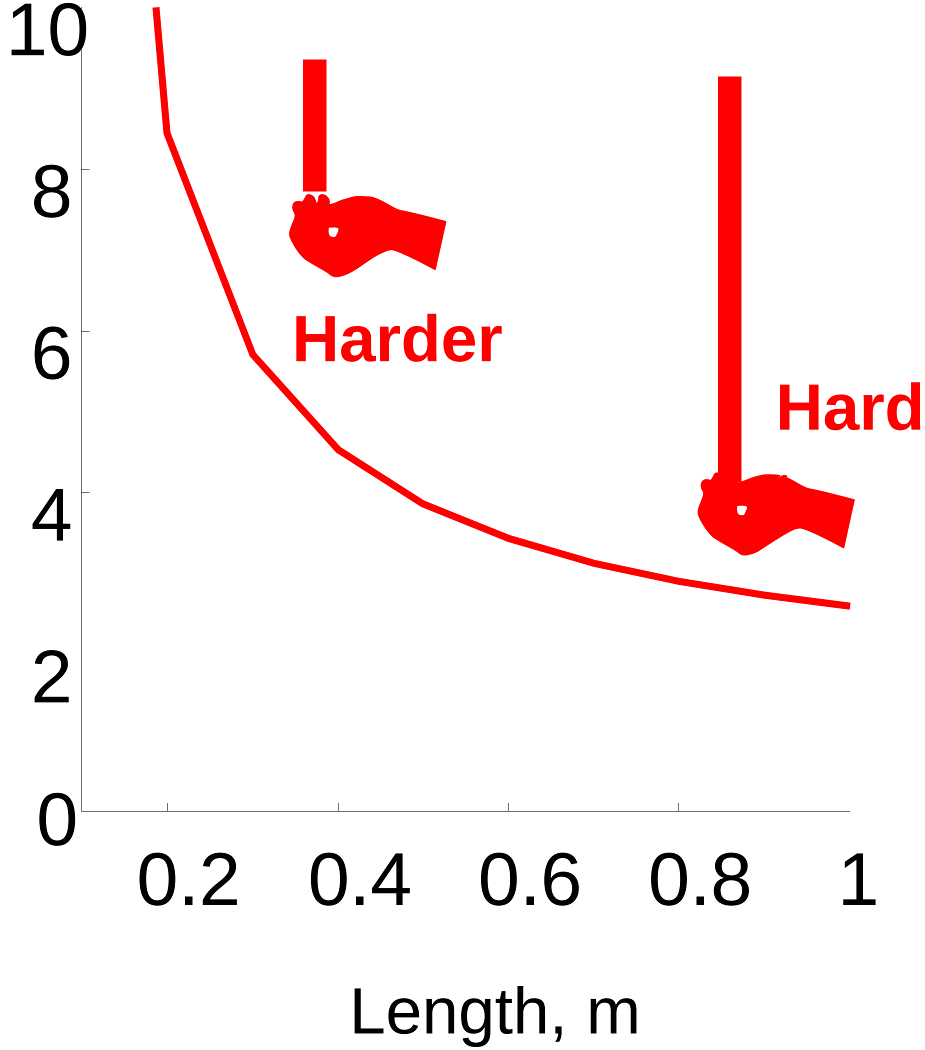
...

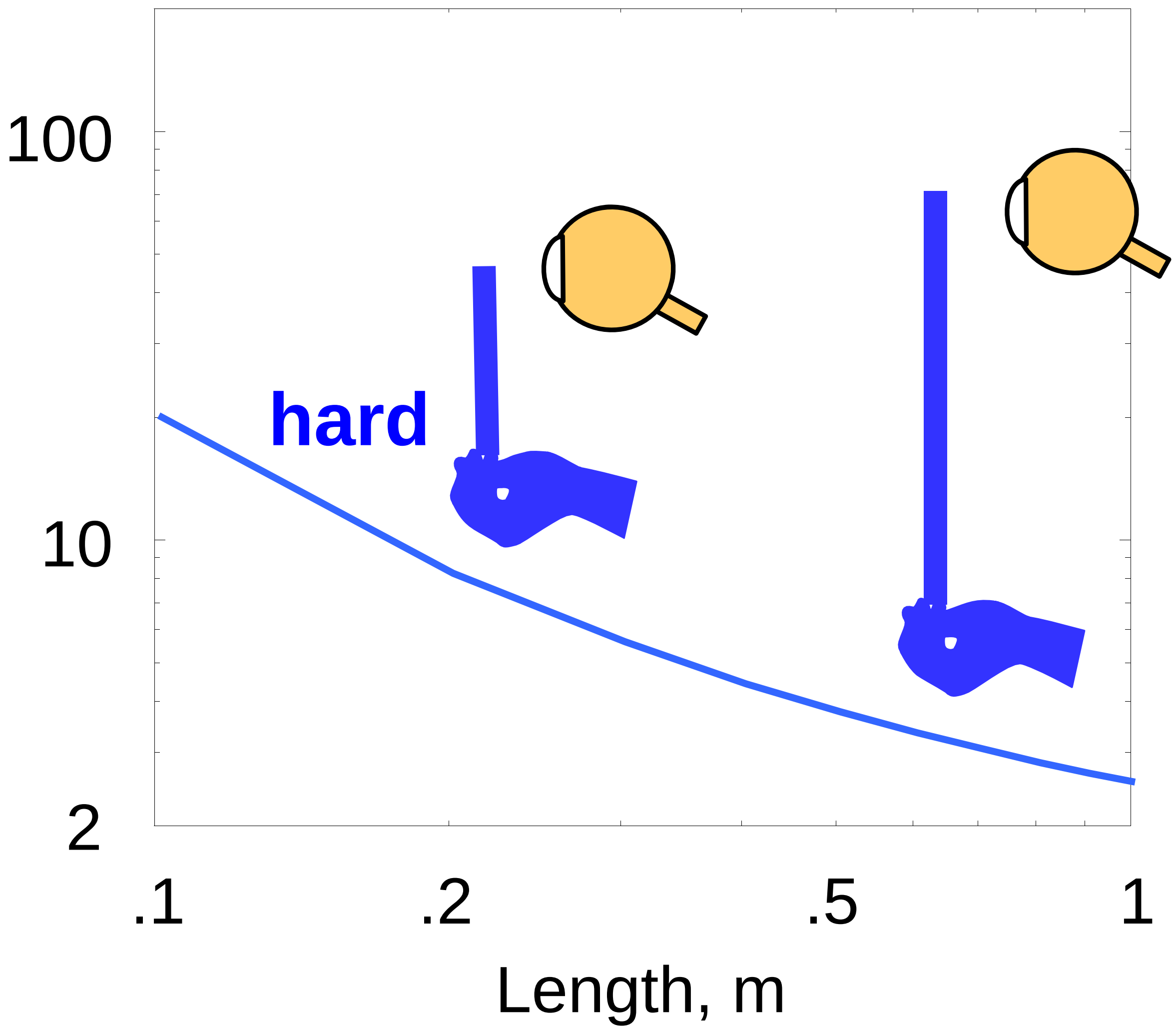


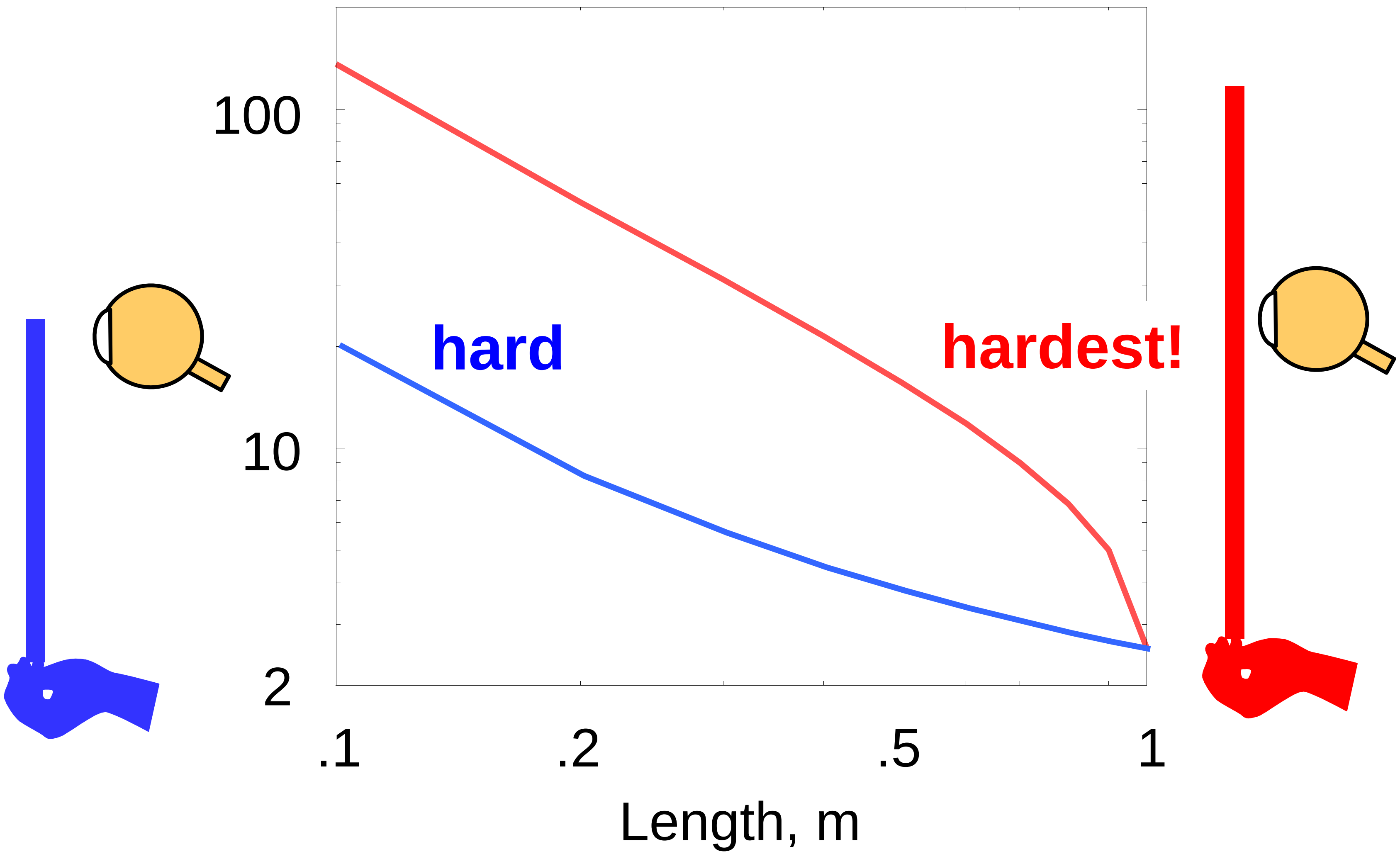


$$\exp(p\tau)$$

Theory & Data

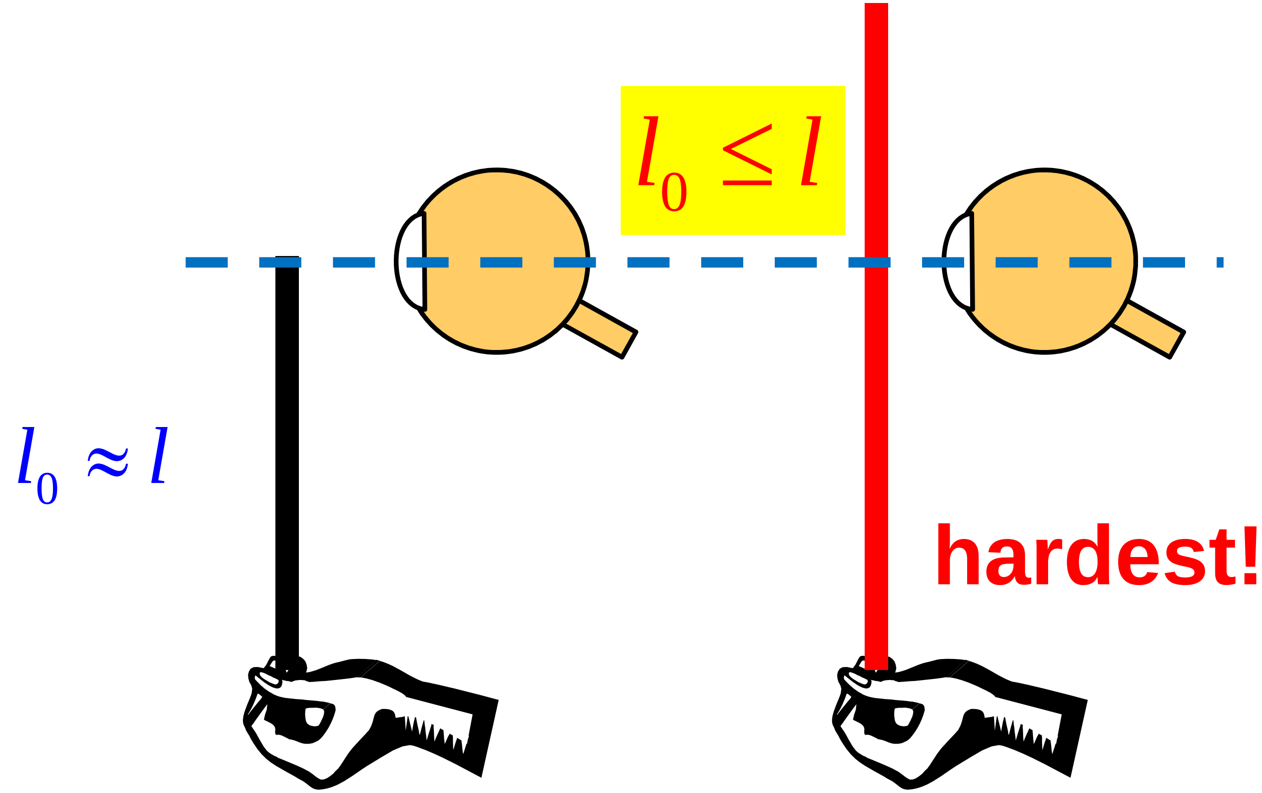



100
10
2
hard
.1
.2
.5
1
Length, m

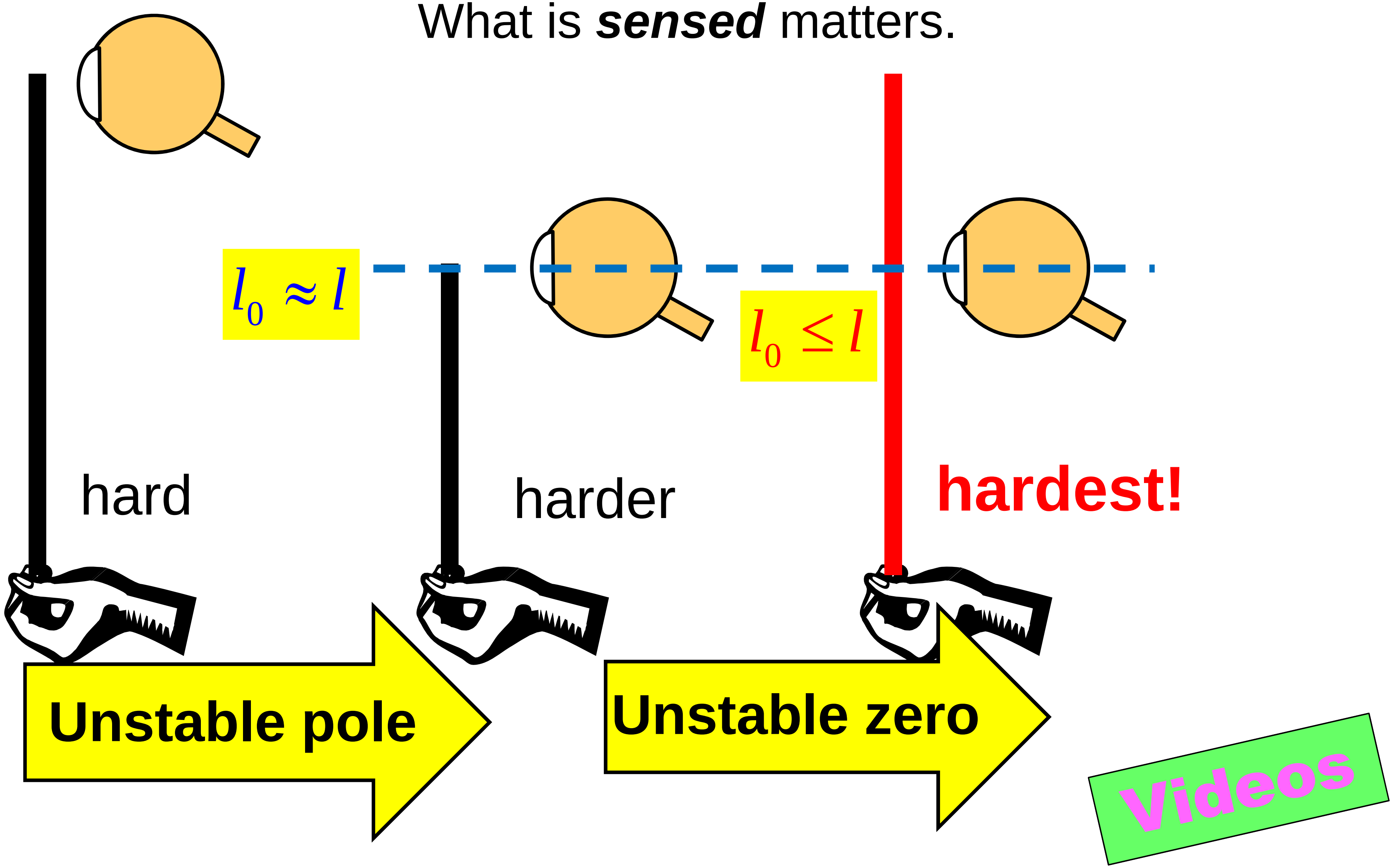

100
10
2
.1
.2
.5
1
Length, m
hard
hardest!

$$\left.\begin{matrix}\exp\left(\int \ln|T|\right) \\ \|T\|_\infty\end{matrix}\right\} \geq \exp(p\tau)\left|\frac{z+p}{z-p}\right| \geq \exp(p\tau)$$

ODE model



What is ***sensed*** matters.

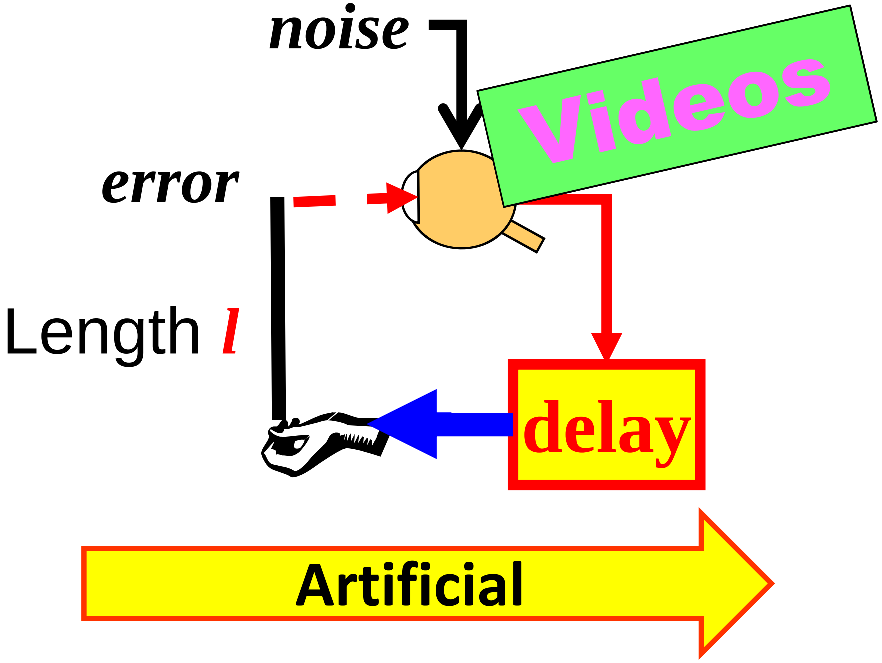


Lower

delayed
reflexes
small disturb
large error
need speed

unstable(real)
distributed
local

unconscious
automatic



$$\min_{L} \sup_{\|w\|_\infty \le 1} \|x\|_\infty = \sum_{i=1}^{T_S} \left|a^{i-1}\right|$$

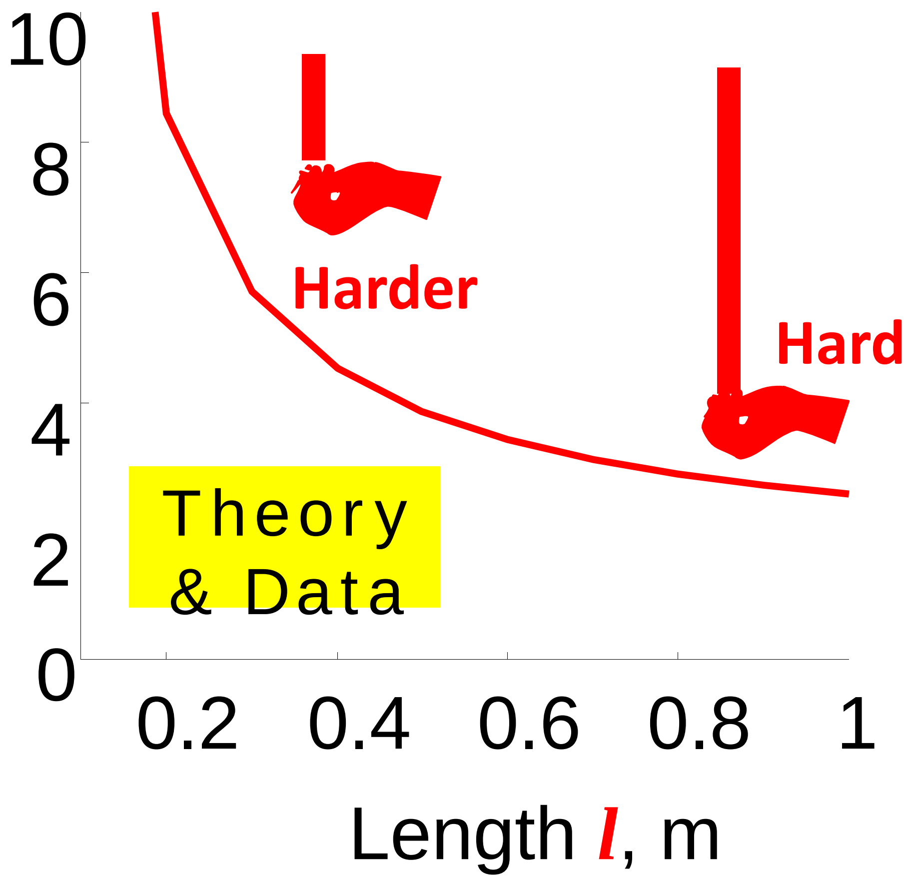
High

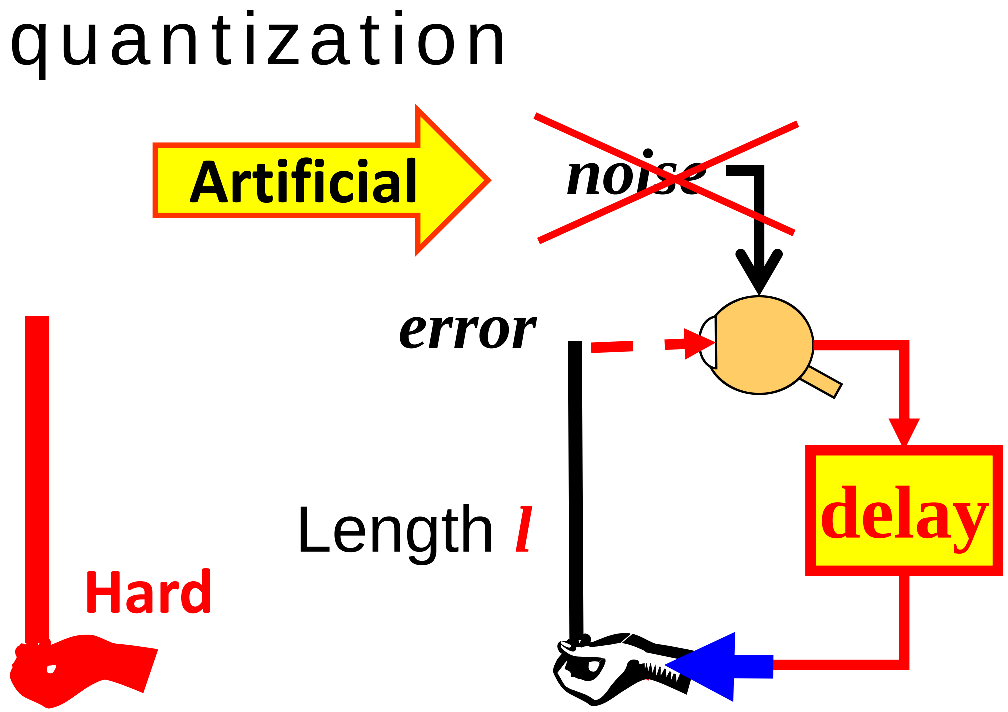
~~advanced~~
~~planning~~
~~large disturb~~
~~small error~~
~~need accuracy~~

~~stable(virtual)~~
centralized
global

conscious
deliberate

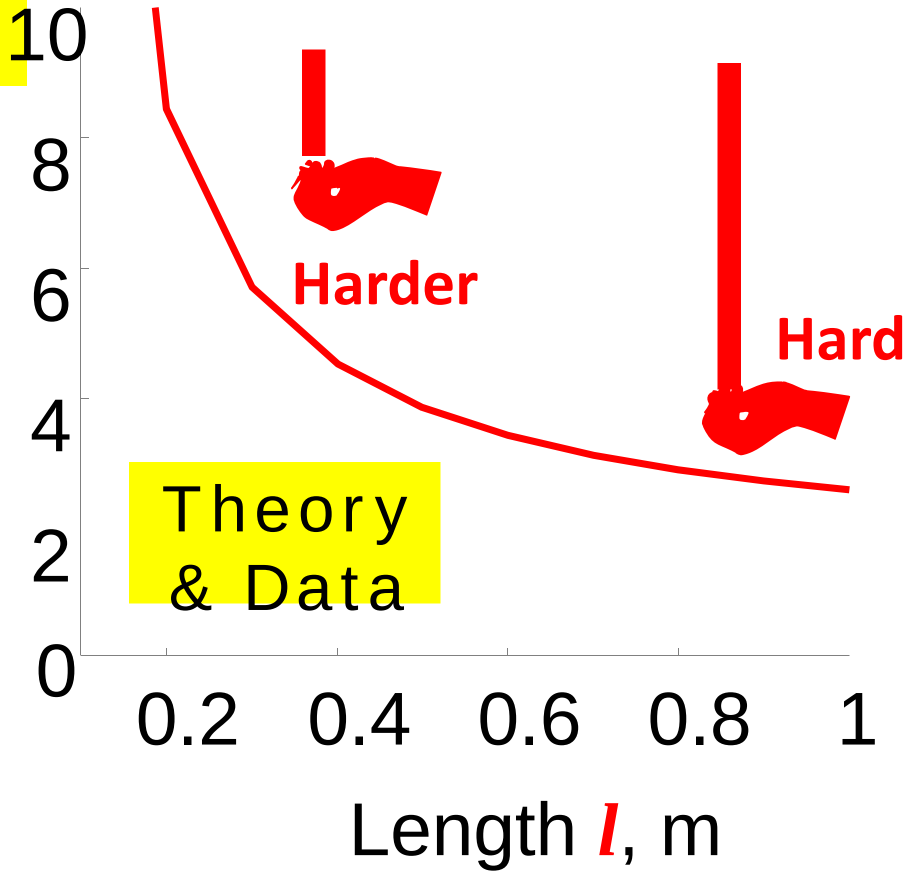
$$\sum_{i=1}^{T_S} \left| a^{i-1} \right|$$

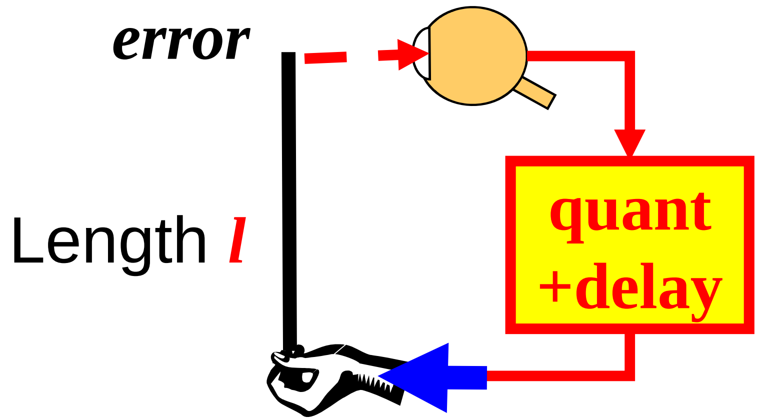




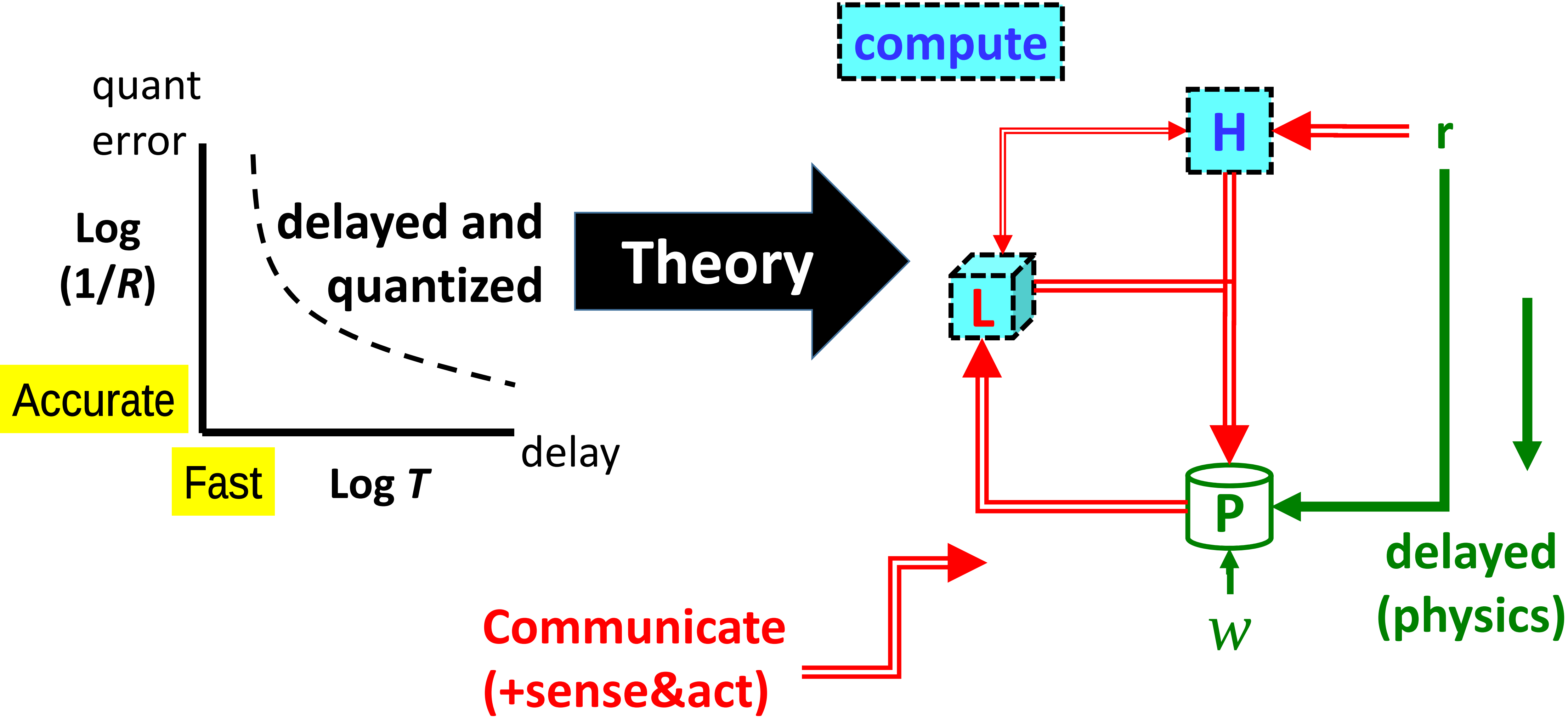
Delay + quantization

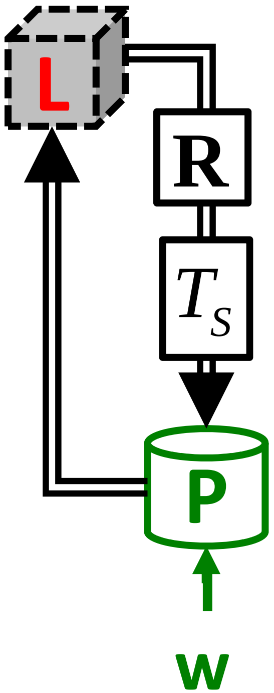
$$\left|a^T\right|\left(2^R-|a|\right)^{-1}$$





Speed vs Accuracy





R = R bits/time

Delay ($T_s > 0$)

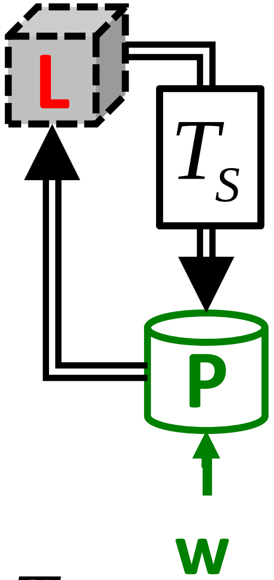
Delay + quantization

Full information

$$u(t) = f\big(x(0:t), r(0:t), u(0:t-1)\big)$$

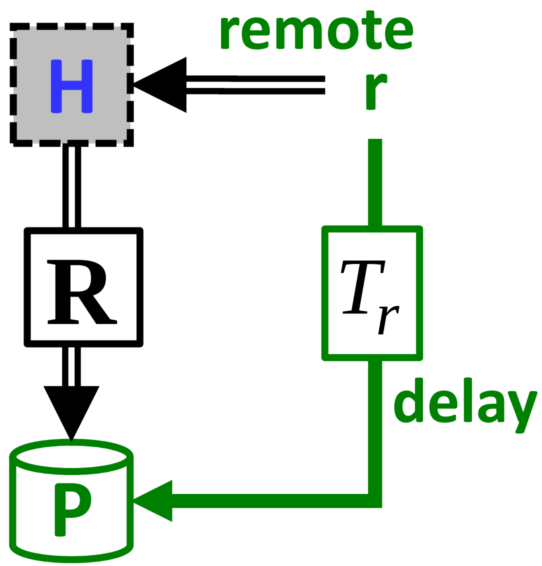
$$x(t+1) = ax(t) - \mathbf{R}\big[u(t-T_s)\big] + w(t)$$

quant delay



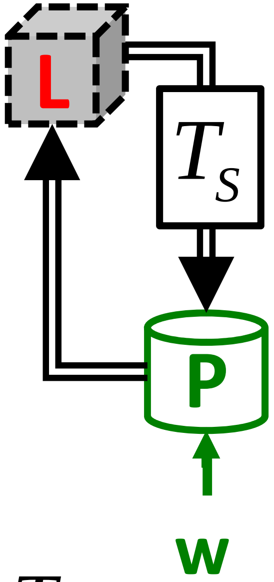
$$\sum_{i=1}^{T_S} \left| a^{i-1} \right|$$

delay



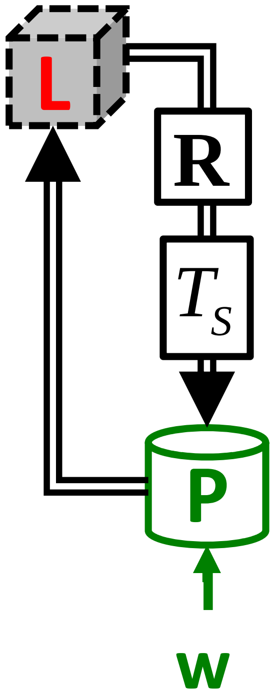
$$\left(2^R - |\,a\,|\right)^{-1}$$

quant



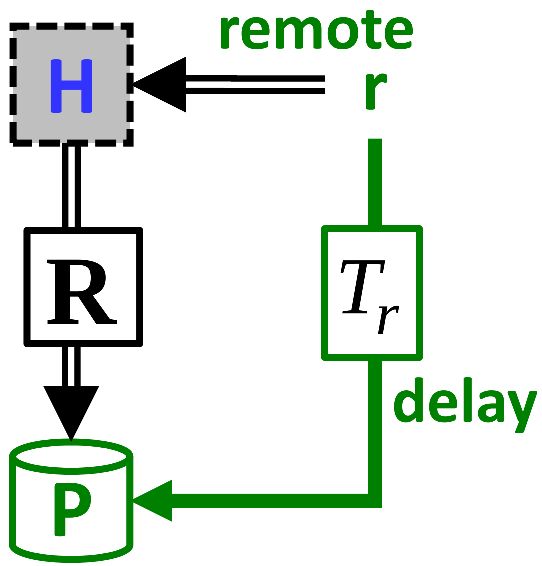
$$\sum_{i=1}^{T_s} \left| a^{i-1} \right|$$

delay



$$\sum_{i=1}^{T_s} \left| a^{i-1} \right| \quad + \quad \left| a^{T_s} \right| \left(2^R - |a| \right)^{-1}$$

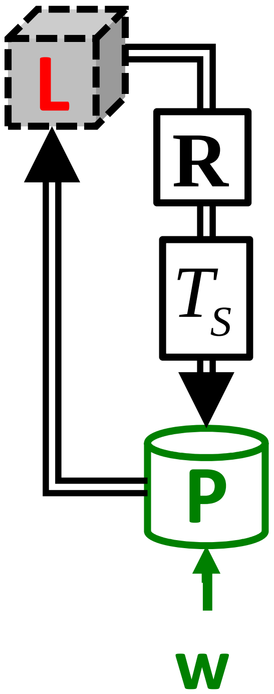
delay delay+quant



$$\left(2^R - |a| \right)^{-1}$$

quant

Delay + quantization



$\mathbf{R} = R$ bits/time

Delay ($T_s > 0$)

Full information

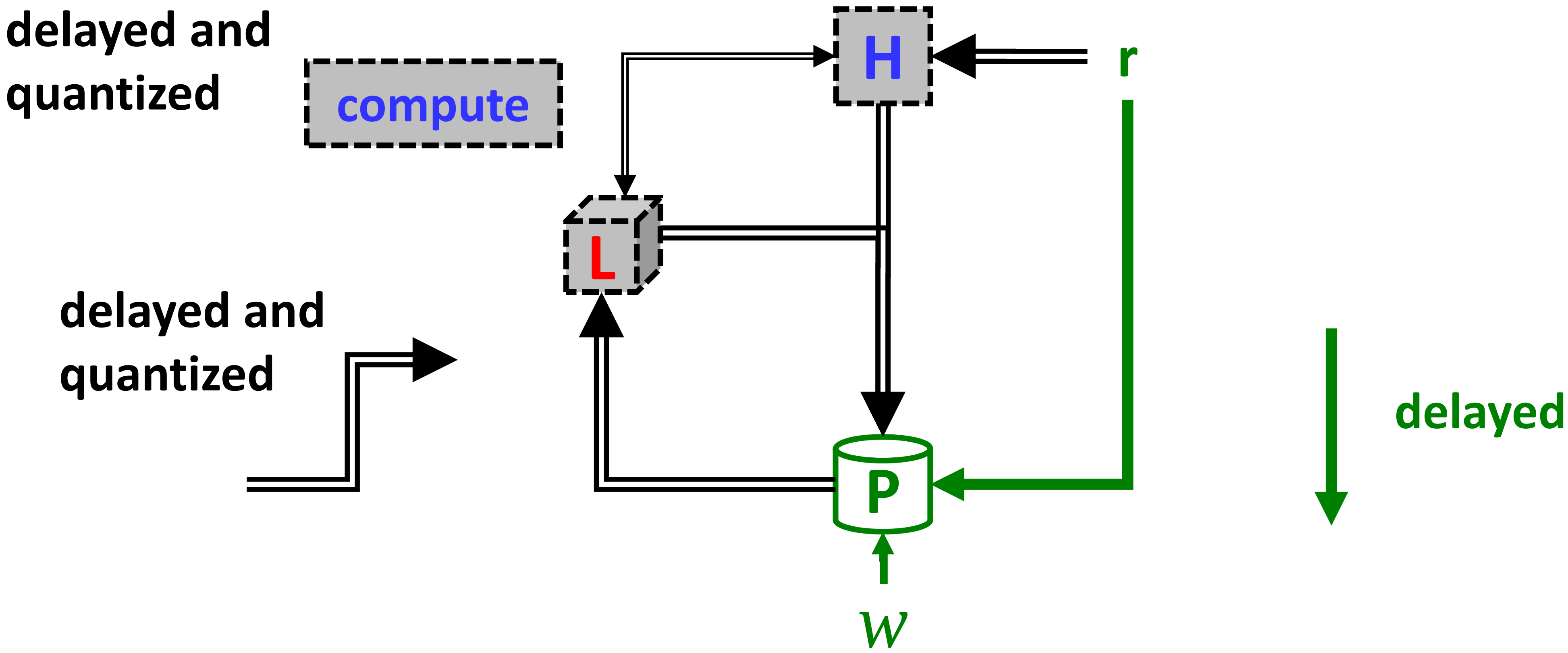
$$u(t) = f\big(x(0:t), r(0:t), u(0:t-1)\big)$$

$$x(t+1) = ax(t) - \mathbf{R}\big[u(t - T_s)\big] + w(t)$$

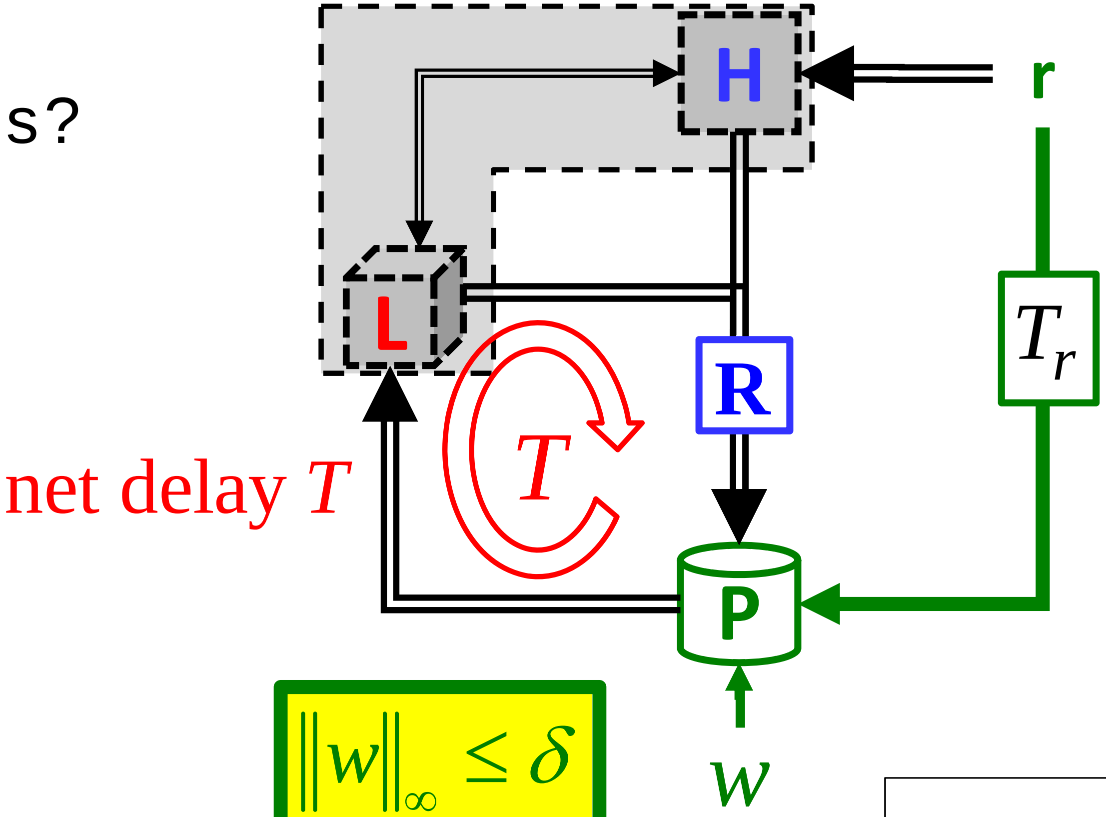
quant delay

$$\min_{L,\ R} \ \sup_{\|w\|_\infty \le 1} \| x \|_\infty = \underbrace{\sum_{i=1}^{T_s} \left|a^{i-1}\right|}_{\text{delay}} + \underbrace{\left|a^{T_s}\right| \left(2^R - |a|\right)^{-1}}_{\text{delay+quant}}$$

G. Nair et al., (2007) Feedback control under data rate constraints: An overview


delayed and
quantized
compute
H
r
L
delayed and
quantized
delayed
P
w

Assumptions?



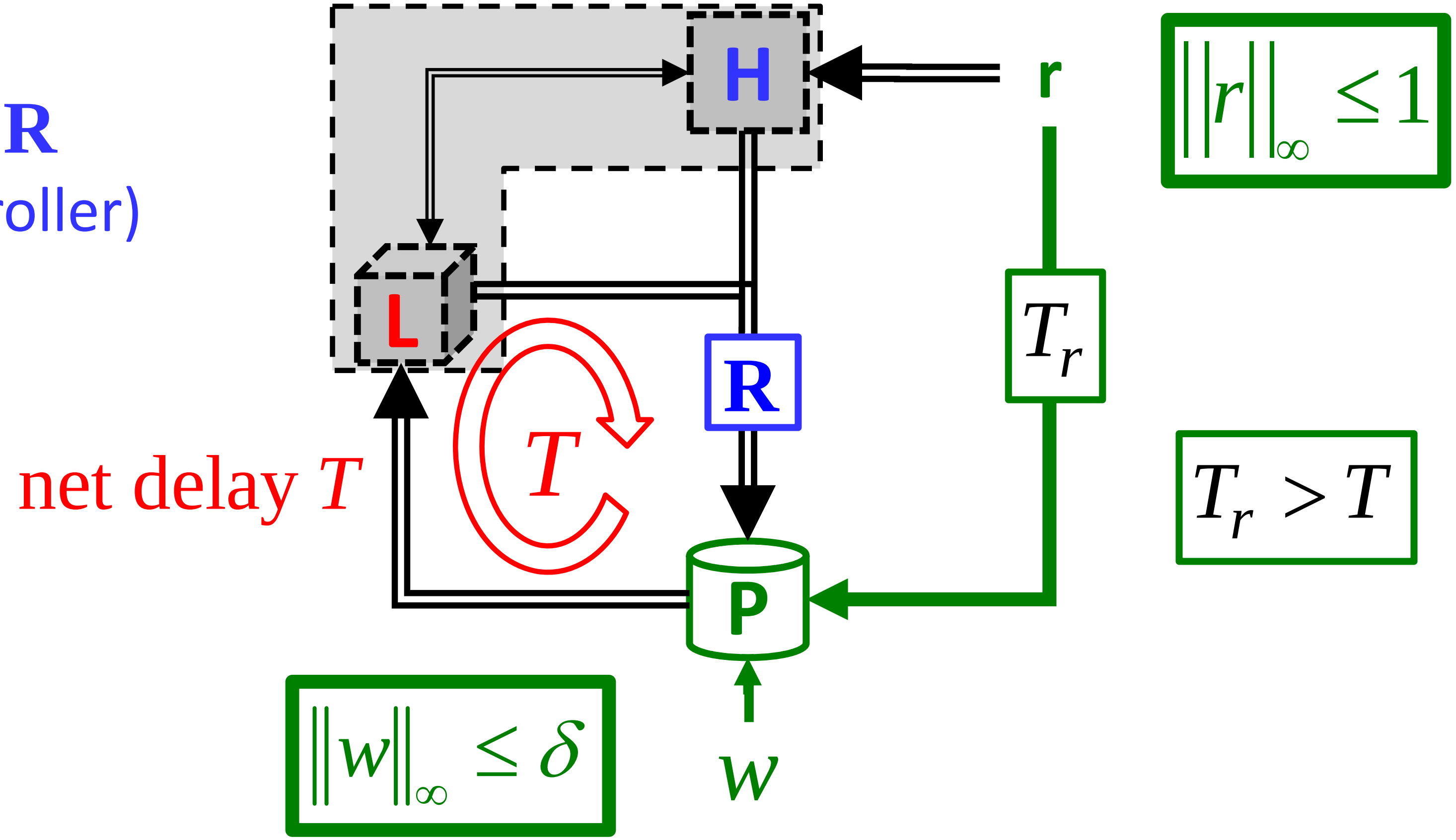
$$\|r\|_\infty \le 1$$

$$T_r > T$$

$$\|w\|_\infty \le \delta$$

$$\min_{L,\ R} \ \sup_{\|w\|_\infty \le 1} \|x\|_\infty$$

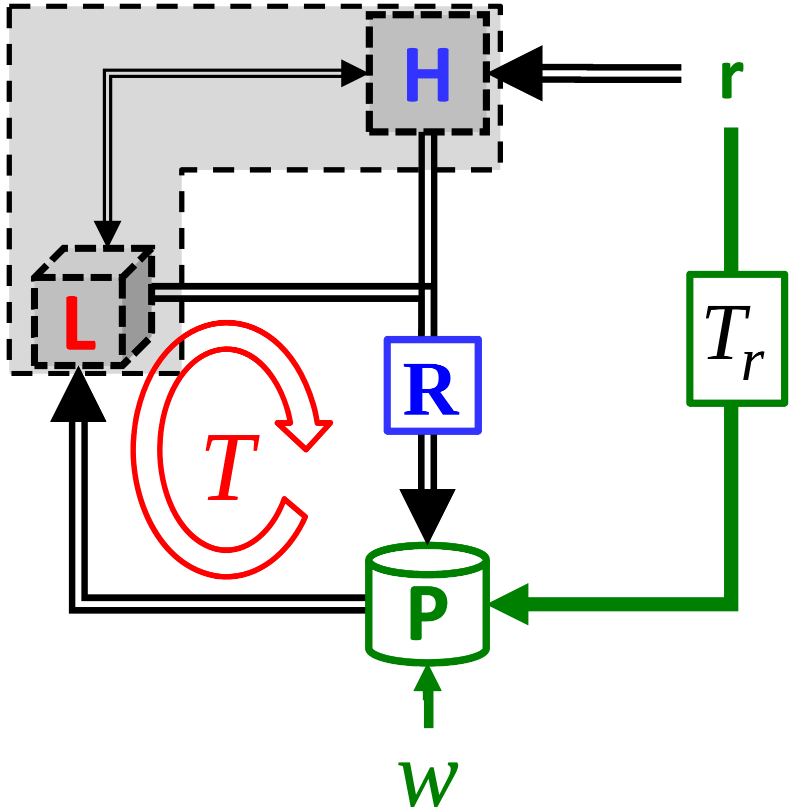
One quantizer **R**
(and one controller)



$$\min\max\|x\|_\infty = \delta\left(\sum_{i=1}^{T}\left|a^{i-1}\right| + \left|a^{T}\right|\left(2^{R}-|a|\right)^{-1}\right) + \left(2^{R}-|a|\right)^{-1}$$

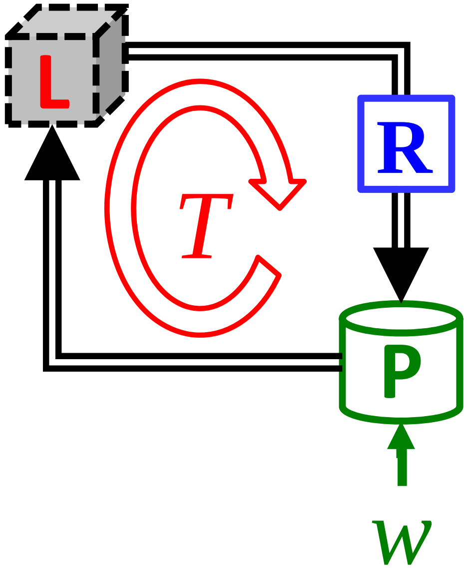
delay **delay+quant** **quant**

One quantizer
(one controller)



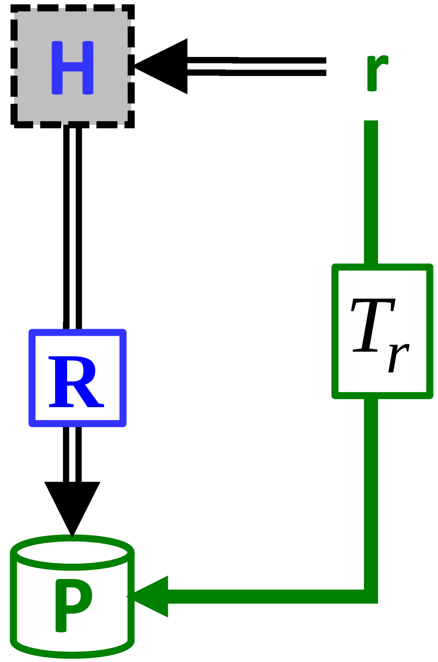
$$\delta\left(\sum_{i=1}^{T}\left|a^{i-1}\right|+\left|a^{T}\right|\left(2^{R}-|a|\right)^{-1}\right)\quad+\left(2^{R}-|\,a\,|\right)^{-1}$$

delay　　　delay+quant　　　quant



$$\|w\|_\infty \le \delta$$

Need $\delta << 1$



$$\delta\left(\sum_{i=1}^{T}\left|a^{i-1}\right| + \left|a^{T}\right|\left(2^{R} - |a|\right)^{-1}\right) \qquad +\left(2^{R} - |a|\right)^{-1}$$

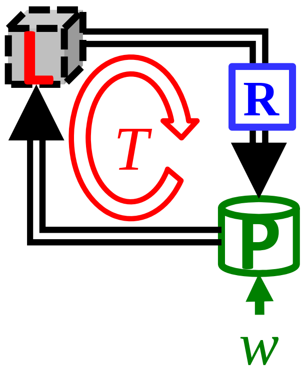
delay — delay+quant — quant

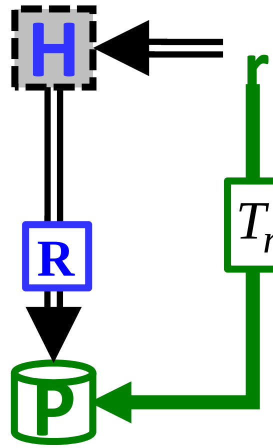
$$\{a \geq 1, T \to \infty\}$$

$$\Rightarrow \sum_{i=1}^{T} \left| a^{i-1} \right| \to \infty$$

$$R \to \infty$$

$$\Rightarrow \left(2^R - |a|\right)^{-1} \to 0$$





$$\delta\left(\sum_{i=1}^{T} \left|a^{i-1}\right| + \left|a^{T}\right| \left(2^{R} - |a|\right)^{-1}\right) + \left(2^{R} - |a|\right)^{-1}$$

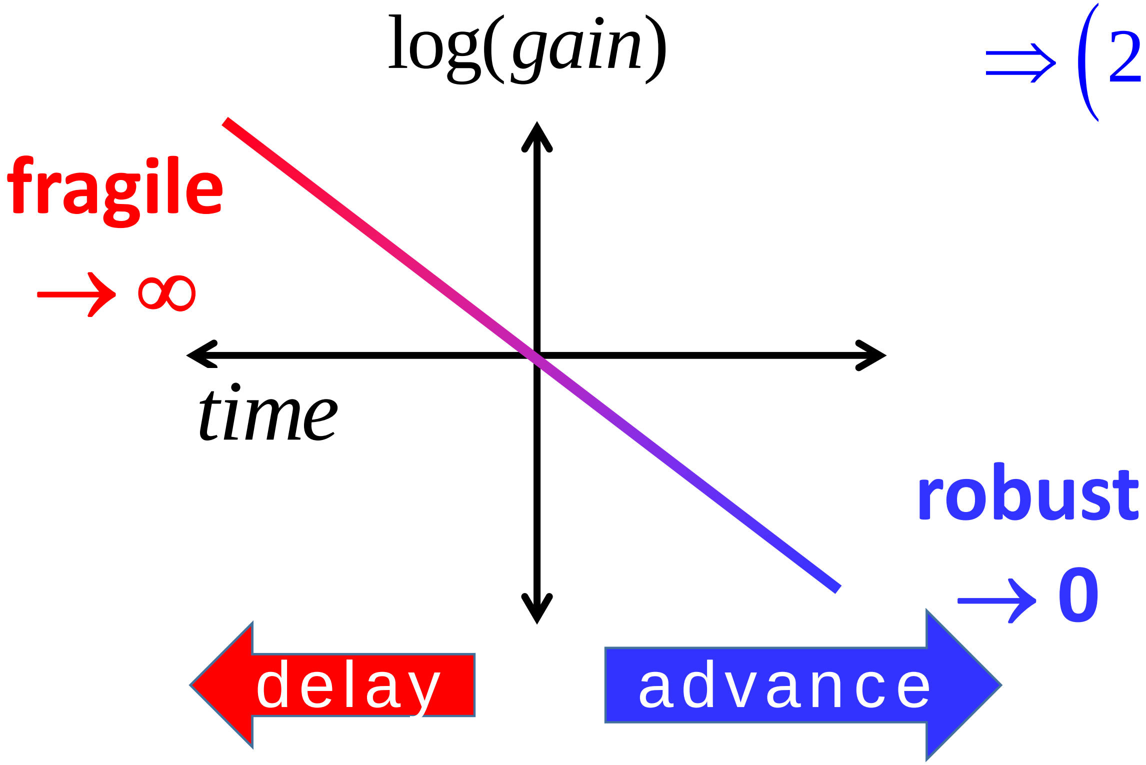
delay **delay+quant** **quant**

$$\{a \ge 1, T \to \infty\}$$

$$\Rightarrow \sum_{i=1}^{T} \left|a^{i-1}\right| \to \infty$$

$$R \to \infty$$

$$\Rightarrow \left(2^{R} - |a|\right)^{-1} \to 0$$

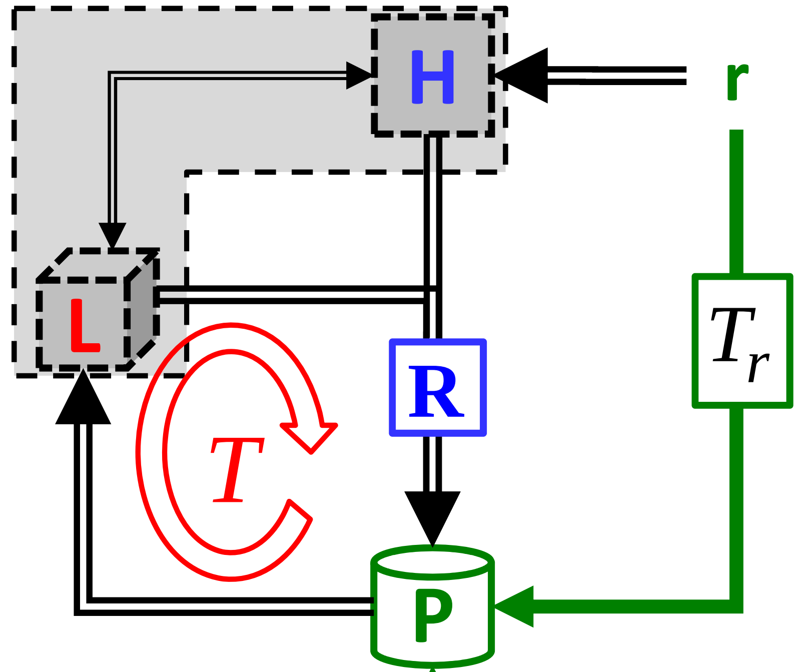


$$\delta\left(\sum_{i=1}^{T} \left|a^{i-1}\right| + \left|a^{T}\right| \left(2^{R} - |a|\right)^{-1}\right) + \left(2^{R} - |a|\right)^{-1}$$

delay **delay+quant** **quant**

Large literature

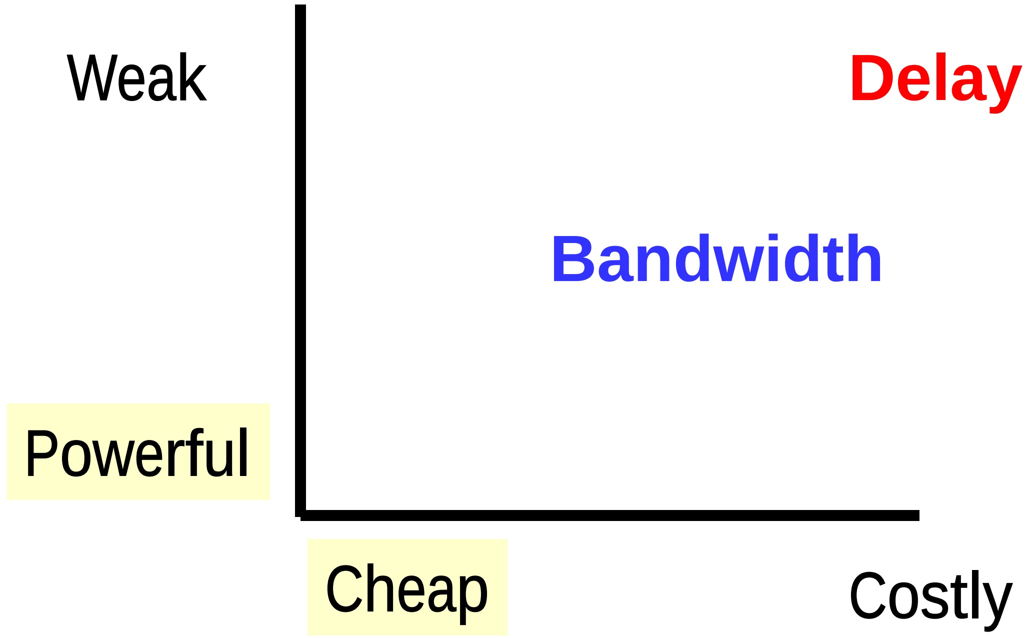
- speed/accuracy tradeoffs
- Fitts' law
- sensorimotor, optimal, and robust control (e.g. Wolpert)



consistent with

delay   delay+quant   quant

$$\delta\left(\sum_{i=1}^{T}\left|a^{i-1}\right| + \left|a^{T}\right|\left(2^{R} - |a|\right)^{-1}\right) \quad + \left(2^{R} - |a|\right)^{-1}$$

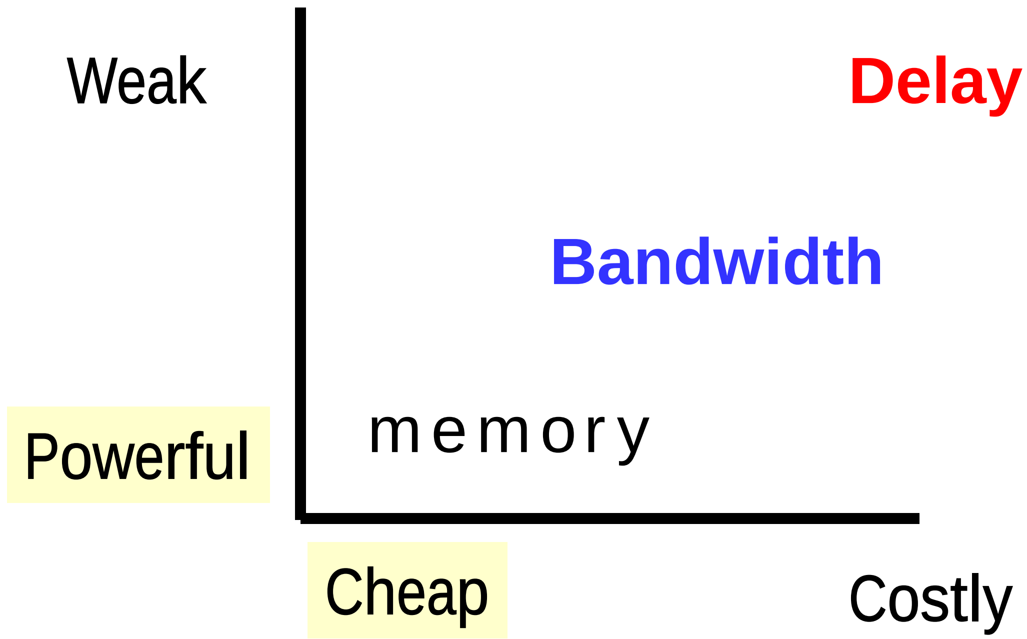


exponential **dependence on *R* (good) and *T* (bad)**

$$\delta\left(\sum_{i=1}^{T}\left|a^{i-1}\right| + \left|a^{T}\right|\left(2^{R}-|a|\right)^{-1}\right) + \left(2^{R}-|a|\right)^{-1}$$

Ptime<<NPtime*

Nptime<<
Pspace=NPspace

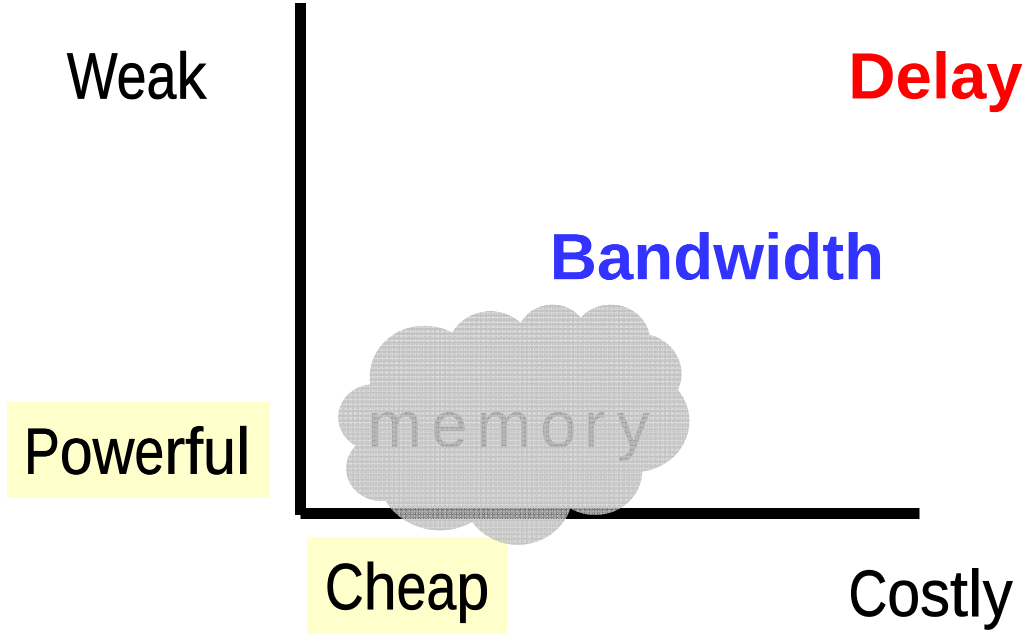


$$\delta\left(\sum_{i=1}^{T}\left|a^{i-1}\right|+\left|a^{T}\right|\left(2^{R}-|a|\right)^{-1}\right)+\left(2^{R}-|a|\right)^{-1}$$

* we think

Ptime<<NPtime*

Nptime<<
Pspace=NPspace



$$\delta\left(\sum_{i=1}^{T}\left|a^{i-1}\right|+\left|a^{T}\right|\left(2^{R}-|a|\right)^{-1}\right)+\left(2^{R}-|a|\right)^{-1}$$

* we think

Issues

$$\min_{\boxed{L}\ \boxed{R}} \sup_{\|w\|_\infty \le \delta} \|x\|_\infty = \sum_{i=1}^{T_S} \left|a^{i-1}\right| + \left|a^{T_S}\right| \left(2^R - |a|\right)^{-1}$$

Robustness: $\sup_{\|w\|_\infty \le \delta} \|x\|_\infty$ *vs* $\sup_{\|w\|_2 \le \delta} \|x\|_2$ *vs* $E\left(|x|^2\right)$

l_1 $\qquad h_\infty \qquad h_2$

crash

Issues

$$\min \sup_{\|w\|_\infty \le \delta} \|x\|_\infty = \sum_{i=1}^{T_S} \left|a^{i-1}\right| + \left|a^{T_s}\right| \left(2^R - |a|\right)^{-1}$$

L R

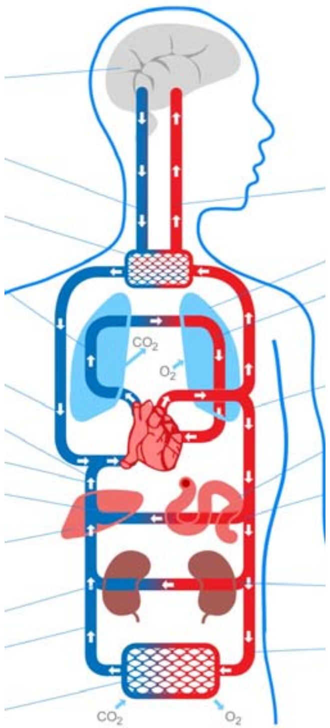
$$\sup_{\|w\|_\infty \le \delta} \|x\|_\infty$$

crash

Issues

$$\min_{\boxed{L}\ \boxed{R}} \sup_{\|w\|_\infty \le \delta} \|x\|_\infty = \sum_{i=1}^{T_S} \left|a^{i-1}\right| + \left|a^{T_S}\right| \left(2^R - |a|\right)^{-1}$$

$$\sup_{\|w\|_\infty \le \delta} \|x\|_\infty \ vs \ \sup_{\|w\|_2 \le \delta} \|x\|_2 \ vs \ E\left(|x|^2\right)$$



videos