A regenerative braking control strategy for electric vehicle with four in-wheel motors

Wei Xu[2], Haiyan Zhao [1,2], Bingtao Ren [2], Hong Chen* [1,2]

1. State Key Laboratory of Automotive Simulation and Control, Changchun, 130025, P. R. China
E-mail: chenh@jlu.edu.cn

2. Jilin University Department of Control Science and Engineering,Changchun, 130025, P. R. China
E-mail: wei.xu.jlu@hotmail.com

Abstract: Regenerative braking control technology of electric vehicle plays a vital role in automotive energy-saving and environmental protection. Actually, there are two important aspects included in regenerative braking control. First and foremost is to maintain the vehicle safety during the braking process, and secondly is to maximize the energy recovery and minimize the energy consumption as far as possible. This paper proposes a regenerative braking control strategy to meet the above two aspects. In this research, the electric vehicle is assumed to keep straight line driving with a driver. First, according to the desired braking torques of the driver during braking process, the brake torque on front and rear axle respectively are allocated based on the tire load ratio, which makes sure that maximizing the use of tire adhesion during deceleration. Second, in order to deal with multi-objections and constraints for maximizing the energy recovery and minimizing the energy consumption, an model predictive controller is designed to distribute the brake torque between the hydraulic brake mode and the electric motor brake mode. In the end, the effectiveness of the proposed strategy is verified through the simulations of the electric vehicle model with four individually driven in-wheel motors based on Matlab/Simulink and AMESim software.

Key Words: electric vehicle, regenerative braking, model predictive control, optimal control

1 Introduction

In order to improve the energy efficiency, energy conservation and ecological environment, the research and development of electric vehicles have become a big concern of the public in recent years. Pure electric vehicle with four independent drive wheels is especially investigated extensively, which thanks to the four wheels can be controlled not only independently, but precisely. Because the torque of the four in-wheel electric motor can be measured and controlled accurately [1].

Due to the limitation of battery and motor technology, the high costs and short driving distance seriously hampered the development of electric vehicle [2]. However, regenerative braking system could convert the kinetic energy and potential energy of vehicle to electrical energy stored in energy storage device for the next driving. Therefore, regenerative braking has become the key technology of electric vehicles by reducing energy consumption and extending driving distance in the present research.

The braking mode can be divided into fully hydraulic braking system, fully electric braking system and hybrid braking system. The traditional friction braking system, i.e. fully hydraulic braking system makes the vehicle decelerates or stops with friction plates by converting the kinetic energy of vehicle to thermal energy dissipated. This braking mode has not at all ability of implementing the energy recovery. Fully electric braking system can recover energy during the entire braking process [3]. It indeed bring about a very fast and accurate brake torque control system with only motors as the actuators for the braking process, but what shortcoming of this braking mode is that the brake torque by electric motors is limited than the traditional hydraulic braking mode, so that would cause a result of insufficient braking force while high strength braking force is required. In addition, fully electric braking mode demands a higher-level energy storage system. The last one is the hybrid braking system, which implements deceleration or braking process by both hydraulic braking mode and electric motors braking mode. It well compensates for the disadvantages of the previous traditional friction braking system and fully electric braking system. Therefore, hybrid braking mechanism is employed in this research. The configuration diagram of hybrid braking system is shown in Fig. 1.

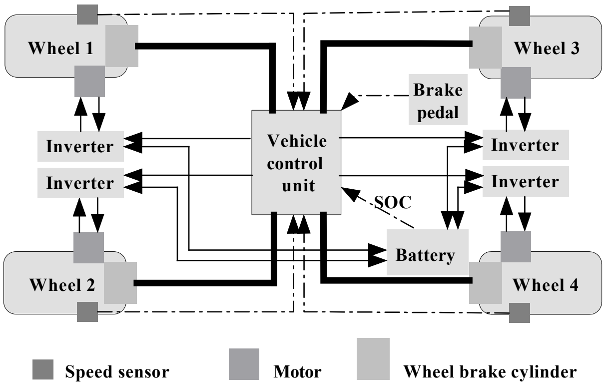

Fig. 1: Hybrid braking system

Considering the hybrid braking mode, the key topic of this approach is how to distribute the hydraulic brake torque and the electric motor brake torque under the premise of baking safety. Many scholars have done some research on this issue. Stratis Kanarachos and Mohsen Alirezaei proposed an allocation controller, which maximizes the regenerative braking energy through the state-dependent Riccati equation control technique and vehicle state estimation [4], where tire saturation and motor constraints are considered. The allocation of braking force of a regenerative braking system is devel-

---

This work was supported by the 973 Program (No.2012CB821202), the National Natural Science Foundation of China (No.61503149, No.61520106008, No.U1564207).

oped based on fuzzy logic control method in [5]. And a nonlinear model predictive controller for regenerative braking system is presented in literature [6], the controller improves the energy recovery by distributing the front and rear brake torque respectively. But the shortcoming is that it did not allocate the brake torque between the electric motor braking mode and the hydraulic braking mode, which limiting the efficiency of energy recovery.

In this paper, a study on the regenerative braking distribute issue using model predictive control method for a electric vehicle with four in-wheel motor is presented. The cost function considered in this research include three aspects. First is to tracking the driver's expectation. Second is the energy change of electric motors, the copper loss is especially considered in this research. The last is the energy consumption of hydraulic braking mode. Model predictive control method can efficiently deal with the multi-constraints online optimization problems included in the regenerative braking control system. Compared with general optimal control method, the model predictive control method employs finite horizon scrolling optimization strategy, not a global optimization objective. At each sample time, the model predictive control might solve the optimization problem to obtain the control vectors [7, 8]. In the end, the simulation verification of the presented regenerative braking strategy is carried out based on Matlab/Simulink and AMESim software.

The remaining parts of this article is organized as follows. The regenerative braking control system model is introduced in the second section. The third section presents the allocation scheme of brake torque for front and rear wheels to obtain maximum tire adhesion force, and the distribution strategy of hydraulic brake torque and the electric motor brake torque to ensure energy optimization. Simulation results and analyzes of the presented regenerative braking controller are given in the forth section, and the conclusions are proposed in the fifth section.

2 Regenerative Braking Control System Model

In this section a brief description of the regenerative braking control system model is introduced. It includes four main sections, vehicle dynamics model, tire model, motor and battery model.

2.1 Vehicle Dynamics Model

As lateral performance is neglected in this study, considering the vehicle dynamics model adopted here has five degrees of freedom for longitudinal motion and the rotational movement of the four wheels. A single wheel braking model is shown in Fig. 2. Assuming that the forces on the left and the right tires are the same, then according to Newtons second law [9], the equations of vehicle dynamic behavior in longitudinal and the rotational directions can be expressed as

$$M\dot{v} = -F_{xf} - F_{xr} - f_{air} - f_{roll}, \tag{1}$$

$$J\dot{\omega}_{wi} = R_e F_{xi} - T_{bi}, i = f, r, \tag{2}$$

where M and v represent the vehicle mass and the longitudinal velocity, F_{xf}, F_{xr} are the longitudinal tire-road friction force of the front and rear wheels. f_{air} and f_{roll} are respectively the air resistance and the rolling resistance of the electric vehicle which are concerned as zero in this paper. J is the wheel rotation inertia, ω_{wi} are the front and rear wheel longitudinal rotational speed in radians per second, R_e is the effective rolling radius in meters of the tire, and T_{bi} are the front and rear brake torque of the wheels.

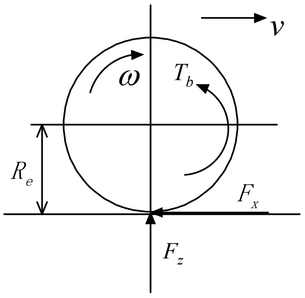

Fig. 2: Single wheel braking model

The tire-road friction force has a relationship with the normal force F_{zi} supported by the wheel, which is described with the friction coefficient μ as given in equation (3). Whereas the friction coefficient μ is the function of the slip ratio κ.

$$F_{xi} = \mu_i(\kappa)F_{zi}. \tag{3}$$

2.2 Tire Model

As the support and transfer unit between the road and vehicle, the tire characteristics have a direct impact on vehicle state. The Magic Formula model developed by Pacejka [10], which contains multi-parameters and exactly describes the tire characteristics, is widely used in the literatures [11]. In Magic Formula model, the longitudinal tire fore is expressed as a nonlinear function of the normal force F_{zi} and the slip ratio κ_i, which is shown in the following equation,

$$\begin{aligned} F_{xi} = 2D_x \sin\{C_x \arctan[B_x\kappa_i - E_x \\ (B_x\kappa_i - \arctan(B_x\kappa_i))]\}, \end{aligned} \tag{4}$$

where B_x, C_x, D_x, E_x respectively represent the stiffness, shape, peak and curvature factor, the value of them can be calculated through model parameters. The longitudinal slip ratio was introduced as an input variable instead of the braking or driving force, which denotes the speed difference between the wheel and the vehicle. The relationship between them is shown in equation (5).

$$\kappa_i = \frac{\omega_{wi}R_e - v}{v}, \tag{5}$$

The normal force F_{zi} can calculated from the deceleration a_x, which is shown in equations (6) and (7),

$$F_{zf} = \frac{M(l_r g - h a_x)}{l_f + l_r}, \tag{6}$$

$$F_{zr} = \frac{M(l_f g + h a_x)}{l_f + l_r}, \tag{7}$$

where g is the acceleration of gravity, h is the distance of the center of vehicle mass to ground, l_f and l_r are the the longitudinal distance from center of gravity to front and rear axis.

2.3 Motor and Battery Model

We assume that each in-wheel motor has a torque controller. As the electric machine is reversible, it works either as a motor or as a generator. The ability of energy recovery during braking process is associated with the maximum motor brake torque of the motor under the current rotary velocity. In this research, the maximum motor rotary velocity can achieve $9000\,\mathrm{rpm}$, the maximum brake torque is $118\,\mathrm{Nm}$, and the gear ratio g_0 is 5.

Another key element affects the ability of energy recovery is the motor-to-battery efficiency η [6], which can be described by

$$\eta = \frac{U_c I_c}{T_{mf}\omega_{mf} + T_{mr}\omega_{mr}}, \tag{8}$$

where U_c and I_c represent the battery charging voltage and current during the regenerative braking process, and the charging power of the battery P_c can be described in equation (9). T_{mf} and T_{mr} are respectively the absolute value of the motor brake torque, ω_{mf} and ω_{mr} are the electric motor angular velocity of front and rear wheel, which can described in equation (10).

$$P_c = U_c I_c, \tag{9}$$

$$\omega_{mi} = g_0 \omega_{wi}, \tag{10}$$

According to DC motor performance, the relation between motor current I_a and torque T_m can be described as

$$I_a = \frac{T_m}{C_T \phi} = \alpha T_m, \tag{11}$$

where C_T is the motor torque constant, and ϕ is the motor pole magnetic flux, which is assumed a constant in this research. Then the energy consumption of the motor resistance R, i.e. copper loss can be described as follows.

$$P_{loss} = \alpha^2 R T_m^2. \tag{12}$$

3 Regenerative Braking Controller Design

This section propose a brake torque allocation controller for regenerative braking system of electric vehicle with four in-wheel motors. The regenerative braking system include several modules, brake torque demand calculation module, brake torque distribution module, brake torque tracking module, and battery management unit. The structure of regenerative braking control system is shown in Fig. 3. In this article, we focus on the research of brake torque distribution module. This module mainly consists of two parts work, one aspect is to allocate the brake torque on front and rear axis, and the other is distributing the brake torque between hydraulic braking mode and electric braking mode. Note that the brake torque demand calculation and tracking module, and battery management unit are assumed useful and beneficial, we will conduct research on these issues in the future.

3.1 Braking Torque Allocation of Front and Rear Wheels

Safety is the premise of energy recovery of the whole braking process. Experience has shown that, vehicle would be unstable if rear wheel locks before front wheel. A shock rotation or tail flick might appear while vehicle is driven in high speed on low friction coefficient road. In order to ensure vehicle in a safe state, a large adhesion between tire and road is expected. As the mutative tire load ratio of front and rear axis have the staple impact on tire-road adhesion, we consider allocating the brake torque based on the mutative tire load ratio of front and rear axis.

According to the previously mentioned tire model, we can easily get the mutative tire load ratio of front and rear axis based on normal force equations.

$$\frac{F_{zf}}{F_{zr}} = \frac{l_r g - h a_x}{l_f g + h a_x}. \tag{13}$$

Therefore, the brake torque coefficient of front and rear axis K can be described as

$$K = \frac{F_{zf}}{F_{zr}} = \frac{T_{mf}}{T_{mr}} = \frac{T_{hf}}{T_{hr}}. \tag{14}$$

3.2 Torque Allocation of Hydraulic Brake and Electric Brake Based on MPC

In this section, a model predictive controller is designed for torque allocation of hydraulic brake and electric brake. First, the controller design model is given based on vehicle dynamics equations. Then the cost function is established for the purpose of braking safety and energy Optimization, next the control constraints are introduced. Finally, the approach for solving the cost function is presented.

According to the current measurement information obtained, model predictive control method solve the finite horizon open loop optimization problems online at each sampling time, and the first element of the obtained control vectors from optimal solution would be applied to the electric vehicle model. On the basis of the new measurements, this procedure is repeated at next sampling time in optimization process [12, 13].

A. Controller design model

According to the vehicle dynamics model, the state equation for hydraulic and electric braking distribution can be established. The state and control variables are defined as follows,

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \omega_{mf} \\ \omega_{mr} \end{bmatrix},$$

$$U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} T_m \\ T_h \end{bmatrix}.$$

Then based on equation (2), (3), (10), the state equations can be described as follows,

$$\dot{x}_1 = \frac{g_0}{2J}\left[\mu_f F_{zf} R_e - \frac{K}{1+K}(g_0 u_1 + u_2)\right], \tag{15a}$$

$$\dot{x}_2 = \frac{g_0}{2J}\left[\mu_r F_{zr} R_e - \frac{1}{1+K}(g_0 u_1 + u_2)\right]. \tag{15b}$$

Note that, brake torque T_m and T_h are both defined positive. K is the ratio of front wheel brake torque to rear wheel brake torque, which is defined in the above section.

B. Cost function

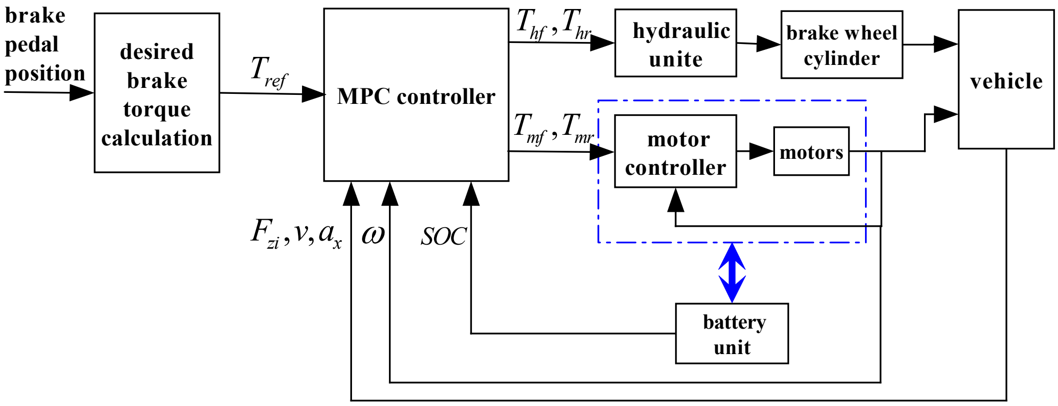

Fig. 3: Structure of regenerative braking control system

Due to the goal that recovering as much energy as possible by distributing the brake torque between hydraulic and electric brake, several cost function terms are need to be established. Actually, the braking mode switching is determined by the characteristics and constraints of the electric motor. First, in order to meeting the demand of total required brake torque and maximizing energy recovery, the electric motor braking mode is implemented as far as possible. If motors have achieved the maximum torque corresponding the current motor speed, then hydraulic braking mode compensate the remaining part of total required braking demand. Secondly, as the vehicle velocity decreases, energy recovery power of electric motors would be lower than its internal copper loss power, then the motors can not produce regenerative current. Therefor, the electric motor braking mode is completely converted into hydraulic braking mode while the motor speed achieve the certain value.

There are three cost function terms considered here, the unified objective of them are to be minimum in the current predictive horizon. First, the sum of motor brake torque and hydraulic brake torque is demanded to track the desired total brake torque T_{ref}, which is given from the diver to ensure the safety. We assumed that the required brake torque is calculated in advance and precisely. The form of this cost term J_1 is described as equation (16), in which e is the brake torque error. The objective of this term is to make the sum of the total brake torque error square from current time t to $t+pT_s$ as small as possible, p represents the predictive horizon and T_s represents the sampling time.

$$\begin{aligned} J_1 &= \int_t^{t+pT_s} e^2 dt \\ &= \int_t^{t+pT_s} (g_0 T_m + T_h - T_{ref})^2 dt. \end{aligned} \tag{16}$$

Secondly, for the purpose of energy optimization, we should maximizing the energy recovery and reducing the energy consumption as far as possible. Then based on the motor and battery model equations (8), (11), (12), as given in the second section, the second cost function J_2 is defined as equation (17), which represents the sum of charging power P_c. It is equal to the difference of copper loss power P_{loss} and energy recovery power P_m. Actually, what this cost function implies is the energy condition of electric motor braking mode. The ideal work mode switching of hybrid braking mode during braking process is that, if the energy recovery power is lager than the copper loss power, then implement the motor braking mode. Conversely, implement the hydraulic braking mode.

$$\begin{aligned} J_2 &= \int_t^{t+pT_s} -P_c dt \\ &= \int_t^{t+pT_s} (P_{loss} - P_m) dt \\ &= \int_t^{t+pT_s} [\alpha^2 R T_m^2 - \eta(\frac{K\omega_{mf} T_m}{K+1} + \frac{\omega_{mr} T_m}{K+1})] dt. \end{aligned} \tag{17}$$

Furthermore, in order to achieve the aims of saving energy and protecting the environment, the less energy consumption from hydraulic braking mode, the better. As a result, the third cost function J_3 is defined in equation (18), in which P_h represents the hydraulic braking power. This cost function represents the sum of hydraulic braking power, it implies the energy consumption of hydraulic braking mode at four tires.

$$\begin{aligned} J_3 &= \int_t^{t+pT_s} P_h dt \\ &= \int_t^{t+pT_s} [\frac{K\omega_{mf} T_h}{g_0(K+1)} + \frac{\omega_{mr} T_h}{g_0(K+1)}] dt. \end{aligned} \tag{18}$$

Due to the control requirements of vehicle safety and maximizing energy recovery, the three cost functions for the overall performance are demanded coordinate. So, according to the previously mentioned cost functions, the integral cost function is defined as follows,

$$\begin{aligned} \min J[u(\cdot)] &= pJ_1 + qJ_2 + sJ_3 \\ &= \int_t^{t+pT_s} [q(g_0 u_1 + u_2 - T_{ref})^2 \\ &+ p(\alpha^2 R u_1^2 - \frac{\eta u_1 (K x_1 + x_2)}{K+1}) \\ &+ \frac{s u_2 (K x_1 + x_2 u_2)}{g_0(K+1)}] dt, \end{aligned} \tag{19}$$

where q, p, s represent the weighting coefficients that determine the significance of each cost terms.

C. Constraints

Due to the limitation of the in-wheel motors, the maximum absolute value of the brake torque for each wheel $T_{b,max}$ is 118Nm, which can be described as follows.

$$|T_{mi}| \le 2T_{b,max}. \tag{20}$$

D. Model predictive control law

Due to the difficulty to get an analytical expression for the optimization problem, numerical solution approaches are applied for solving optimization issue online. In this research, fmincon function in Matlab optimization tool box is used to solve the above cost function.

4 Simulations and Analysis

In this section, the above proposed regenerative braking control strategy is verified through the combined simulation on a electric vehicle model with four in-wheel motors based on Matlab/Simulink and AMESim software. The electric vehicle model parameters are listed in Table 1.

Table 1: Main parameters of electric vehicle

Definition	Symbol	Value
Vehicle mass	M	1500kg
Height of vehicle c.g.	h	0.474m
Axle distance	d	1.45m
Distance from c.g. to front axle	l_f	1.2m
Distance from c.g. to rear axle	l_r	1.2m
Effective radius of the tire	R_e	0.29m
Maximum torque of motor	$T_{m,max}$	118Nm
Maximum power of motor	$P_{m,max}$	26000W
Reducer ratio	g_0	5
Flux of motor	ϕ	0.8Wb

In order to illustrate the efficiency of the presented regenerative braking control strategy, the off-line simulations based on Matlab/Simulink and AMESim software are conducted in NEDC (New European Driving Cycle). In this research, a detailed electric vehicle model for the simulations is established in AMESim software. It mainly includes vehicle dynamics module, suspension system module, Magic Formula tire module, in-wheel motors module, a high power dynamic battery pack module, and a diver module, which provides the desired driving and brake torque. the mechanical steering system module,etc. The simulation time in this driving cycle is 1200 s, and the sampling time T_s is set to 0.01 s, both of the control horizon and predictive horizon of the model predictive control method for allocation of hybrid braking mode is set to 3. Results of the simulation are shown as follows.

Vehicle speed tracking curve is shown in Fig. 4. It can be easily seen from the simulation results that, the actual vehicle speed can well track the controlled vehicle speed, and the average error between the two is 0.03 m/s. This indicates that the proposed regenerative braking controller can ensure vehicle in a stable state corresponding to the driver expectation, not only the driving performance, as well as the braking performance.

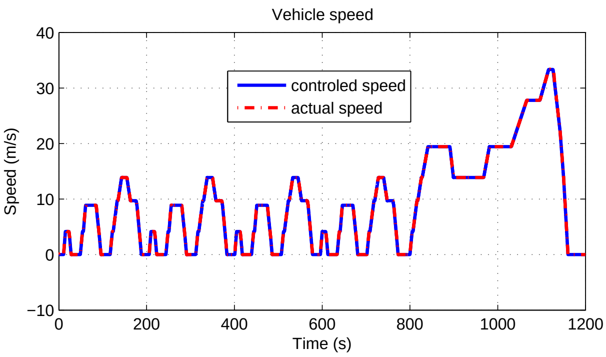

Fig. 4: Vehicle speed tracking graph

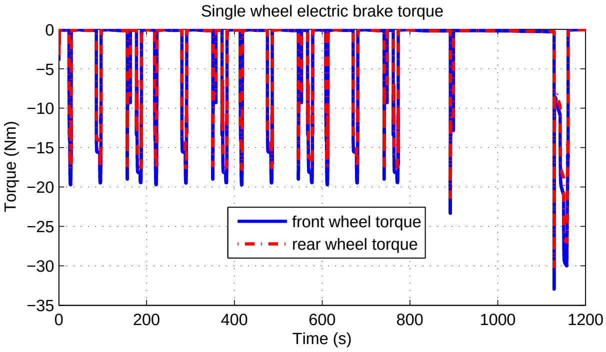

Fig. 5: Single wheel electric brake torque graph

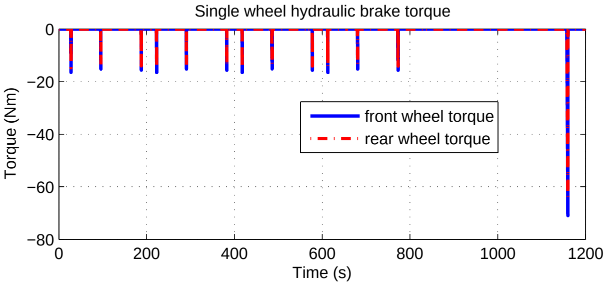

Fig. 6: Single wheel hydraulic brake torque graph

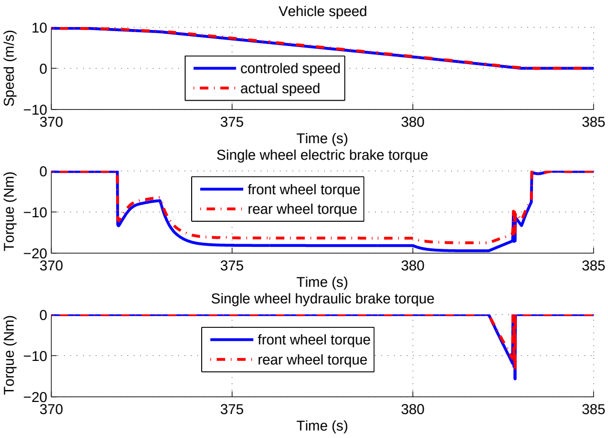

Fig. 7: Torque distribution during braking period

Fig. 5 and Fig. 6 are respectively represents the brake torque on a single front and rear wheel of electric and hydraulic braking mode. Torque distribution during 370 s to 385 s is given in Fig. 7. Note that the brake torque of left and right wheels in this research are the same. it can be seen from these simulation results that, electric motor braking plays a major role in each braking process. Hydraulic braking mode works only when the vehicle speed below a certain value, which in this simulation is about $1\,\mathrm{m/s}$.

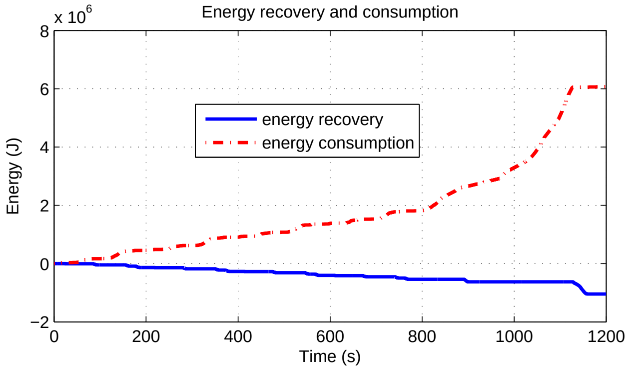

Fig. 8: Energy recovery and consumption graph

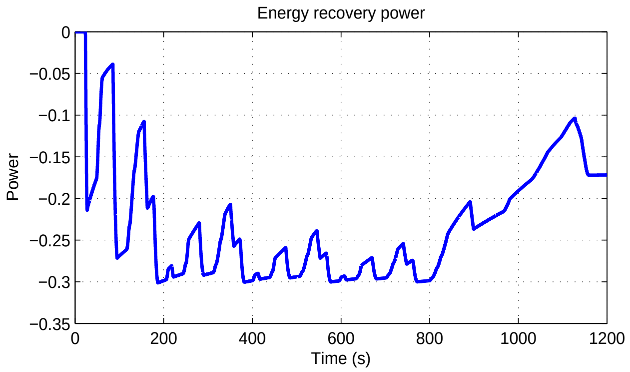

Fig. 9: Energy recovery power graph

The energy consumption and recovery state are simultaneously represented in Fig. 8. It can be seen that energy consumed during the driving process and hydraulic braking process, and recovered during electric motor braking process. The total energy consumption during this driving cycle is about 6.07×10^6 J, and the total energy recovery is about 1.04×10^5 J.

In this paper, the energy recovery efficiency is defined as the ratio between energy recovery and energy consumption,

$$\zeta = \frac{\text{Energy recover}}{\text{Energy consumption}}, \tag{21}$$

and the energy recovery efficiency trend is shown in Fig. 9, it can be seen that the energy recovery efficiency changes with vehicle speed. The total energy recovery efficiency at the end is about 17.2%.

In summary, the simulation results indicate that the regenerative braking control strategy proposed can meet the driver's expectation as well as recovering energy as far as possible.

5 Conclusion

In this research, a regenerative braking control system of electric vehicle with four in-wheel motors for energy conservation and environmental protection is proposed. The research is developed from two aspects. First, in order to ensure the adhesion between tire and road during braking process, a strategy allocating braking torques on front and rear axis respectively is proposed based on front and rear mutative tire load ratio. Second, for the purpose of maximizing the energy recovery, a controller is designed based on model predictive control method to distribute the braking torques between the hydraulic braking mode and the electric motor braking mode. In the end, the simulation results of the electric vehicle model with four individually driven in-wheel motors based on Matlab/Simulink and AMESim software illustrate that the presented regenerative braking control approach has a good effectiveness on both tracking driver's expectation and energy recovery. The next work is to design an integrated controller which takes the safety constraint of tire slip ratio into account.

References

[1] K. Maeda, H. Fujimoto, and Y. Hori, "Four-wheel driving-force distribution method for instantaneous or split slippery roads for electric vehicle," *Automatika-Journal for Control, Measurement, Electronics, Computing and Communications*, vol. 54, no. 1, p. 103C113, 2013.

[2] B. Wang, J. Xu, and B. Cao, "A novel multimode hybrid energy storage system and its energy management strategy for electric vehicles," *Journal of Power Sources*, vol. 281, pp. 432–443, 2015.

[3] G. Xu, K. Xu, C. Zheng, X. Zhang, and T. Zahid, "Fully electrified regenerative braking control for deep energy recovery and safety maintaining of electric vehicles," *IEEE Transactions on Vehicular Technology*, pp. 1–13, 2015.

[4] S. Kanarachos, M. Alirezaei, S. Jansen, and J. P. Maurice, "Control allocation for regenerative braking of electric vehicles with an electric motor at the front axle using the state-dependent riccati equation control technique," *Journal of Automobile Engineering*, vol. 228, no. 2, pp. 129–143, 2014.

[5] X. Nian, F. Peng, and H. Zhang, "Regenerative braking system of electric vehicle driven by brushless DC motor," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 10, pp. 5798–5808, 2014.

[6] X. Huang and J. Wang, "Model predictive regenerative braking control for lightweight electric vehicles with in-wheel motors," *Journal of Automobile Engineering*, vol. 226, no. 9, pp. 1220–1232, 2012.

[7] H. Chen, *Model Predictive Control*, 1st ed., ser. System and Control Series. Beijing, China: Science Press, 2013.

[8] B. Ren, H. Chen, H. Zhao, and L. Yuan, "MPC-based yaw stability control in in-wheel-motored ev via active front steering and motor torque distribution," *Mechatronics*, 2015(online).

[9] R. de Castro, R. E. Araujo, and D. Freitas, "Wheel slip control of evs based on sliding mode technique with conditional integrators," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 8, pp. 3256–3271, 2013.

[10] H. B. Pacejka, *Tyre and Vehicle Dynamics*, second edition ed. London, UK: Elsevier, 2005.

[11] W. Liu, H. He, and J. Peng, "Driving control research for longitudinal dynamics of electric vehicles with independently driven front and rear wheels," *Mathematical Problems in Engineering*, 2013.

[12] H. Zhao, B. Ren, H. Chen, and W. Deng, "Model predictive control allocation for stability improvement of four-wheel drive electric vehicles in critical driving condition," *Control Theory and Applications, IET*, vol. 9, no. 18, pp. 2688–2696, 2015.

[13] V. S. L. Efstathios, S. Efstathios, "Rear wheel torque vectoring model predictive control with velocity regulation for electric vehicles," *Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility*, vol. 53, no. 11, pp. 1555–1579, 2015.