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Characteristic Modeling Theory and Its Applications in Rendezvous and Docking

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1. Introduction

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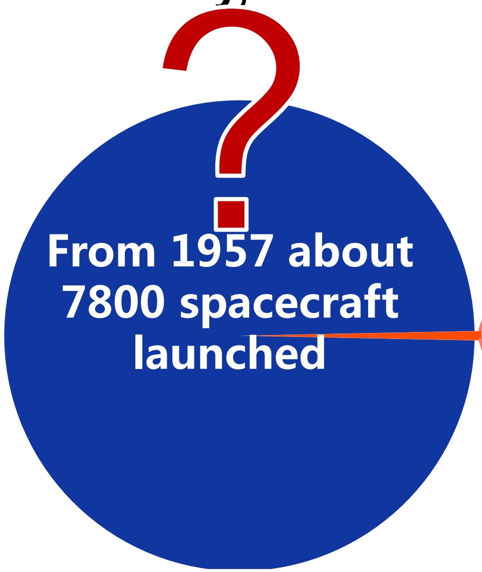
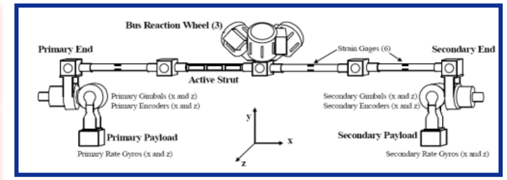
1.1 Motivation

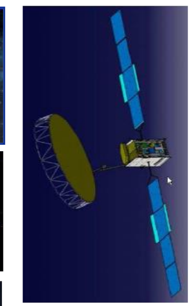
In industry, 97% PID.

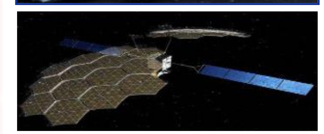


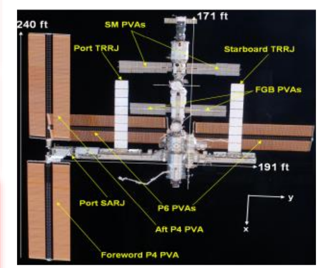
- Traditional control >98.5%
- Modern control <1.5%

1995 USA	MACE	H∞control system identification
1998 USA	MACE II	adaptive structural control
1994 Japan	ETS-VI	modal identification
2009 Japan	ETS-VIII	u-synthesis control gain scheduling
2003+ France	TAS-F SBus4000	H∞control u-synthesis
2006 USA	ISS	pseudo spectral solution
2010 USA	FASTSAT -HSV01	asymptotic periodic LQR
1999+ China	SZ-1/ SZ-10	all-coefficient adaptive control
2010 China	SZ-8/ SZ-10	golden-section phase plane adaptive control









I. Yamaguchi, T. Kida, T. Kasai, “Experimental demonstration of LSS system identification by eigensystem realization algorithm,” ACC, 1995

1. Introduction

1.1 Motivation

Analytical Model	**Obtaining an accurate model in engineering practice is usually unrealistic.**	**Characteristic Model-based Adaptive Control (CMAC)**
High-order Controller	**For uncertain high-order systems, the existing methods are often complex and difficult to apply in practice.**	
On-site Trial and Error	**Most control methods need tuning on-line repeatedly, limiting their applications.**	

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1.2 Development of CMAC

Sum of all-coefficients equals to one (1979-1986)

Golden-section control （1986-1992）

Characteristic model （1992- ）

$$\lim_{T\to 0}\left(\sum_i f_i + \sum_i g_i\right) = 1$$

Adaptive estimation

$$u_G(k) = -[l_1 \hat{f}_1 y(k) + l_2 \hat{f}_2 y(k-1)]/(\hat{g}_0 + \lambda)$$

$$l_1 = 0.382,$$

$$l_2 = 0.618$$

Stability

$$y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k)$$

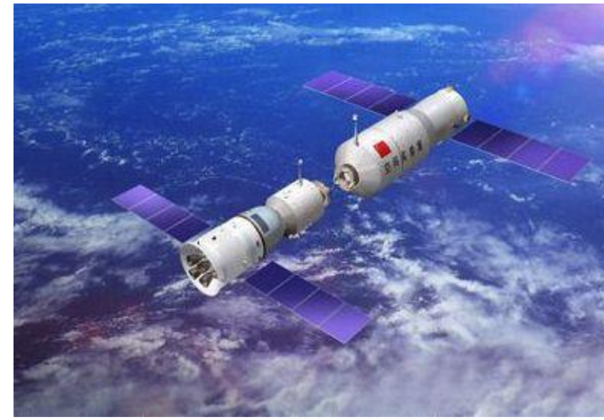
$$D_S : \begin{cases} f_1 \in [1.4331, 1.9974] \\ f_2 \in [-0.9999, -0.5134] \\ f_1 + f_2 \in [0.9196, 0.9999] \\ g_0 \in [0.003, 0.3] \end{cases}$$

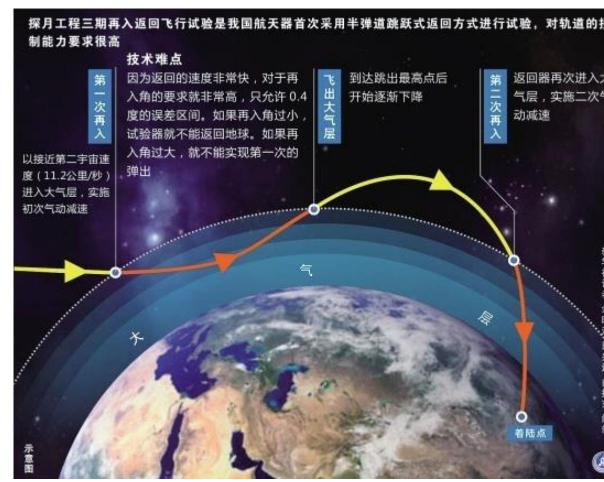
Low-order controller design

1.2 Development of CMAC

✓ State of the art

- **Engineering applications (11 classes of systems)**
  - **Aerospace control**
  - **Industrial control**

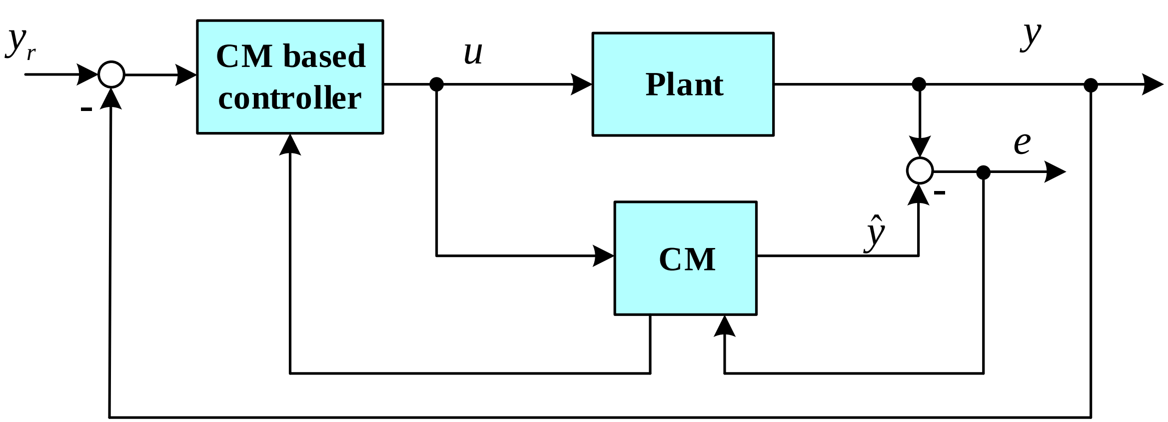




1.2 Development of CMAC

✓ State of the art

- ➢ Theoretical study
  - Characteristic modeling
  - Stability analysis



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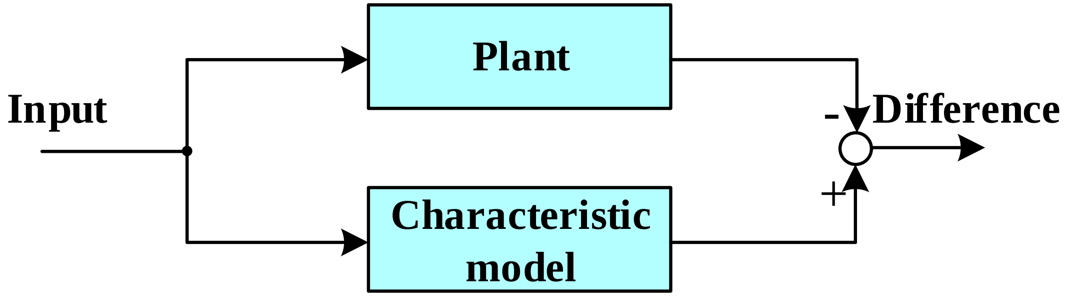
2.1 Definition of Characteristic Model

Characteristic Model

According to the actual plant dynamic properties, environmental features, and control performance requirements, a model is derived for facilitating controller design. It is required that under the same control input, the difference between the outputs of this model and the original plant equals to zero at the steady state, and maintains in an allowed range in the transient process.



2. CMAC Method

2.1 Definition of Characteristic Model

✓ Plant dynamic properties

- **Characteristic variables——inputs, outputs, etc.**
- **Characteristic parameters——time delay, system gain, relative degree, coefficients, etc.**
- **Analytical, graphical, or logical description involving characteristic variables and parameters, e.g.,**

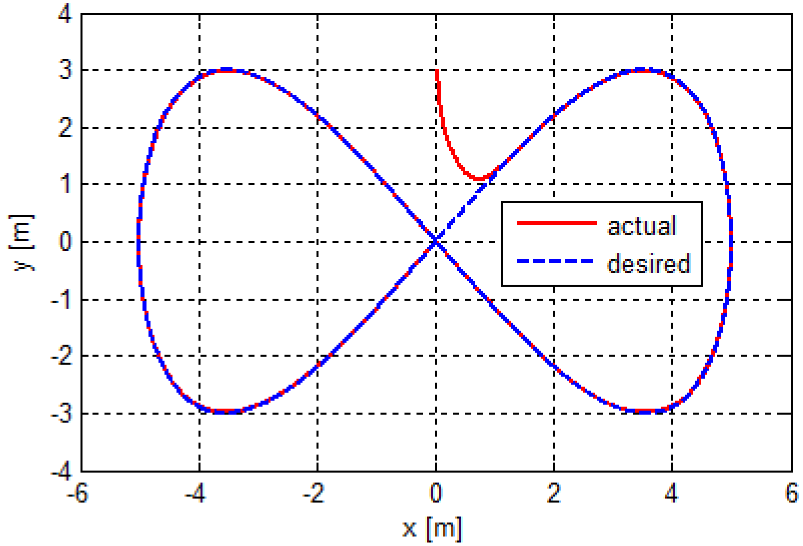
$$y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k) + g_1(k)u(k-1)$$

2. CMAC Method

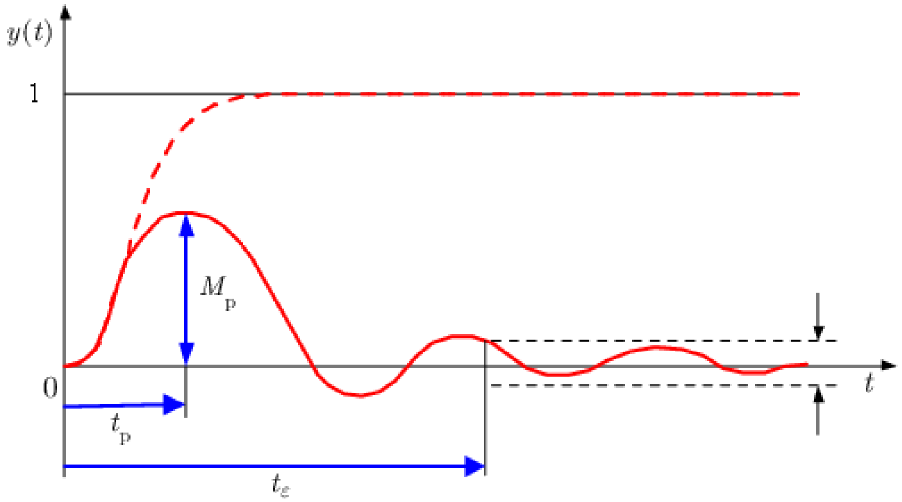
2.1 Definition of Characteristic Model

✓ Control performance requirements

- ➢ **Tracking**



- ➢ **Maintaining**



2.2 Categories of Characteristic Models

✓ **Deterministic characteristic model**

The plants that can be described by mathematical equations, including linear/nonlinear, time-invariant/time-varying, etc. can be modeled by low-order difference equations.

✓ **Intelligent characteristic model**

The plants that cannot be explicitly described by mathematical equations, e.g., industrial process systems, social economic systems, can be modeled by synthetic descriptions, e.g., graphical/logical description.

2. CMAC Method

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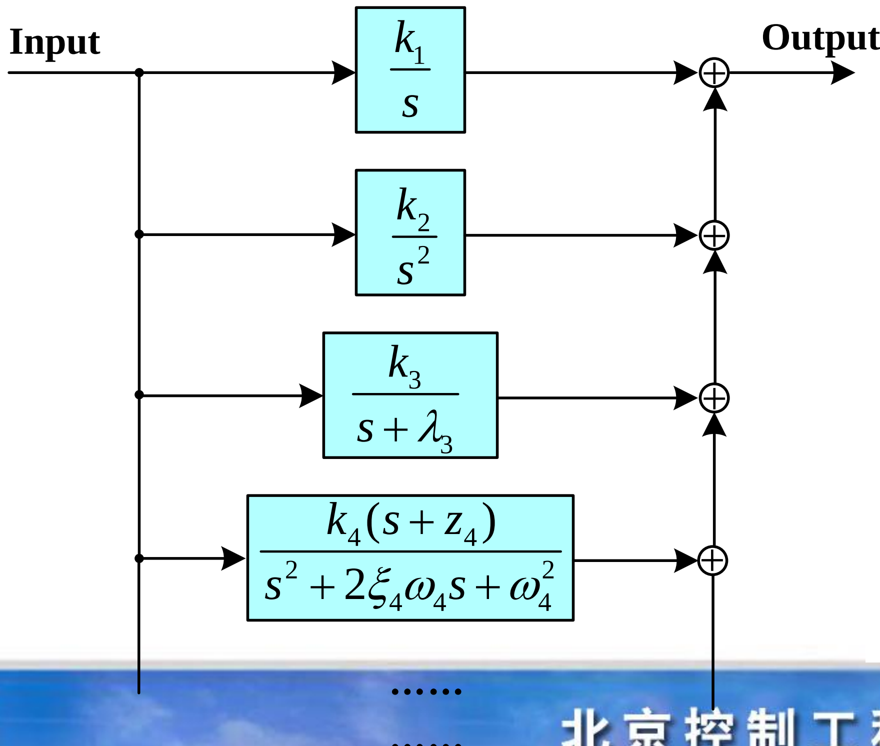
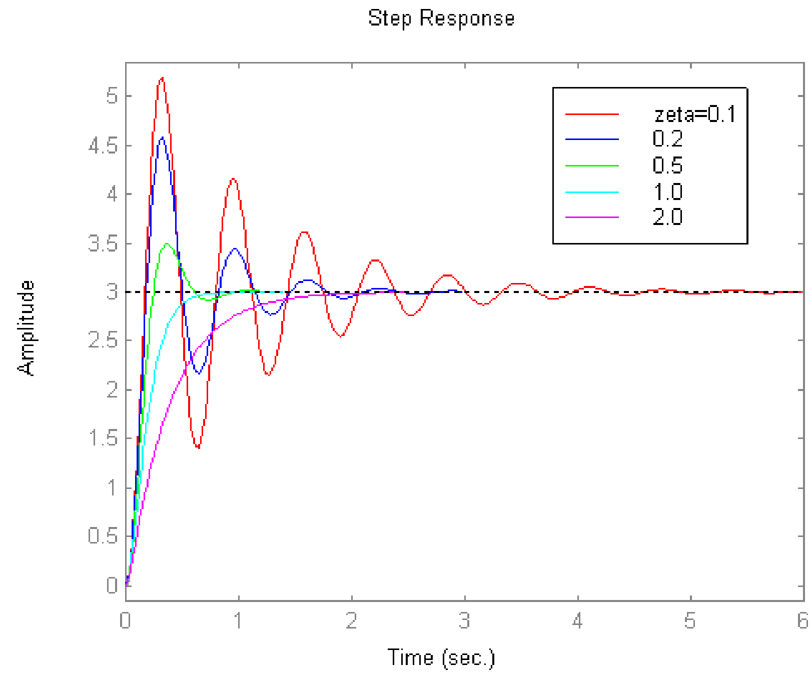
2.3 Mechanism of Characteristic Modeling

✓ Structure perspective

Linear system is composed of a series of components with the order not being greater than 2.

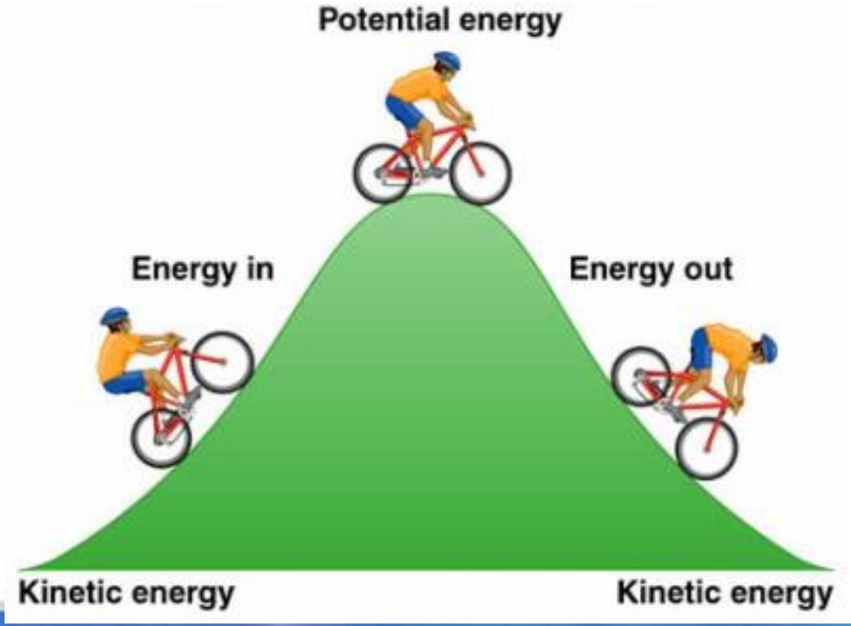


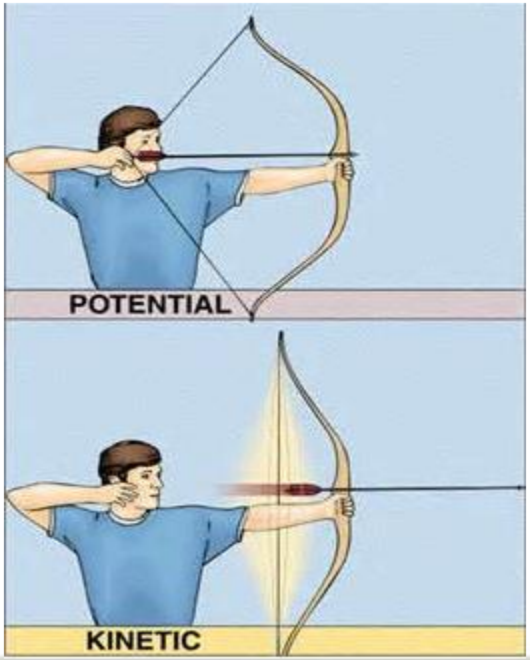


2.3 Mechanism of Characteristic Modeling

✓ Energy perspective

Mechanical energy has two forms: potential energy and kinetic energy. Control process may be regarded as a kind of energy transformation.





2. CMAC Method

2.3 Mechanism of Characteristic Modeling

✓ Mathematical foundation

High-order differential/difference equations can be equivalent to low-order time-varying differential/difference equations.

High-order equation

$\Updownarrow$

Low-order time-varying equation

As a result, establishing a characteristic model for a practical plant to be controlled is feasible.

2. CMAC Method

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2.4 Second-order Characteristic Model

✓ Plant

Consider the LTI plant:

$$G(s)=\frac{b_m s^m + b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n + a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}$$

which can be decomposed into the following form

$$G(s)=\frac{k_{v,1}}{s}+\frac{k_{v,2}}{s^2}+\sum_{i=1}^{p}\frac{k_{a,i}}{s+\lambda_i}+\sum_{i=1}^{p}\frac{k_{b,i}}{(s+\lambda_i)^2}+\sum_{i=1}^{q}\left(\frac{k_{p+i}}{s+\lambda_{p+i}}+\frac{\bar{k}_{p+i}}{s+\bar{\lambda}_{p+i}}\right)$$

where $k_{v,1},k_{v,2},k_{a,i},k_{b,i},\lambda_i (i=1,\cdots,p)$ **are real,** $k_{p+i},\lambda_{p+i}(i=1,\cdots,q)$ **are complex.**

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2.4 Second-order Characteristic Model

✓ Characteristic model

When the control requirement is position keeping or tracking, its characteristic model can be described by

$$y(k+1)=f_1(k)y(k)+f_2(k)y(k-1)+g_0(k)u(k)+g_1(k)u(k-1)$$

As the plant is stable or contains integral component:

- **The coefficients are slowly time-varying;**
- **The range of the coefficients can be determined beforehand;**
- **The output of CM becomes arbitrarily close to that of the plant as the sampling period decreases;**
- **The sum of the coefficients at steady state is equal to 1 if the static gain is one**

$$f_1(\infty)+f_2(\infty)+g_0(\infty)+g_1(\infty)=1$$

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2.4 Second-order Characteristic Model

✓ Range of characteristic parameters

In practice, we have

$$g_0 \in [0.003, 0.3], \qquad |g_1| \le g_0$$

➢ **Stable plant**

$$D_S : \begin{cases} f_1 \in [1.4331, 1.9974] \\ f_2 \in [-0.9999, -0.5134] \\ f_1 + f_2 \in [0.9196, 0.9999] \end{cases}$$

$$\frac{T}{T_{\min}} \in [1/10, 1/3]$$

➢ **Unstable plant**

$$D_N : \begin{cases} f_1 \in [1.9844, 2.2663] \\ f_2 \in [-1.2840, -1] \\ f_1 + f_2 \in [0.9646, 1] \end{cases}$$

$$\frac{T}{T_{\min}} \in [1/10, 1/4]$$

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2.5 Parameter Estimation

✓ Projected gradient algorithm

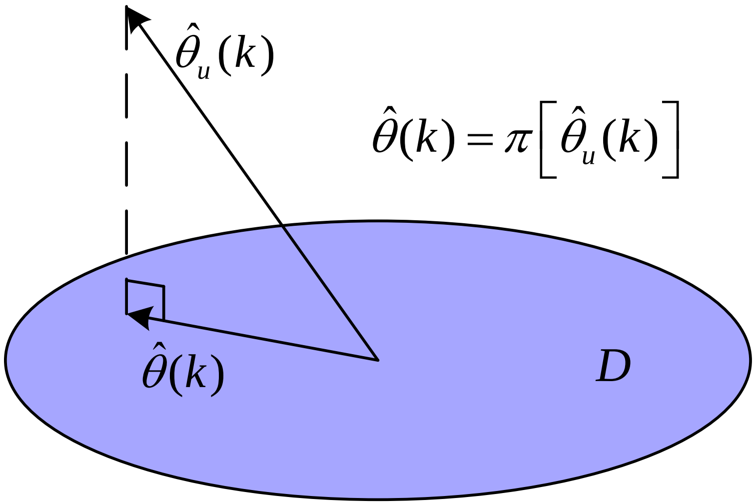
$$\begin{cases} \hat{\theta}_u(k) = \hat{\theta}(k-1) + \dfrac{\lambda_1 \phi(k-1)(y(k) - \phi^T(k-1)\hat{\theta}(k-1))}{\lambda_2 + \phi^T(k-1)\phi(k-1)} \\ \hat{\theta}(k) = \pi\left[\hat{\theta}_u(k)\right] \end{cases}$$

$$\phi(k) = \left[y(k), y(k-1), u(k), u(k-1)\right]^T$$

$$\theta(k) = \left[f_1(k), f_2(k), g_0(k), g_1(k)\right]^T$$

$$\hat{\theta}(k) = \left[\hat{f}_1(k), \hat{f}_2(k), \hat{g}_0(k), \hat{g}_1(k)\right]^T$$

$$\lambda_1, \lambda_2 > 0$$



$\pi[\cdot]$——Orthogonal projector

2. CMAC Method

2.6 Adaptive Controller

$$u(k) = u_0(k) + u_G(k) + u_D(k) + u_I(k)$$

- **Maintaining/tracking control**

$$u_0(k) = \frac{y_r(k+1) - \hat{f}_1(k)y_r(k) - \hat{f}_2(k)y_r(k-1) - \hat{g}_1(k)u_0(k-1)}{\hat{g}_0(k)}$$

- **Golden-section control**

$$u_G(k) = \frac{l_1\hat{f}_1(k)e(k) + l_2\hat{f}_2(k)e(k-1) - \hat{g}_1(k)u_G(k-1)}{\hat{g}_0(k)}$$

where $e(k) = y_r(k) - y(k),\ \ l_1 = 0.382,\ \ l_2 = 0.618.$

2.6 Adaptive Controller

- **Logical-integral control**

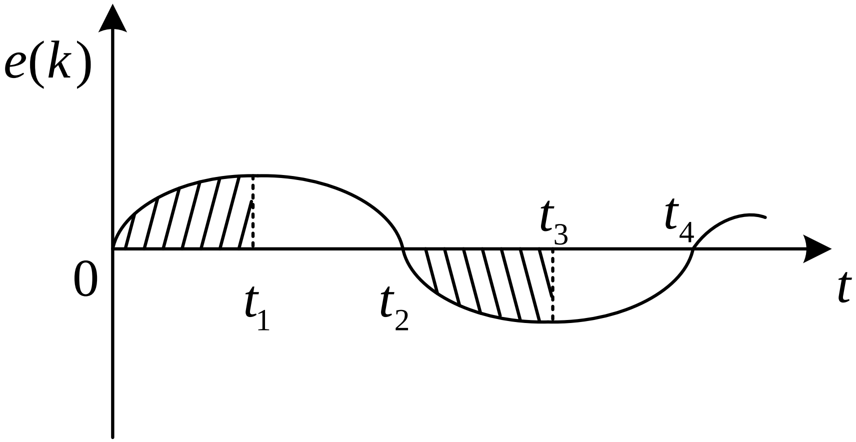
$$u_I(k) = cu_I(k-1) + k_I e(k),$$

$$k_I = \begin{cases} k_{I1}, & \text{if} \quad e(k)\left[e(k)-e(k-1)\right]-\Delta \le 0 \\ k_{I2}, & \text{if} \quad e(k)\left[e(k)-e(k-1)\right]-\Delta > 0 \end{cases}$$

where $k_{I2} > k_{I1} > 0,$

$\Delta > 0,$

$c \in \{-1, 0, 1\}$



2.6 Adaptive Controller

➢ **Logical-derivative control**

$$u_D(k) = -k_d(e(k),\dot{e}(k))\, f(\dot{e}(k))^{e_d(e(k),\dot{e}(k))}$$

where $k_d(e(k),\dot{e}(k)),\ e_d(e(k),\dot{e}(k))$ **is the equivalent gain and chosen according to the demand of velocity feedback, e.g.,**

$$k_d = \begin{cases} 2\,\mathrm{sgn}(\dot{e}(k)), & \forall k-m \le \tau \le k, |e(\tau)| \le r \\ 2(0.5|e(k)|+c_1)^{-1/2}\,\mathrm{sgn}(\dot{e}(k)), & \exists k-m \le \tau \le k, |e(\tau)| > r \end{cases}$$

$$f(\dot{e}(k)) = |\dot{e}(k)|, \qquad e_d = \rho\left(\max_{k-n\le\tau\le k}(\dot{e}(k))\right).$$

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3. Applications

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Aerospace applications	RVD of Shenzhou-8 with Tiangong-1	BICE
	Reentry of Shenzhou spacecraft	BICE
	Reentry of Chang’e 5 test spacecraft	BICE
	Spacecraft transient thermal current	BICE
Industrial applications	Continuous sterilization in fermentation	BCT Co. Ltd
	Aluminum electrolysis	BCT Co. Ltd
	Disposal of sewage	BJUT
	Hydraulic cauldron	BICE
	Stove of refiner	BICE
	Beer fermentation	HEBUST
	Papermaking	HBIA & BICE
	Steel glowing furnace	BICE
	Hydroturbine	BICE
Laboratory research	Flexible rotor active magnetic bearing	UVa, USA
	Fourier transformation spectrometer	HZUST
	Servo system	NUST
	Retractable flexible solar panels	BICE

3. Applications

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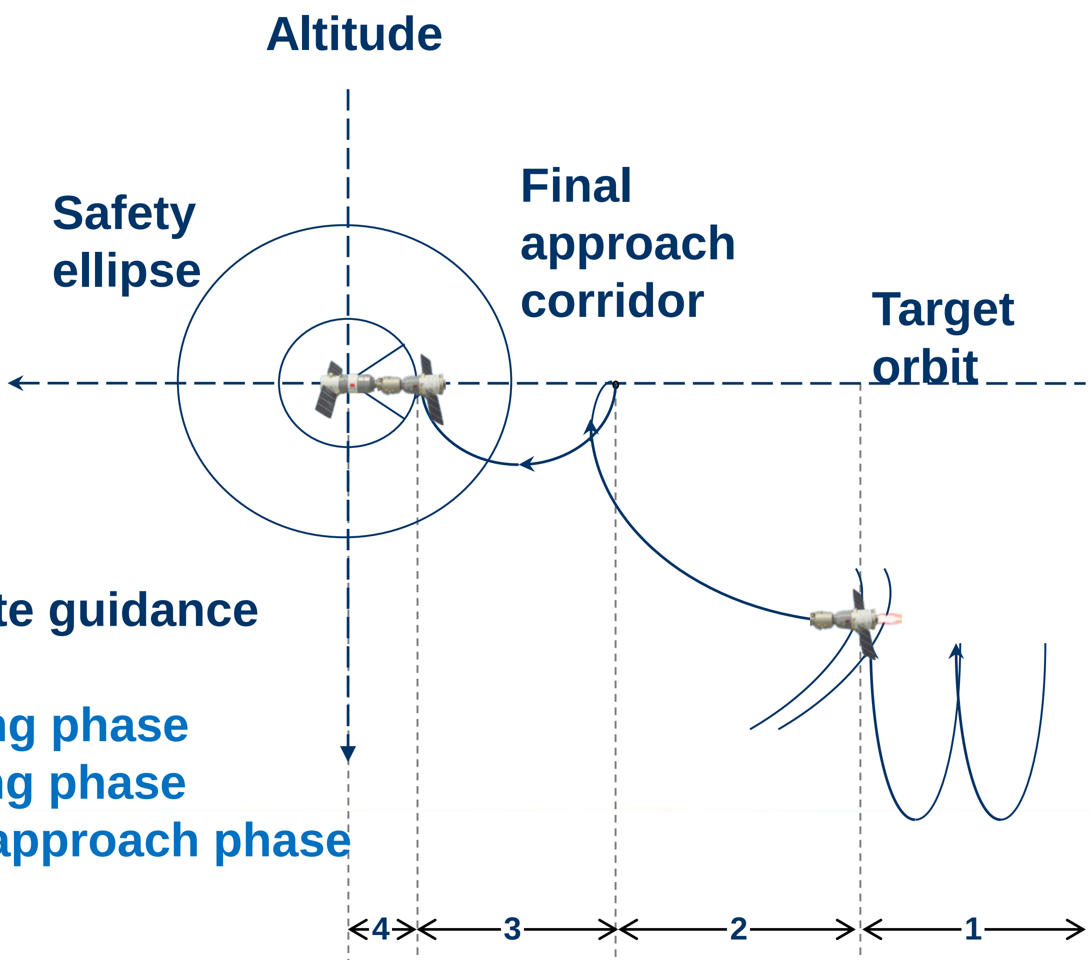
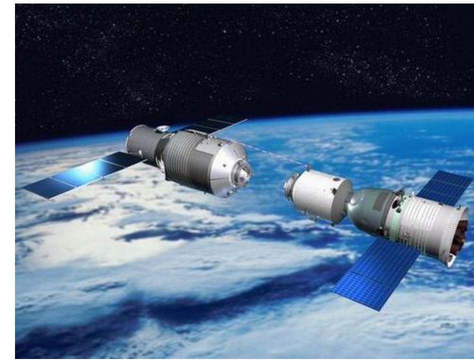
3.1 Aerospace—Rendezvous and Docking (RVD)

✓ RVD Process



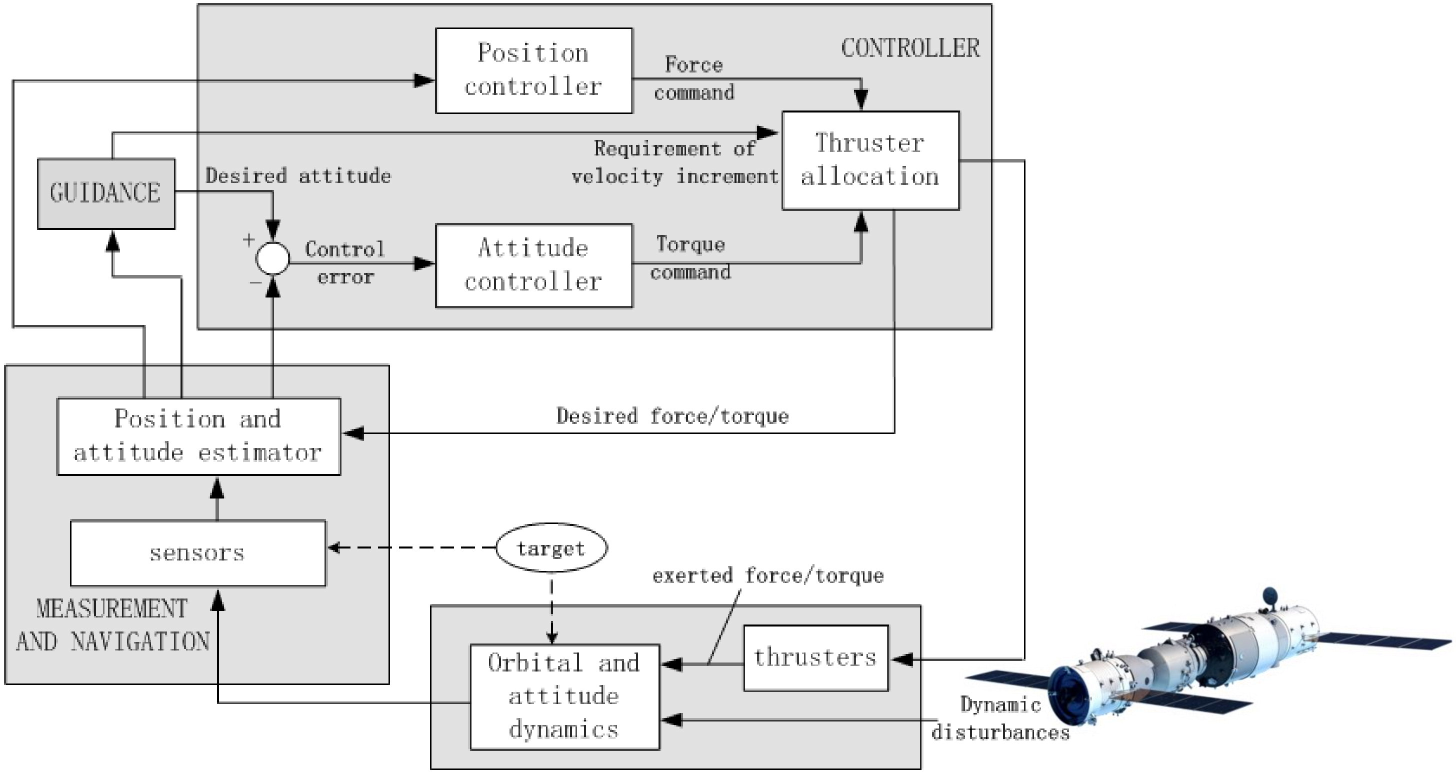


1. Remote guidance phase
2. Homing phase
3. Closing phase
4. Final approach phase

3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ Guidance navigation and control system

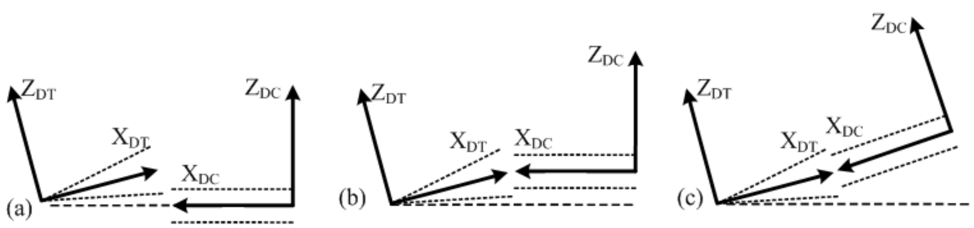


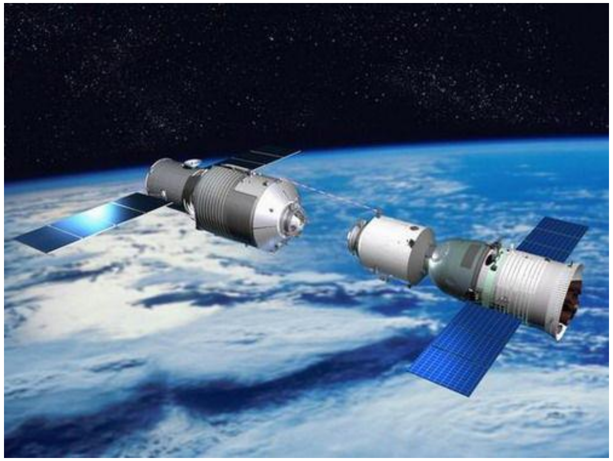
3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ Challenges

- **Kinematic coupling between the rotation and translation in the final approach phase**
- **Dynamic coupling between the attitude and position control caused by thruster installation**
- **Thruster plume disturbance**
- **Flexibility of solar panel**
- **Large time delay**





3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ Characteristic modeling

Coupling items

$$M\ddot{X} + F_{trs}\ddot{\eta}_{rs} + F_{tls}\ddot{\eta}_{ls} = P_s$$ (1) **Translation**

$$\boxed{I_s\dot{\omega}_s + \tilde{\omega}_s I_s \omega_s} + \boxed{F_{sls}\ddot{\eta}_{ls} + F_{srs}\ddot{\eta}_{rs} + R_{asls}\dot{\omega}_{als} + R_{asrs}\dot{\omega}_{ars}} = T_s$$ (2) **Rotation**

$$I_{als}\dot{\omega}_{als} + F_{als}\ddot{\eta}_{ls} + R_{asls}^T\dot{\omega}_s = T_{als}$$ (3.a) **Left SADA**

$$I_{ars}\dot{\omega}_{ars} + F_{ars}\ddot{\eta}_{rs} + R_{asrs}^T\dot{\omega}_s = T_{ars}$$ (3.b) **Right SADA**

$$\ddot{\eta}_{ls} + 2\xi_{ls}\Omega_{als}\dot{\eta}_{ls} + \Omega_{als}^2\eta_{ls} + F_{tls}^T\ddot{X} + F_{sls}^T\dot{\omega}_s + F_{als}^T\dot{\omega}_{als} = 0$$ (4.a) **Left Solar Panel**

$$\ddot{\eta}_{rs} + 2\xi_{rs}\Omega_{ars}\dot{\eta}_{rs} + \Omega_{ars}^2\eta_{rs} + F_{trs}^T\ddot{X} + F_{srs}^T\dot{\omega}_s + F_{ars}^T\dot{\omega}_{ars} = 0$$ (4.b) **Right Solar Panel**

$$\boxed{\theta(k+1) = \bar{F}_1\theta(k) + \bar{F}_2\theta(k-1) + \bar{G}_1 T(k) + \bar{G}_2 T_d(k)}$$

Characteristic Variables

$$\dot{\theta} = T(\theta)\omega_s$$

Characteristic Variables

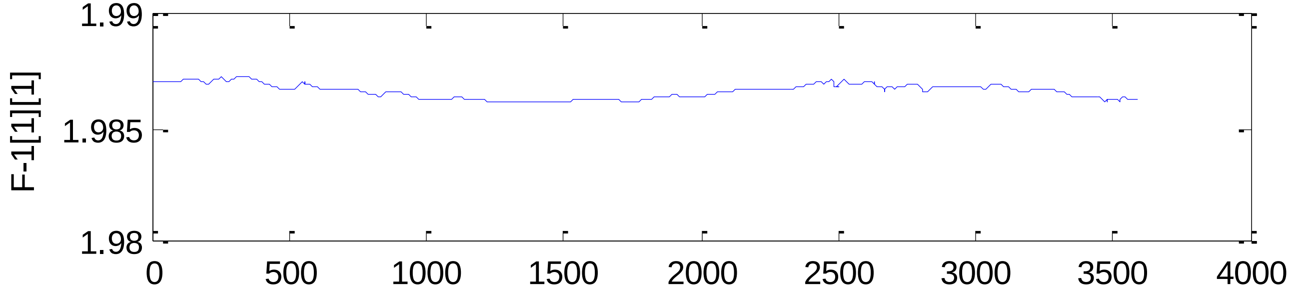
$T(k)$:Control Torques

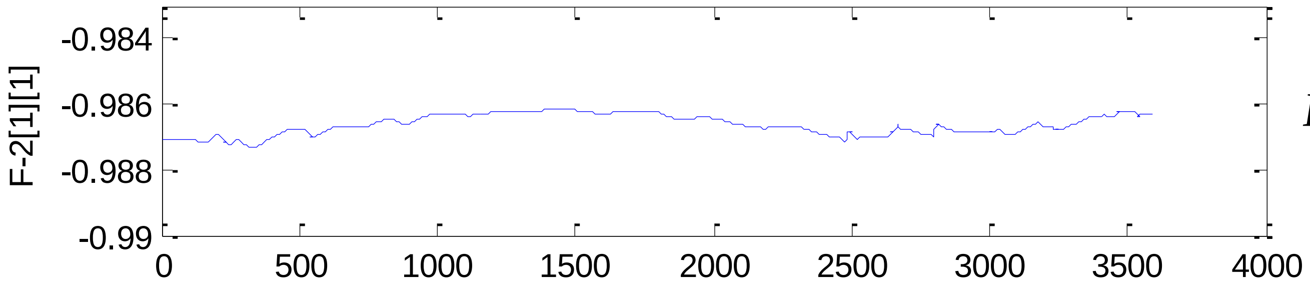
$T_d(k)$: Disturbances (SADA-driven Torques)

3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ **True values of characteristic parameters**





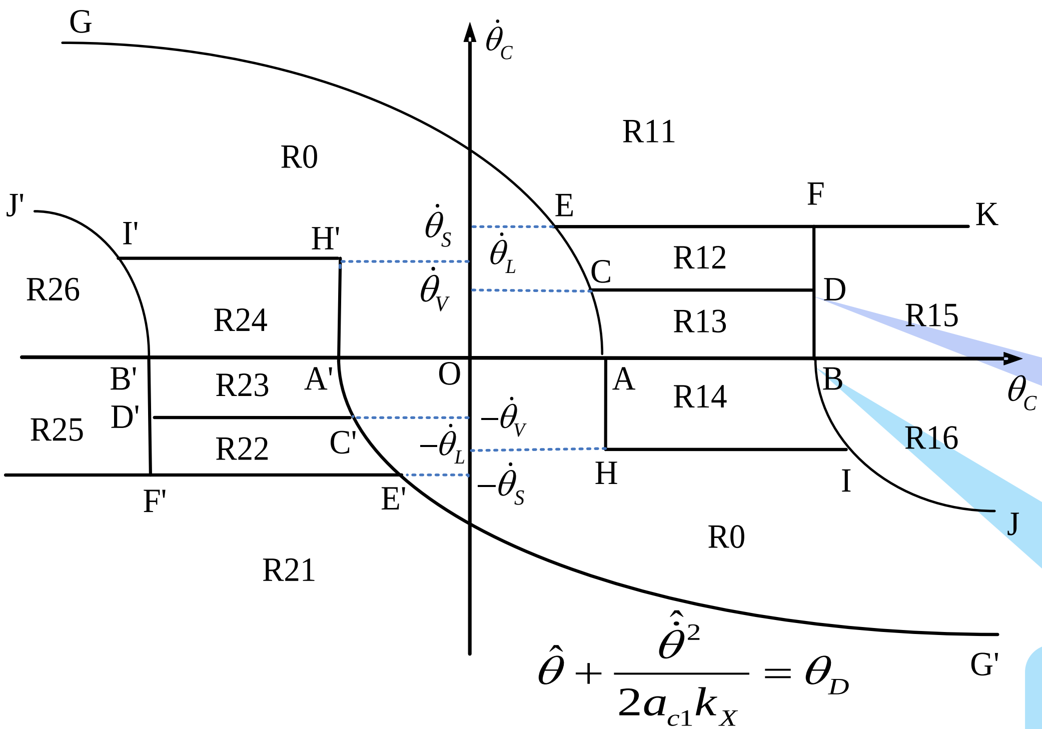
$$D_S:\begin{cases} f_1 \in [1.4331, 1.9974] \\ f_2 \in [-0.9999, -0.5134] \\ f_1 + f_2 \in [0.9196, 0.9999] \end{cases}$$

Simulation illustrates that the characteristic parameters are slowly time-varying and located in D_S.

3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ Phase-plane control



R11: high-thruster turn-on region
R12: low-thruster turn-on region
R13: low-thruster stepping region
R14: low-thruster outer sliding resistance region
R15, R16: high-thruster velocity restricted region
R0: turn-off region

$$T_{N1} = \frac{|\dot{\theta}_c|}{a_{c2}} = 0.618\Delta T \qquad a_{c2} = \dot{\theta}_V / (0.618\Delta T)$$

$$T_{N2} = K_j \frac{|\theta_c|}{a_{c2}} = 0.382\Delta T$$

$$K_j = 0.382 a_{c2} \Delta T / \theta_B = 0.618 \dot{\theta}_v / \theta_B$$

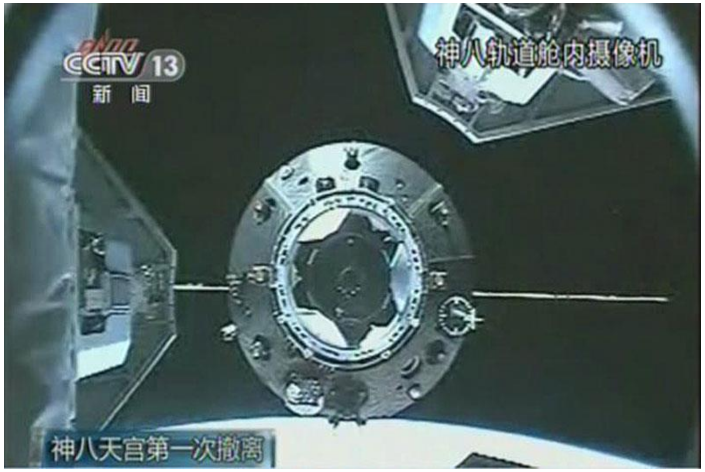
Calculate the control parameters instead of trial and error.

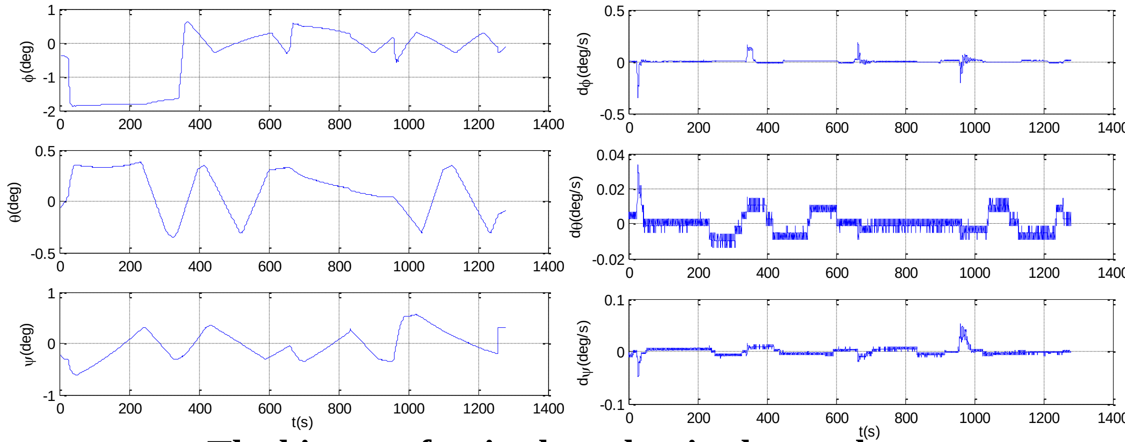
3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ In-orbit flight data validation

In Nov. 2011, June 2012 and June 2013, the manned and unmanned spacecraft Shenzhou-8, Shenzhou-9 and Shenzhou-10 have successfully completed automatic RVD with spacecraft Tiangong-1.





The history of attitude and attitude angular velocity of an RVD operation

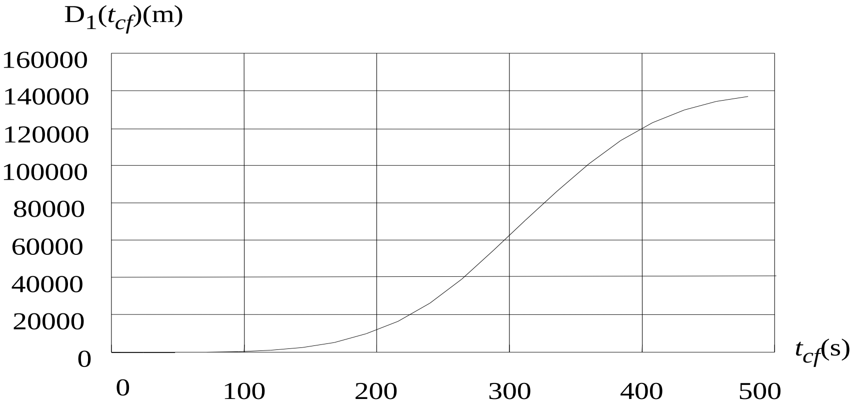
3. Applications

3.1 Aerospace—Reentry of Shenzhou Spacecraft

The return module enters the atmosphere and returns to the Earth safely by decreasing its speed with the atmosphere drag.

✓Dynamic characteristics

- High-order nonlinear system
- Small lift-to-drag ratio and weak controllability
- Strong uncertainty of atmosphere density and aerodynamic parameters
- System gain changes greatly, more than 1000 times.



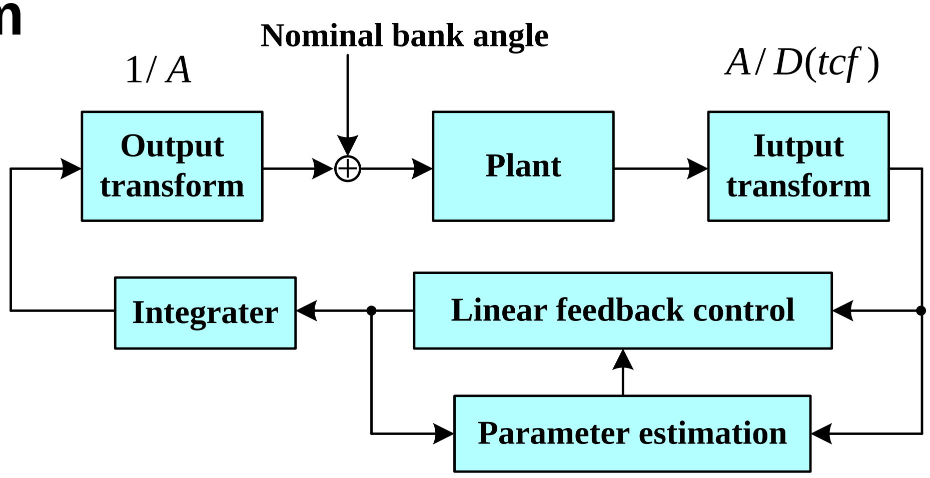
3.1 Aerospace—Reentry of Shenzhou Spacecraft

✓ Adaptive reentry control

- **Input and output transformation**
- **Parameter estimation-projected gradient algorithm**

$$y(k+1)=f_1(k)y(k)+f_2(k)y(k-1)+g_0(k)u(k)$$

$$\begin{cases} f_1(k)\in[1.4331, 1.9974] \\ f_2(k)\in[-0.9999,\ -0.5134] \\ f_1(k)+f_2(k)\in[0.920, 0.995] \\ g_0(k)\in[0.003, 0.65] \end{cases}$$



Block diagram of adaptive reentry control system

3. Applications

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3.1 Aerospace—Reentry of Shenzhou Spacecraft

- **Linear feedback control**

$$u_L(k) = -(L_1 \hat{f}_1(k) y(k) + L_2 \hat{f}_2(k) y(k-1)) / \hat{g}_0(k)$$

$$0 < L_1, L_2 < 1$$

- **Integral control**

$$u(k) = u(k-1) + u_L(k)$$

- **Total control quantity limitation**

$$u(k) \in [A(\cos\bar{\gamma} - \cos(\gamma_0(t))), A(\cos\underline{\gamma} - \cos(\gamma_0(t)))]$$

γ_0 **is the nominal value,** $\underline{\gamma} = 10^{\circ}, \bar{\gamma} = 125^{\circ}$.

Spacecraft Shenzhou-1 ~ Shenzhou-10 all have successfully returned to the Earth.

3. Applications

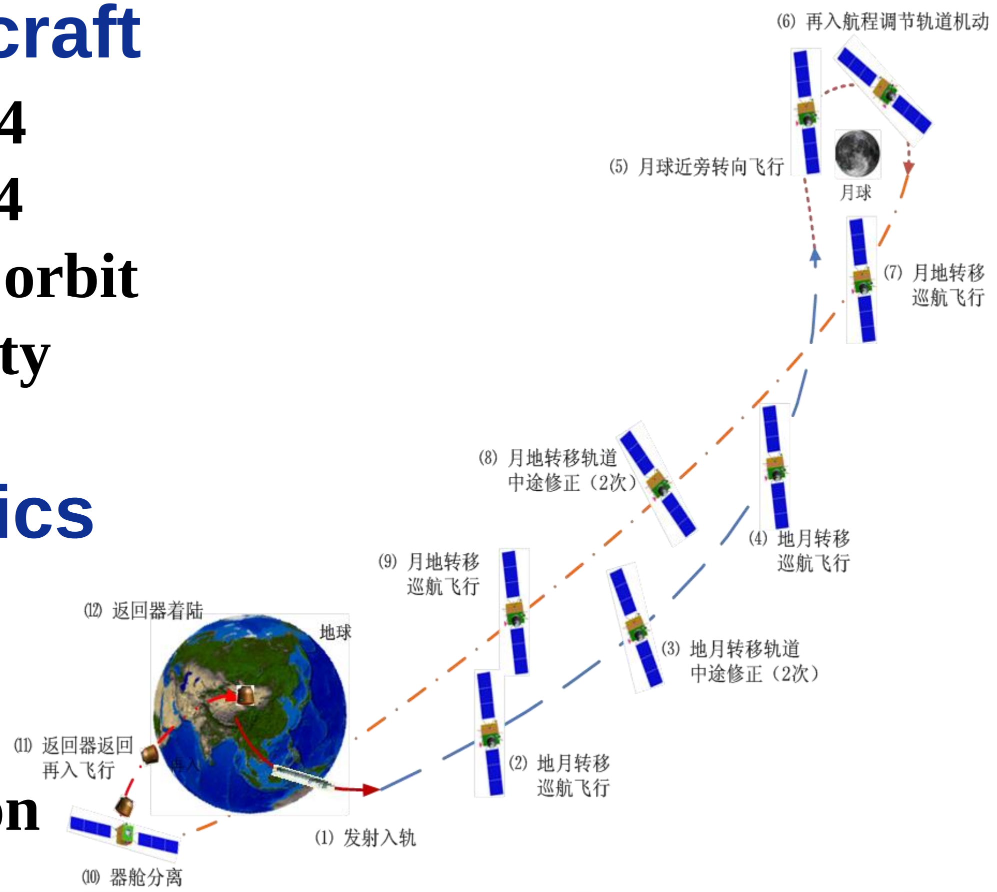
3.1 Aerospace-Reentry of Chang’e 5 test spacecraft

✓ Chang’e 5 test spacecraft

- Launch date: 23 Oct. 2014
- Landing date: 1 Nov. 2014
- Skip reentry from Lunar orbit with the 2nd cosmic velocity

✓ Dynamic characteristics

- Nonlinearity
- Strong uncertainty
- Great parameter variation



3. Applications

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3.1 Aerospace-Reentry of Chang'e 5 test spacecraft

✓ Controller design

- **First-order characteristic model**

$$y(k+1) = f(k)y(k) + g(k)u(k)$$

$$f(k) \in [0.2, 0.99],\ g(k) \in [0.003, 1]$$

- **Parameter estimation: projected gradient algorithm with filter**

$$\hat{\theta}'(k) = \Pi\left[\hat{\theta}(k-1) + \frac{\lambda_1 \phi(k)}{\lambda_2 + \phi^T(k)\phi(k)}\left(y(k) - \phi^T(k)\hat{\theta}(k-1)\right)\right]$$

$$\hat{\theta}(k) = F\hat{\theta}'(k) + (1-F)\hat{\theta}(k-1),\ 0 \le F \le 1$$

Π --orthogonal projector

- **Adaptive control**

Linear feedback control

$$u_L(k) = -L\hat{f}(k)y(k)/\hat{g}(k),$$
$$0 < L < 1$$

Integral control

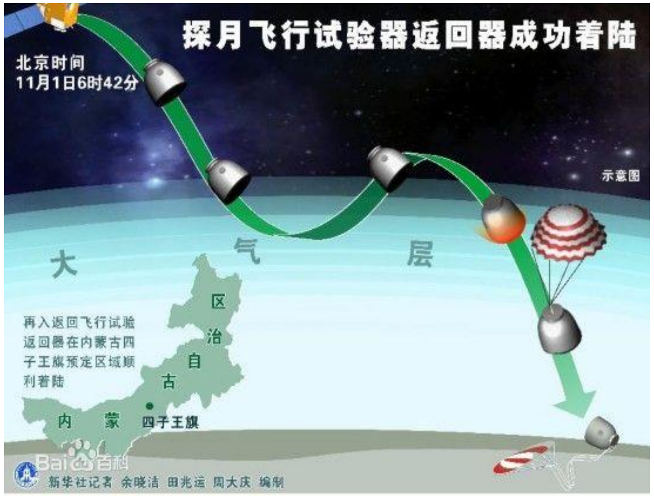
$$u(k) = u(k-1) + u_L(k)$$

M, Sun, Z. Li, L. Yu, Adaptive iterative learning feature first-order dynamic system models and linear servo system control, JSSM, *32*(6), 666-682, 2012.

3. Applications

3.1 Aerospace-Reentry of Chang’e 5 test spacecraft

✓ Flight validation

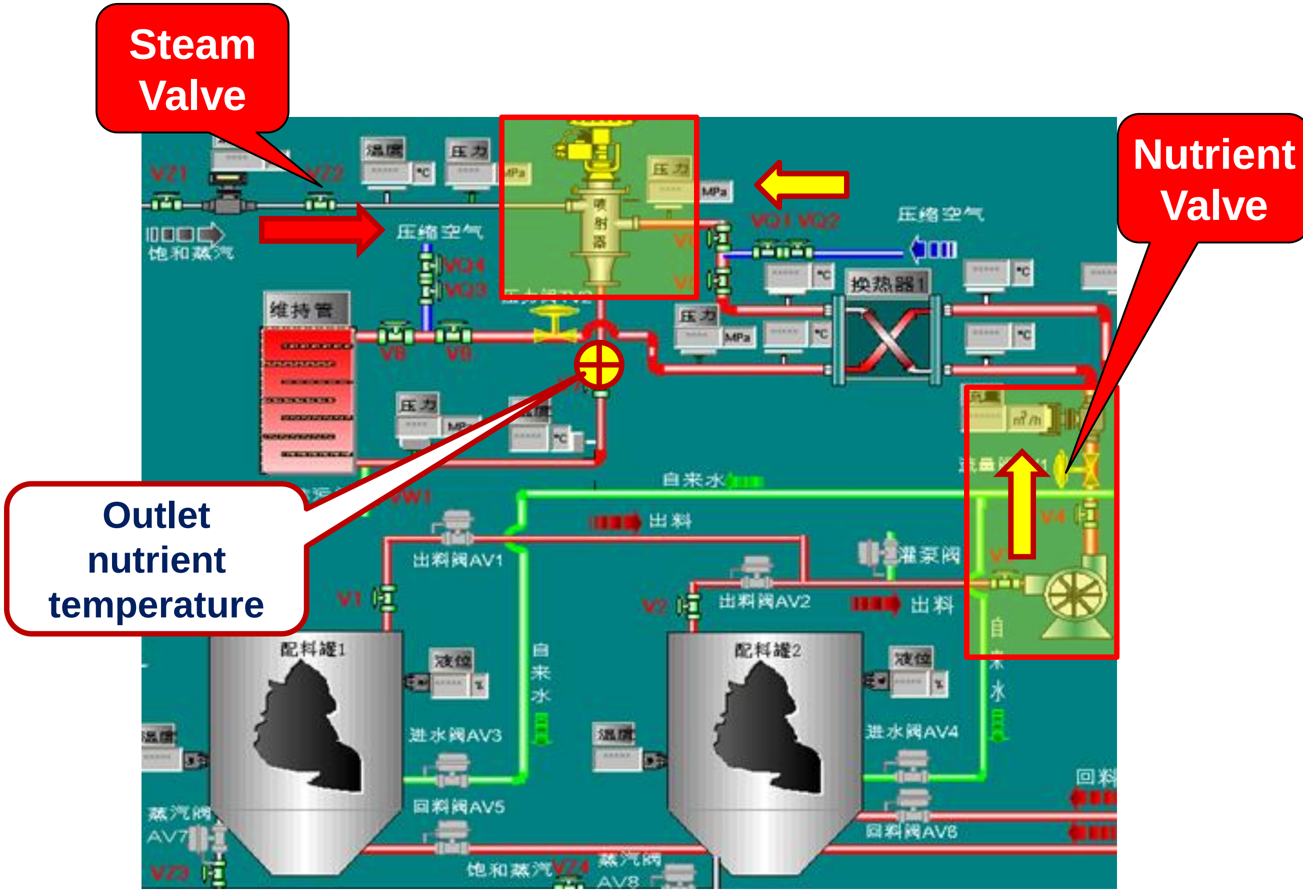


- ➢ **A first-order characteristic model was successfully applied for the first time.**

- ➢ **The control accuracy has reached the leading level in the world.**

3. Applications

3.2 Industry—Continuous Sterilization in Fermentation



- Heat-exchange process
- Input: steam flow quantity
- Output: outlet nutrient temperature of the steam ejector

Reference: Yong Wang，Xin Liu. Characteristic model based iterative learning adaptive control for continuous sterilization in the fermentation, *Industrial Control Computer Annual Conference*, 10.31,2014,Chongqing.

3. Applications

中国航天

3.2 Industry—Continuous Sterilization in Fermentation

✓Plant characteristics

- Temperature **regulating** system with a relative degree of **one**
- Subject to some **fast time-varying disturbances**
- **Strong coupling** between steam and liquid
- The control gain is affected by disturbances.

$$\dot{x} = -2k(t)x + g(-x, d)u$$

Disturbances

3. Applications

3.2 Industry—Continuous Sterilization in Fermentation

✓Controller design

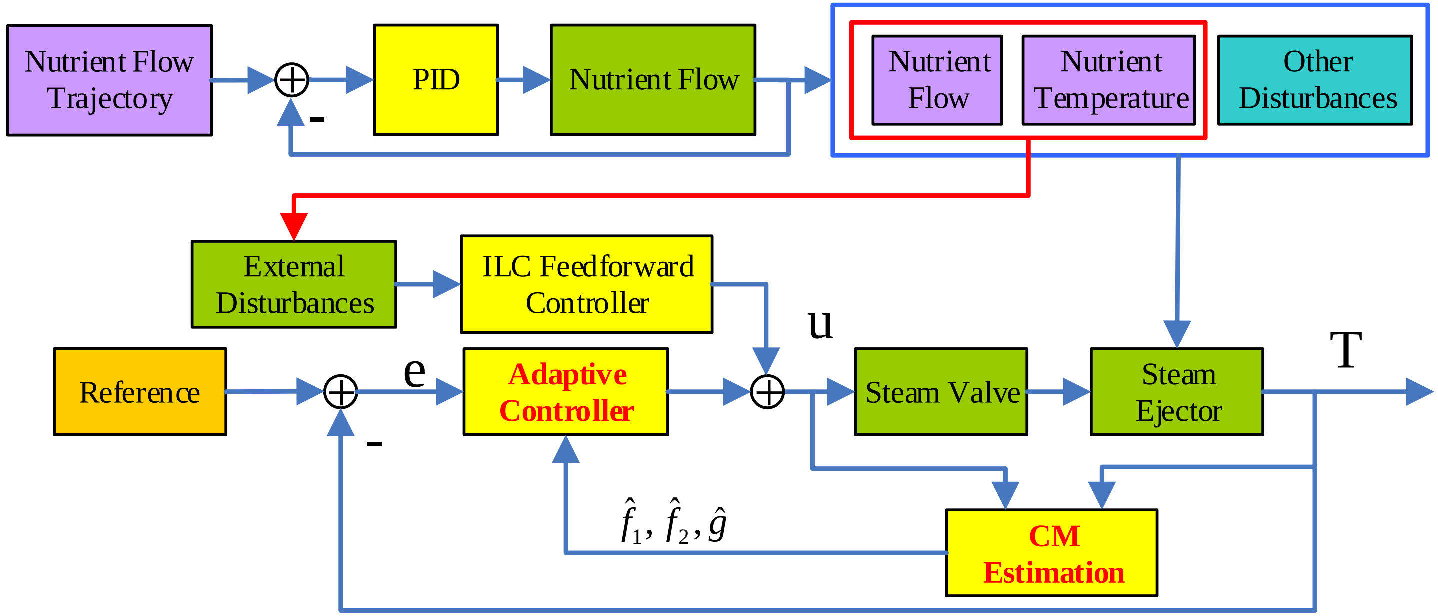
- **Characteristic model**

$$z(k+1) = f_1(k)z(k) + g_0(k)u(k)$$

- **Adaptive control**
- **Feedforward control**

Take disturbances as inputs to construct the feedforward lookup table, and use iterative learning control to improve it.

Nutrient Flow	Nutrient Temperature	Control value
15	78	0.45
13	75	0.40

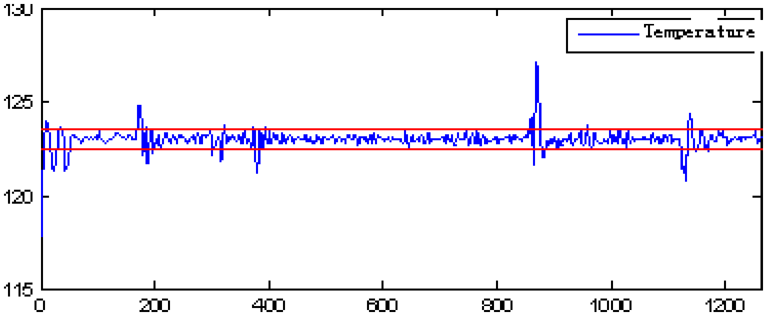


3. Applications

3.2 Industry—Continuous Sterilization in Fermentation

✓Control results

- Steady state error : ±0.5℃
- Transient error : ±3℃
- Settling time is greatly decreased from 20 minutes to less than 2 minutes.
- The steam consumption is greatly decreased, and the quality of the nutrient after sterilization is significantly improved.



Sterilization temperature history

3. Applications

3.2 Industry—Aluminum Electrolysis

- **Electrochemistry reaction: alumina is reduced to metal aluminum**
- **Input: alumina feeding time interval**
- **Output: concentration of alumina**

✓Plant characteristics

- **Concentration regulating system with a relative degree of one**
- **Hard to directly measure the controlled outputs**
- **Large time delay, strong nonlinearity and uncertainty**
- **Multiple equilibria**

Aluminum Electrolysis

Reference: Yingchun Wang, Changfu Geng, Hongxin Wu. An adaptive fuzzy controller and its application in the process control of aluminum electrolysis, *Aerospace Control*, 2001, 4: 22-28.

3. Applications

3.2 Industry—Aluminum Electrolysis

✓ Controller design

- **Characteristic model of a medium variable and the unmeasured output**

$$z(k) = h_1(\bullet)[y_h(k) - y_{h0}]^2 + R_0 + \xi_1$$

- **Characteristic model of the medium variable and the control input in the neighborhood of the operating point**

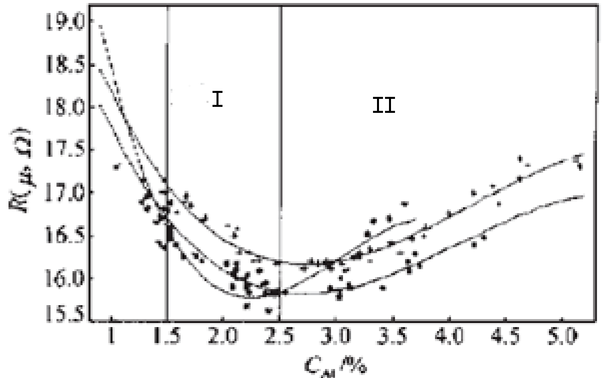
$$z(k+1) = f_1(k)z(k) + g_0(k)u(k) + \Delta_1^{'}(k)$$

- **Guidance controller**

  Guide system to the neighborhood of the desired operating point.

- **Feedback controller**

  Adaptive fuzzy controller based on the CM



Operating area

3. Applications

3.2 Industry—Aluminum Electrolysis

✓Control results




INDEX	PAST	PRESENT
Anode effect controlled percentage	**60%**	**more than 80%**
Current efficiency	**91.5%**	**93.3%**
Electricity saving		**315 kwh/t**
Production increment per electro bath		**8.473 t**

OUTLINE

4. Theoretical Results

- ✓ **Characteristic Modeling**
  - ➢ LTI systems
  - ➢ LTV systems
  - ➢ NLTI systems

- ✓ **Stability Analysis**
  - ➢ Stability of CM + golden-section adaptive controller (GSAC)
  - ➢ Robust stability of CM + GSAC
  - ➢ Stability of the original plant + GSAC

4. Theoretical Results

4.1 Characteristic Modeling-LTI systems

Consider the plant

$$y = G(s)u \qquad (4.1.1)$$

where $G(s) \in R^{p \times q}$ **is a rational transfer function matrix, and** $u \in R^q$ **and** $y \in R^p$ **represent inputs and outputs respectively.**

Assumption 1

There exists a sampling controller such that the outputs can track the constant reference signals.

Assumption 2

The sampling period *h* is small enough.

4. Theoretical Results

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4.1 Characteristic Modeling-LTI systems

Theorem 4.1.1

Under Assumptions 1 and 2， the characteristic model of plant (4.1.1) can be described by the time-varying difference equation

$$y_i(k+1) = f_{i1}(k)y_i(k) + f_{i2}(k)y_i(k-1) + g_{i0}^T(k)u(k) + g_{i1}^T(k)u(k-1), i = 1, 2, \cdots, p$$

where $f_{i1}(k), f_{i2}(k) \in R$ **and** $g_{i0}(k), g_{i1}(k) \in R^q$ **are slowly time-varying,**

$$\begin{cases} f_{i1}(k) \in [2+2h\lambda_{\min}, 2], f_{i2}(k) \in [-1, -(1+2h\lambda_{\min})], & \text{when } \lambda_{\min} < \lambda_{\max} < 0 \\ f_{i1}(k) \in [2, 2+2h\lambda_{\max}], f_{i2}(k) \in [-(1+2h\lambda_{\max}), -1], & \text{when } \lambda_{\max} > \lambda_{\min} > 0 \\ f_{i1}(k) \in [2+2h\lambda_{\min}, 2+2h\lambda_{\max}], f_{i2}(k) \in [-(1+2h\lambda_{\max}), -(1+2h\lambda_{\min})], & \text{otherwise} \\ g_{i0}(k) = -g_{i1}(k) + O(h^2) = O(h) \end{cases}$$

Furthermore， when there are not the poles with zero real parts， the modeling error is $O(h)$ **in the transient process, and the outputs of the CM and plant (4.1.1) are equal in the steady stage； in general cases, the modeling errors are** $O(h)$ **and** $O(h^2)$ **in the transient process and steady stage respectively.**

B. Meng, H. Wu, A unified proof of the characteristic model of linear time-invariant systems, SIC-E, 37(10), 1258~1271, 2007.

4. Theoretical Results

4.1 Characteristic Modeling-LTV systems

Consider the plant

$$\begin{aligned} & y^{(n)}(t)+a_{n-1}(t)y^{(n-1)}(t)+\cdots+a_2(t)y^{(2)}(t)+a_1(t)y^{(1)}(t)+a_0(t)y(t) \\ & =b_m(t)y^{(m)}(t)+\cdots+b_2(t)u^{(2)}(t)+b_1(t)u^{(1)}(t)+b_0(t)u(t) \quad (m<n) \end{aligned} \tag{4.1.2}$$

Assumption 1

The plant (4.1.2) is controllable， and $y(t), \dot{y}(t)$ and $\ddot{y}(t)$ are measurable.

Assumption 2

$u(t)$**and** $y(t)$**, and their derivatives are bounded.**

Assumption 3

$a_i(t), b_j(t), i=0,\cdots,n-1,\ j=0,\cdots,m,$ **and their derivatives are bounded, when** $t\to\infty$**, they are constants, and**

$$\left|a_i(t+\Delta t)-a_i(t)\right|<M_1\Delta t,\ \left|b_j(t+\Delta t)-b_j(t)\right|<M_2\Delta t$$

where M_1 **and** M_2 **are constants.**

4. Theoretical Results

4.1 Characteristic Modeling-LTV systems

Theorem 4.1.2

When the control requirement is position keeping or tracking, then the characteristic model of the plant (4.1.2) can be described by the following time-varying difference equation

$$y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k) + g_1(k)u(k-1) \quad \textbf{(4.1.3)}$$

provided that Assumptions 1-3 hold and the sampling period satisfies certain conditions. Furthermore, if the plant (4.1.2) is stable, then the output error between (4.1.3) and (4.1.2) is kept in a permitted range under the same input, and the range of the coefficients of (4.1.3) can be determined beforehand.

H. Wu, J. Hu, Y. Xie, Characteristic model based intelligent adaptive control, Beijing Science and Technology Press, 2009.

4. Theoretical Results

4.1 Characteristic Modeling-NLTI systems

Consider the plant

$$\dot{x}(t) = f(x,u) \qquad \textbf{(4.1.4)}$$

Assumption 1

The plant (4.1.4) is SISO, and the order of $u(t)$ in (4.1.4) is less than 2 and $u(t)$ is bounded.

Assumption 2

$f(x,u)$ **is differentiable and its derivatives are bounded, and** $f(0,0)=0$.

Assumption 3

$\left| f(x(t+\Delta t),u(t+\Delta t)) - f(x(t),u(t)) \right| < M\Delta t,$ **where** $M>0$ **is constant.**

4. Theoretical Results

4.1 Characteristic Modeling-NLTI systems

Theorem 4.1.3

Under Assumptions 1-3， if the sampling period satisfies certain conditions, then the characteristic model of the plant (4.1.4) can be described by the following time-varying difference equation

$$y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k) + g_1(k)u(k-1) \quad \textbf{(4.1.5)}$$

Furthermore， if the plant (4.1.4) is stable, then the output error between (4.1.5) and (4.1.4) is kept in a permitted range under the same input, and the range of the coefficients of (4.1.5) can be determined beforehand.

H. Wu, J. Hu, Y. Xie, Characteristic model based intelligent adaptive control, Beijing Science and Technology Press, 2009.

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4.2 Stability Analysis

- ✓ **Stability of CM + GSAC**
  - ➢ **Time-invariant/Time-varying**
  - ➢ **SISO/MIMO**
- ✓ **Robust stability of CM + GSAC**
  - ➢ **Additive unstructured uncertainty**
  - ➢ **Multiplicative unstructured uncertainty**
- ✓ **Stability of original plant + GSAC**
  - ➢ **Nonlinear uncertain plants with a relative degree of two**

4. Theoretical Results

4.2 Stability Analysis

✓ Stability of CM + GSAC

Second-order CM	**SISO**	**Time-invariant**	$y(k+1)=f_1y(k)+f_2y(k-1)+g_0u(k)$ **(4.2.1)**
		Time-varying	$y(k+1)=f_1(k)y(k)+f_2(k)y(k-1)+g_0(k)u(k)$
	MIMO	**Time-invariant**	$\begin{cases} y_1(k+1)=f_{11}y_1(k)+f_{12}y_1(k-1)+h_1y_2(k)+g_1u_1(k) \\ y_2(k+1)=f_{12}y_2(k)+f_{22}y_2(k-1)+g_2u_2(k) \end{cases}$
		Time-varying	$\begin{cases} y_1(k+1)=f_{11}(k)y_1(k)+f_{12}(k)y_1(k-1)+h_1(k)y_2(k)+g_1(k)u_1(k) \\ y_2(k+1)=f_{12}(k)y_2(k)+f_{22}(k)y_2(k-1)+g_2(k)u_2(k) \end{cases}$

4. Theoretical Results

4.2 Stability Analysis

✓ Stability of CM + GSAC

- **SISO time-invariant CM**

LTI second-order plant with unknown parameters

$$y(k+1) = f_1 y(k) + f_2 y(k-1) + g_0 u(k)$$

$$\left(f_1, f_2\right) \in D, \ \ 0.003 \le g_0 \le 0.3$$

- **Golden-section adaptive control**

$$u(k) = -\left[l_1 \hat{f}_1 y(k) + l_2 \hat{f}_2 y(k-1)\right] \Big/ \hat{g}_0$$

$$l_1 = 0.382, \ \ l_2 = 0.618, \ \ \left(\hat{f}_1, \hat{f}_2\right) \in D$$

4. Theoretical Results

中国航天

4.2 Stability Analysis

➢ **Stability results (4.2.1)**

$$D_S : \begin{cases} f_1 \in [1.4331, 1.9974] \\ f_2 \in [-0.9999, -0.5134] \\ f_1 + f_2 \in [0.9196, 0.9999] \end{cases}$$

LTI second-order plant with unknown parameters	Open-loop stable	Open-loop unstable
Coefficient range	$(f_1, f_2) \in D_S$, $0.003 \le g_0 \le 0.3$	$(f_1, f_2) \in D_N$, $0.003 \le g_0 \le 0.3$
Control parameters	$(\hat{f}_1, \hat{f}_2) \in D_S$	$(\hat{f}_1, \hat{f}_2) \in D_N$
Stability condition	$0.5 g_0 \le \hat{g}_0 < \infty$	$0.5 g_0 \le \hat{g}_0 \le 2 g_0$

$$D_N : \begin{cases} f_1 \in [1.9844, 2.2663] \\ f_2 \in [-1.2840, -1] \\ f_1 + f_2 \in [0.9646, 1] \end{cases}$$

Y. C. Xie and H. X. Wu, The application of the golden-section in adaptive robust controller design, ACTA Automatica Sinica, 18(2): 177-185, 1992.

4. Theoretical Results

4.2 Stability Analysis

✓ **Stability of CM + GSAC**

Second-order CM	**SISO**	**Time-invariant**	$y(k+1)=f_1y(k)+f_2y(k-1)+g_0u(k)$
		Time-varying	$y(k+1)=f_1(k)y(k)+f_2(k)y(k-1)+g_0(k)u(k)$ **(4.2.2)**
	MIMO	**Time-invariant**	$\begin{cases} y_1(k+1)=f_{11}y_1(k)+f_{12}y_1(k-1)+h_1y_2(k)+g_1u_1(k) \\ y_2(k+1)=f_{12}y_2(k)+f_{22}y_2(k-1)+g_2u_2(k) \end{cases}$
		Time-varying	$\begin{cases} y_1(k+1)=f_{11}(k)y_1(k)+f_{12}(k)y_1(k-1)+h_1(k)y_2(k)+g_1(k)u_1(k) \\ y_2(k+1)=f_{12}(k)y_2(k)+f_{22}(k)y_2(k-1)+g_2(k)u_2(k) \end{cases}$

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4.2 Stability Analysis

✓ Stability of CM + GSAC

- **SISO time-varying CM**

$$y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k)$$

$$\left(f_1(k), f_2(k) \right) \in D_S, \ 0 < g_{\min} \le g_0(k) \le g_{\max}$$

- **Golden-section adaptive control**

$$u(k) = -\left[l_1 \hat{f}_1(k)y(k) + l_2 \hat{f}_2(k)y(k-1) \right] \Big/ \hat{g}_0(k)$$

where $l_1 = 0.382, \ l_2 = 0.618,$ **and** $\left\{ \hat{f}_1(k), \hat{f}_2(k), \hat{g}_0(k) \right\}$ **are estimated by using the projected gradient algorithm.**

4. Theoretical Results

4.2 Stability Analysis

- **Stability result**

Theorem 4.2.2

The closed-loop system composed of the SISO time-varying CM and the GSAC is asymptotically stable if

Lyapunov Stability Theory

$$\begin{cases} 0 < \beta \le g_0(k)/\hat{g}_0(k) \le 2, \\ \dfrac{-m_1(k+1) - \sqrt{m_1^2(k+1) + 4m_0(k+1) - \varepsilon_0}}{2} < p_{12}(k) - p_{12}(k+1) < \dfrac{-m_1(k+1) + \sqrt{m_1^2(k+1) + 4m_0(k+1) - \varepsilon_0}}{2}, \end{cases}$$

where ε_0 is a small constant,

$$\begin{cases} m_0(k) = \left([1+\alpha_2(k)]^2 - \alpha_1^2(k)\right)[1 - p_{11}(k)][p_{11}(k) - \alpha_2^2(k)]\big/[1+\alpha_2(k)]^2, \;\; m_1(k) = 2\alpha_1(k)[p_{11}(k) - \alpha_2^2(k)]\big/[1+\alpha_2(k)], \\ \alpha_1(k) = l_1 \hat{f}_1(k) g_0(k)\big/\hat{g}_0(k) - f_1(k), \;\; \alpha_2(k) = l_2 \hat{f}_2(k) g_0(k)\big/\hat{g}_0(k) - f_2(k), \\ p_{11}(k) = 1/(1+\varepsilon), \;\; p_{12}(k) = [1 + (1+\varepsilon)\alpha_2(k)]\alpha_1(k)/[(1+\varepsilon)(1+\alpha_2(k))^2], \;\; p_{22}(k) = 1, \end{cases}$$

in which $\varepsilon > 0$ satisfies: when $\bar{m}_2(k) \ne 0$, $0 < \varepsilon < \min\left\{1\big/\left(\alpha_2^2(k) + \Delta\right) - 1, \; \left(\bar{m}_1(k) + \sqrt{\bar{m}_1^2(k) + 4\bar{m}_0(k)\bar{m}_2(k) - \varepsilon_1}\right)\big/\left(2\bar{m}_2(k)\right)\right\}$,

when $\bar{m}_2(k) = 0$, $0 < \varepsilon < 1\big/(\alpha_2^2(k) + \Delta) - 1$, where $\varepsilon_1, \Delta > 0$ are small constants, $\bar{m}_0(k) = [1+\alpha_2(k)]^2[1 + \alpha_2^2(k) - \alpha_1^2(k)]$,

$\bar{m}_1(k) = [1+\alpha_2(k)][(1+\alpha_2(k))^3 - 2\alpha_1^2(k)\alpha_2(k)]$, $\bar{m}_2(k) = \alpha_1^2(k)\alpha_2^2(k)$

C. Z. Qi, Study on TDRS multi-variable adaptive control method, Ph.D. dissertation, Beijing Institute of Control Engineering, Beijing, China, 1999.

4. Theoretical Results

4.2 Stability Analysis

✓ Stability of CM + GSAC

Second-order CM	**SISO**	**Time-invariant**	$y(k+1) = f_1 y(k) + f_2 y(k-1) + g_0 u(k)$
		Time-varying	$y(k+1) = f_1(k) y(k) + f_2(k) y(k-1) + g_0(k) u(k)$
	MIMO	**Time-invariant**	$\begin{cases} y_1(k+1) = f_{11} y_1(k) + f_{12} y_1(k-1) + h_1 y_2(k) + g_1 u_1(k) \\ y_2(k+1) = f_{12} y_2(k) + f_{22} y_2(k-1) + g_2 u_2(k) \end{cases}$ **(4.2.3)**
		Time-varying	$\begin{cases} y_1(k+1) = f_{11}(k) y_1(k) + f_{12}(k) y_1(k-1) + h_1(k) y_2(k) + g_1(k) u_1(k) \\ y_2(k+1) = f_{12}(k) y_2(k) + f_{22}(k) y_2(k-1) + g_2(k) u_2(k) \end{cases}$

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4.2 Stability Analysis

✓ Stability of CM + GSAC

➢ **MIMO time-invariant CM**

$$\begin{cases} y_1(k+1) = f_{11}y_1(k) + f_{12}y_1(k-1) + h_1 y_2(k) + g_1 u_1(k) \\ y_2(k+1) = f_{21}y_2(k) + f_{22}y_2(k-1) + g_2 u_2(k) \end{cases}$$

$$\left(f_{i1}, f_{i2} \right) \in D_S,\ g_i \in \left[g_{i\min}, g_{i\max} \right],\ h_1 \in \left[h_{\min}, h_{\max} \right],\ i = 1, 2$$

➢ **Golden-section adaptive control**

$$\begin{cases} u_1(k) = -\left[l_1 \left(\hat{f}_{11}(k)y_1(k) + \hat{h}_1(k)y_2(k) \right) + l_2 \hat{f}_{12}(k)y_1(k-1) \right] \Big/ \hat{g}_1(k) \\ u_2(k) = -\left[l_1 \hat{f}_{21}(k)y_2(k) + l_2 \hat{f}_{22}(k)y_2(k-1) \right] \Big/ \hat{g}_2(k) \end{cases}$$

where $l_1 = 0.382,\ l_2 = 0.618,$ **and** $\left\{ \hat{f}_{i1}(k), \hat{f}_{i2}(k), \hat{g}_i(k), \hat{h}_1(k) \right\}$ **are estimated by using the projected gradient algorithm.**

4. Theoretical Results

中国航天

4.2 Stability Analysis

- **Stability result**

Theorem 4.2.3

The closed-loop system composed of the MIMO time-invariant CM and the GSAC is asymptotically stable if

$$\begin{cases} 0<\beta\le g_0(k)/\hat{g}_0(k)\le 2, \\ \dfrac{-M_{11}(k+1)-\sqrt{M_{11}^2(k+1)+4M_{10}(k+1)-\varepsilon_0}}{2}<p_{10}(k)-p_{10}(k+1) \\ \qquad <\dfrac{-M_{11}(k+1)+\sqrt{M_{11}^2(k+1)+4M_{10}(k+1)-\varepsilon_0}}{2}, \\ \dfrac{-M_{21}(k+1)-\sqrt{M_{21}^2(k+1)+4M_{20}(k+1)M_{22}(k+1)-\varepsilon_0}}{2M_{22}(k+1)}<p_{20}(k)-p_{20}(k+1) \\ \qquad <\dfrac{-M_{21}(k+1)+\sqrt{M_{21}^2(k+1)+4M_{20}(k+1)M_{22}(k+1)-\varepsilon_0}}{2M_{22}(k+1)}, \\ H^2(k)=\left[l_1\hat{h}_1(k)g_1/\hat{g}_1(k)-h_1\right]^2<M_{h1}(k)/M_{h2}(k), \end{cases}$$

where ε_0 is a small constant,

Lyapunov Stability Theory

4. Theoretical Results

4.2 Stability Analysis

- **Stability result**

$$\begin{cases} M_{10}(k)=\left([1+\alpha_{12}(k)]^2-\alpha_{11}^2(k)\right)[p_{22}(k)-p_{11}(k)][p_{11}(k)-p_{22}(k)\alpha_{12}^2(k)]\big/[1+\alpha_{12}(k)]^2,\ \ M_{11}(k)=2\alpha_{11}(k)[p_{11}(k)-p_{22}(k)\alpha_{12}^2(k)]/[1+\alpha_{12}(k)], \\ M_{20}(k)=\left\{\left([1+\alpha_{22}(k)]^2-\alpha_{21}^2(k)\right)[p_{44}(k)-p_{33}(k)]M_{22}(k)-M_{h2}(k)H^2(k)+p_{10}^2(k-1)H^2(k)\right\}[p_{33}(k)-p_{44}(k)\alpha_{22}^2(k)]\big/[1+\alpha_{22}(k)]^2, \\ M_{21}(k)=2\alpha_{21}(k)M_{22}(k)\left[p_{33}(k)-p_{44}(k)\alpha_{22}^2(k)\right]\big/[1+\alpha_{22}(k)],\ M_{22}(k)=-[p_{10}(k-1)-p_{10}(k)]^2-M_{11}(k)[p_{10}(k-1)-p_{10}(k)]+M_{10}(k), \\ M_{h1}(k)=\left([1+\alpha_{22}(k)]^2-\alpha_{21}^2(k)\right)M_{22}(k)\big/[1+\alpha_{22}(k)]^2,\ M_{h2}(k)=[p_{22}(k)-p_{11}(k)][p_{22}(k)p_{11}(k)-p_{10}^2(k)]+p_{10}^2(k), \\ \alpha_{i1}(k)=l_1\hat{f}_{i1}(k)g_i/\hat{g}_i(k)-f_{i1},\ \ \alpha_{i2}(k)=l_2\hat{f}_{i2}(k)g_i(k)/\hat{g}_i(k)-f_{i2},\ p_{i0}(k)=[1+(1+\varepsilon_i)\alpha_{i2}(k)]\alpha_{i1}(k)/[(1+\varepsilon_i)(1+\alpha_{i2}(k))^2],\ i=1,2, \\ p_{11}(k)=1/(1+\varepsilon_1),\ \ p_{22}(k)=p_{44}(k)=1,\ p_{33}(k)=1/(1+\varepsilon_2), \end{cases}$$

where $\varepsilon_1>0$ satisfies: when $m_{12}(k)\neq 0,\ 0<\varepsilon_1<\min\left\{1\big/\left(\alpha_{12}^2(k)+\Delta_1\right)-1,\ \left(m_{11}(k)+\sqrt{m_{11}^2(k)+4m_{10}(k)m_{12}(k)}\right)\big/\left(2m_{12}(k)\right)\right\}$,

when $m_{12}(k)=0,\ 0<\varepsilon_1<1/(\alpha_{12}^2(k)+\Delta_1)-1$; Moreover, $\varepsilon_2>0$ satisfies: when $m_{22}(k)\neq 0$,

$M_{h2}(k)H^2(k)/[M_{h1}(k)-M_{h2}(k)H^2(k)]<\varepsilon_2<\min\left\{1\big/\left(\alpha_{22}^2(k)+\Delta_1\right)-1,\ \left(m_{21}(k)+\sqrt{m_{21}^2(k)+4m_{20}(k)m_{22}(k)}\right)\big/\left(2m_{22}(k)\right)\right\}$,

when $m_{22}(k)=0$, $0<\varepsilon_2<1/(\alpha_{22}^2(k)+\Delta_2)-1$, in which $\Delta_i>0$ are small constants， $m_{i0}(k)=[1+\alpha_{i2}(k)]^2[(1+\alpha_{i2}(k))^2-\alpha_{i1}^2(k)]$,
$m_{i1}(k)=[1+\alpha_{i2}(k)][(1+\alpha_{i2}(k))^3-2\alpha_{i1}^2(k)\alpha_{i2}(k)]$, $m_{i2}(k)=\alpha_{i1}^2(k)\alpha_{i2}^2(k)$, $i=1,2$.

C. Z. Qi, Study on TDRS multi-variable adaptive control method, Ph.D. dissertation, Beijing Institute of Control Engineering, Beijing, China, 1999.

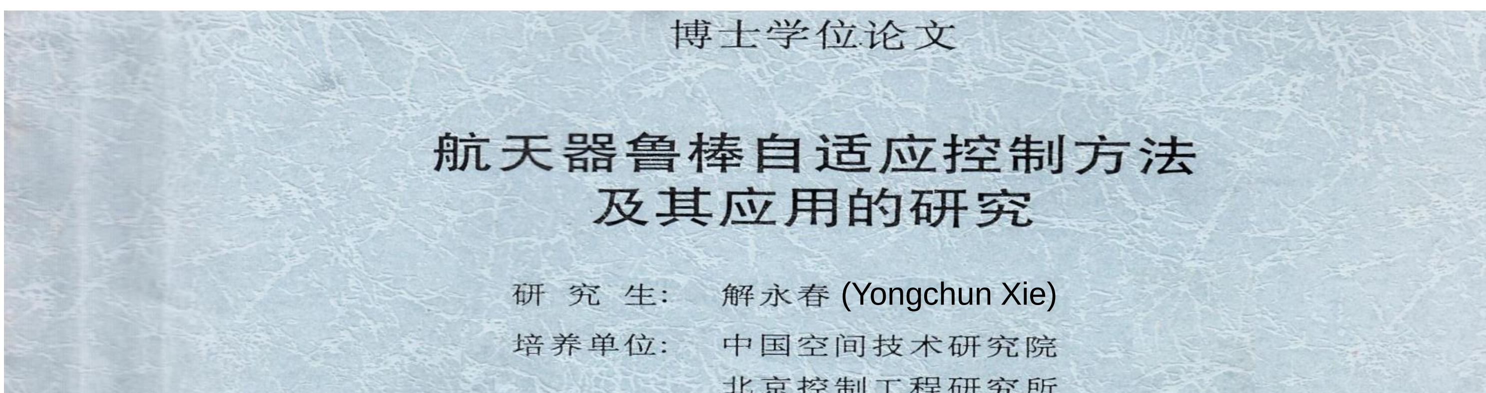
4. Theoretical Results

4.2 Stability Analysis

✓ Robust stability of CM + GSAC

Additive uncertainty $G_{\text{add}} = \{G : G = G_0 + \Delta E\}$			
LTI systems		LTV systems	
Regulation problem	Tracking problem	Regulation problem	Tracking problem

Multiplicative uncertainty $G_{\text{mul}} = \{G : G = G_0(1 + E\Delta)\}$			
LTI systems		LTV systems	
Regulation problem	Tracking problem	Regulation problem	Tracking problem


博士学位论文

航天器鲁棒自适应控制方法
及其应用的研究

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4. Theoretical Results

4.2 Stability Analysis

✓ Systems with additive uncertainty

- **LTI systems:** $G_{\text{add}} = \{G : G = G_0 + \Delta E\}$

$$G_0 = g_0 z^{-1} \big/ \left(1 - f_1 z^{-1} - f_2 z^{-2}\right), \;\; \left(f_1, f_2\right) \in D_s, \; g_0 \in \left[g_{\min}, g_{\max}\right];$$

G can be rewritten as

$$y(k) = \phi^T(k-1)\theta + \eta(k) + w(k),$$

where $\theta = [f_1, f_2, g_0]^T$

$$\phi(k-1) = [y(k-1), y(k-2), u(k-1)]^T$$

unmodeled dynamics:

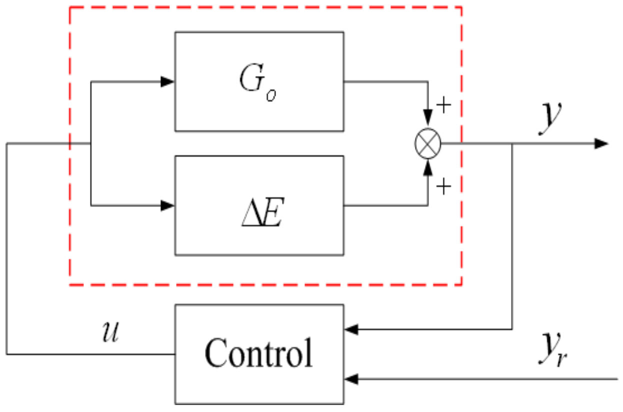
$$\eta(k) = \left(1 - f_1 z^{-1} - f_2 z^{-2}\right)\Delta E u(k)$$

and disturbance $w(k)$

- **Operator Δ: strictly causal, stable, linear and** $\|\Delta\| < 1$;
- **Operators** E, E^{-1}**: causal, stable, and linear;**
- **Unmodeled dynamics** $\eta(k)$ **: there exist** $\varepsilon_\eta \geq 0$ **and** $\sigma_\eta \in [0,1)$, **such that**

$$|\eta(k)| \leq \varepsilon_\eta \sup_{0 \leq \tau \leq k} \left\{ e^{-\sigma_\eta (k-\tau)} v\left[\phi(\tau)\right] \right\}, \;\; 0 \leq v(x) \leq \|x\|;$$

- **Disturbance:** $|w(k)| \leq d.$



4. Theoretical Results

4.2 Stability Analysis

✓ A modified golden-section adaptive control

$$K: \quad u(k) = -\frac{1}{r_0}\left[l_1 \hat{f}_1(k) y(k) + l_2 \hat{f}_2(k) y(k-1)\right] \Big/ \hat{g}_0(k)$$

$$r_0 = \begin{cases} 1, & \text{if } \alpha = g_{\max} / g_{\min} \le 2 \\ \alpha/2, & \text{if } \alpha > 2 \end{cases}$$

✓ Robust stability result

Theorem 4.2.4

The closed-loop systems composed of the LTI systems G_{add} and the above modified GSAC K have the following properties:

1） G_0 can be stabilized by control K;

2）G_{add} can be uniformly stabilized by K iff $\inf_{k \in Z^+} \left\| EK\left(I - G_0 K\right)^{-1} S_k \right\| < 1$.

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4.2 Stability Analysis

✓ Stability of original plant + GSAC

➢ **Plant (SISO)**

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = \pi(z,\xi) + g(z,\xi)u \\ \dot{\xi} = \theta(z,\xi) \\ y = z_1 \end{cases} \qquad \textbf{(4.2.4)}$$

- **The relative degree is 2, and** $\xi \in R^{n-2}$ **is internal dynamic variable;**
- $\pi(z,\xi), \theta(z,\xi)$ **are unknown and locally Lipschitz;**
- $g(z,\xi)$ **is bounded with known bounds, and** $g(z,\xi) > 0$;
- **Zero dynamics** $\dot{\xi} = \theta(0,\xi)$ **is exponentially stable.**

4. Theoretical Results

4.2 Stability Analysis

- **Control objective**

Design $\{u(kT),\ k\geq 0\}$ **so that** $\lim_{t\to\infty} y(t)\to 0.$

- **Characteristic model**

$$y(k+1)=f_1(k)y(k)+f_2(k)y(k-1)+g_0(k)u(k)$$

$$\left(f_1(k),\ f_2(k), g_0(k)\right)\in D,$$

$$D=\left\{(a_1,a_2,a_3)\in R^3\ \middle|\ \begin{array}{l}|a_1-2|\leq\varepsilon_1(y,T)\\ |a_2+1|\leq\varepsilon_2(y,T)\\ 0<\varepsilon_3(y,T^2)\leq a_3\leq\varepsilon_4(y,T^2)\end{array}\right\}$$

- **Golden-section adaptive control**

$$u(k)=-\frac{l_1\hat{f}_1(k)y(k)+l_2\hat{f}_2(k)y(k-1)}{\hat{g}_0(k)}$$

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4.2 Stability Analysis

- **Stability result**

Theorem 4.2.5

Consider the closed-loop system composed of the original plant (4.2.4) and the golden-section adaptive control, if persistent excitation (PE) condition holds, then the above closed-loop system is exponentially stable.

Y. Wang, Studies on Characteristic Model Based Attitude Control of Hypersonic Vehicles, Ph. D. Dissertation, Beijing Institute of Control Engineering, Beijing, China, 2012.

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5. Conclusions

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5.1 Summary of CMAC

Analytical Model	First-order/second-order characteristic model	Weak model dependence
High-order Controller	Low-order controller	Simple and easy to be realized
On-site Trial and Error	Golden-section control	Control parameters can be designed without trial and error. System stability can be ensured in the transient process.

5. Conclusions

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5.2 Open Problems

- Characteristic model
  - Intelligent CM
- Parameter estimation
  - Estimation of characteristic parameters relevant to system states
- Stability
  - Closed-loop stability considering logical integral and logical derivative control
  - Stability for the original systems with relative degree greater than 2

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