Characteristic Modeling Theory and Its Applications in Rendezvous and Docking

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1. Introduction

1.1 Motivation

In industry, 97% PID.

- Traditional control >98.5%
- Modern control <1.5%

From 1957 about 7800 spacecraft launched

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<tr>
<th>Year</th>
<th>Country</th>
<th>System</th>
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<td>1995</td>
<td>USA</td>
<td>MACE</td>
<td>H∞-control system identification</td>
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<td>1998</td>
<td>USA</td>
<td>MACE II</td>
<td>adaptive structural control</td>
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<td>1994</td>
<td>Japan</td>
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<td>2009</td>
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<td>ETS-VIII</td>
<td>u-synthesis control gain scheduling</td>
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<td>2003+</td>
<td>France</td>
<td>TAS-F</td>
<td>H∞-control u-synthesis</td>
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<tr>
<td>2006</td>
<td>USA</td>
<td>ISS</td>
<td>pseudo spectral solution</td>
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<tr>
<td>2010</td>
<td>USA</td>
<td>FASTSAT-HSV01</td>
<td>asymptotic periodic LQR</td>
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<td>1999+</td>
<td>China</td>
<td>SZ-1/ SZ-10</td>
<td>all-coefficient adaptive control</td>
</tr>
<tr>
<td>2010</td>
<td>China</td>
<td>SZ-8/ SZ-10</td>
<td>golden-section phase plane adaptive control</td>
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## 1. Introduction

### 1.1 Motivation

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<th>Analytical Model</th>
<th>Obtaining an accurate model in engineering practice is usually unrealistic.</th>
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<td>High-order Controller</td>
<td>For uncertain high-order systems, the existing methods are often complex and difficult to apply in practice.</td>
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<td>On-site Trial and Error</td>
<td>Most control methods need tuning on-line repeatedly, limiting their applications.</td>
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</table>

Characteristic Model-based Adaptive Control (CMAC)
1. Introduction

1.2 Development of CMAC

Sum of all-coefficients equals to one (1979-1986)

Golden-section control (1986-1992)

Characteristic model (1992-)

Adaptive estimation

\[ \lim_{T \to 0} \left( \sum_i f_i + \sum_i g_i \right) = 1 \]

Stability

\[ u_G(k) = -[l_1 \hat{f}_1 y(k) + l_2 \hat{f}_2 y(k-1)]/(\hat{g}_0 + \lambda) \]

\[ l_1 = 0.382, \quad l_2 = 0.618 \]

\[ y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k) \]

Low-order controller design

\[ \begin{align*}
D_S : \\
& f_1 \in [1.4331, 1.9974] \\
& f_2 \in [-0.9999, -0.5134] \\
& f_1 + f_2 \in [0.9196, 0.9999] \\
& g_0 \in [0.003, 0.3]
\end{align*} \]
1. Introduction

1.2 Development of CMAC

✓ State of the art

- Engineering applications (11 classes of systems)
  - Aerospace control
  - Industrial control
1. Introduction

1.2 Development of CMAC

✓ State of the art

➢ Theoretical study
  • Characteristic modeling
  • Stability analysis
OUTLINE

1. Introduction
2. CMAC Method
3. Applications
4. Theoretical Results
5. Conclusions
2. CMAC Method

2.1 Definition of Characteristic Model

**Characteristic Model**

According to the actual plant dynamic properties, environmental features, and control performance requirements, a model is derived for facilitating controller design. It is required that under the same control input, the difference between the outputs of this model and the original plant equals to zero at the steady state, and maintains in an allowed range in the transient process.
2. CMAC Method

2.1 Definition of Characteristic Model

✓ **Plant dynamic properties**

➢ **Characteristic variables** —— inputs, outputs, etc.

➢ **Characteristic parameters** —— time delay, system gain, relative degree, coefficients, etc.

➢ **Analytical, graphical, or logical description involving characteristic variables and parameters**, e.g.,

\[
y(k + 1) = f_1(k)y(k) + f_2(k)y(k - 1) + g_0(k)u(k) + g_1(k)u(k - 1)
\]
2. CMAC Method

2.1 Definition of Characteristic Model

✓ Control performance requirements

- Tracking
- Maintaining
2.2 Categories of Characteristic Models

✓ **Deterministic characteristic model**

The plants that *can be described by mathematical equations*, including linear/nonlinear, time-invariant/time-varying, etc. can be modeled by **low-order difference equations**.

✓ **Intelligent characteristic model**

The plants that *cannot be explicitly described* by mathematical equations, e.g., industrial process systems, social economic systems, can be modeled by **synthetic descriptions**, e.g., graphical/logical description.
2. CMAC Method

2.3 Mechanism of Characteristic Modeling

✓ **Structure perspective**

Linear system is composed of a series of components with the order not being greater than 2.

\[
\begin{align*}
\frac{k_1}{s} & \\
\frac{k_2}{s^2} & \\
\frac{k_3}{s + \lambda_3} & \\
\frac{k_4(s + z_4)}{s^2 + 2\xi_4\omega_4s + \omega_4^2} & \\
\end{align*}
\]
2. CMAC Method

2.3 Mechanism of Characteristic Modeling

✓ **Energy perspective**

Mechanical energy has two forms: potential energy and kinetic energy. Control process may be regarded as a kind of energy transformation.
As a result, establishing a characteristic model for a practical plant to be controlled is feasible.

2.3 Mechanism of Characteristic Modeling

✓ Mathematical foundation

High-order differential/difference equations can be equivalent to low-order time-varying differential/difference equations.

As a result, establishing a characteristic model for a practical plant to be controlled is feasible.
2. CMAC Method

2.4 Second-order Characteristic Model

✓ Plant

Consider the LTI plant:

\[
G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}
\]

which can be decomposed into the following form

\[
G(s) = \frac{k_{v,1}}{s} + \frac{k_{v,2}}{s^2} + \sum_{i=1}^{p} \frac{k_{a,i}}{s + \lambda_i} + \sum_{i=1}^{p} \frac{k_{b,i}}{(s + \lambda_i)^2} + \sum_{i=1}^{q} \left( \frac{k_{p+i}}{s + \lambda_{p+i}} + \frac{\bar{k}_{p+i}}{s + \bar{\lambda}_{p+i}} \right)
\]

where \( k_{v,1}, k_{v,2}, k_{a,i}, k_{b,i}, \lambda_i (i = 1, \cdots, p) \) are real, \( k_{p+i}, \lambda_{p+i} (i = 1, \cdots, q) \) are complex.
2. CMAC Method

2.4 Second-order Characteristic Model

✓ **Characteristic model**

When the control requirement is position keeping or tracking, its characteristic model can be described by

\[ y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k) + g_1(k)u(k-1) \]

As the plant is stable or contains integral component:

- The coefficients are slowly time-varying;
- The range of the coefficients can be determined beforehand;
- The output of CM becomes arbitrarily close to that of the plant as the sampling period decreases;
- The sum of the coefficients at steady state is equal to 1 if the static gain is one

\[ f_1(\infty) + f_2(\infty) + g_0(\infty) + g_1(\infty) = 1 \]
2. CMAC Method

2.4 Second-order Characteristic Model

✓ Range of characteristic parameters

In practice, we have

\[ g_0 \in [0.003, 0.3], \quad |g_1| \leq g_0 \]

➢ Stable plant

\[ D_S : \begin{cases} f_1 \in [1.4331, 1.9974] \\ f_2 \in [-0.9999, -0.5134] \\ f_1 + f_2 \in [0.9196, 0.9999] \end{cases} \]

\[ \frac{T}{T_{\text{min}}} \in [1/10, 1/3] \]

➢ Unstable plant

\[ D_N : \begin{cases} f_1 \in [1.9844, 2.2663] \\ f_2 \in [-1.2840, -1] \\ f_1 + f_2 \in [0.9646, 1] \end{cases} \]

\[ \frac{T}{T_{\text{min}}} \in [1/10, 1/4] \]
2. CMAC Method

2.5 Parameter Estimation

✓ Projected gradient algorithm

\[
\begin{align*}
\hat{\theta}_u(k) &= \hat{\theta}(k-1) + \frac{\lambda_1 \phi(k-1)(y(k) - \phi^T(k-1)\hat{\theta}(k-1))}{\lambda_2 + \phi^T(k-1)\phi(k-1)} \\
\hat{\theta}(k) &= \pi \left[ \hat{\theta}_u(k) \right] \\
\phi(k) &= [y(k), y(k-1), u(k), u(k-1)]^T \\
\theta(k) &= [f_1(k), f_2(k), g_0(k), g_1(k)]^T \\
\hat{\theta}(k) &= [\hat{f}_1(k), \hat{f}_2(k), \hat{g}_0(k), \hat{g}_1(k)]^T \\
\lambda_1, \lambda_2 &> 0
\end{align*}
\]

\[\hat{\theta}(k) = \pi \left[ \hat{\theta}_u(k) \right] \quad \pi[\cdot] \text{— Orthogonal projector} \]
2. CMAC Method

2.6 Adaptive Controller

\[ u(k) = u_0(k) + u_G(k) + u_D(k) + u_I(k) \]

- **Maintaining/tracking control**

\[ u_0(k) = \frac{y_r(k+1) - \hat{f}_1(k)y_r(k) - \hat{f}_2(k)y_r(k-1) - \hat{g}_1(k)u_0(k-1)}{\hat{g}_0(k)} \]

- **Golden-section control**

\[ u_G(k) = \frac{l_1\hat{f}_1(k)e(k) + l_2\hat{f}_2(k)e(k-1) - \hat{g}_1(k)u_G(k-1)}{\hat{g}_0(k)} \]

where \( e(k) = y_r(k) - y(k) \), \( l_1 = 0.382 \), \( l_2 = 0.618 \).
2.6 Adaptive Controller

➤ Logical-integral control

\[ u_i(k) = cu_i(k-1) + k_ie(k), \]

\[ k_i = \begin{cases} 
  k_{i1}, & \text{if } e(k)[e(k) - e(k-1)] - \Delta \leq 0 \\
  k_{i2}, & \text{if } e(k)[e(k) - e(k-1)] - \Delta > 0
\end{cases} \]

where \( k_{i2} > k_{i1} > 0, \)

\( \Delta > 0, \)

\( c \in \{-1, 0, 1\} \)
2. CMAC Method

2.6 Adaptive Controller

➤ Logical-derivative control

\[ u_D(k) = -k_d(e(k), \dot{e}(k)) f(\dot{e}(k)) e_d(e(k), \dot{e}(k)) \]

where \( k_d(e(k), \dot{e}(k)), e_d(e(k), \dot{e}(k)) \) is the equivalent gain and chosen according to the demand of velocity feedback, e.g.,

\[
k_d = \begin{cases} 
2 \text{sgn}(\dot{e}(k)), & \forall k - m \leq \tau \leq k, |e(\tau)| \leq r \\
2(0.5|e(k)| + c_1)^{-1/2} \text{sgn}(\dot{e}(k)), & \exists k - m \leq \tau \leq k, |e(\tau)| > r 
\end{cases}
\]

\[ f(\dot{e}(k)) = |\dot{e}(k)|, \quad e_d = \rho \left( \max_{k-n \leq \tau \leq k} (\dot{e}(k)) \right). \]
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3. Applications
4. Theoretical Results
5. Conclusions
## 3. Applications

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<td>Retractable flexible solar panels</td>
<td>BICE</td>
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3. Applications

3.1 Aerospace—Rendezvous and Docking (RVD)

✓ RVD Process

1. Remote guidance phase
2. Homing phase
3. Closing phase
4. Final approach phase

Altitude

Safety ellipse

Final approach corridor

Target orbit

1 2 3 4
3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ Guidance navigation and control system

![Diagram of Guidance, Navigation, and Control System]

- Guidance
- Navigation
- Control
- Attitude
- Thrusters
- Dynamic disturbances

3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ Challenges

- Kinematic coupling between the rotation and translation in the final approach phase
- Dynamic coupling between the attitude and position control caused by thruster installation
- Thruster plume disturbance
- Flexibility of solar panel
- Large time delay
3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ Characteristic modeling

\[
M \ddot{X} + F_{trs} \ddot{\eta}_{rs} + F_{tls} \ddot{\eta}_{ls} = P_s
\]

\[
I_s \ddot{\omega}_s + \dddot{\omega}_s I_s \omega_s + F_{sls} \dddot{\eta}_{ls} + F_{srs} \dddot{\eta}_{rs} + R_{als} \dot{\omega}_{als} + R_{ars} \dot{\omega}_{ars} = T_s
\]

\[
I_{als} \dot{\omega}_{als} + F_{als} \dddot{\eta}_{ls} + R_{als}^T \dot{\omega}_s = T_{als}
\]

\[
I_{ars} \dot{\omega}_{ars} + F_{ars} \dddot{\eta}_{rs} + R_{ars}^T \dot{\omega}_s = T_{ars}
\]

\[
\dddot{\eta}_{ls} + 2\xi_{ls} \Omega_{als} \dddot{\eta}_{ls} + \Omega_{als}^2 \eta_{ls} + F_{tls} \dddot{\dot{X}} + F_{sls} \dddot{\dot{\theta}} + F_{als} \dot{\omega}_{als} = 0
\]

\[
\dddot{\eta}_{rs} + 2\xi_{rs} \Omega_{ars} \dddot{\eta}_{rs} + \Omega_{ars}^2 \eta_{rs} + F_{trs} \dddot{\dot{X}} + F_{srs} \dddot{\dot{\theta}} + F_{ars} \dot{\omega}_{ars} = 0
\]

\[
\theta(k+1) = \bar{F}_1 \theta(k) + \bar{F}_2 \theta(k-1) + \bar{G}_1 T(k) + \bar{G}_2 T_d(k)
\]

\[T(k) : \text{Control Torques}\]
\[T_d(k) : \text{Disturbances (SADA-driven Torques)}\]
3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ True values of characteristic parameters

Simulation illustrates that the characteristic parameters are slowly time-varying and located in $D_S$. 

$$D_S: \begin{cases} f_1 \in [1.4331, 1.9974] \\ f_2 \in [-0.9999, -0.5134] \\ f_1 + f_2 \in [0.9196, 0.9999] \end{cases}$$
3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ Phase-plane control

R11: high-thruster turn-on region
R12: low-thruster turn-on region
R13: low-thruster stepping region
R14: low-thruster outer sliding resistance region
R15, R16: high-thruster velocity restricted region
R0: turn-off region

Calculate the control parameters instead of trial and error.

\[ T_{N1} = \frac{\dot{\theta}}{a_{c2}} = 0.618\Delta T \quad a_{c2} = \dot{\theta}_v / (0.618\Delta T) \]

\[ T_{N2} = K_j \frac{a_{c2}}{\theta_B} = 0.382\Delta T \]

\[ K_j = 0.382a_{c2}\Delta T / \theta_B = 0.618 \frac{\dot{\theta}_v}{\theta_B} \]
3. Applications

3.1 Aerospace—Rendezvous and Docking

✓ In-orbit flight data validation

In Nov. 2011, June 2012 and June 2013, the manned and unmanned spacecraft Shenzhou-8, Shenzhou-9 and Shenzhou-10 have successfully completed automatic RVD with spacecraft Tiangong-1.

The history of attitude and attitude angular velocity of an RVD operation
3. Applications

3.1 Aerospace—Reentry of Shenzhou Spacecraft

The return module enters the atmosphere and returns to the Earth safely by decreasing its speed with the atmosphere drag.

✓ Dynamic characteristics

- High-order nonlinear system
- Small lift-to-drag ratio and weak controllability
- Strong uncertainty of atmosphere density and aerodynamic parameters
- System gain changes greatly, more than 1000 times.
3. Applications

3.1 Aerospace—Reentry of Shenzhou Spacecraft
✓ Adaptive reentry control

- Input and output transformation
- Parameter estimation-projected gradient algorithm

\[
y(k + 1) = f_1(k)y(k) + f_2(k)y(k - 1) + g_0(k)u(k)
\]

\[
\begin{align*}
f_1(k) &\in [1.4331, 1.9974] \\
f_2(k) &\in [-0.9999, -0.5134] \\
f_1(k) + f_2(k) &\in [0.920, 0.995] \\
g_0(k) &\in [0.003, 0.65]
\end{align*}
\]

Block diagram of adaptive reentry control system
3. Applications

3.1 Aerospace—Reentry of Shenzhou Spacecraft

- **Linear feedback control**
  \[ u_L(k) = -\left( L_1 \hat{f}_1(k) y(k) + L_2 \hat{f}_2(k) y(k-1) \right) / \hat{g}_0(k) \]
  \[ 0 < L_1, L_2 < 1 \]

- **Integral control**
  \[ u(k) = u(k-1) + u_L(k) \]

- **Total control quantity limitation**
  \[ u(k) \in [A(\cos \bar{\gamma} - \cos(\gamma_0(t))), A(\cos \gamma - \cos(\gamma_0(t)))] \]
  \[ \gamma_0 \text{ is the nominal value, } \gamma = 10^\circ, \bar{\gamma} = 125^\circ. \]

Spacecraft Shenzhou-1 ~ Shenzhou-10 all have successfully returned to the Earth.
3. Applications

3.1 Aerospace-Reentry of Chang’e 5 test spacecraft

✓ Chang’e 5 test spacecraft
  - Launch date: 23 Oct. 2014
  - Landing date: 1 Nov. 2014
  - Skip reentry from Lunar orbit with the 2nd cosmic velocity

✓ Dynamic characteristics
  - Nonlinearity
  - Strong uncertainty
  - Great parameter variation
3. Applications

3.1 Aerospace-Reentry of Chang’e 5 test spacecraft

✓ Controller design

- First-order characteristic model
  \[ y(k+1) = f(k)y(k) + g(k)u(k) \]
  \[ f(k) \in [0.2, 0.99], g(k) \in [0.003,1] \]

- Parameter estimation: projected gradient algorithm with filter
  \[ \hat{\theta}'(k) = \Pi \left[ \hat{\theta}(k-1) + \frac{\lambda_1 \phi(k)}{\lambda_2 + \phi^T(k) \phi(k)} \left( y(k) - \phi^T(k) \hat{\theta}(k-1) \right) \right] \]
  \[ \hat{\theta}(k) = F \hat{\theta}'(k) + (1-F) \hat{\theta}(k-1), 0 \leq F \leq 1 \]
  \[ \Pi -- \text{orthogonal projector} \]

- Adaptive control

  Linear feedback control
  \[ u_L(k) = -L \hat{f}(k) y(k)/\hat{g}(k), \]
  \[ 0 < L < 1 \]

  Integral control
  \[ u(k) = u(k-1) + u_L(k) \]

3. Applications

3.1 Aerospace-Reentry of Chang’e 5 test spacecraft

✓ Flight validation

- A first-order characteristic model was successfully applied for the first time.

- The control accuracy has reached the leading level in the world.
3. Applications

3.2 Industry—Continuous Sterilization in Fermentation

- Heat-exchange process
- Input: steam flow quantity
- Output: outlet nutrient temperature of the steam ejector

3. Applications

3.2 Industry—Continuous Sterilization in Fermentation

✓ Plant characteristics

- Temperature regulating system with a relative degree of one
- Subject to some fast time-varying disturbances
- Strong coupling between steam and liquid
- The control gain is affected by disturbances.

\[ \dot{x} = -2k(t)x + g(-x, d)u \]
3. Applications

3.2 Industry—Continuous Sterilization in Fermentation

Controller design

- Characteristic model
  \[ z(k+1) = f_1(k)z(k) + g_0(k)u(k) \]
- Adaptive control
- Feedforward control

Take disturbances as inputs to construct the feedforward lookup table, and use iterative learning control to improve it.

<table>
<thead>
<tr>
<th>Nutrient Flow</th>
<th>Nutrient Temperature</th>
<th>Control value</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>78</td>
<td>0.45</td>
</tr>
<tr>
<td>13</td>
<td>75</td>
<td>0.40</td>
</tr>
</tbody>
</table>

![Diagram of control system with Nutrient Flow Trajectory, PID, Nutrient Flow, Nutrient Flow Estimation, Adaptive Controller, ILC Feedforward Controller, External Disturbances, Reference, u, Steam Valve, Steam Ejector, CM Estimation, \( f_1, f_2, \hat{g} \).]
3. Applications

3.2 Industry—Continuous Sterilization in Fermentation

✓ Control results

- Steady state error: ±0.5°C
- Transient error: ±3°C
- Settling time is greatly decreased from 20 minutes to less than 2 minutes.
- The steam consumption is greatly decreased, and the quality of the nutrient after sterilization is significantly improved.
3. Applications

3.2 Industry—Aluminum Electrolysis

- Electrochemistry reaction: alumina is reduced to metal aluminum
- Input: alumina feeding time interval
- Output: concentration of alumina

✓ Plant characteristics

- Concentration regulating system with a relative degree of one
- Hard to directly measure the controlled outputs
- Large time delay, strong nonlinearity and uncertainty
- Multiple equilibria

3. Applications

3.2 Industry—Aluminum Electrolysis

✓ Controller design

- Characteristic model of a medium variable and the unmeasured output
  \[ z(k) = h_1(\cdot)[y_h(k) - y_{h0}]^2 + R_0 + \xi \]

- Characteristic model of the medium variable and the control input in the neighborhood of the operating point
  \[ z(k+1) = f_1(k)z(k) + g_0(k)u(k) + \Delta_1(k) \]

- Guidance controller
  Guide system to the neighborhood of the desired operating point.

- Feedback controller
  Adaptive fuzzy controller based on the CM

Operating area
## 3. Applications

### 3.2 Industry—Aluminum Electrolysis

☑ Control results

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<tr>
<th>INDEX</th>
<th>PAST</th>
<th>PRESENT</th>
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<tr>
<td>Anode effect controlled percentage</td>
<td>60%</td>
<td>more than 80%</td>
</tr>
<tr>
<td>Current efficiency</td>
<td>91.5%</td>
<td>93.3%</td>
</tr>
<tr>
<td>Electricity saving</td>
<td></td>
<td>315 kwh/t</td>
</tr>
<tr>
<td>Production increment per electro bath</td>
<td></td>
<td>8.473 t</td>
</tr>
</tbody>
</table>
4. Theoretical Results

✓ Characteristic Modeling
  ➢ LTI systems
  ➢ LTV systems
  ➢ NLTI systems

✓ Stability Analysis
  ➢ Stability of CM + golden-section adaptive controller (GSAC)
  ➢ Robust stability of CM + GSAC
  ➢ Stability of the original plant + GSAC
4. Theoretical Results

4.1 Characteristic Modeling-LTI systems

Consider the plant

\[ y = G(s)u \]  \hspace{1cm} (4.1.1)

where \( G(s) \in R^{p \times q} \) is a rational transfer function matrix, and \( u \in R^q \) and \( y \in R^p \) represent inputs and outputs respectively.

**Assumption 1**

There exists a sampling controller such that the outputs can track the constant reference signals.

**Assumption 2**

The sampling period \( h \) is small enough.
4. Theoretical Results

4.1 Characteristic Modeling-LTI systems

Theorem 4.1.1

Under Assumptions 1 and 2, the characteristic model of plant (4.1.1) can be described by the time-varying difference equation

\[ y_i(k+1) = f_{i1}(k)y_i(k) + f_{i2}(k)y_i(k-1) + g^T_{i0}(k)u(k) + g^T_{i1}(k)u(k-1), \ i = 1, 2, \ldots, p \]

where \( f_{i1}(k), f_{i2}(k) \in R \) and \( g_{i0}(k), g_{i1}(k) \in R^q \) are slowly time-varying,

\[
\begin{align*}
    f_{i1}(k) &\in [2 + 2h\lambda_{\text{min}}, 2], f_{i2}(k) \in [-1, -(1+2h\lambda_{\text{min}})], & \text{when } \lambda_{\text{min}} < \lambda_{\text{max}} < 0 \\
    f_{i1}(k) &\in [2, 2 + 2h\lambda_{\text{max}}], f_{i2}(k) \in [-(1+2h\lambda_{\text{max}}), -1], & \text{when } \lambda_{\text{max}} > \lambda_{\text{min}} > 0 \\
    f_{i1}(k) &\in [2 + 2h\lambda_{\text{min}}, 2 + 2h\lambda_{\text{max}}], f_{i2}(k) \in [-(1+2h\lambda_{\text{max}}), -(1+2h\lambda_{\text{min}})], & \text{otherwise}
\end{align*}
\]

\[ g_{i0}(k) = -g_{i1}(k) + O(h^2) = O(h) \]

Furthermore, when there are not the poles with zero real parts, the modeling error is \( O(h) \) in the transient process, and the outputs of the CM and plant (4.1.1) are equal in the steady stage; in general cases, the modeling errors are \( O(h) \) and \( O(h^2) \) in the transient process and steady stage respectively.

4. Theoretical Results

4.1 Characteristic Modeling-LTV systems

Consider the plant

\[ y^{(n)}(t) + a_{n-1}(t)y^{(n-1)}(t) + \cdots + a_2(t)y^{(2)}(t) + a_1(t)y^{(1)}(t) + a_0(t)y(t) \]
\[ = b_m(t)y^{(m)}(t) + \cdots + b_2(t)u^{(2)}(t) + b_1(t)u^{(1)}(t) + b_0(t)u(t) \quad (m < n) \]  

Assumption 1

The plant (4.1.2) is controllable, and \( y(t) \), \( \dot{y}(t) \) and \( \ddot{y}(t) \) are measurable.

Assumption 2

\( u(t) \) and \( y(t) \), and their derivatives are bounded.

Assumption 3

\( a_i(t), b_j(t), i = 0, \ldots, n-1, j = 0, \ldots, m \), and their derivatives are bounded, when \( t \to \infty \), they are constants, and

\[ |a_i(t + \Delta t) - a_i(t)| < M_1 \Delta t, \quad |b_j(t + \Delta t) - b_j(t)| < M_2 \Delta t \]

where \( M_1 \) and \( M_2 \) are constants.
When the control requirement is position keeping or tracking, then the characteristic model of the plant (4.1.2) can be described by the following time-varying difference equation:

\[ y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k) + g_1(k)u(k-1) \quad (4.1.3) \]

provided that Assumptions 1-3 hold and the sampling period satisfies certain conditions. Furthermore, if the plant (4.1.2) is stable, then the output error between (4.1.3) and (4.1.2) is kept in a permitted range under the same input, and the range of the coefficients of (4.1.3) can be determined beforehand.
4. Theoretical Results

4.1 Characteristic Modeling-NLTI systems

Consider the plant

\[ \dot{x}(t) = f(x,u) \]  \hspace{1cm} (4.1.4)

**Assumption 1**

The plant (4.1.4) is SISO, and the order of \( u(t) \) in (4.1.4) is less than 2 and \( u(t) \) is bounded.

**Assumption 2**

\( f(x,u) \) is differentiable and its derivatives are bounded, and \( f(0,0) = 0 \).

**Assumption 3**

\[ |f(x(t+\Delta t),u(t+\Delta t)) - f(x(t),u(t))| < M \Delta t, \text{ where } M > 0 \text{ is constant.} \]
4. Theoretical Results

4.1 Characteristic Modeling-NLTI systems

**Theorem 4.1.3**

Under Assumptions 1-3, if the sampling period satisfies certain conditions, then the characteristic model of the plant (4.1.4) can be described by the following time-varying difference equation

\[ y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k) + g_1(k)u(k-1) \]  

(4.1.5)

Furthermore, if the plant (4.1.4) is stable, then the output error between (4.1.5) and (4.1.4) is kept in a permitted range under the same input, and the range of the coefficients of (4.1.5) can be determined beforehand.

4. Theoretical Results

4.2 Stability Analysis

✓ Stability of CM + GSAC
  - Time-invariant/Time-varying
  - SISO/MIMO

✓ Robust stability of CM + GSAC
  - Additive unstructured uncertainty
  - Multiplicative unstructured uncertainty

✓ Stability of original plant + GSAC
  - Nonlinear uncertain plants with a relative degree of two
### 4. Theoretical Results

#### 4.2 Stability Analysis

✓ Stability of CM + GSAC

<table>
<thead>
<tr>
<th>Second-order CM</th>
<th>SISO</th>
<th>Time-invariant</th>
<th>( y(k+1) = f_1 y(k) + f_2 y(k-1) + g_0 u(k) ) (4.2.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time-varying</td>
<td>( y(k+1) = f_1(k) y(k) + f_2(k) y(k-1) + g_0(k) u(k) )</td>
<td></td>
</tr>
<tr>
<td>MIMO</td>
<td>Time-invariant</td>
<td>( \begin{cases} y_1(k+1) = f_{11} y_1(k) + f_{12} y_1(k-1) + h_1 y_2(k) + g_1 u_1(k) \ y_2(k+1) = f_{12} y_2(k) + f_{22} y_2(k-1) + g_2 u_2(k) \end{cases} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Time-varying</td>
<td>( \begin{cases} y_1(k+1) = f_{11} y_1(k) + f_{12} y_1(k-1) + h_1(k) y_2(k) + g_1(k) u_1(k) \ y_2(k+1) = f_{12} y_2(k) + f_{22} y_2(k-1) + g_2(k) u_2(k) \end{cases} )</td>
<td></td>
</tr>
</tbody>
</table>
4. Theoretical Results

4.2 Stability Analysis

✓ **Stability of CM + GSAC**

- **SISO time-invariant CM**

\[ y(k + 1) = f_1 y(k) + f_2 y(k - 1) + g_0 u(k) \]

\[ (f_1, f_2) \in D, \quad 0.003 \leq g_0 \leq 0.3 \]

- **Golden-section adaptive control**

\[ u(k) = -\left[l_1 \hat{f}_1 y(k) + l_2 \hat{f}_2 y(k - 1)\right]/\hat{g}_0 \]

\[ l_1 = 0.382, \quad l_2 = 0.618, \quad \left(\hat{f}_1, \hat{f}_2\right) \in D \]
4. Theoretical Results

4.2 Stability Analysis

- Stability results (4.2.1)

<table>
<thead>
<tr>
<th>LTI second-order plant with unknown parameters</th>
<th>Open-loop stable</th>
<th>Open-loop unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient range ((f_1, f_2)) (\in D_S), (0.003 \leq g_0 \leq 0.3)</td>
<td>((f_1, f_2)) (\in D_N), (0.003 \leq g_0 \leq 0.3)</td>
<td></td>
</tr>
<tr>
<td>Control parameters ((\hat{f}_1, \hat{f}_2)) (\in D_S)</td>
<td>((\hat{f}_1, \hat{f}_2)) (\in D_N)</td>
<td></td>
</tr>
<tr>
<td>Stability condition (0.5g_0 \leq \hat{g}_0 &lt; \infty)</td>
<td>(0.5g_0 \leq \hat{g}_0 \leq 2g_0)</td>
<td></td>
</tr>
</tbody>
</table>

\[ D_S : \begin{cases} f_1 \in [1.4331, 1.9974] \\ f_2 \in [-0.9999, -0.5134] \\ f_1 + f_2 \in [0.9196, 0.9999] \end{cases} \]

\[ D_N : \begin{cases} f_1 \in [1.9844, 2.2663] \\ f_2 \in [-1.2840, -1] \\ f_1 + f_2 \in [0.9646, 1] \end{cases} \]

4. Theoretical Results

4.2 Stability Analysis

✓ Stability of CM + GSAC

<table>
<thead>
<tr>
<th></th>
<th>Time-invariant</th>
<th>Time-varying</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Second-order CM</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SISO</td>
<td>$y(k + 1) = f_1y(k) + f_2y(k - 1) + g_0u(k)$</td>
<td>$y(k + 1) = f_1(k)y(k) + f_2(k)y(k - 1) + g_0(k)u(k)$ (4.2.2)</td>
</tr>
<tr>
<td>MIMO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time-invariant</td>
<td>$\begin{cases} y_1(k + 1) = f_{11}y_1(k) + f_{12}y_1(k - 1) + h_1y_2(k) + g_1u_1(k) \ y_2(k + 1) = f_{12}y_2(k) + f_{22}y_2(k - 1) + g_2u_2(k) \end{cases}$</td>
<td></td>
</tr>
<tr>
<td>Time-varying</td>
<td>$\begin{cases} y_1(k + 1) = f_{11}(k)y_1(k) + f_{12}(k)y_1(k - 1) + h_1(k)y_2(k) + g_1(k)u_1(k) \ y_2(k + 1) = f_{12}(k)y_2(k) + f_{22}(k)y_2(k - 1) + g_2(k)u_2(k) \end{cases}$</td>
<td></td>
</tr>
</tbody>
</table>
4. Theoretical Results

4.2 Stability Analysis

✓ Stability of CM + GSAC

➢ SISO time-varying CM

\[ y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k) \]

\[ (f_1(k), f_2(k)) \in D_S, \quad 0 < g_{\min} \leq g_0(k) \leq g_{\max} \]

➢ Golden-section adaptive control

\[ u(k) = -\left[ l_1 \hat{f}_1(k)y(k) + l_2 \hat{f}_2(k)y(k-1) \right]/\hat{g}_0(k) \]

where \( l_1 = 0.382, \ l_2 = 0.618, \) and \( \{ \hat{f}_1(k), \hat{f}_2(k), \hat{g}_0(k) \} \) are estimated by using the projected gradient algorithm.
4. Theoretical Results

4.2 Stability Analysis

Stability result

Theorem 4.2.2

The closed-loop system composed of the SISO time-varying CM and the GSAC is asymptotically stable if

\[
0 < \beta \leq \frac{g_0(k)}{\hat{g}_0(k)} \leq 2,
\]

\[
-m_1(k+1) - \frac{m_0^2(k+1) + 4m_0(k+1) - \varepsilon_0}{2} < p_{12}(k) - p_{12}(k+1) < -m_1(k+1) + \frac{m_0^2(k+1) + 4m_0(k+1) - \varepsilon_0}{2}
\]

where \(\varepsilon_0\) is a small constant,

\[
\begin{align*}
m_0(k) &= (1 + \alpha_2(k))^2 - \alpha_1^2(k)) [1 - p_{11}(k)] [p_{11}(k) - \alpha_2^2(k)]/[1 + \alpha_2(k)]^2, \\
m_1(k) &= 2\alpha_1(k) [p_{11}(k) - \alpha_2^2(k)]/[1 + \alpha_2(k)], \\
\alpha_1(k) &= l_1f_1(k) \hat{g}_0(k) - f_1(k), \\
\alpha_2(k) &= l_2f_2(k) g_0(k) / \hat{g}_0(k) - f_2(k), \\
p_{11}(k) &= 1/(1 + \varepsilon), \\
p_{12}(k) &= [1 + (1 + \varepsilon)\alpha_2(k)\alpha_1(k)]/[1 + (1 + \alpha_2(k))^2], \\
p_{22}(k) &= 1,
\end{align*}
\]

in which \(\varepsilon > 0\) satisfies: when \(\tilde{m}_2(k) \neq 0\), \(0 < \varepsilon < \min \left\{ \frac{1}{(\alpha_2^2(k) + \Delta) - 1}, \frac{v_1(k) + \sqrt{\tilde{m}_0^2(k) + 4\tilde{m}_0(k)\tilde{m}_2(k) - \varepsilon_1}}{2\tilde{m}_2(k)} \right\},\)

when \(\tilde{m}_2(k) = 0\), \(0 < \varepsilon < 1/(\alpha_2^2(k) + \Delta) - 1\), where \(\varepsilon_1, \Delta > 0\) are small constants, \(\tilde{m}_0(k) = [1 + \alpha_2(k)]^2 [1 + \alpha_2^2(k) - \alpha_1^2(k)], \)

\(\tilde{m}_1(k) = [1 + \alpha_2(k)] [(1 + \alpha_2(k))^3 - 2\alpha_1^2(k)\alpha_2(k)], \tilde{m}_2(k) = \alpha_1^2(k)\alpha_2^2(k)\)

## 4. Theoretical Results

### 4.2 Stability Analysis

- **Stability of CM + GSAC**

<table>
<thead>
<tr>
<th>Second-order CM</th>
<th>SISO</th>
<th>Time-invariant</th>
<th>$y(k+1) = f_1y(k) + f_2y(k-1) + g_0u(k)$</th>
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<td></td>
</tr>
<tr>
<td></td>
<td>Time-invariant</td>
<td>$\begin{cases} y_1(k+1) = f_{11}y_1(k) + f_{12}y_1(k-1) + h_1y_2(k) + g_1u_1(k) \ y_2(k+1) = f_{12}y_1(k) + f_{22}y_2(k-1) + g_2u_2(k) \end{cases}$ (4.2.3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Time-varying</td>
<td>$\begin{cases} y_1(k+1) = f_{11}(k)y_1(k) + f_{12}(k)y_1(k-1) + h_1(k)y_2(k) + g_1(k)u_1(k) \ y_2(k+1) = f_{12}(k)y_2(k) + f_{22}(k)y_2(k-1) + g_2(k)u_2(k) \end{cases}$</td>
<td></td>
</tr>
</tbody>
</table>
4. Theoretical Results

4.2 Stability Analysis

✓ Stability of CM + GSAC

- **MIMO time-invariant CM**
  \[
  \begin{align*}
  y_1(k+1) &= f_{11}y_1(k) + f_{12}y_1(k-1) + h_1y_2(k) + g_1u_1(k) \\
  y_2(k+1) &= f_{21}y_2(k) + f_{22}y_2(k-1) + g_2u_2(k)
  \end{align*}
  \]

  \((f_{i1}, f_{i2}) \in D_S, \ g_i \in \left[ g_{\text{min}}, g_{\text{max}} \right], \ h_i \in \left[ h_{\text{min}}, h_{\text{max}} \right], \ i = 1, 2\)

- **Golden-section adaptive control**
  \[
  \begin{align*}
  u_1(k) &= -\left[ l_1 \left( \hat{f}_{11}(k)y_1(k) + \hat{h}_1(k)y_2(k) \right) + l_2 \hat{f}_{12}(k)y_1(k-1) \right] / \hat{g}_1(k) \\
  u_2(k) &= -\left[ l_1 \hat{f}_{21}(k)y_2(k) + l_2 \hat{f}_{22}(k)y_2(k-1) \right] / \hat{g}_2(k)
  \end{align*}
  \]

  where \( l_1 = 0.382, \ l_2 = 0.618 \), and \( \{ \hat{f}_{ij}(k), \hat{f}_{ij}(k), \hat{g}_i(k), \hat{h}_i(k) \} \) are estimated by using the projected gradient algorithm.
4. Theoretical Results

4.2 Stability Analysis

- Stability result

**Theorem 4.2.3**

The closed-loop system composed of the MIMO time-invariant CM and the GSAC is asymptotically stable if

\[
0 < \beta \leq \frac{g_0(k)}{\hat{g}_0(k)} \leq 2,
\]

\[
\begin{align*}
-\frac{M_{11}(k + 1) - \sqrt{M_{11}^2(k + 1) + 4M_{10}(k + 1) - \varepsilon_0}}{2} &< p_{10}(k) - p_{10}(k + 1) \\
-\frac{M_{21}(k + 1) - \sqrt{M_{21}^2(k + 1) + 4M_{20}(k + 1)M_{22}(k + 1) - \varepsilon_0}}{2M_{22}(k + 1)} &< p_{20}(k) - p_{20}(k + 1) \\
\end{align*}
\]

\[
\frac{H^2(k) = \left[ l_1 \hat{h}_1(k) g_1 / \hat{g}_1(k) - h_1 \right]^2}{< M_{h1}(k) / M_{h2}(k),}
\]

where \( \varepsilon_0 \) is a small constant.

Lyapunov Stability Theory
4. Theoretical Results

4.2 Stability Analysis

- Stability result

\[
M_{i0}(k) = \left( [1 + \alpha_{i2}(k)] - \alpha_{i1}^2(k) \right) \left[ p_{22}(k) - p_{11}(k) \right] \left[ p_{11}(k) - p_{22}(k) \alpha_{i2}^2(k) \right] / \left[ 1 + \alpha_{i2}(k) \right]^2,
\]

\[
M_{i1}(k) = 2\alpha_{i1}(k) \left[ p_{11}(k) - p_{22}(k) \alpha_{i2}^2(k) \right] / \left[ 1 + \alpha_{i2}(k) \right]^2,
\]

\[
M_{20}(k) = \left( [1 + \alpha_{22}(k)] - \alpha_{21}^2(k) \right) \left[ p_{44}(k) - p_{33}(k) \right] M_{22}(k) - M_{h2}(k) H^2(k) + p_{i0}(k - 1) H^2(k) \left[ p_{33}(k) - p_{44}(k) \alpha_{22}^2(k) \right] / \left[ 1 + \alpha_{22}(k) \right]^2,
\]

\[
M_{21}(k) = 2\alpha_{21}(k) M_{22}(k) \left[ p_{33}(k) - p_{44}(k) \alpha_{22}^2(k) \right] / \left[ 1 + \alpha_{22}(k) \right],
\]

\[
M_{h1}(k) = \left( [1 + \alpha_{h2}(k)] - \alpha_{h1}^2(k) \right) M_{22}(k) / \left[ 1 + \alpha_{22}(k) \right]^2,
\]

\[
M_{h2}(k) = \left[ p_{22}(k) - p_{11}(k) \right] \left[ p_{22}(k) p_{11}(k) - p_{i0}^2(k) \right] + p_{i0}^2(k),
\]

\[
\alpha_{i1}(k) = l_{1i} \hat{f}_{i1}(k) g_{i1}(k) / \hat{g}_{i1}(k) - f_{i1}, \quad \alpha_{i2}(k) = l_{2i} \hat{f}_{i2}(k) g_{i2}(k) / \hat{g}_{i2}(k) - f_{i2}, \quad p_{i0}(k) = [1 + (1 + \epsilon_i) \alpha_{i2}(k) \alpha_{i1}(k)] / [(1 + \epsilon_i)(1 + \alpha_{i2}(k)^2)], \quad i = 1, 2,
\]

where \( \epsilon_i > 0 \) satisfies: when \( m_{i2}(k) \neq 0, \quad 0 < \epsilon_i < \min \left\{ 1 / (\alpha_{i2}(k) + \Delta_i) - 1, \left( m_{i1}(k) + \sqrt{m_{i1}^2(k) + 4m_{i0}(k)m_{i2}(k)} \right) / (2m_{i2}(k)) \right\},
\]
when \( m_{i2}(k) = 0, \quad 0 < \epsilon_i < 1 / (\alpha_{i2}(k) + \Delta_i) - 1; \) Moreover, \( \epsilon_2 > 0 \) satisfies: when \( m_{22}(k) \neq 0,
\]

\[
M_{h2}(k) H^2(k) / [M_{h1}(k) - M_{h2}(k) H^2(k)] < \epsilon_2 < \min \left\{ 1 / (\alpha_{22}(k) + \Delta_2) - 1, \left( m_{21}(k) + \sqrt{m_{21}^2(k) + 4m_{20}(k)m_{22}(k)} \right) / (2m_{22}(k)) \right\},
\]
when \( m_{22}(k) = 0, \quad 0 < \epsilon_2 < 1 / (\alpha_{22}(k) + \Delta_2) - 1, \) in which \( \Delta_i > 0 \) are small constants, \( m_{i0}(k) = [1 + \alpha_{i2}(k)]^2 [1 + (1 + \alpha_{i2}(k))^2 - \alpha_{i1}^2(k)],
\]

\[
m_{i1}(k) = [1 + \alpha_{i2}(k)][(1 + \alpha_{i2}(k))^3 - 2\alpha_{i1}^2(k)\alpha_{i2}(k)], \quad m_{i2}(k) = \alpha_{i1}^2(k)\alpha_{i2}^2(k), \quad i = 1, 2.
\]

4. Theoretical Results

4.2 Stability Analysis

✓ Robust stability of CM + GSAC

<table>
<thead>
<tr>
<th>Additive uncertainty</th>
<th>Multiplicative uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_{\text{add}} = { G : G = G_0 + \Delta E } )</td>
<td>( G_{\text{mul}} = { G : G = G_0 (1 + E \Delta) } )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LTI systems</th>
<th>LTV systems</th>
</tr>
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<tbody>
<tr>
<td>Regulation problem</td>
<td>Tracking problem</td>
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</table>

博士学位论文

航天器鲁棒自适应控制方法及其应用的研究

研究生：解永春 (Yongchun Xie)

培养单位：中国空间技术研究院

北京控制工程研究所
4. Theoretical Results

4.2 Stability Analysis

Systems with additive uncertainty

- LTI systems: $G_{\text{add}} = \{ G : G = G_0 + \Delta E \}$
  
  $G_0 = g_0 z^{-1} / \left(1 - f_1 z^{-1} - f_2 z^{-2}\right)$, $(f_1, f_2) \in D_s$, $g_0 \in [g_{\text{min}}, g_{\text{max}}]$;

- Operator $\Delta$: strictly causal, stable, linear and $\| \Delta \| < 1$;

- Operators $E, E^{-1}$: causal, stable, and linear;

- Unmodeled dynamics $\eta(k)$: there exist $\varepsilon_\eta \geq 0$ and $\sigma_\eta \in [0,1)$, such that
  
  $|\eta(k)| \leq \varepsilon_\eta \sup_{0 \leq \tau \leq k} \left\{ e^{-\sigma_\eta (k-\tau)} \nu[\phi(\tau)] \right\}$, $0 \leq \nu(x) \leq \|x\|;$

- Disturbance: $|w(k)| \leq d.$

$G$ can be rewritten as

$y(k) = \phi^T (k-1) \theta + \eta(k) + w(k)$,

where $\theta = [f_1, f_2, g_0]^T$

$\phi(k-1) = [y(k-1), y(k-2), u(k-1)]^T$

unmodeled dynamics:

$\eta(k) = (1 - f_1 z^{-1} - f_2 z^{-2}) \Delta E u(k)$

and disturbance $w(k)$
4. Theoretical Results

4.2 Stability Analysis

✓ A modified golden-section adaptive control

\[
K: \quad u(k) = -\frac{1}{r_0} \left[ l_1 \hat{f}_1(k) y(k) + l_2 \hat{f}_2(k) y(k-1) \right] / \hat{g}_0(k)
\]

\[
r_0 = \begin{cases} 
1, & \text{if } \alpha = g_{\max} / g_{\min} \leq 2 \\
\alpha/2, & \text{if } \alpha > 2 
\end{cases}
\]

✓ Robust stability result

**Theorem 4.2.4**

The closed-loop systems composed of the LTI systems \( G_{\text{add}} \) and the above modified GSAC \( K \) have the following properties:

1) \( G_0 \) can be stabilized by control \( K \);

2) \( G_{\text{add}} \) can be uniformly stabilized by \( K \) iff

\[
\inf_{k \in \mathbb{Z}^+} \left\| E K (I - G_0 K)^{-1} S_k \right\| < 1.
\]
4. Theoretical Results

4.2 Stability Analysis

✓ Stability of original plant + GSAC

➢ Plant (SISO)

\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= \pi(z, \xi) + g(z, \xi)u \\
\dot{\xi} &= \theta(z, \xi) \\
y &= z_1
\end{align*}
\]

(4.2.4)

• The relative degree is 2, and \( \xi \in R^{n-2} \) is internal dynamic variable;
• \( \pi(z, \xi), \theta(z, \xi) \) are unknown and locally Lipschitz;
• \( g(z, \xi) \) is bounded with known bounds, and \( g(z, \xi) > 0 \);
• Zero dynamics \( \dot{\xi} = \theta(0, \xi) \) is exponentially stable.
4. Theoretical Results

4.2 Stability Analysis

➤ Control objective

\[ \text{Design } \{u(kT), \ k \geq 0\} \text{ so that } \lim_{t \to \infty} y(t) \to 0. \]

➤ Characteristic model

\[ y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g_0(k)u(k) \]

\[ (f_1(k), f_2(k), g_0(k)) \in D, \]

\[ D = \left\{ (a_1, a_2, a_3) \in \mathbb{R}^3 \left| \begin{align*}
|a_1 - 2| &\leq \varepsilon_1(y, T) \\
|a_2 + 1| &\leq \varepsilon_2(y, T) \\
0 &< \varepsilon_3(y, T^2) \leq a_3 \leq \varepsilon_4(y, T^2) \end{align*} \right. \right\} \]

➤ Golden-section adaptive control

\[ u(k) = -\frac{l_1 \hat{f}_1(k)y(k) + l_2 \hat{f}_2(k)y(k-1)}{\hat{g}_0(k)} \]
4. Theoretical Results

4.2 Stability Analysis

- Stability result

**Theorem 4.2.5**

Consider the closed-loop system composed of the original plant (4.2.4) and the golden-section adaptive control, if persistent excitation (PE) condition holds, then the above closed-loop system is exponentially stable.

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Y. Wang, Studies on Characteristic Model Based Attitude Control of Hypersonic Vehicles, Ph. D. Dissertation, Beijing Institute of Control Engineering, Beijing, China, 2012.
5. Conclusions

5.1 Summary of CMAC

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5. Conclusions

5.2 Open Problems

- **Characteristic model**
  - Intelligent CM

- **Parameter estimation**
  - Estimation of characteristic parameters relevant to system states

- **Stability**
  - Closed-loop stability considering logical integral and logical derivative control
  - Stability for the original systems with relative degree greater than 2
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