Fault detection of Boolean control networks

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- potential games (Cheng, Automatica 2014).

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Genes can be modeled as binary devices that exhibit two transcriptional states: active or inactive ("expressed" or not).

Even more, the state of a gene depends on the activation status of other genes, and such an interaction can be described by means of logical function.

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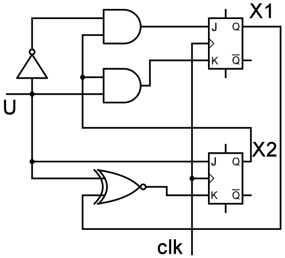
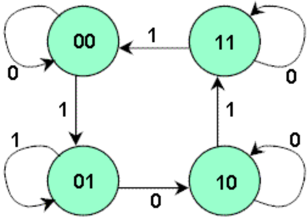


Sequential logic circuits (2)

If we denote by $X_1(t)$ and $X_2(t)$ the logical states of the upper and lower flip-flops at time t, and by $U(t)$ the Boolean input at time t, then the update of the pair (X_1, X_2) depending on the value of U follows the state diagram:



Sequential logic circuits (3)

This corresponds to the following Boolean control network:

$$\begin{aligned} X_1(t+1) &= [(X_1(t) \vee X_2(t)) \wedge \bar{U}(t)] \vee [X_1(t) \wedge \bar{X}_2(t) \wedge U(t)], \\ X_2(t+1) &= [(\bar{X}_1(t) \vee \bar{X}_2(t)) \wedge U(t)] \vee [X_1(t) \wedge X_2(t) \wedge \bar{U}(t)]. \end{aligned}$$

Assume that this circuit is part of a complex logical network whose output value Y depends at every time t from the variables $X_1(t)$ and $X_2(t)$ and from other logical state variables.

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How can we detect such faults?

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As a countermeasure, various antioxidant response elements (ARE) are activated.

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In [Sridharan et al., BMC Genomics 2012] the following Boolean control network has been proposed for the oxidative stress response:

Oxidative stress response (3)

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\begin{aligned}
X_1(t+1) &= \bar{X}_6(t) \wedge U(t), \\
X_2(t+1) &= \bar{X}_1(t) \wedge [X_2(t) \vee X_4(t)], \\
X_3(t+1) &= X_1(t) \wedge \bar{X}_6(t), \\
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X_5(t+1) &= \bar{X}_1(t), \\
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where $X_1 = ROS,\ X_2 = Keap1,\ X_3 = PKC,\ X_4 = Nrf2,\ X_5 = Bach1,\ X_6 = ARE,\ U = Stress$, and small Maf proteins are always assumed to be expressed (SMP=1).

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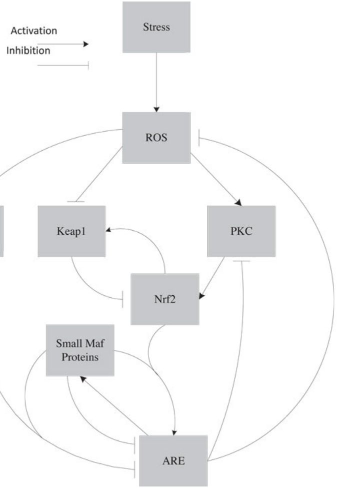

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What are the output (measures) we can take, and how can we use them to detect a fault?

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It extends to a bijective correspondence between $\mathcal{B}^n$ and $\mathcal{L}_{2^n}$ through:

$$X = \begin{bmatrix} X_1 & X_2 & \dots & X_n \end{bmatrix}^\top \longleftrightarrow \mathbf{x} = X_1^v \ltimes X_2^v \ltimes \dots \ltimes X_n^v.$$

BCNs: from logic to algebraic representations (1)

A Boolean control network (BCN) is described by the following equations

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Algebraic representation [D. Cheng]: if we represent the Boolean vectors by means of their “canonical equivalent", the BCN (1) can be described as

$$\begin{aligned} \mathbf{x}(t+1) &= L \ltimes \mathbf{u}(t) \ltimes \mathbf{x}(t), \qquad t \in \mathbb{Z}_+, \\ \mathbf{y}(t) &= H\mathbf{x}(t) \end{aligned} \tag{2}$$

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where $L \in \mathcal{L}_{2^n \times 2^{(n+m)}}$ and $H \in \mathcal{L}_{2^p \times 2^n}$. In the following

$$N := 2^n, \qquad M := 2^m, \qquad P := 2^p.$$

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Assumptions:
A1) The BCN exhibits only two possible configurations:
a non-faulty (NF) and a faulty (F) one.
A2) The fault affects only the state-update, not the output measurements.

The scenario (2)

Accordingly, we represent the non-faulty BCN as in (2) and the faulty one as

$$\begin{aligned} \mathbf{x}(t+1) &= L^{(F)} \ltimes \mathbf{u}(t) \ltimes \mathbf{x}(t), \\ \mathbf{y}(t) &= H \ltimes \mathbf{x}(t) = H\mathbf{x}(t), \qquad t \in \mathbb{Z}_+, \end{aligned} \tag{3}$$

with $L^{(F)} \in \mathcal{L}_{N \times NM}$.

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We introduce the fault signal $(\mathbf{f}(t))_{t\in\mathbb{Z}_+}$, taking values in $\mathcal{L}_2$, and assume that $\mathbf{f}(t) = \delta_2^1$ corresponds to the non-faulty BCN and $\mathbf{f}(t) = \delta_2^2$ to the faulty one.

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with $L^{(F)} \in \mathcal{L}_{N \times NM}$.

We introduce the fault signal $(\mathbf{f}(t))_{t \in \mathbb{Z}_+}$, taking values in $\mathcal{L}_2$, and assume that $\mathbf{f}(t) = \delta_2^1$ corresponds to the non-faulty BCN and $\mathbf{f}(t) = \delta_2^2$ to the faulty one. So, the overall BCN dynamics is

$$\begin{aligned} \mathbf{x}(t+1) &= \tilde{L} \ltimes \mathbf{f}(t) \ltimes \mathbf{u}(t) \ltimes \mathbf{x}(t), \\ \mathbf{y}(t) &= H \ltimes \mathbf{x}(t) = H\mathbf{x}(t), \qquad t \in \mathbb{Z}_+. \end{aligned} \tag{4}$$

where $\tilde{L} := \begin{bmatrix} L & L^{(F)} \end{bmatrix} \in \mathcal{L}_{N \times 2NM}$.

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Another assumption: A3) the BCN cannot autonomously recover from a fault: once the fault signal switches from δ_2^1 to δ_2^2, it cannot switch back to δ_2^1. So, a fault acting at time $\bar{t}$ is described by a step function

$$\mathbf{f}(t) = \begin{cases} \delta_2^1, & 0 \le t < \bar{t}; \\ \delta_2^2, & t \ge \bar{t}, \end{cases}$$

where $\bar{t} = +\infty$ in case no fault affects the BCN.

Meaningful fault (1)

Let $\mathbf{x}(t; \mathbf{x}_0, \mathbf{u}(\cdot), \mathbf{f}(\cdot))$ and $\mathbf{y}(t; \mathbf{x}_0, \mathbf{u}(\cdot), \mathbf{f}(\cdot))$ denote the state and output vectors of the BCN (4) at time t, when it starts from $\mathbf{x}(0) = \mathbf{x}_0$ and the input and fault sequences are $\mathbf{u}(\cdot)$ and $\mathbf{f}(\cdot)$, respectively.

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A fault taking place at time $\bar{t}$, for certain values of $\bar{\mathbf{x}} := \mathbf{x}(\bar{t}) \in \mathcal{L}_N$ and $\mathbf{u}(t), t \geq \bar{t}$, may not reveal itself, independently of how we choose the output measurements.

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Indeed, the state trajectory generated by the faulty BCN (3) starting from $\bar{\mathbf{x}}$ at $t = \bar{t}$, under the effect of $\mathbf{u}(\cdot)$, may coincide with the state trajectory that the non-faulty BCN (2) generates in the same conditions.

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Definition 1 Given a state $\mathbf{x}_0 \in \mathcal{L}_N$ and an input $(\mathbf{u}(t))_{t\in\mathbb{Z}_+}$, a fault sequence $(\mathbf{f}(t))_{t\in\mathbb{Z}_+}$ induces a meaningful fault for the BCN if the state trajectory $(\mathbf{x}(t;\mathbf{x}_0,\mathbf{u}(\cdot),\mathbf{f}(\cdot)))_{t\in\mathbb{Z}_+}$ is different from the state trajectory $(\mathbf{x}(t;\mathbf{x}_0,\mathbf{u}(\cdot),\,\delta_2^1))_{t\in\mathbb{Z}_+}$.

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Meaningful fault sequences are the only ones we may hope to detect, by making use of the input and output trajectories.

Possible problems

On-line fault detection: the BCN, undergoing normal working conditions, is subject to an arbitrary input, and we want to understand, based on its input-output behavior, if a fault has occurred.

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On-line fault detection: Preliminaries

We introduce the set of the admissible input/output trajectories of the non-faulty BCN:

$$\begin{aligned} \mathfrak{B}_{uy} \;:=\; & \{(\mathbf{u}(t), \mathbf{y}(t))_{t\in\mathbb{Z}_+} : (\mathbf{u}(t))_{t\in\mathbb{Z}_+} \in (\mathcal{L}_M)^{\mathbb{Z}_+}, \text{and} \\ & \exists \mathbf{x}_0 \in \mathcal{L}_N \text{ s.t. } \mathbf{y}(t) = \mathbf{y}(t; \mathbf{x}_0, \mathbf{u}(\cdot), \delta_2^1)\}. \end{aligned} \tag{5}$$

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$\mathfrak{B}_{uy}$ is the set of all pairs $(\mathbf{u}(t), \mathbf{y}(t))_{t\in\mathbb{Z}_+}$ such that $(\mathbf{y}(t))_{t\in\mathbb{Z}_+}$ is the output trajectory generated by (2) corresponding to some initial state $\mathbf{x}(0) = \mathbf{x}_0 \in \mathcal{L}_N$ and to the input $(\mathbf{u}(t))_{t\in\mathbb{Z}_+}$.

Detectability of a meaningful fault (1)

Definition 2 Given a BCN (4), a state $\mathbf{x}_0 \in \mathcal{L}_N$, an input $(\mathbf{u}(t))_{t\in\mathbb{Z}_+}$, and a (meaningful) fault sequence $(\mathbf{f}(t))_{t\in\mathbb{Z}_+}$,

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Detectability of a meaningful fault (2)

We define:

$$X^* := \{\mathbf{x}^* \in \mathcal{L}_N : \exists\ \mathbf{u}^* \in \mathcal{L}_M \ \text{s.t.} L \ltimes \mathbf{u}^* \ltimes \mathbf{x}^* \neq L^{(F)} \ltimes \mathbf{u}^* \ltimes \mathbf{x}^*\},$$

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and, for every $\mathbf{x}^* \in X^*$,

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Finally, $\mathcal{UY}^*$ denotes the set of input/output trajectories $(\mathbf{u}(t), \mathbf{y}^{(F)}(t)) \in (\mathcal{L}_M \times \mathcal{L}_P)^{\mathbb{Z}_+}$ generated by the faulty BCN (3) corresponding to some $\mathbf{x}(0) = \mathbf{x}^* \in X^*$ and to some input $(\mathbf{u}(t))_{t\in\mathbb{Z}_+}$ with $\mathbf{u}(0) \in U^*(\mathbf{x}^*)$.

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i) for every initial condition $\mathbf{x}_0 \in \mathcal{L}_N$ and every input sequence $(\mathbf{u}(t))_{t\in\mathbb{Z}_+}$, every fault that is meaningful (for the specific choice of $\mathbf{x}_0$ and $\mathbf{u}$) is also detectable (the on-line fault detection problem is solvable);

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ii) $\mathcal{U}\mathcal{Y}^* \cap \mathfrak{B}_{uy} = \emptyset$.

Graph-theoretic characterization (1)

The idea: to introduce a graph that is able to keep in parallel the state-transitions in the non-faulty BCN and in the faulty one, starting from any pair of states and corresponding to any input sequence:

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The idea: to introduce a graph that is able to keep in parallel the state-transitions in the non-faulty BCN and in the faulty one, starting from any pair of states and corresponding to any input sequence: the NF-F (non-faulty-faulty) directed graph.

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The NF-F (non-faulty-faulty) directed graph is defined as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, C_0, C_1)$, where

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Proposition 2 Given the BCN (4), let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, C_0, C_1)$ be the associated NF-F directed graph. The on-line fault detection problem is solvable if and only if

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each path in $\mathcal{G}$ endowed with the properties:

P1) it starts from some vertex pair $(\mathbf{x}_0, \mathbf{x}^*) \in \mathcal{L}_N \times X^*$;
P2) the first arc of the path (outgoing from $(\mathbf{x}_0, \mathbf{x}^*)$) is labeled by some $\mathbf{u}^* \in U^*(\mathbf{x}^*)$;

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enters the class C_0 in a finite number of steps.

Off-line fault detection: Preliminaries

Proposition 3 The following facts are equivalent:
i) the off-line fault detection problem is solvable;
ii) there exist $T \in \mathbb{Z}_+$ and an input $\hat{\mathbf{u}}(t), t \in [0, T-1]$, taking values in $\mathcal{L}_M$, such that the two sets of output trajectories

$$\begin{aligned}
\hat{\mathcal{Y}}|_{[0,T]} &:= \{(\mathbf{y}(t))|_{[0,t]} : \exists\, \mathbf{x}_0 \in \mathcal{L}_N \text{ s.t.} \\
&\qquad \mathbf{y}(t) = \mathbf{y}(t; \mathbf{x}_0, \hat{\mathbf{u}}(\cdot), \delta_2^1), \forall\, t \in [0, T]\} \\
\hat{\mathcal{Y}}^{(F)}|_{[0,T]} &:= \{(\mathbf{y}(t))|_{[0,t]} : \exists\, \mathbf{x}_0 \in \mathcal{L}_N \text{ s.t.} \\
&\qquad \mathbf{y}(t) = \mathbf{y}(t; \mathbf{x}_0, \hat{\mathbf{u}}(\cdot), \delta_2^2), \forall\, t \in [0, T]\}
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i) the off-line fault detection problem is solvable;
ii) for every vertex $v = (\delta_N^i, \delta_N^j) \in C_1$ there is path in the NF-F graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, C_0, C_1)$ from v to some vertex belonging to C_0.

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But, assuming that such conditions hold, how do we practically detect whether a fault occurred in either one of the previous set-ups?

Let $\mathbf{X}_{\tau}^{NF}$ be the set of the states that the non-faulty BCN (2) can reach at time τ, under the effect of the input sequence $\mathbf{u}(t), t \in [0, \tau - 1]$, meanwhile generating the output $\mathbf{y}(t), t \in [0, \tau]$.

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A fault has occurred if and only if there is τ such that $\mathbf{X}_\tau^{NF} = \emptyset$.

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By proceeding in this way, we obtain the sequence of sets $\mathbf{X}_\tau^{NF}$, whose cardinality decreases with τ. If for some τ we have $\mathbf{X}_\tau^{NF} = \emptyset$, a fault has occurred.

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By proceeding in this way, we obtain the sequence of sets $\mathbf{X}_\tau^{NF}$, whose cardinality decreases with τ. If for some τ we have $\mathbf{X}_\tau^{NF} = \emptyset$, a fault has occurred.

Note: at every t, the state δ_N^i is compatible with a given output sample $\mathbf{y}(t) \in \mathcal{L}_P$ if and only if $[H^\top \mathbf{y}(t)]_i = 1$.

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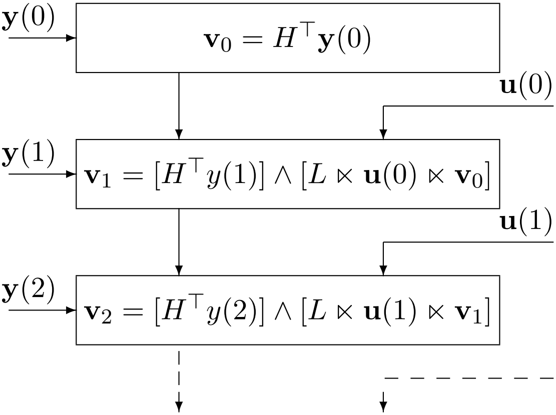

Fig. 1: Flowchart corresponding to the Algorithm.

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- For the off-line fault detection problem:

* If solvable, how can we find the most efficient (= the shortest input test sequence)?

Thanks for your attention!

Questions?