Applications of Real-time Optimization Algorithm in Modeling and Control of Automotive Powertrains

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Outline

- On going projects at Sophia
- Real-time Optimization (RHC)
  - Case Study I: Combustion Engine Control
  - Case Study II: Energy Management of HEVs
  - Case Study III: Real-time D-Optimization
- Future Challenges
Issues and Facility at SOPHIA

Projects

Industrial projects
- Advanced control technologies for the next generation of combustion engines, ’05 ~, Toyota.
- Energy management strategy with traffic information ‘11 ~, Nissan
- Optimization algorithm for ECU Calibration, ‘12 ~, Hitachi

National projects
- Engine efficiency improvement by transient control with dynamical boundary control KAKENHI(B), ’14 ~
- Model-based transient control of combustion engines, KAKENHI(B), No. 23360186), ‘11~’13
- Development of high efficiency engines, JST JKC Project, ’13~
Issues (engine)

-- CPS-based engine management
-- Real-time optimization
-- Probability-constrained optimal control
-- Statistical method for boundary detection
-- Stochastic logical control of Residual Gas Fraction
-- Knock probability control with likelihood estimation
-- Cycle-to-cycle behavior
-- Extremal seeking
-- D-optimization for calibration

Background: Transient and Hardware in Loop

-- Controller model + Real Engine
-- ECU + Engine Model
-- ECU Calibration based on Engine Model

To get engine physics right, engine-in-the-loop system is effective
Real-time operating is always in Transient operation.
Controller calibration at static modes does not satisfies real-world.
Engine-in-The-Loop System

MicroAutoBox II

Individual Fuel (P/D), SA, VVTin, VVTex, Throttle

dSPACE DS1006

AD-Phoenix (A&D)

In Motion/Car Maker
Real-time Optimization (RHC)

Model-based Optimal Control

- Model of system dynamics under control
  \[
  \frac{dx}{dt} = f(x, u), \quad x(0) = x_0
  \]

- Cost function along trajectories
  \[
  J(u) = \int_0^T R(x, u)dt
  \]

- Control Constraint \( u \in U \)

Optimization problem with dynamical constraint

\[
\min_{u \in U} J(u) \quad \text{Subject to} \quad \frac{dx}{dt} = f(x, u), \quad x(0) = x_0
\]
Pontryagin’s Maximum Principle

- **Cost function**
  \[ J(u) = \int_0^T R(x, u) dt + g(x(T)) \]

- **Constraint condition**
  \[ \dot{x} = f(x, u), \quad x(0) = x_0 \]
  \[ u \in \mathcal{U} \]

- If \( u^*(t) \in \mathcal{U} \) is the solution, then
  \[ \dot{x}^* = \mathcal{H}_p(x^*, p^*, u^*), \quad x^*(0) = x_0 \]
  \[ p^* = -\mathcal{H}_x(x^*, p^*, u^*), \quad p^*(T) = g_p(x(T)) \]
  \[ \mathcal{H}(x^*, p^*, u^*) = \max \mathcal{H}(x^*, p^*, u), \quad \forall t \leq T \]

Hamiltonian:
\[ \mathcal{H}(x, p, u) = f(x, u)p + R(x, u), \quad x, p \in \mathbb{R}^m, \quad u \in \mathcal{U} \]

Real-time Optimization: Receding Horizon control

\[ \min_u J = \int_t^{t+T} R(x(\tau), u(\tau)) d\tau \]

Subject to:
\[ \begin{cases} \frac{dx}{d\tau} = f(x, u) \\ u \in \mathcal{U}, \quad x \in X \end{cases} \]
Numerical Solver: Iterative Algorithm

Discretization of the optimality condition (N-steps $N\Delta T = T$)

$$x_{k+1}^* (t) = x_k^* (t) + f[x_k^* (t), u_k^* (t)] \Delta T, \quad x_0^* (t) = x(t)$$

$$\lambda_k^* (t) = \lambda_{k+1}^* (t) + H_x^T [x_k^* (t), x_d (t), u_c^* (t), \lambda_{k+1}^* (t), \mu_k^* (t)] \Delta T, \quad \lambda_N^* = \frac{\partial \phi(\gamma_N)}{\partial \gamma_N}$$

$$\tilde{C} [x_k^* (t), u_c^* (t)] = 0$$

$$H_{uc} [x_k^* (t), x_d (t), u_c^* (t), \lambda_{k+1}^* (t), \mu_k^* (t)] = 0 \quad (k = 0, 1, 2, \ldots, N)$$

$$H = L[x(t), x_d (t), u(t)] - ra'(t) + \lambda^T(t) f(x(t), u(t)) + u^T(t) \tilde{C}(x(t), u(t))$$

Find: \textbf{C/GMRES Algorithm, by T. Ohtsuka}

CASE Study 1: Torque Control of Gasoline Engine

Thanks to M. Kang (Candidate PhD)
Motivation: Transient Behavior

- Intake path, fuel path and mechanical dynamics cause transient phenomenon
- Transient behavior presents imbalance of mass, thermal energy, etc.
- With uncertainties such as nonlinearity, stochastic properties, etc., due to the stochastic characteristics in combustion event.

Transient Behavior in Combustion Events

- Transient response of CA50
- Transient response of in-cylinder pressure
- Engine Throttle
- In-cylinder Pressure (bar)
- Volume of Cylinder (m$^3$)
- Engine speed (RPM)
- Time (s)
- Crank angle (degree)
- Constant Load Mode: Load=76Nm, Average Cycle=3
Physics and Control-oriented Model

Mean-value Transient Model

- **Air path dynamics (MVM)**

  \[ \dot{p}_{in} = \frac{RT_{in}}{V_{in}} \{ \dot{m}_{th} - \dot{m}_{cyl} \} \]

  \[ \dot{m}_{th} = c_{d} \cdot A(\phi) \cdot \sqrt{2p} \cdot \sqrt{p_{in} - p_{out}} \]

  \[ \dot{m}_{cyl} = p_{in} \cdot \lambda \cdot \frac{V_{cyl}}{4\pi} \]

- **Rotation speed dynamics**

  \[ \dot{\omega} = \frac{1}{J} (\tau_{e} - \tau_{f} - \tau_{L}) \]

- **Torque Generation Model (Static)**

  \[ \tau_{e} = g_{1}(\omega) + g_{2}(\omega)p_{in} \]

  \[ g_{1}(\omega) = p_{1}\omega^{2} + p_{2}\omega + p_{3} \]

  \[ g_{2}(\omega) - d\omega = p_{4}\omega^{2} - d\omega + p_{6} \]

  Mean value : 0.0047 g/s

  Variance : 0.6935
Real-time Optimization: Torque Tracking

- Control objective

\[ u^*(t) = \arg \min J(u^*) = \arg \min \left\{ \int_0^T \left[ r_d(\tau) - r_e(\tau) \right]^2 + r_2 u(\tau)^2 d\tau \right\}, \]

subject to

a) Dynamic equations

\[
\begin{align*}
\dot{\rho}_m & = a \omega \cdot \dot{p}_m + b \cdot \left( 1 - \frac{\cos(\phi)}{\cos(\phi_t)} \right) \cdot \sqrt{P_m - \rho_m} \\
\dot{e} & = \tau_d - \tau_e \\
\dot{\phi} & = r_3 e + r_4 u
\end{align*}
\]

b) Torque output equation

\[ \tau_e = f(p_m, \omega) = q_1(\omega) p_m + q_2(\omega) \]

c) Necessary constraints

\[
\begin{align*}
P_{\text{min}} & \leq P_m & \leq P_{\text{max}} \\
u_{\text{min}} & \leq u & \leq u_{\text{max}}
\end{align*}
\]

Torque Tracking

- **Original RHC parameters:**
  - Predictive time: 0.3s
  - Control period: 0.01s
  - Predictive period: 0.01s
  - Predictive steps: 30

- **Improved RHC parameters:**
  - Predictive time: 1s
  - Control period: 0.01s
  - Predictive period: 0.01s
  - Predictive steps: 30
  - Integral Gain: 0.6
## Experiment (Speed Tracking)

- **MPC parameters:**
  - Predictive time: 1s
  - Control period: 0.01s
  - Predictive step: 100
  - Integral Gain: 3

- **PID control parameters:**
  - $k_p = 0.01$
  - $k_i = 0.008$
  - $k_d = 0.002$

- **Load torque:**
  - Random step change

<table>
<thead>
<tr>
<th>Tracking error</th>
<th>Mean value</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>0.21543</td>
<td>1460.8549</td>
</tr>
<tr>
<td>MPC</td>
<td>0.045444</td>
<td>1038.7143</td>
</tr>
</tbody>
</table>

## Comparison between PID control and RHC control (Enlarged figure)
Driving cycle tests

Tracking error
- Mean value: -0.0783 Nm
- Variance: 23.9715

CASE Study II:
Energy Management for HEVs

*Thanks to Dr. J. Zhang, Shunichi Hara(M.S.)*
Energy Management Problem

Goal of Energy Management: Minimizing Consumption

\[ \text{Demand Decision} \quad \tau_{\text{engine}}, \tau_{\text{motor}} \quad \Rightarrow \quad \min \int_{L} m_f(s)ds \]

Total power must satisfies the demanded power by driver.
Rule-based and Optimization

- **Rule-based approach**
  High efficiency operating point of each power device under the constraint of total power demand

- **Optimization-based Approach**
  \[ \text{power}_{\text{engine}}, \text{power}_{\text{motor}} \rightarrow \min \int L m_f(s) ds \]

  -- Off-line optimization
  -- Fuel or Equivalent Consumption
  -- Prior Route Information

Brief Review

- **Rule-based Approach**
  Based on efficiency property of the devices

- **Optimization-based Approach**
  [A] Dynamical Programming
  Off-line optimization (DP, Stochastic DP, Pontryagin’s Principle)
  Global optimal solution regarding to a priori known driving cycle

  [B] ECMS (Equivalent Consumption Minimization Strategy)
  Ref. G. Rizzoni, IFAC Workshop EPCSM, 2012, C. Musardo et al., EJC, V11, 2005

  [C] Real-time Optimization
  Look-ahead optimal control, Receding Horizon Control
  Ref. H. Borhan et al., IEEE CST, V20, 2012
Modeling

Input
- $T_e$: engine torque
- $T_m$: motor torque
- $T_g$: generator torque
- $T_b$: braking torque

Output
- $v$: Speed
- $SoC$: State of Charge
- $f$: Fuel Consumption

Dynamical Model in Nonlinear State Equation Form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = f(x_1, x_2, u, p_1, p_2) = \begin{bmatrix} \frac{9fP_m}{R_w} - M_g(\mu, \cos p_2 + \sin p_2) - \frac{1}{2} \rho A C_d x_2^2 \\ -U_{m}\omega_0(x_2) + \sqrt{U_{m}^2(x_2) - 4R_e(x_2) \left( \frac{\eta_m T_m^*}{R_w} x_1 + \eta_m^2 T_e^*(x_1, p_1) \omega_0(x, u) \right)} \end{bmatrix}$$

state Variable: $x = [v_s \ SoC]^T$
control input: $u = \alpha$
external constraint: $p = [T_{drive}, \ \theta]^T$

$T_m^* = T_{\text{max}} (\omega_m)$

$\omega_s = \alpha \omega_{\text{max}}, \ \alpha \in (0, 1]$
Real-time Optimization

\[
\min J = \int_{t}^{t+T} m_f(x_1(t'), u(t'), \dot{p}_1) dt'
\]

Subject to:

\[
\begin{align*}
\dot{x} &= f(x, u, p) \\
u_{\text{min}}(x_1) &\leq u \leq 1 \\
x_{2\text{min}} &\leq x_2 \leq x_{2\text{max}} \\
\omega_g^* &= \frac{R_c + R_e}{R_e} \omega_g \left( \frac{R_c}{R_e} \omega_{\text{emax}} \right) - \frac{R_c}{R_e} \omega_{\text{gmin}} \geq \omega_{\text{gmin}}
\end{align*}
\]

Simulations Validation with GT-Simulator provided by JSAE-SICE benchmark problem

*Parameter from Y. Yasui, JSAE-SICE Benchmark Problem 2

Standard city driving cycle

Driving cycle with part of highway driving

Thanks to IDAJ for supporting GT-SUIT
Standard city driving cycle (regular driving)

Acceleration performance is weak due to the limitation of the engine power when the emergency mode is activated.
- **Standard city driving cycle (Jam driving)**

- **Comparisons (regular driving)**

  by the proposed control scheme

  by the benchmark example control scheme
Testing Results on GT-SUIT

<table>
<thead>
<tr>
<th></th>
<th>Receding Horizon Control Algorithm</th>
<th>Example Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Final SoC[-]</td>
<td>Fuel Consumption[g]</td>
</tr>
<tr>
<td>Standard City Driving</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular</td>
<td>0.4942</td>
<td>398.95</td>
</tr>
<tr>
<td>Jam</td>
<td>0.4858</td>
<td>496.78</td>
</tr>
<tr>
<td>Driving with Highway</td>
<td>0.4709</td>
<td>567.7</td>
</tr>
</tbody>
</table>

Realtimeness Testing on dSPACE

<table>
<thead>
<tr>
<th></th>
<th>MicroAutoBox II</th>
<th>DS1104</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor</td>
<td>900 MHz</td>
<td>250MHz</td>
</tr>
<tr>
<td>$\delta\tau = 0.01$</td>
<td>real-time execution fail</td>
<td>real-time execution fail</td>
</tr>
<tr>
<td>$\delta\tau = 0.02$</td>
<td>real-time execution success; $|\mathcal{F}|_{\text{max}} = 1.12$</td>
<td>real-time execution fail</td>
</tr>
<tr>
<td>$\delta\tau = 0.05$</td>
<td>real-time execution success; $|\mathcal{F}|_{\text{max}} = 1.85$</td>
<td>real-time execution success; $|\mathcal{F}|_{\text{max}} = 1.85$</td>
</tr>
</tbody>
</table>

CASE Study III: Real-time D-Optimization

 Thanks to Mitsuru Toyoda (Mater Course)
Model Identification: Least-Squares Estimation

- **Model: linearity**
  \[ y(k) = \Phi^T(k)\theta, \quad \Phi^T(k) = [f_1(k), \ldots, f_n(k)] \]
  \[ f_i(k) = f_i(y(k), y(k-1), \ldots, y(k-m), u(k-1), \ldots, u(k-h)) \]

- **Parameter Identification**
  \[ J(\hat{\theta}) = \sum_{k=1}^{N} (y(k) - \Phi^T(k)\hat{\theta})^2 \]
  \[ \hat{\theta} = \arg\min J(\hat{\theta}) \]

- **Solution**
  \[ \hat{\theta} = (\Phi^T(N)\Phi(N))^{-1}\Phi^T(N)Y(N), \quad \Phi(N) = \begin{bmatrix} P_{y}\phi_T(1) \\ P_{y}\phi_T(2) \\ \vdots \\ P_{y}\phi_T(k) \end{bmatrix}, \quad Y(N) = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(k) \end{bmatrix}, \]

Experiment Design: D-Optimization

- **With the estimation**
  \[ y(k) = x^T(k)\hat{\theta} + \epsilon(k) \]

- **Statistical property**
  \[ \mathbb{E} [\hat{\theta}] = \mathbb{E} [(\Phi^T\Phi)^{-1}\Phi^T(y) + L_s] = \theta \]  
  \[ \text{Variance} \]
  \[ \text{Expectation} \]
  \[ \operatorname{var} [\hat{\theta}] = \mathbb{E} [(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T] = \sigma^2 (\Phi^T(N)\Phi(N))^{-1} \]

- **D-Optimality**
  \[ L_s(k) = [\epsilon(0) \epsilon(1) \ldots \epsilon(k)]^T, \quad \sigma = \operatorname{var} [\epsilon(k)] \]
  \[ \max \|\Phi^T(N)\Phi(N)\| \Rightarrow \min \{\operatorname{var} [\hat{\theta}]\} \]

*Wald(1943), Kiefer, Wolfowita (1959).*
**D-Optimization: Find optimal input for identification**

- Fisher information matrix

\[
M(k, \theta, u) = \sum_{i=1}^{N} \left\{ \left( \frac{\partial f(x_i, i, \theta)}{\partial \theta} \right)^T R^{-1}(i) \left( \frac{\partial f(x_i, i, \theta)}{\partial \theta} \right) \right\}
\]

\[
f(x, i, \theta) = x^T(k) \hat{\theta}
\]

\[
cov \left( \hat{\theta} \right) = M^{-1}
\]

**D-Optimality**

to reject covariance of estimated parameter, design optimal input signal in the sense of

\[
u(k) = \arg \max_{u(k) \in \Omega} \left( \det(M) \right), \quad k = 1, 2, \ldots, N
\]

**Example: Linear Second-order System**

- **Plant**

\[
y(t) = x_1 y(t) + x_0 y(t) + b_0 u(t) + \gamma(t)
\]

- **Discretization**

\[
y(k) = \tilde{a}_1 y(k - 1) + \tilde{a}_2 y(k - 2) + \tilde{b}_2 u(k - 2), \quad \begin{cases} \tilde{a}_1 = 1.94, \\ \tilde{a}_2 = -0.9408, \\ \tilde{b}_2 = 4E - 4 \end{cases}
\]

- **Real-time D-optimization**

\[
\max J(u) \quad \text{With constraint} \quad \begin{align*}
y(k) &= \hat{\theta}_1 y(k - 1) + \hat{\theta}_2 y(k - 2) + \hat{\theta}_3 u(k - 2) \\
|y(k)| &\leq 200, \\
|y'(k)| &\leq 200, \\
|y_{(k)}| &\leq 50
\end{align*}
\]

\[
J(u) = -\sum_{i=1}^{n_r} \left( y_i(k) \right) y_i(k) + \left[ y_i(k + \delta) \right] y_i(k), \quad x = \begin{bmatrix} x(t) \mid y(t) \end{bmatrix}
\]

\[
x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}
\]

\[
\delta = \begin{bmatrix} \delta(t) \\ \delta(t) \end{bmatrix}
\]

\[
\delta(t) = \begin{bmatrix} \delta_1(t) \\ \delta_2(t) \end{bmatrix}
\]
Concluding Remarks

- Stochastic property
- Model-free control strategy
- Real-time Optimization with constraint
- Multi-core ECU