Applications of Real-time Optimization Algorithm in Modeling and Control of Automotive Powertrains

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Outline

- On going projects at Sophia
- Real-time Optimization (RHC)
  - > Case Study I: Combustion Engine Control
  - > Case Study II: Energy Management of HEVs
  - > Case Study III: Real-time D-Optimization
- Future Challenges

Issues and Facility at SOPHIA

Projects

Industrial projects

- Advanced control technologies for the next generation of combustion engines,'05～, *Toyota*.
- Energy management strategy with traffic information '11 ～, *Nissan*
- Optimization algorithm for ECU Calibration, '12～, *Hitachi*

National projects

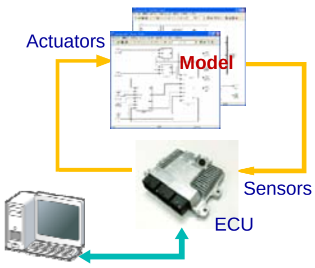
- Engine efficiency improvement by transient control with dynamical boundary control *KAKENHI(B)*, '14 ～
- Model-based transient control of combustion engines, *KAKENHI(B)*, No. 23360186), '11-' 13
- Development of high efficiency engines, *JST* JKC Project, ' 13～

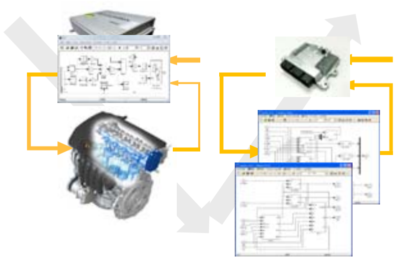
Issues (engine)

-- CPS-based engine management
-- Real-time optimization
-- Probability-constrained optimal control
-- Statistical method for boundary detection
-- Stochastic logical control of Residual Gas Fraction
-- Knock probability control with likelihood estimation
-- Cycle-to-cycle behavior
-- Extremal seeking
-- D-optimization for calibration

Background: Transient and Hardwar in Loop

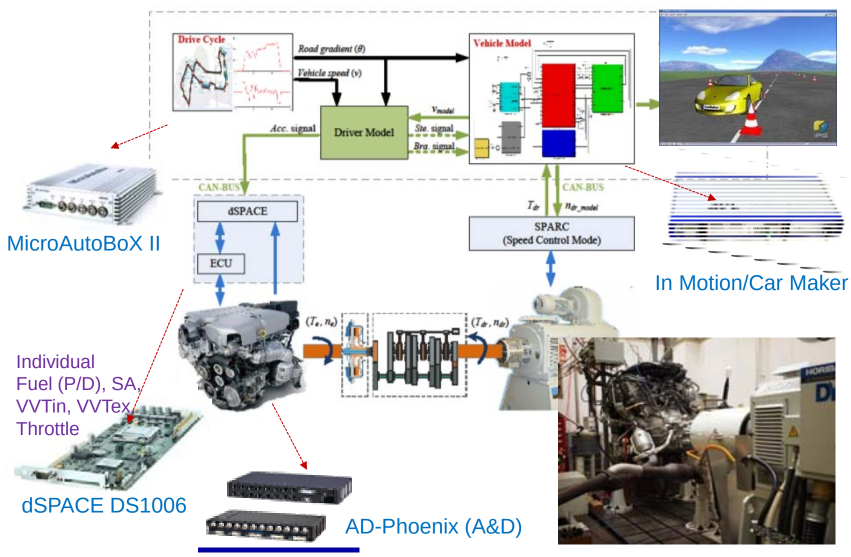
-- Controller model + Real Engine
-- ECU + Engine Model
-- ECU Calibration based on Engine Model





- To get engine physics right, engine-in-the-loop system is effective
- Real-time operating is always in Transient operation.
- Controller calibration at static modes does not satisfies real-world.

Engine-in-The-Loop System



Real-time Optimization (RHC)

Model-based Optimal Control

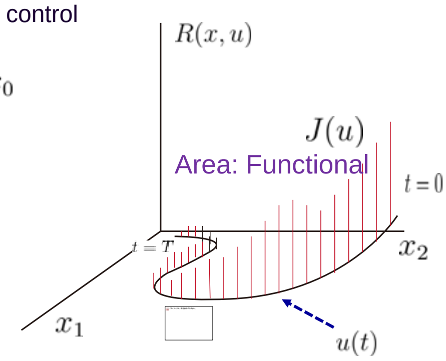
- Model of system dynamics under control

$$\frac{dx}{dt} = f(x,u), \quad x(0) = x_0$$

- Cost function along trajectories

$$J(u) = \int_0^T R(x,u)dt$$

- Control Constraint $u \in \mathcal{U}$



Optimization problem with dynamical constraint

$$\min_{u \in \mathcal{U}} J(u) \quad \text{Subject to} \quad \frac{dx}{dt} = f(x,u), \quad x(0) = x_0$$

Pontryagin’s Maximum Principle

Lev Semenovich Pontryagin
1908-1988

- Cost function

$$J(u) = \int_0^T R(x,u)dt + g(x(T))$$

- Constraint condition

$$\dot{x} = f(x,u), \quad x(0) = x_0$$
$$u \in \mathcal{U}$$

- If $u^*(t) \in \mathcal{U}$ is the solution, then

$$\dot{x}^* = \mathcal{H}_p(x^*, p^*, u^*), \quad x^*(0) = x_0$$
$$\dot{p}^* = -\mathcal{H}_x(x^*, p^*, u^*), \quad p^*(T) = g_x(x(T))$$
$$\mathcal{H}(x^*, p^*, u^*) = \max \mathcal{H}(x^*, p^*, u), \quad \forall t \le T$$

$$\text{Mapping}: \quad t \to \mathcal{H}(x^*, p^*, u^*) \quad \text{constant}$$

Hamiltonian

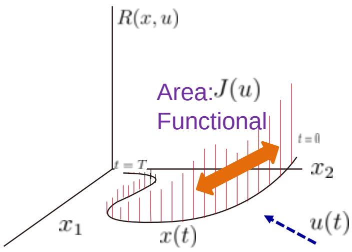
$$\mathcal{H}(x,p,u) = f(x,u)p + R(x,u), \quad x,p \in R^n, \; u \in \mathcal{U}$$

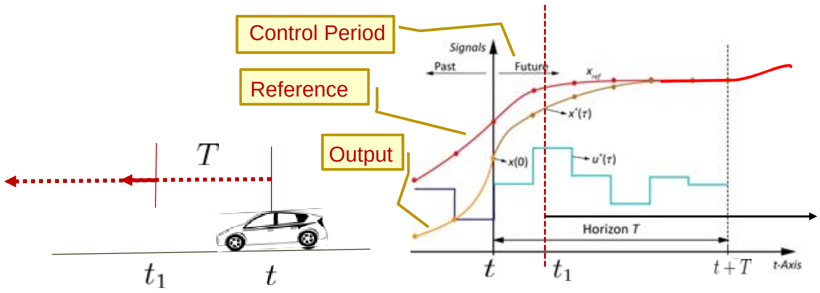
Real-time Optimization: Receding Horizon control

$$\min_u J = \int_t^{t+T} R(x(\tau), u(\tau))d\tau$$

Subject to :

$$\begin{cases} \frac{dx}{dt} = f(x,u) \\ u \in U, \quad x \in X \end{cases}$$





Numerical Solver: Iterative Algorithm

Discretization of the optimality condition (N-steps $N\Delta T = T$)

$$x_{k+1}^*(t) = x_k^*(t) + f[x_k^*(t), u_k^*(t)]\Delta T, \quad x_0^*(t) = x(t)$$

$$\lambda_k^*(t) = \lambda_{k+1}^*(t) + H_x^T\left[x_k^*(t), x_{d_k}(t), u_{c_k}^*(t), \lambda_{k+1}^*(t), \mu_k^*(t)\right]\Delta T, \lambda_N^* = \frac{\partial \phi(x_N^*)}{\partial x_N}$$

$$\bar{C}\left[x_k^*(t), u_{c_k}^*(t)\right] = 0$$

$$H_{u_c}\left[x_k^*(t), x_{d_k}(t), u_{c_k}^*(t), \lambda_{k+1}^*(t), \mu_k^*(t)\right] = 0 \qquad (k = 0, 1, 2, , N)$$

$$H = L[x(t), x_d(t), u(t)] - ru'(t) + \lambda^T(t) f(x(t), u(t)) + \mu^T(t)\bar{C}[x(t), u_c(t)]$$

Find:

C/GMRES Algorithm, by T. Ohtsuka

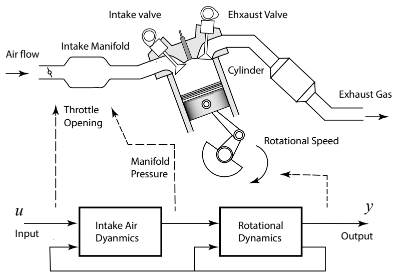
$$U(t) = [u_{c_0}$$

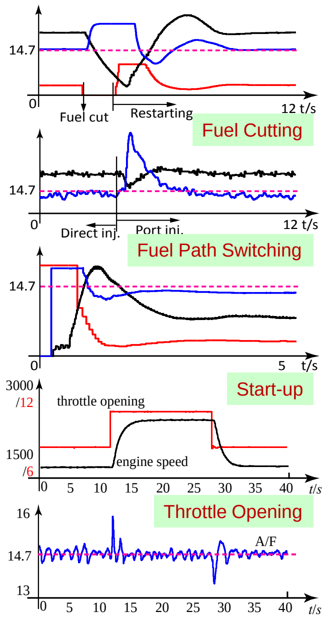
CASE Study 1:
Torque Control of Gasoline Engine

Thanks to M. Kang (Candidate PhD)

Motivation:Transient Behavior

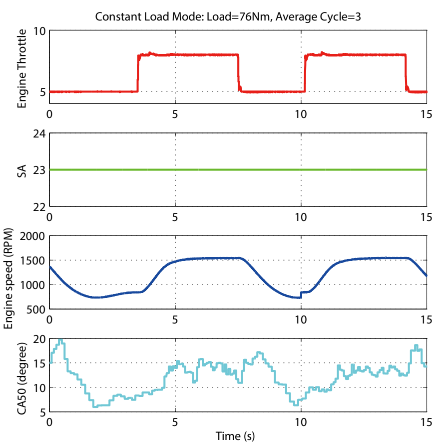
- Intake path, fuel path and mechanical dynamics cause transient phenomenon
- Transient behavior presents imbalance of mass, thermal energy, etc.
- With uncertainties such as nonlinearity, stochastic properties, etc, due to the stochastic characteristics in combustion event.



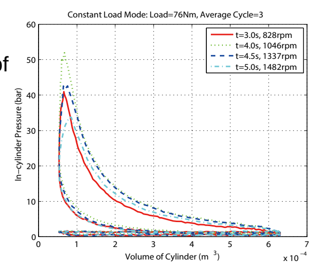


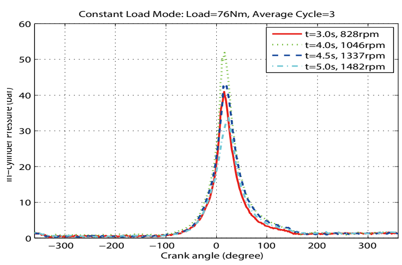
Transient Behavior in Combustion Events

Transient response of CA50

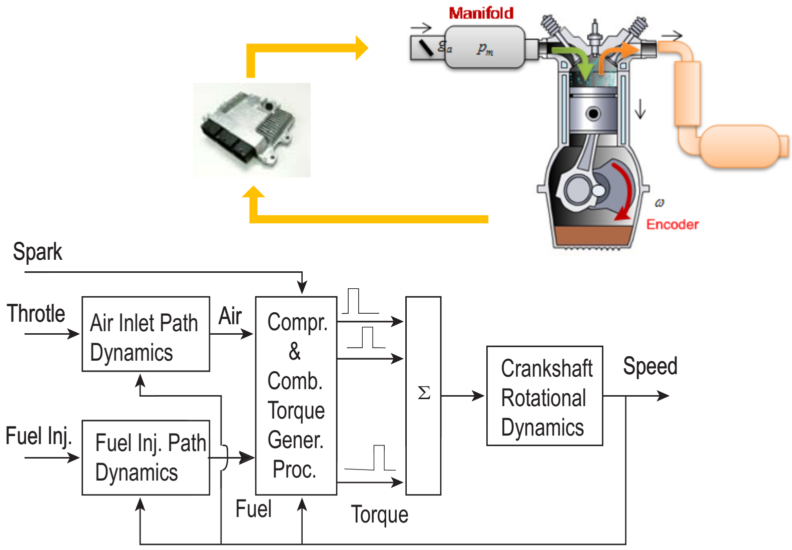


Transient response of in-cylinder pressure





Physics and Control-oriented Model



Mean-value Transient Model

- **Air path dynamics(MVM)**

$$\dot{p}_m = \frac{RT_m}{V_m}\{\dot{m}_{th} - \dot{m}_{cyl}\}$$

$$\dot{m}_{th} = c_d \cdot A(\phi) \cdot \sqrt{2\rho} \cdot \sqrt{p_{in} - p_{out}}$$
$$\dot{m}_{cyl} = p_m \cdot \lambda \cdot \frac{V_d \omega}{4\pi}$$

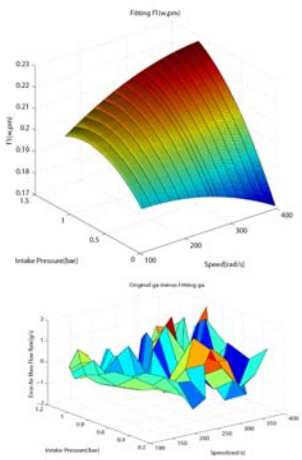
- **Rotation speed dynamics**

$$\dot{\omega} = \frac{1}{J}(\tau_e - \tau_f - \tau_L)$$

- **Torque Generation Model (Static)**

$$\tau_e = g_1(\omega) + g_2(\omega) p_m$$

$$g_1(\omega) = p_1\omega^2 + p_2\omega + p_3$$
$$g_2(\omega) - d\omega = p_4\omega^2 - d\omega + p_6$$



Mean value : 0.0047 g/s
Variance : *0.6935*

Real-time Optimization: Torque Tracking

■ Control objective

$$u^*(t) = \arg\min J(u^*) = \arg\min \left\{ \int_t^{t+T} [r_1(\tau_d(\tau) - \tau_e(\tau))^2 + r_2 u(\tau)^2] d\tau \right\},$$

subject to

a) Dynamic equations

$$\begin{cases} \dot{p}_m = a_0 \cdot \omega \cdot p_m + b_0 \cdot (1 - \frac{\cos(\phi)}{\cos(\phi_0)}) \cdot \sqrt{p_{in} - p_m} \\ \dot{e} = \tau_d - \tau_e \end{cases}$$

$$\phi = r_3 e + r_4 u$$

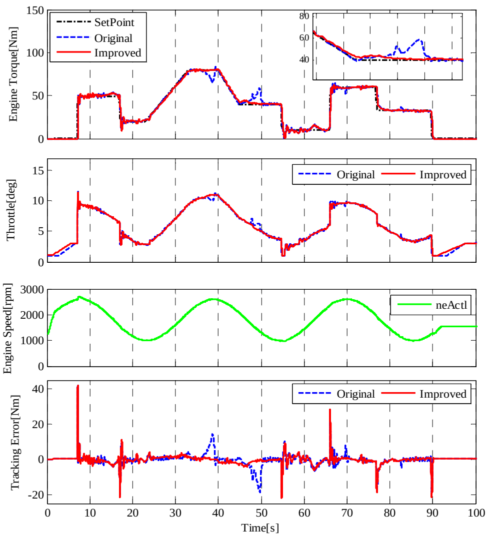
b) Torque output equation

$$\tau_e = f(p_m, \omega) = g_1(\omega) p_m + g_2(\omega)$$

c) Necessary constraints

$$\begin{cases} p_{m\,\min} \le p_m \le p_{m\,\max} \\ u_{\min} \le u \le u_{\max} \end{cases}$$

Torque Tracking



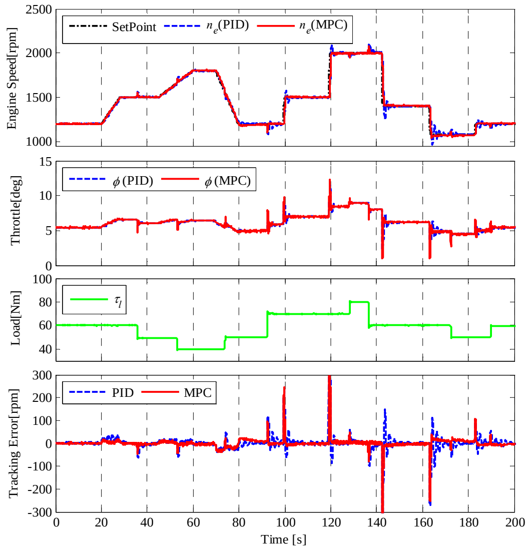
Original RHC parameters:

Predictive time: 0.3s
Control period: 0.01s
Predictive period: 0.01s
Predictive steps: 30

Improved RHC parameters:

Predictive time: 1s
Control period: 0.01s
Predictive period: 0.01s
Predictive steps: 30
Integral Gain: 0.6

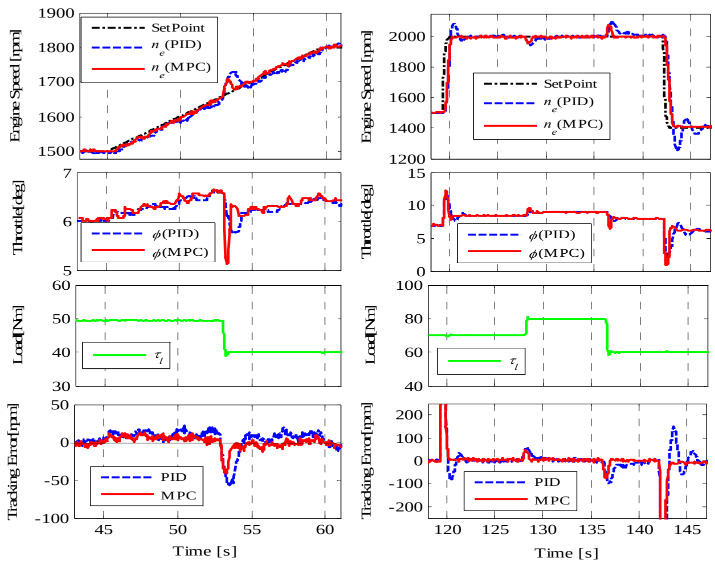
Experiment (Speed Tracking)



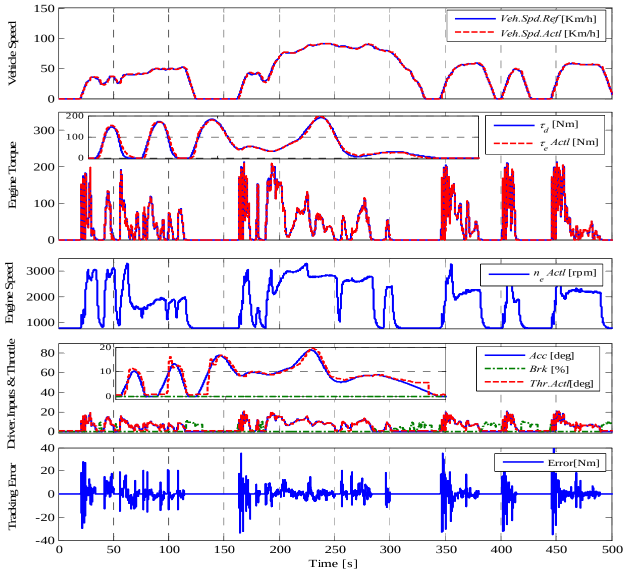
- **MPC parameters:**
  - Predictive time: 1s
  - Control period: 0.01s
  - Predictive period: 0.01s
  - Predictive steps: 100
  - Integral Gain: 3
- **PID control parameters:**
  - $k_p = 0.01;$
  - $k_i = 0.008;$
  - $k_d = 0.002.$
- **Load torque:**
  - Random step change

Tracking error	Mean value	variance
PID	0.21543	1460.6545
MPC	0.045444	1038.7143

Comparison between PID control and RHC control (Enlarged figure)



Driving cycle tests



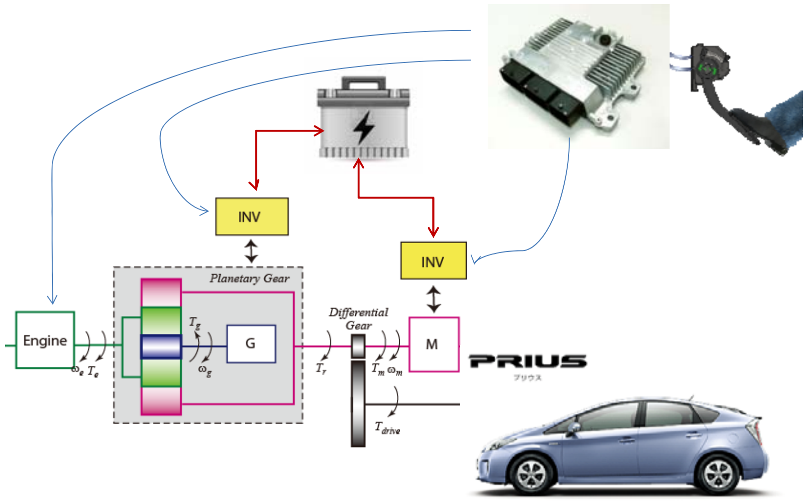
Tracking error

- Mean value: -0.0783 Nm
- Variance: 23.9715

CASE Study II: Energy Management for HEVs

Thanks to Dr. J. Zhang, Shunichi Hara(M.S.)

Energy Management Problem



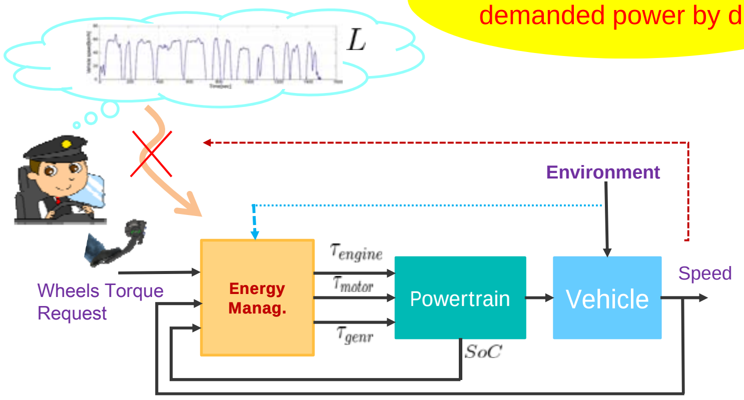
Goal of Energy Management: Minimizing Consumption

Demand Decision T_{engine}, T_{motor} ⇒ $\min \int_L m_f(s)ds$

Energy Management Problem

Off-line (Here-and-Now)

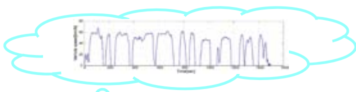
Total power must satisfies the demanded power by driver



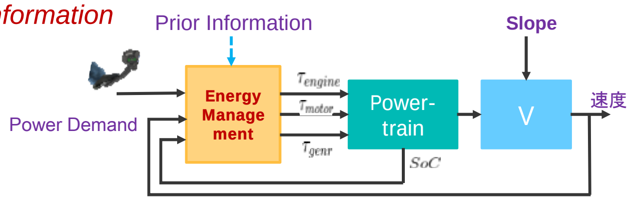
Rule-based and Optimization

- Rule-based approach
  High efficiency operating point of each power device under the constraint of total power demand
- Optimization-based Approach

$$power_{engine}, power_{motor} \Rightarrow \min \int_L m_f(s)ds$$



--Off-line optimization
--Fuel or Equivalent Consumption
--Prior Route Information



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Brief Review

- Rule-based Approach

Based on efficiency property of the devices
Ref. L. Serrao, S. Onori, G. Rizzoni, ASME JDSMC, Vol133, 2011

- Optimization-based Approach

[A] Dynamical Programming

Off-line optimization (DP, Stochastic DP, Pontriyagin's Principle)
Global optimal solution regarding to a priori known driving cycle
Ref. O. Sundstrom, et al,Oil&Gas ST, V65,2010; S. Moura, et al., IEEE CST, V19,2011

[B] ECMS（ **Equivalent Consumption Minimization Strategy** ）

Ref. G. Rizzoni, IFAC Workshop EPCSM, 2012, C.Musardo et al., EJC, V11, 2005

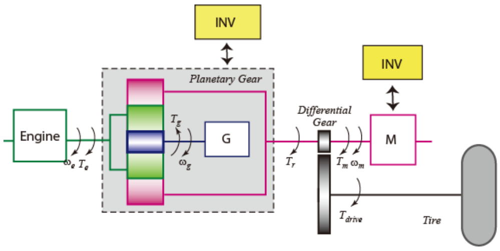
[C] Real–time Optimization

Look-ahead optimal control, Receding Horizon Control
Ref. H.Borhan et al., IEEE CST, V20,2012

Modeling

Input

- τ_e engine torque
- τ_m motor torque
- τ_g generator torque
- τ_b braking torque



Output

- v Speed
- SoC
- f Fuel Consumption

□ power balancing

$$\omega_g R_s + \omega_m R_r = \omega_e (R_r + R_s)$$

Dynamical Model in Nonlinear State Equation Form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = f(x_1, x_2, u, p_1, p_2) = \begin{bmatrix} \frac{\eta_f p_1}{R_w} - Mg(\mu_r \cos p_2 + \sin p_2) - \frac{1}{2}\rho A C_d x_1^2 \\ \frac{-U_{oc}(x_2) + \sqrt{U_{oc}^2(x_2) - 4R_b(x_2)\left(\eta_m^k T_m^*(x_1)\frac{g_f}{R_w}x_1 + \eta_g^k T_g^*(x_1, p_1)\omega_g^*(x_1, u)\right)}}{2Q_{bmax}R_b(x_2)} \end{bmatrix}$$

state Variable: $x = [v_s \ \mathrm{SoC}]^T$

control input: $u = \alpha$

external constraint: $p = [T_{drive}, \ \theta]^T$

$$T_m^* = T_{mmax}^*(\omega_m)$$

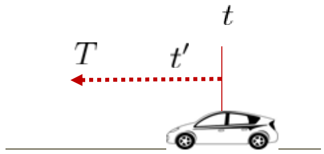
$$\omega_e^* = \alpha \omega_{emax}, \quad \alpha \in (0, 1]$$

Real-time Optimization

$$\min_u J = \int_t^{t+T} \dot{m}_f(x_1(t'), u(t'), p_1)dt'$$

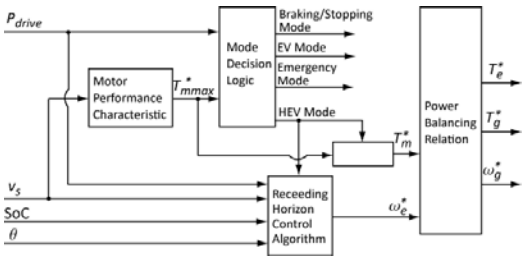
Subject to :

$$\begin{cases} \dot{x} = f(x, u, p) \\ u_{min}(x_1) \leq u \leq 1 \\ x_{2min} \leq x_2 \leq x_{2max} \end{cases}$$



$$\omega_g^* = \frac{R_r + R_s}{R_s}\alpha\omega_{emax} - \frac{R_r}{R_s}\frac{g_f}{R_{tire}}v_s \geq \omega_{gmin}$$

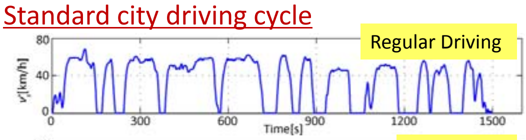
Constraint of power split by driver-demand has been embedded

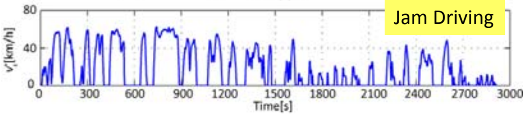


Simulations Validation with GT-Simulator provided by JSAE-SICE benchmark problem

**Parameter from Y. Yasui, JSAE-SICE Benchmark Problem 2*

Standard city driving cycle





slope θ=0;

ω_{emax}=4000[rpm],

ω_{gmin}=2000[rpm];

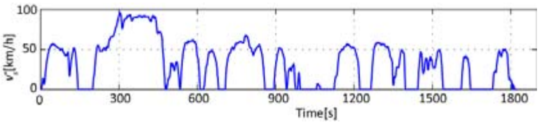
SoC(0)=0.9;

SoC^{I}_{min}=0.4, SoC^{II}_{min}=0.5

T=5[s], $\delta\tau$=0.01[s];

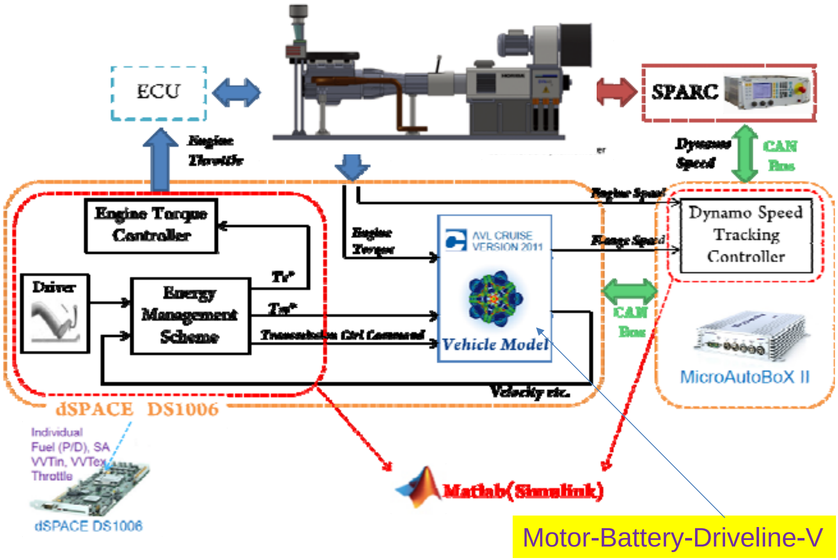
($\delta\tau$:Control Period)

Driving cycle with part of highway driving

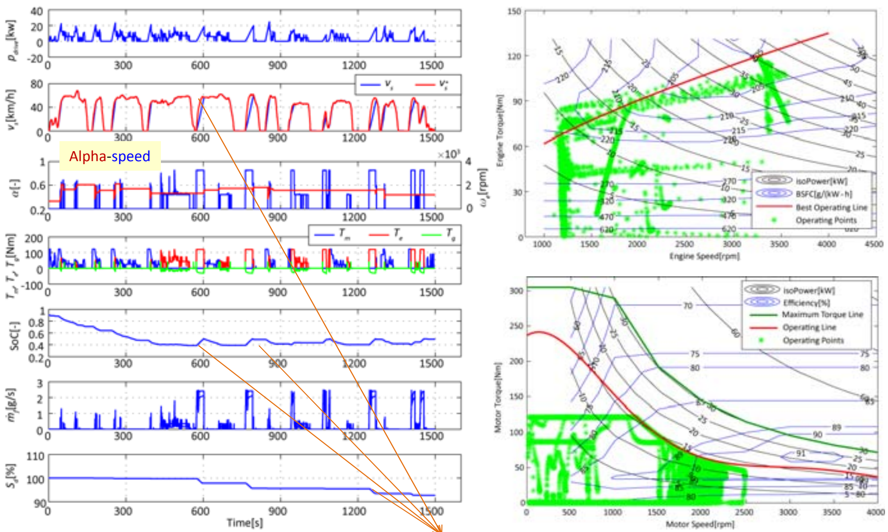


Thanks to IDAJ for supporting GT-SUIT

Engine-in-The-Loop HEV

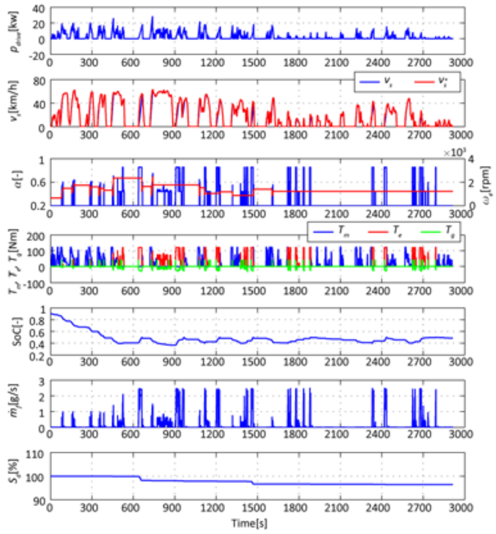


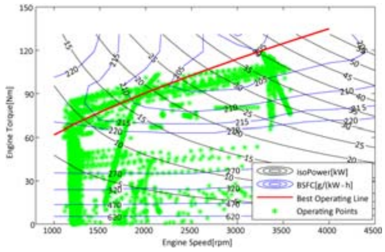
- Standard city driving cycle (regular driving)

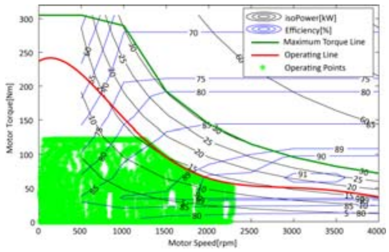


Acceleration performance is weak due to the limitation of the engine power when the emergency mode is activated

Standard city driving cycle (Jam driving)



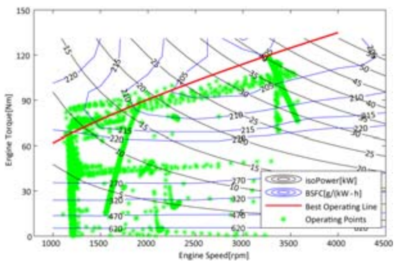


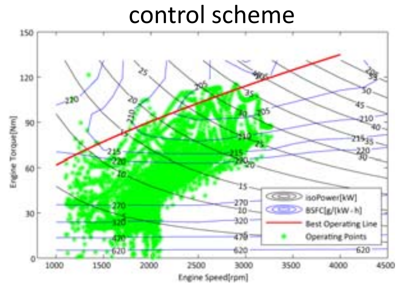


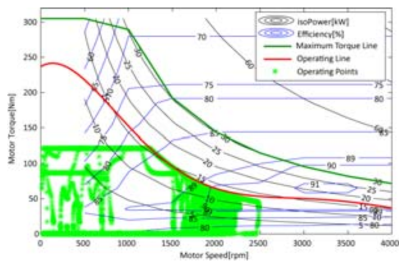
Comparisons (regular driving)

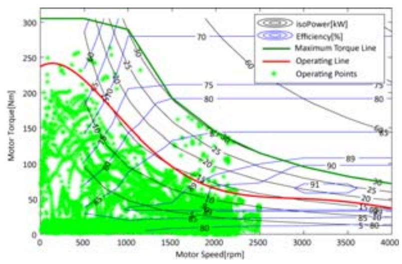
by the proposed control scheme

by the benchmark example control scheme









Testing Results on GT-SUIT

		Receding Horizon Control Algorithm			Example Algorithm		
		Final SoC[-]	Fuel Consumption[g]	S_d[%]	Final SoC[-]	Fuel Consumption[g]	S_d[%]
Standard City Driving	Regular	0.4942	398.95	92.7	0.5203	570.5	100
	Jam	0.4858	496.78	96.5	0.5066	909.6	
Driving with Highway		0.4709	567.7	91.8	0.5087	764.8	

Realtimeness Testing on dSPACE

	MicroAutoBox II	DS1104
Processor	900 MHz	250MHz
$\delta\tau = 0.01$	real-time execution fail	real-time execution fail
$\delta\tau = 0.02$	real-time execution success; $\|\mathcal{F}\|_{max} = 1.12$	real-time execution fail
$\delta\tau = 0.05$	real-time execution success; $\|\mathcal{F}\|_{max} = 1.85$	real-time execution success; $\|\mathcal{F}\|_{max} = 1.85$

CASE Study III: Real-time D-Optimization

Thanks to Mitsuru Toyoda (Mater Course)

Model Identification: Least-Squares Estimation

- **Model: linearity**

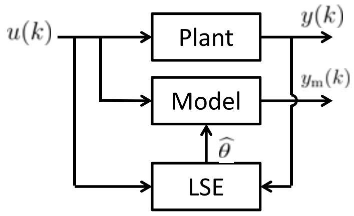
$$y(k) = \Phi^T(k)\theta, \quad \Phi^T(k) = [f_1(k), \cdots, f_n(k)]$$

$$f_i(k) = f_i(y(k), y(k-1), , y(k-m), u(k-1), \cdots, u(k-h))$$

- **Parameter Identification**

$$J\left(\hat{\theta}\right) = \sum_{k=1}^{N}\left(y(k) - \Phi^T(k)\hat{\theta}\right)^2$$

$$\hat{\theta} = \arg\min \ J\left(\hat{\theta}\right)$$



- **Solution**

$$\hat{\theta} = \left(\Phi^T(N)\Phi(N)\right)^{-1}\Phi^T(N)Y(N), \quad \Phi(N) = \begin{bmatrix} Phi^T(1) \\ \Phi^T(2) \\ \vdots \\ \Phi^T(k) \end{bmatrix}, Y(k) = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(k) \end{bmatrix},$$

Experiment Design: D-Optimization

- With the estimation

$$y(k) = x^T(k)\hat{\theta} + \epsilon(k)$$

- Statistical property

$$\mathbb{E}\left[\hat{\theta}\right] = \mathbb{E}\left[\left(\Phi^{\mathrm{T}}\Phi\right)^{-1}\Phi^{\mathrm{T}}(\Phi\theta + I_\epsilon\right] = \theta$$ Expectation

$$\mathrm{var}\left[\hat{\theta}\right] = \mathbb{E}\left[\left(\hat{\theta} - \theta\right)\left(\hat{\theta} - \theta\right)^{\mathrm{T}}\right] = \sigma^2\left(\Phi^{\mathrm{T}}(N)\Phi(N)\right)^{-1}$$ Variance

$$I_\epsilon(k) = [\epsilon(0) \ \epsilon(1) \ \cdots \ \epsilon(k)]^T \qquad \sigma = \mathrm{var}\{\epsilon(k)\}$$

- D-Optimality

$$\max\left\|\Phi^{\mathrm{T}}(N)\Phi(N)\right\| \quad \Rightarrow \quad \min\{\mathrm{var}\left[\hat{\theta}\right]\}$$

Wald(1943), Kiefer, Wolfowita (1959).

D-Optimization: Find optimal input for identification

■ Fisher information matrix

$$M(k,\theta,u)=\sum_{i=1}^{k}\left\{\left(\frac{\partial f\left(x(i),i,\theta\right)}{\partial\theta}\right)^{\mathrm{T}}R^{-1}(i)\left(\frac{\partial f\left(x(i),i,\theta\right)}{\partial\theta}\right)\right\}$$

$$f(x,i,\theta)=x^{T}(k)\hat{\theta}$$

$$\operatorname{cov}\left(\widehat{\theta}\right)=M^{-1}$$

■ D-Optimality

to reject covariance of estimated parameter, design optimal input signal in the sense of

$$u(k)=\arg\max_{u(k)\in\Omega}\left(\det(M)\right),k=1,2,\ldots,N$$

Example: Linear Second-order System

□ Plant

$$\ddot{y}(t)=a_1\dot{y}(t)+a_0y(t)+b_0u(t)+\gamma(t)$$

□ Discretization

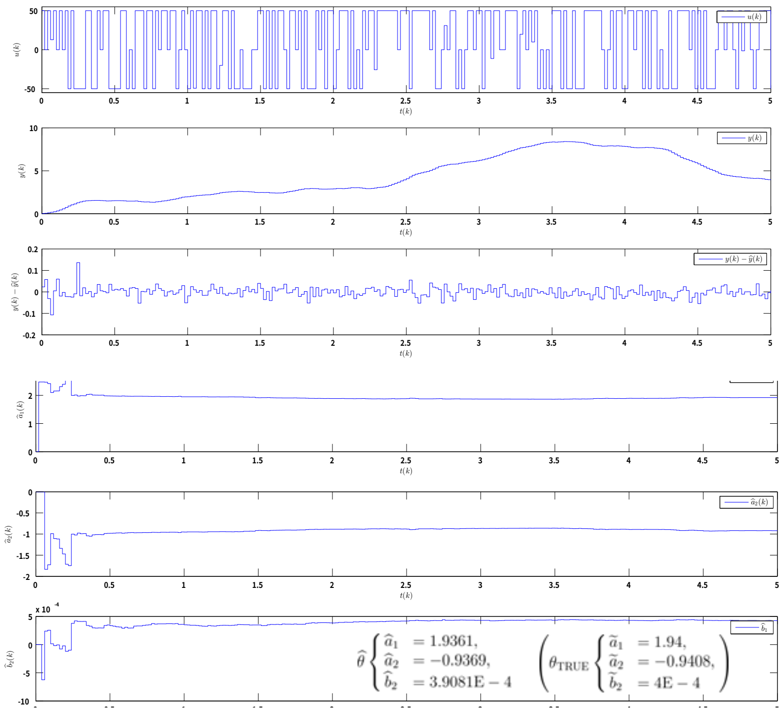
$$y(k)=\widetilde{a}_1y(k-1)+\widetilde{a}_2y(k-2)+\widetilde{b}_2u(k-2),\qquad\begin{cases}\widetilde{a}_1&=1.94,\\\widetilde{a}_2&=-0.9408,\\\widetilde{b}_2&=4\mathrm{E}-4\end{cases}$$

□ Real-time D-optinization)

$\max J(u)$ With constraint

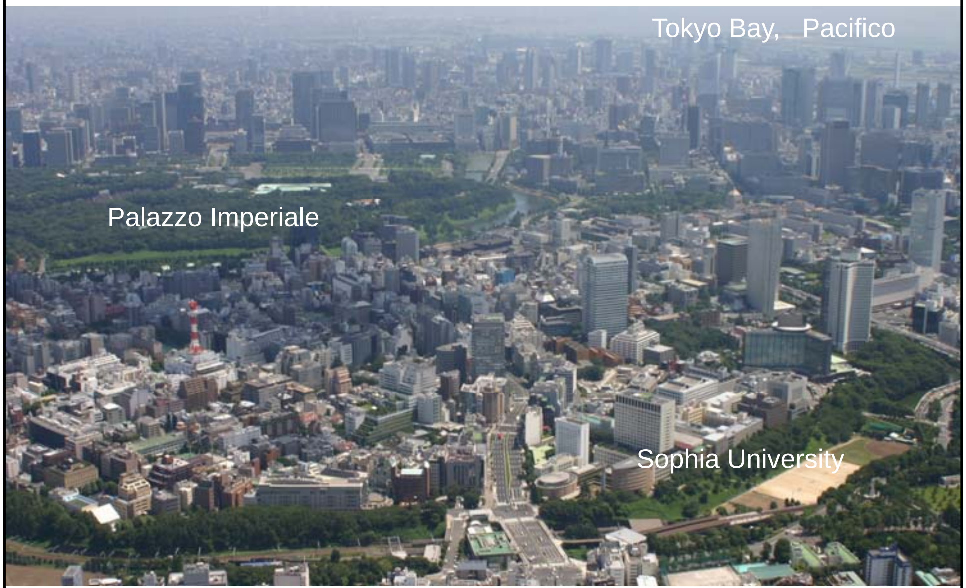
$$y(k)=\hat{\theta}_1y(k-1)+\hat{\theta}_{2k}y(k-2)+\hat{\theta}_{3k}u(k-2)$$
$$|y(k)|\le 200,$$
$$|y'(kt)|=\left|\frac{y(k+1)-y(k)}{T_s}\right|\le 200,$$
$$|u(k)|\le 50$$

$$J(u)=-\sum_{i=1}^{n_h}x_p^{\mathrm{T}}(k+i)M^{-1}(k)x_p(k+i)-\left[x_p(k+n_h)M^{-1}(k)x_p(k+n_h)\right]_{u=0},x_p=\begin{bmatrix}x_2(t)T_s+x_1(t)\\x_1(t)\\u(t)\end{bmatrix}$$



Concluding Remarks

- **Stochastic property**
- **Model-free control strategy**
- **Real-time Optimization with constraint**
- **Multi-core ECU**



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