不连续控制系统的未来发展与挑战

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报告提纲

不连续控制系统的定义
不连续控制系统的类别
不连续控制系统的性质
不连续控制系统中存在的问题
不连续控制系统的未来发展及挑战

1. 不连续控制系统的定义

Consider the following Ordinary Differential Equation (ODE) that describes a typical control system

$$\dot{x} = f(x) + b(x)u$$

where commonly, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, m < n. In here, f(x) and b(x) are smooth, and the control u(x) is smooth as well.

AND

The term f(x) + b(x)u(x) satisfies the Lipschitz condition to ensure the existence and uniqueness of the solutions.

For discontinuous control, it is of the form (simplest form)

$$u = \begin{cases} u^+ & s(x) > 0\\ u^- & s(x) < 0 \end{cases}$$

where $u^+ \neq u^-$.

2. 不连续控制系统的类别

- Discontinuous control is everywhere ...typical types are
 - Sliding mode control
 - Switching control
 - -Fuzzy control
 - Optimal control
 - State vector control
 - Impulsive control
 - Control in event-triggered systems

Vector control



Fuzzy control

-

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A typical fuzzy control system

For $\dot{x} = f(x) + b(x)u(x)$ Rule 1: IF x_1 is A_{11} and x_2 is A_{12} ... x_n is A_{1n} THEN $u = u_1$ Rule 2: IF x_1 is A_{21} and x_2 is A_{22} ... x_n is A_{2n} THEN $u = u_2$

Rule m: IF x_1 is A_{m1} and x_2 is A_{m2} x_n is A_{mn} THEN $u = u_m$

where A_{ij} is fuzzy value

Optimal control

Consider the 2nd order dynamical system

$$\dot{x}_1 = x_2,; \quad \dot{x}_2 = u; \quad |u| \le 1$$

The time optimal control is

$$u = \operatorname{sgn}[\Xi(x)]$$

$$\Xi(x) = \begin{cases} \xi(x) = x_1 + \frac{1}{2}x_2|x_2|; & \xi(x) \neq 0\\ & x_2; & \xi(x) = 0 \end{cases}$$

Impulsive control

A typical Impulsive control system

$$\dot{x} = f(x) + b(x,t)$$
 $t \in (t_{k-1}, t_k]$

$$\Delta x = cu(t_k), \ t = t_k, \ k = 0,1,2,...$$

The impulsive control occurs at $t = t_k$.

Control in event-triggered systems

A typical event triggered system

For $\dot{x} = Ax + bu$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ $u = kx(t_k)$, $t \in [t_k, t_{k+1})$

The event time $t = t_k$ is determined by an event-trigger $e(t) = x(t_k) - x(t)$. If $\gamma^2 ||e||^2 = ||x||^2$, then $x(t_k^+) = x(t)$ such that $e(t_k^+) = 0$.

Switching control



For $\dot{x} = f(x) + b(x)u(x)$

 $u = u_i(x)$ if condition i is satisfied

Sliding mode control

Consider single-input control system

$$\dot{x} = f(x) + b(x)u$$

- 1. Define a switching manifold which prescribe the desirable properties s(x)
- 2. Design a discontinuous control u(x)

$$u = \begin{cases} u^+ & s(x) > 0\\ u^- & s(x) < 0 \end{cases}$$

such that

$$\lim_{s \to 0^+} \dot{s} < 0, \ and \ \lim_{s \to 0^-} \dot{s} > 0$$

Note: This is not the same as $s\dot{s} < 0$ which is often wrongly used!

Robustness in SMC systems

• Consider a single input single output (SISO) system

$$\dot{x} = f(x) + g(x)u + \xi(x,t)$$

- where x is the state, u is the control and ζ represents uncertainties and disturbance, f and g are smooth functions.
- When an ideal sliding mode is created, we have

$$\dot{s} = 0, s = 0$$

• There exists a virtual control, called equivalent control,

$$u_{eq} = -\left(\frac{\partial s}{\partial x}g(x)\right)^{-1}\left(\frac{\partial s}{\partial x}(f(x) + \xi(x,t))\right)$$

When the matching condition is satisfied

$$\dot{x} = \left(I - g(x)\left(\frac{\partial s}{\partial x}g(x)\right)^{-1}\frac{\partial s}{\partial x}\right)f(x)$$

No uncertainty nor disturbance is involved!

3. 不连续控制系统的性质 – 可利用的优势

• The benefits of switching are enormous ... for example,

 $\ddot{y} + a_1 \dot{y} + a_{01} y = 0$ system 1 $\ddot{y} + a_1 \dot{y} + a_{01} y = 0$ system 2

 $-a_1 > 0$ the systems are both asymptotically stable.

- $-a_1 = 0$ the systems are both marginally stable.
- $-a_1 < 0$ the systems are both unstable.

稳定系统可切换成不稳定系统



不稳定系统可切换成稳定系统



不连续控制类别之间的关系

• The relationship between these various discontinuous controls is ambiguous

Consider a double integrator given by

$$\ddot{y} = u(t)$$

Consider the feedback control

u = -ky(t)

where k>0. If we take the Lyapunov function

$$V(y) = \frac{1}{2}(\dot{y}^2 + ky^2)$$

Then

$$\dot{V} = 0 \implies V = c, \ c > 0$$



For $0 < k_1 < l < k_2$

However, if we choose

$$u = \begin{cases} -k_1 y & if \ y\dot{y} < 0 \\ -k_2 y & otherwise \end{cases}$$



and a new Lyapunov function

Then
$$V(y) = \frac{1}{2}(\dot{y}^2 + y^2)$$

$$\dot{V} = y\dot{y} + \dot{y}\ddot{y} = \dot{y}(y+u) = \begin{cases} y\dot{y}(1-k_1) & \text{if } y\dot{y} < 0\\ y\dot{y}(1-k_2) & \text{if } y\dot{y} > 0 \end{cases}$$

It is a switching control!

However, if we choose a switching line

 $s = \dot{y} + cy$

The control is chosen as

$$u = -\operatorname{sgn}(s) = \begin{cases} -1 & \text{if } s > 0\\ 1 & \text{if } s < 0 \end{cases}$$

n, when $c|\dot{v}| < 1$

Then _|**y**|

 $s\dot{s} = s(c\dot{y} + \ddot{y}) = s(c\dot{y} - sgn(s)) = |s|(c|\dot{y}|-1) < 0$



When s=0 is reached,

That is,

$$\dot{y} = -cy, \ c > 0 \quad y(t) = \exp(-ct)y(0) \rightarrow 0, \ for \ t \rightarrow \infty$$



It is a local sliding mode control inside a global switching control!

Relation between discontinuous control and finite time control ...

Consider the 2nd dynamical system

$$\dot{x}_1 = x_2,; \quad \dot{x}_2 = u; \quad |u| \le 1$$

Time optimal control

$$u = \operatorname{sgn}[\Xi(x)]$$

$$\Xi(x) = \begin{cases} \xi(x) = x_1 + \frac{1}{2}x_2|x_2|; & \xi(x) \neq 0\\ & x_2; & \xi(x) = 0 \end{cases}$$

Take very large odd integer q and p, we have

$$x_1 + \frac{1}{2}x_2^{p/q} \approx x_1 + \frac{1}{2}x_2|x_2| = \xi(x).$$

Switching control can approximate finite time control!

Finite control versus linear control (asymptotical convergence)

• For the 1st order dynamics

$$\dot{x}_1 = -\alpha x_1 - \beta x_1^{q/p}$$

The Jacobian is

$$J = \frac{\partial \dot{x}_1}{\partial x_1} = -\alpha - \frac{\beta q}{p x_1^{\frac{p-q}{q}}} \qquad J \to -\infty \quad \text{when} \quad x_1 \to 0^+$$

Exact reaching time

$$t^s = \frac{p}{\alpha(p-q)} (\ln(\alpha x_1(0)^{(p-q)/p} + \beta) - \ln\beta)$$

- With $\alpha = 1$, $\beta = 1$, at $t^s = 1.0396999999990$,
- For p=3, q=1, $x_1(t^s) = 0.00000009178540$ finite control!
- For p=1, q=1, $x_1(t^s) = 0.12500519281775$ linear control!

Finite time stability versus asymptotical time stability

- Continuous-Time ODE based
- · Many new methods: e.g. Hinf,
- Fundamentally the Liptschiz condition must be satisfied
- Asymptotical stability
- The problem with asymptotical stability ...
 - -It requires 'enormous' control efforts to improve precision
 - -Lets take a very simple example:

$$\dot{x} = u, \qquad u = \begin{cases} -\lambda x \\ -\lambda x^{1/3}, \qquad \lambda > 0 \end{cases}$$



Precision and robustness properties - 1

• If there is a disturbance, $|\xi| < \varepsilon > 0$, such that

•
$$\dot{x} = u + \xi$$
, $u = \begin{cases} -\lambda x \\ -\lambda x^{1/3} \end{cases}$, $\lambda > 0$

Then

For
$$u = -\lambda x$$
, $|x(\infty)| < \frac{\varepsilon}{\lambda}$.
For $u = -\lambda x^{1/3}$, $|x(\infty)| < \left(\frac{\varepsilon}{\lambda}\right)^3$

So, for $\lambda > \varepsilon$, $\frac{\varepsilon}{\lambda} \ll \left(\frac{\varepsilon}{\lambda}\right)^3$. Less steady state error if using a non-smooth control!

Precision and robustness properties - 2

 If implemented digitally with a sample period h, and assuming we use Euler approximation, then

•
$$x(k+1) = x(k) + hu(k), \ u(k) = \begin{cases} -\lambda x(k) \\ \frac{1}{3}(k) \end{cases}, \ \lambda > 0$$

Then

For $u = -\lambda x$, $|x(\infty)| < h$.

For $u = -\lambda x^{1/3}$, $|x(\infty)| < \max\left(\left(\frac{\lambda h}{2}\right)^{3/2}, h\right)$ so to maintain the required fast convergence speed while retaining the required accuracy, $\lambda < 2h^{-1/2}$

Some typical finite-time control systems

• (Bhat & Bernstein, 2001)

$$\dot{x_1} = x_2, \qquad \dot{x}_2 = u, \qquad u = -x_2^{\frac{1}{3}} - \left(x_1 + \frac{3}{5}x_2^{5/3}\right)^{1/3}$$

• (Levant, 2001)

$$\dot{x_1} = x_2, \qquad \dot{x}_2 = u, \qquad u = -\alpha \, sgn(x_2 + |x_1|^{\frac{1}{2}} sgn(x_1))$$

• (Yu & Man, 1996; Feng, Yu & Man, 2002)

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad u = -\alpha \, sgn(s) - c\gamma \dot{x}_2^{2-\frac{1}{\gamma}},$$

 $s = x_1 + \frac{1}{c} x_2^{\frac{1}{\gamma}}, \quad 0 < \gamma < 1$

• (Hong, Xu & Huang, 2002)

 $\dot{x}_1 = x_2, \ \dot{x}_2 = -c_1 sgn x_1 |x_1|^{\alpha_1} - c_1 sgn x_2 |x_2|^{\alpha_2}$

The terminal sliding mode concept

- The dynamical performance of a SMC system is determined by the prescribed switching manifolds. The most commonly used switching manifolds are linear hyperplanes which guarantee only asymptotic stability.
- Nonlinear switching manifolds can be created purposefully to improve performance. Terminal sliding mode (TSM) is a nonlinear dynamics that provides finite time mechanism (Man & Yu 1994, Yu and Man, 1996, Feng et al, 2001).

Singularity problem ...

Consider $s = \beta x_1^{q/p}(t) + \dot{x}_1(t), \beta > 0$ Take V=0.5s². Then

$$\dot{V} = s\dot{s} = s(\ddot{x}_1 + \frac{q}{p}\beta x_1^{q/p-1}\dot{x}_1)$$

Since q / p < 1, singularity occurs before sliding mode realised! $\dot{x}_1(t) = -\beta x_1^{q/p}(t)$ $\dot{V} = s\dot{s} = s(\ddot{x}_1 + \frac{q}{p}\beta x_1^{q/p-1}\dot{x}_1) = s(\ddot{x}_1 - \frac{q}{p}\beta^2 x_1^{2q/p-1})$

During the sliding mode,

No singularity during the sliding mode if

½<q/p<1

Nonsingular TSM

In order to overcome the singularity problem during the reaching phase, a nonsingular TSM is proposed

$$s(t) = x_1(t) + \frac{1}{\beta} \dot{x}_1^{p/q}(t)$$

Take V=0.5s². Then

$$\dot{V} = s\dot{s} = s(\dot{x}_1 + \frac{p}{q}\dot{x}_1^{p/q-1}\ddot{x}_1)$$

Since p/q > 1, there is NO singularity.

3. 不连续控制系统的性质 - 从空间(状态) 的角度看

• Typical discontinuities are

- Switching between different dynamics

- Sliding mode control
- -Fuzzy control
- Time optimal control
- -Bang-bang control

– Jumping in systems states

- Impulsive control
- Control in event-triggered systems

3. 不连续控制系统的性质 – 从时间(切换频率)的角度看

Zeno Phenomena

 The Zeno phenomenon appears when the execution of the discontinuous control system is such that

$$\lim_{i \to \infty} \tau_i = \sum_{i=0}^{\infty} (\tau_{i+1} - \tau_i) = \tau_{\infty} < \infty$$

- where τ_∞ (the Zeno time) is a right accumulation point for the time constants sequence

$$(\tau_{i+1} - \tau_i) \mathop{\longrightarrow}\limits_{i \to \infty} 0$$

- The switching frequency tends to be infinite!
- Definitions
 - -Chattering Zeno: $\exists J > 0, \forall j > J, (\tau_{i+1} \tau_i) = 0$
 - -Genuinely Zeno: $\exists J > 0, \forall j > J, (\tau_{i+1} \tau_i) > 0$

Bouncing ball ...

Zeno behavior occurs when there are an infinite number of discrete transitions in a finite amount of time.



Impact of switching frequency ...

The frequency influences significantly the behaviour of dynamics – even methodology may differ significantly due to frequency range

- Low frequency many existing methodologies can be used by 'piecingtogether' various 'smooth' subsystems – in time or in state.
- Medium frequency same as Low frequency though presents challenges of using Lyapunov theory, e.g. piece-wise Lyapunov function. Various causes: deliberate medium frequency such as switched control systems; time-delay due to digitization, etc.
- High frequency tends to violate usual 'smooth' dynamics based approaches, may need to use drastically different method such as Fillipov theory!

Low/Medium frequency – impact on discontinuous control systems

Consider a second order system

$$\dot{x}_{1} = x_{2} \qquad u = \begin{cases} a^{+}x_{1} & x_{1}s > 0 \\ a^{-}x_{1} & x_{1}s < 0 \end{cases}$$

where

$$b > 0, c > 0, s = cx_1 + x_2$$

Equivalently,

$$\ddot{x} + b\dot{x} + a^{\mp}x = 0$$

Discretization for control



Discretization (emulation) design

- Controller designed based in continuous-time systems then digitized for implementation
- Discrete-time design
 - Controller design in discrete-time based on discrete-time system models





Phase plane portrait for '-' (s<0, a-<0)



Near continuous-time behavior ...

Some interesting discretization behaviors



Discrete-time behavior...

Discrete-time bifurcating behavior...

Is it true that a 'small enough' sampling period will not cause chaotic motions in control systems?



With b=-4.1, α =4.1, c=1, according to the upper bound formulae (Potts & Yu, 1991), the maximum *H* is 0.0016!

Another example

$$\dot{x}_1 = x_2 \qquad u = -\operatorname{sgn}(s)$$

$$\dot{x}_2 = u$$

where $c > 0, \ s = cx_1 + x_2$ Equivalently, $\ddot{x} \pm 1 = 0$

4 I. 2 т ı. ı 0 ı. -2 1 -4 -8 -6 -2 0 2 6 8 -4 4

Fig. 1. Continuous time system, $c_1 = 3$, initial point $(x_1, x_2) = (7, 1.2)$,

Some interesting behaviors



Fig. 4. Effects of discretization with different time steps, $c_1 = 3$, (a) h = 0.1, trajectory converges to a period-4 orbit, (b) h = 0.2, trajectory converges to a period-10 orbit, (c) h = 0.3, trajectory converges to a complex period-48 orbit, (c) h = 0.3, period-24 orbit of type (19), $(x_1, x_2) = (0.1, -1.5)$



Fig. 2. Short cycles for h = 0.1, $c_1 = 3$

n	8	x_2	x_1
2	+	-1/20	[-7/600, 1/60)
4	++	-1/10	[0, 1/100)
6	+-++	-7/60	[7/1800, 19/1800)
6	++-+	-1/12	[-7/1800, 1/360)
8	+-++	-1/8	[7/1200, 13/1200)
8	++-+-+	-3/40	[-7/1200, -1/1200)
10	+-+-++	-13/100	[7/1000, 11/1000)
10	+-++	-11/100	[7/3000, 19/3000)
10	++++-+	-9/100	[-7/3000, 1/600)
10	++-+-+	-7/100	[-7/1000, -3/1000)
12	+-+-++++	-2/15	[7/900, 1/90]
12	++-+-+-+	-1/15	[-7/900, -1/225)
14	+-+-++++	-19/140	[1/120, 47/4200)
14	+-++	-17/140	[1/200, 11/1400)
14	+-++++	-3/28	[1/600, 19/4200)
14	++++++-+	-13/140	[-1/600, 1/840)
14	++-+-++-+-+	-11/140	[-1/200, -3/1400)
14	++-+-+-+-+	-9/140	[-1/120, -23/4200)
16	+-+-++++	-11/80	[7/800, 9/800)
16	+-++++	-9/80	[7/2400, 13/2400)
16	++++-++-++-++	-7/80	[-7/2400, -1/2400)
16	++-+-+-+-++-+-++-++-++-++-++-++-++-++	-1/16	[-7/800, -1/160)
18	+-+-+++++++++++++++++++++++++++++++++	-5/36	[49/5400, 61/5400)
18	+-++++-++++++++++++++++++++++++++++++	-23/180	[7/1080, 47/5400)
18	+-++++++	-19/180	[7/5400, 19/5400)
18	++++++++-+	-17/180	[-7/5400, 1/1080)
18	++-+-++-++-+-+-+-+-+-+-+-+-+-+-+-+-+-+	-13/180	[-7/1080, -23/5400)
18	++-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+	-11/180	[-49/5400, -37/5400)

TABLE I

Short cycles for $h = 0.1, c_1 = 3$

Event triggered systems also have complex behaviors ...



Phase-plane portraits of an event triggered system (a) $\gamma = 10.2$; (b) $\gamma = 0.9$; (c) $\gamma = 0.82$.

Low frequency - Stability of switching control systems

For $\dot{x} = f(x) + b(x)u(x)$, If there exist Lyapunov function $V_p, p \in P$, two class K_{∞} functions α_1 and α_2 , and a positive number ρ_0 , such that

$$\begin{aligned} &\alpha_1(|x|) \le V_p \le \alpha_2(|x|) \\ &\frac{\partial V_p}{\partial x} (f(x) + b(x)u(x)) \le -2\rho_0 V_p(x) \\ &V_p(x) \le \mu V_q(x), \, \forall p, q \in P \end{aligned}$$

Then switched control system is globally asymptotically stable for every switching signal with average dwell time

$$\tau > \frac{\log \mu}{2\rho_0}$$

[Daniel Liberzon, Switching in Systems and Control, 2003]

High frequency – impact on discontinuous control systems

$$\dot{x} = f(x,t)$$
 is equivalent to $\dot{x} = f_0(x,t)$

Trajectory

$$f^0 = \alpha f^+ + (1 - \alpha) f^-, 0 < \alpha < 1$$



4. 不连续控制系统中存在的问题

- Gaps between methodologies dealing with discontinuous control systems
 - How to analyze and design discontinuous control systems using a unified theoretical framework which can deal with switching frequencies ranging from low to very high?
- Impact of time delay type element on dynamics
 - Lack of effective tools to study rich dynamics of 'periodical' nature of steady states (e.g. limit cycle and finite state cycle)
 - The gap between continuous-time and digitized time-delayed discontinuous control systems ...
- Multiple time scales: Inherent response speeds at different levels
- Chattering is still a problem especially for sliding mode control types

5. 不连续控制系统的未来发展及挑战

The scale and complexity of the industrial systems are growing exponentially

- -Big data environments
- Network environments
- *Ever increasing complexity*
- *Time-scale issues: Inherent response speeds at different levels*
- The impact of scale and complexity on control theories
- Human and machine interactions
- Switching in complex systems may result in much richer behaviors

即便是传统的工业系统也面临严重的挑战。。。

Dynamic (temporal/real-time) nature

Different data/model platforms

□Influenced by resources and demands

□Interactions between machines and people

□Continuous-time and discrete-time

Embedded, semi-automation

Different kinds of decisions (non-structured, semi-structured, structured)



未来智能电网是一个典型的例子。。。



Figure courtesy of Petr Stluka, Honeywell

智能电网的高度复杂性。。。

- High complexity: large-scale, nonlinear, switched, uncertain, networked, multi-agent, multi-objective, inertial versus non-inertial mechanisms
- Bidirectional electricity flows
- Intermittent availability
- Randomness (e.g. Electric Vehicle charging behaviours)
- Massive nodes (e.g. renewable energy sources, smart meters)
- Smart metering data stream mining applications
- Different time scales in operations
- Power quality monitoring and control
- Security and safety issues
- Integration within the Cyber-Physical mega-system framework
- Social behaviours (flocking, swarming, human psychological behaviors)

智能电网的信息与控制问题。。。



Control issues

- Milti-level monitoring and diagnosis – *self awareness*
- Distributed adaptive control self-organising
- Reconfigurability selfhealing
- Big Data Intelligence

The key question is how to handle the sheer size and complexity of smart grids effectively in real time?

Thank you!