西北工業大學
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Modeling and Control of Autonomous Underwater Vehicles

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Outline

1. Introduction
2. Modeling of AUVs
3. Path following Control of a single AUV
4. Formation control of multiple AUVs
5. Conclusions and Remarks

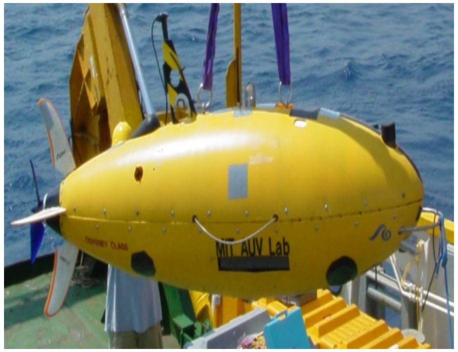
1 Introduction

Unmanned Underwater Vehicles

Remote Controlled Vehicle (ROV)

- Mothership supported, easy power supply
- Real time data transmission
- Underwater manipulation, resource exploration, archaeology, search and rescue
- Cable restricts the motion area

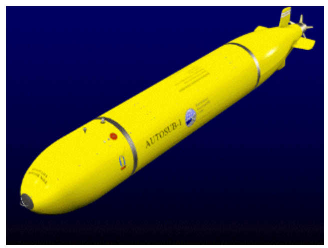
Autonomous Underwater Vehicle (AUV)



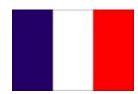
- Autonomous, without mothership supported
- Concealment and can move in large area
- Environment monitoring, mobile reconnaissance and surveillance
- Limited power supply

1 Introduction

Since 1970, nearly 200 AUVs have been developed by 20 countries towards the civil or military applications.












1 Introduction

Domestic overview

- Shenyang Institute of Automation, CAS
- Harbin Engineering University
- Shanghai Jiaotong University
- The institutes affiliated to CSIC
- Northwestern Polytechnical University, etc





1 Introduction

Related technologies

- System design
- Energy and power
- Perception and detection
- Navigation and positioning
- Control and guidance
- Planning and decision making

2 AUV Model

Kinetemics

- Coordinate frames
- Rotation between frames

Dynamics

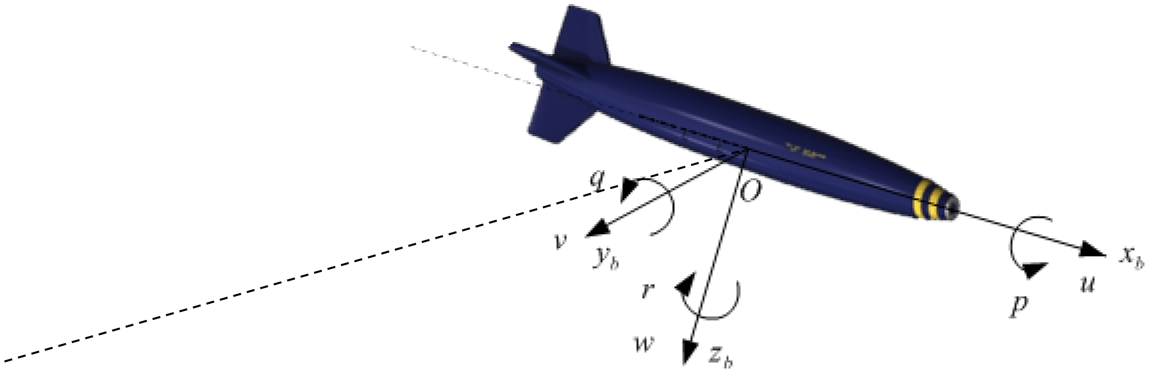
- Momentum theory

The AUV model can be divided into two parts: *kinematics*, which treats only geometrical aspects of motion, and *dynamics*, which is the analysis of the forces causing the motion.

2 AUV Model

Coordinate frame

- Inertial frame {I}
- Earth frame {E}
- NED frame {N}
- Body frame {B}



2 AUV Model

6 Degree of Freedoms

DOF	Definitions	Forces/ Moments	Linear/Angular velocities	Positions/ Euler angles
surge	Motions in the x-direction	X	u	x
sway	Motions in the y-direction	Y	v	y
heave	Motions in the z-direction	Z	w	z
roll	Rotation about the x-axis	K	p	ϕ
pitch	Rotation about the y-axis	M	q	θ
yaw	Rotation about the z-axis	N	r	ψ

Generalized velocity $\nu=\left[\nu_1^T,\nu_2^T\right]^T, \nu_1=[u,v,w]^T, \nu_2=[p,q,r]^T$

Generalized position $\eta=\left[\eta_1^T,\eta_2^T\right]^T, \eta_1=[x,y,z]^T, \eta_2=[\phi,\theta,\psi]^T$

2 AUV Model

Rotation between frames

Inertia frame to Body frame (three times rotation)

$$R_{z,\psi}=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}\quad R_{y,\theta}=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}\quad R_{x,\phi}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi & \cos\phi\end{bmatrix}$$

$$\boldsymbol{R}_n^b(\eta_2)=\boldsymbol{R}_{x,\phi}\boldsymbol{R}_{y,\theta}\boldsymbol{R}_{z,\psi}$$

$$=\begin{bmatrix}\cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta\\ \sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi & \cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta\\ \sin\phi\sin\psi+\cos\phi\cos\psi\sin\theta & \cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi & \cos\phi\cos\theta\end{bmatrix}$$

It is common to assume that the NED frame is an approximate inertial frame by neglecting the Earth rotation.

2 AUV Model

AUV Kinematics

Linear velocity transformation

$$\dot{\eta}_1 = \boldsymbol{R}_b^n(\eta_2)\nu_1$$

$$\boldsymbol{R}_b^n(\eta_2) = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix}$$

Angular velocity transformation

$$\dot{\eta}_2 = \boldsymbol{T}_\eta(\eta_2)\nu_2$$

$$\boldsymbol{T}_\eta(\eta_2) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix}$$

$$J(\eta) = \begin{bmatrix} R_b^n(\eta_2) & 0_{3\times3} \\ 0_{3\times3} & \boldsymbol{T}_\eta(\eta_2) \end{bmatrix}$$

$$\dot{\eta} = J(\eta)\nu$$

2 AUV Model

AUV dynamics

Assumptions

- Rigid body
- NED frame is inertial
- Completely immersed

Body frame: the momentum theorem

$$m\dot{v}_1 + m\dot{v}_2 \times r_G + mv_2 \times v_1 + mv_2 \times (v_2 \times r_G) = \tau_1$$

$$I_0\dot{v}_2 + v_2 \times (I_0 v_2) + mr_G \times (\dot{v}_1 + v_2 \times v_1) = \tau_2$$

m **Mass** I_0 **Moment of inertia**

$r_G = [x_G, y_G, z_G]^T$ **Position of the center of gravity in {B}**

2 AUV Model

AUV dynamics

$$M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = \tau$$

Inertia Matrix

$$\boldsymbol{M}_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m & my_G & -mx_G & 0 \\ 0 & -mz_G & my_G & I_{xx} & -I_{xy} & -I_{xz} \\ mz_G & 0 & -mx_G & -I_{yx} & I_{yy} & -I_{yz} \\ -my_G & mx_G & 0 & -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}$$

Coriolis and centripetal matrix

$$\boldsymbol{C}_{RB}(\nu) = \begin{bmatrix} m\mathrm{S}(\nu_2) & -m\mathrm{S}(\nu_2)\mathrm{S}(\boldsymbol{r}_G) \\ m\mathrm{S}(\boldsymbol{r}_G)\mathrm{S}(\nu_2) & -\mathrm{S}(\boldsymbol{I}_0\nu_2) \end{bmatrix}$$

$\mathrm{S}(\bullet)$ **Cross product operator**

2 AUV Model

AUV dynamics

- Hydrodynamics
  - Added force
  - Viscous force
- Restoring force
- Control force
- Environment force

2 AUV Model

AUV dynamics — *Added force*

$$M_A\dot{\nu}+C_A(\nu)\nu=\tau_A$$

Added inertial matrix

(Symmetric with respect to the vertical plane)

$$M_A=-\begin{bmatrix} X_{\dot u} & X_{\dot v} & X_{\dot w} & X_{\dot p} & X_{\dot q} & X_{\dot r} \\ Y_{\dot u} & Y_{\dot v} & Y_{\dot w} & Y_{\dot p} & Y_{\dot q} & Y_{\dot r} \\ Z_{\dot u} & Z_{\dot v} & Z_{\dot w} & Z_{\dot p} & Z_{\dot q} & Z_{\dot r} \\ K_{\dot u} & K_{\dot v} & K_{\dot w} & K_{\dot p} & K_{\dot q} & K_{\dot r} \\ M_{\dot u} & M_{\dot v} & M_{\dot w} & M_{\dot p} & M_{\dot q} & M_{\dot r} \\ N_{\dot u} & N_{\dot v} & N_{\dot w} & N_{\dot p} & N_{\dot q} & N_{\dot r} \end{bmatrix} \Rightarrow -\begin{bmatrix} X_{\dot u} & 0 & X_{\dot w} & 0 & X_{\dot q} & 0 \\ 0 & Y_{\dot v} & 0 & Y_{\dot p} & 0 & Y_{\dot r} \\ X_{\dot w} & 0 & Z_{\dot w} & 0 & Z_{\dot q} & 0 \\ 0 & Y_{\dot p} & 0 & K_{\dot p} & 0 & K_{\dot r} \\ X_{\dot q} & 0 & Z_{\dot q} & 0 & M_{\dot q} & 0 \\ 0 & Y_{\dot r} & 0 & K_{\dot r} & 0 & N_{\dot r} \end{bmatrix}$$

Added Coriolis matrix

$$C_A(\nu)=\begin{bmatrix} 0_{3\times3} & -S(A_{11}\nu_1+A_{12}\nu_2) \\ -S(A_{11}\nu_1+A_{12}\nu_2) & -S(A_{21}\nu_1+A_{22}\nu_2) \end{bmatrix}$$

$$\boldsymbol{C}_A(\nu)=-\boldsymbol{C}_A^{\mathrm{T}}(\nu)$$

2 AUV Model

AUV dynamics—*Viscous force*

The viscous force

$$f = -\frac{1}{2}\rho C_D(\mathrm{R}_n) D|U_0|U_0$$

6 DOF representation of quadratic drag

$$D_N(\nu)\nu = \begin{bmatrix} |\nu|^T D_1\nu \\ |\nu|^T D_2\nu \\ |\nu|^T D_3\nu \\ |\nu|^T D_4\nu \\ |\nu|^T D_5\nu \\ |\nu|^T D_6\nu \end{bmatrix} \qquad D_i \in R^{6\times 6}$$

- Symmetric configuration
- Ignore the small coupling

$$\boldsymbol{D}_N(\nu) = \mathrm{diag}\{X_u, Y_v, Z_w, K_p, M_q, N_r\} + \mathrm{diag}\{X_{u|u|}|u|, Y_{v|v|}|v|, Z_{w|w|}|w|, K_{p|p|}|p|, M_{q|q|}|q|, N_{r|r|}|r|\}$$

2 AUV Model

AUV dynamics — *Control force*

Control force

$$U = [T_1, T_2, \cdots, T_n]^T$$ ***n thrusters***

$$T = LU$$

$$L = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\ L_{21} & L_{22} & \cdots & L_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ L_{61} & L_{62} & \cdots & L_{6n} \end{bmatrix} \in R^{6\times n}$$

$$L^* = (L^T L)^{-1} L^T$$

Generated by thrusters and rudders

$$\tau_c = \begin{bmatrix} T_f & T_s & T_h & K_{\delta d}\delta_d & M_{\delta e}\delta_e & N_{\delta r}\delta_r \end{bmatrix}^{\mathrm{T}}$$

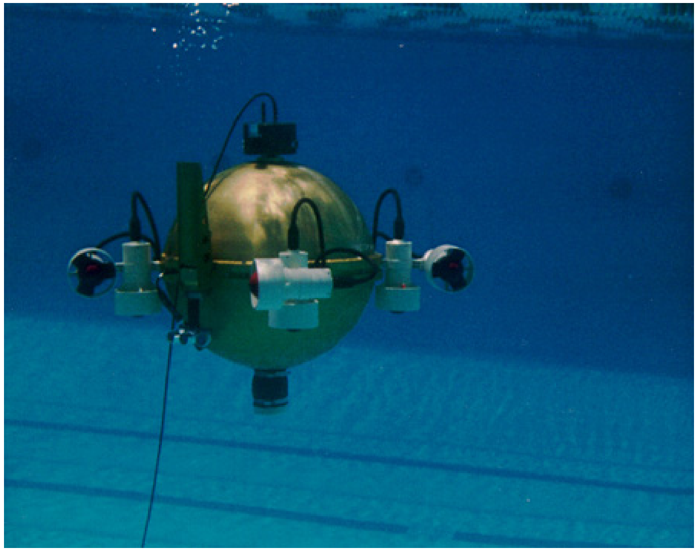
2 AUV Model

AUV dynamics — *Control force*

Fully actuated AUV

Number of Control inputs is not less than the DOFs

ODIN AUV: designed by University of Hawaii: 8 Thrusters



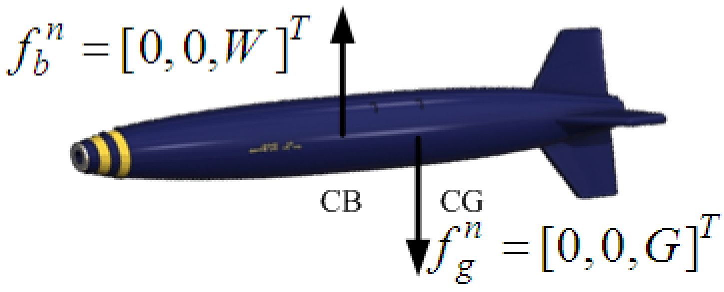
Underactuated AUV

REMUS AUV: controlled by one thruster and three rudders

2 AUV Model

AUV dynamics — *Restoring forces and moments*

$$g(\eta) = -\begin{bmatrix} F_{HS} \\ M_{HS} \end{bmatrix} = \begin{bmatrix} f_G(\eta_2) - f_B(\eta_2) \\ r_G \times f_G(\eta_2) - r_B \times f_G(\eta_2) \end{bmatrix}$$



$$f_G(\eta_2) = R_n^b(\eta_2)\begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix} \qquad f_B(\eta_2) = R_n^b(\eta_2)\begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}$$

W -- Weight

B -- Buoyancy

$r_B = [x_B, y_B, z_B]^T$

Position of the center of buoyancy in {B}

$$g(\eta) = -\begin{bmatrix} -(W-B)\sin\theta \\ (W-B)\cos\theta\sin\phi \\ (W-B)\cos\theta\cos\phi \\ -(y_G W - y_B B)\cos\theta\cos\phi - (z_G W - z_B B)\cos\theta\sin\phi \\ -(z_G W - z_B B)\sin\theta - (x_G W - x_B B)\cos\theta\cos\phi \\ -(x_G W - x_B B)\cos\theta\sin\phi - (y_G W - y_B B)\sin\theta \end{bmatrix}$$

2 AUV Model

AUV dynamics — *Environmental forces*

Currents

$$V_c = V_t + V_{lw} + V_s + V_m + V_{set-up} + V_d$$

V_t *generated by tide*

V_{lw} *generated by wind*

V_s *generated by wave*

V_m *generated by ocean current*

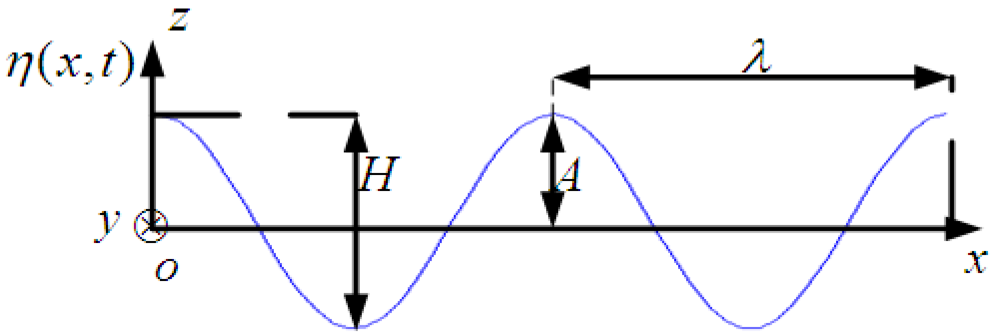
V_{set-up} *generated by the motion of earth surface*

V_d *generated by the temperature variation along with depth*

2 AUV Model

AUV dynamics—*Environmental forces*

Waves

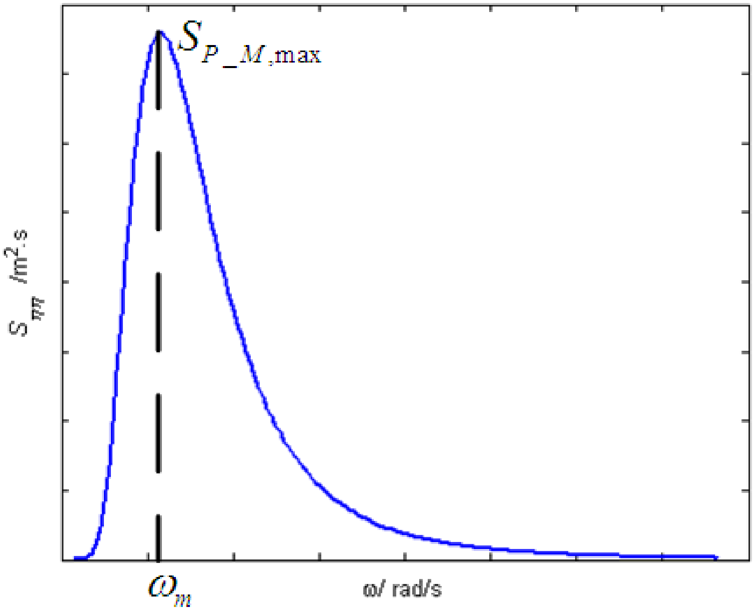


Linear wave

$$\eta(x,t) = A\cos(kx - \omega t + \varepsilon)$$

Stochastic wave

$$\eta(x,y,t) = \mathrm{Re}\iint dA(\omega,\gamma)\exp[-ik(x\cos\gamma + y\sin\gamma) + i\omega t]$$



2 AUV Model

6 DOFs model of AUV

$$M_{RB}\dot{v} + C_{RB}(v)v = -M_A\dot{v} - C_A(v) - D_N(v)v - D_L(v)v - g(\eta) + \tau_E + \tau_c$$

the form of generalized vector

dynamics

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau_E + \tau_c$$

$$M = M_{RB} + M_A$$

$$C(v) = C_{RB}(v) + C_A(v)$$

$$D(v) = D_N(v) + D_L(v)$$

kinematics

$$\dot{\eta} = J(\eta)v$$

- ***Nonlinear***
- ***Coupling between DOFs***
- ***Parameter Uncertainties***

3 Path Following of a single AUV

Outline

3.1 3D Straight-line tracking

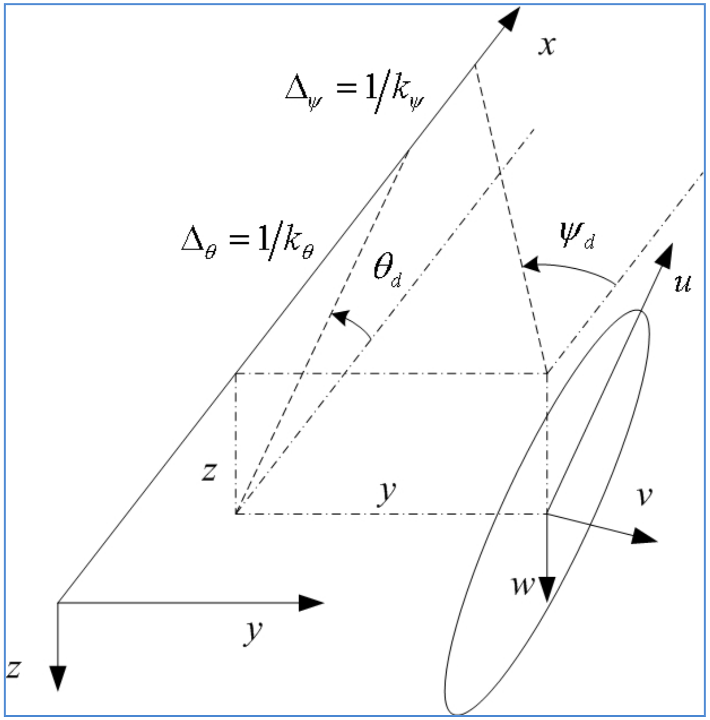
3.2 2D Path following

3.3 3D Path following

In this talk, we focus on the control of underactuated AUVs

3.1 3D Straight-line tracking

Problem Formulation



Assumptions

- The forward speed u_c is a positive constant.
- Simplify the motion model from 6 DOF to 5 DOF by ignoring the roll angle.
- Ignore the non-diagonal and quadratic damping of the inertia and damping matrix.
- The CG and CB are coincide and the net buoyancy is zero.

3.1 3D Straight-line tracking

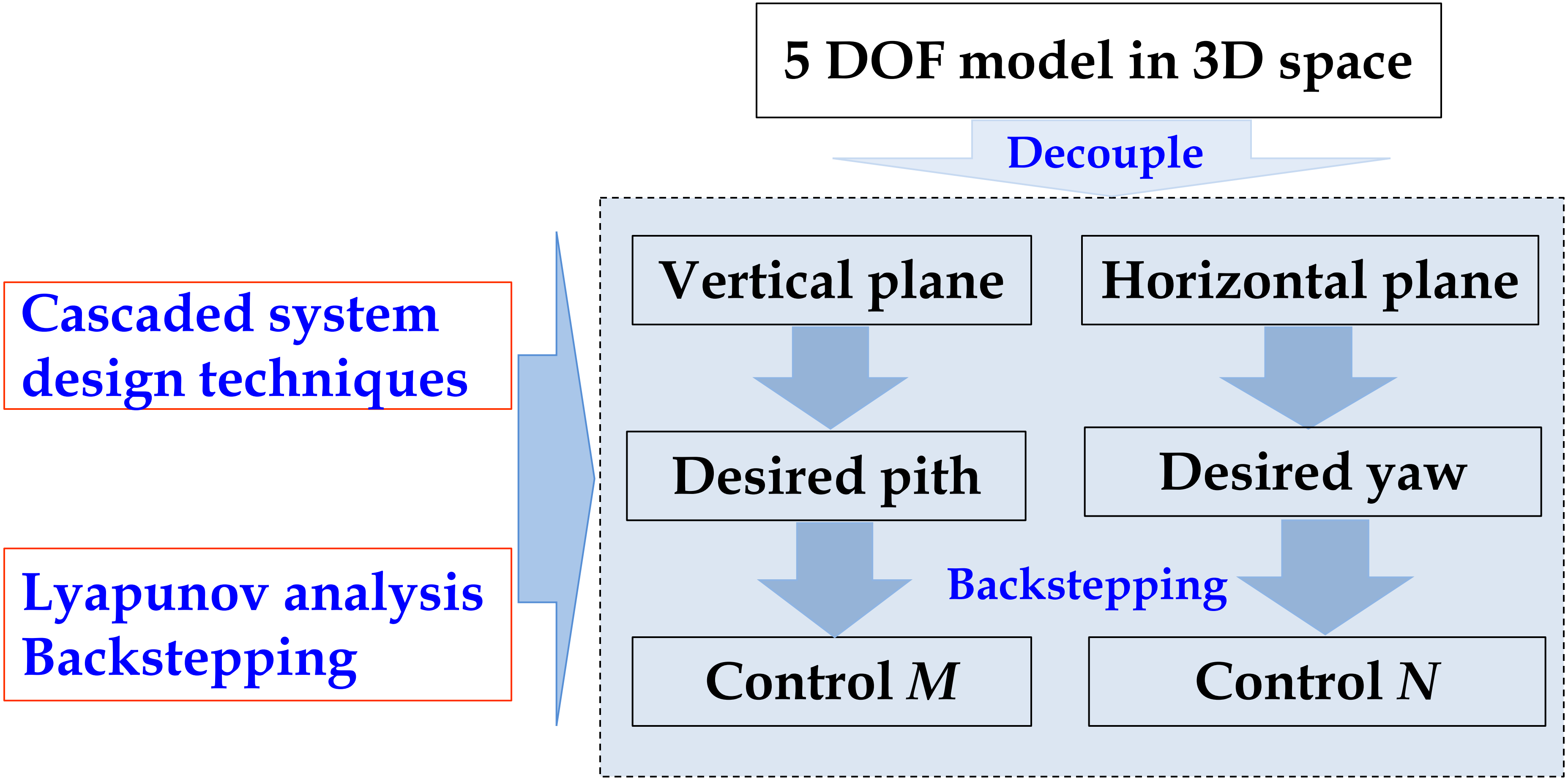
Problem Formulation

Kinematic model	Dynamics model
$\dot{x} = u_c \cos\psi \cos\theta - v \sin\psi + w \cos\psi \sin\theta$ $\dot{y} = u_c \sin\psi \cos\theta + v \cos\psi + w \sin\theta \sin\psi$ $\dot{z} = -u_c \sin\theta + w \cos\theta$ $\dot{\theta} = q$ $\dot{\psi} = \dfrac{1}{\cos\theta} r$	$\dot{v} = -\dfrac{m_{11}}{m_{22}} u_c r - \dfrac{d_{22}}{m_{22}} v$ $\dot{w} = \dfrac{m_{11}}{m_{33}} u_c q - \dfrac{d_{33}}{m_{33}} w$ $\dot{q} = -\dfrac{m_{11} - m_{33}}{m_{55}} u_c w - \dfrac{d_{55}}{m_{55}} q + \dfrac{1}{m_{55}} M$ $\dot{r} = \dfrac{m_{11} - m_{22}}{m_{66}} u_c v - \dfrac{d_{66}}{m_{66}} r + \dfrac{1}{m_{66}} N$

Control objective: Choose suitable controls M and N to make the tracking errors y and z approaches zero exponentially.

3.1 3D Straight-line tracking

Control design overview



3.1 3D Straight-line tracking

Cascaded system review

$$\Sigma \begin{cases} \Sigma_1 : \dot{\mathbf{x}}_1 = \mathbf{f}_1(t, \mathbf{x}_1) + \mathbf{g}(t, \mathbf{x}_1, \mathbf{x}_2)\mathbf{x}_2 \\ \Sigma_2 : \dot{\mathbf{x}}_2 = \mathbf{f}_2(t, \mathbf{x}_2) \end{cases}$$

$\mathbf{x}_1 \in \mathbb{R}^{n_1}$, $\mathbf{x}_2 \in \mathbb{R}^{n_2}$

$\mathbf{f}_1(t, \mathbf{x}_1)$: **continuously differentiable**

$\mathbf{f}_2(t, \mathbf{x}_2)$, $\mathbf{g}(t, \mathbf{x}_1, \mathbf{x}_2)$: **continuous and locally Lipschitz**

Assumption 1: $V(t, \mathbf{x}_1) : \mathbb{R}_{\geq 0} \times \mathbb{R}^{n_1} \to \mathbb{R}$ **is continuous and differentiable.**

Assumption 2: $\mathbf{g}(t, \mathbf{x}_1, \mathbf{x}_2)$ **satisfies** $\left\| \mathbf{g}(t, \mathbf{x}_1, \mathbf{x}_2) \right\| \leq \theta_1(\|\mathbf{x}_2\|) + \theta_2(\|\mathbf{x}_2\|)\|\mathbf{x}_1\|$.

Assumption 3: **Systems** Σ_2 **satisfies** $\int_{t_0}^{\infty} \left\| \mathbf{x}_2(t, t_0, \mathbf{x}_2(t_0)) \right\| \mathrm{d}t \leq \phi\left(\left\| \mathbf{x}_2(t_0) \right\|\right)$.

Theorem: With the Assumptions 1-3, the cascade system $\{\Sigma_1, \Sigma_2\}$ is globally uniformly asymptotically stable (GUAS).

E. Panteley, A. Loría. On Global Uniform Asymptotic Stability of Nonlinear Time-Varing Systems in Cascade. System and Control Letters, 1998, 33(2), 131-138.

3.1 3D Straight-line tracking

Definitions

G$\mathcal{K}$ES: globally $\mathcal{K}$-exponential stable
GES: globally exponential stable

Consider system $\dot{x} = f(t,x) \quad f(t,0) = 0, \forall t > 0$, where $f(t,x)$ is piecewise continuous in t and locally Lipshitz in x.

The system is GES of there exists k>0 and g>0 such that for any initial state $\| x(t) \| \leq \| x(t_0) \| k \exp[-\gamma(t-t_0)]$.

A slightly weaker notion of the GES is G$\mathcal{K}$ES if there exists k>0 and a class of $\mathcal{K}$ function $k(\cdot)$ such that g>0 such that $\| x(t) \| \leq k(\| x(t_0) \|) \exp[-\gamma(t-t_0)]$.

3.1 3D Straight-line tracking

Control design

Decoupled motion model

Horizontal plane

$$\mathbf{x}_H = [y \quad v \quad \psi \quad r]^{\mathrm{T}}$$

$$\Sigma_H : \begin{bmatrix} \dot{y} \\ \dot{v} \\ \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} u_c \sin\psi\cos\theta + v\cos\psi + w\sin\theta\sin\psi \\ -\dfrac{m_{11}}{m_{22}} u_c r - \dfrac{d_{22}}{m_{22}} v \\ \dfrac{1}{\cos\theta} r \\ \dfrac{m_{11} - m_{22}}{m_{66}} u_c v - \dfrac{d_{66}}{m_{66}} r + \dfrac{1}{m_{66}} N \end{bmatrix}$$

Vertical plane

$$\mathbf{x}_V = [z \quad w \quad \theta \quad q]^{\mathrm{T}}$$

$$\Sigma_V : \begin{bmatrix} \dot{z} \\ \dot{w} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -u_c \sin\theta + w\cos\theta \\ \dfrac{m_{11}}{m_{33}} u_c q - \dfrac{d_{33}}{m_{33}} w \\ q \\ -\dfrac{m_{11} - m_{33}}{m_{55}} u_c w - \dfrac{d_{55}}{m_{55}} q + \dfrac{1}{m_{55}} M \end{bmatrix}$$

3.1 3D Straight-line tracking

Control Design – vertical plane

Choose the desired pitch angle $\theta_d = \arctan(k_\theta z), \quad k_\theta > 0$

$$q_d \triangleq \dot{\theta}_d = \frac{k_\theta}{1+k_\theta^2 z^2}\left(-u_c \sin\theta + w\cos\theta\right)$$

Define the error variables $\theta_e = \theta - \theta_d \qquad q_e = q - q_d$

Introduce two equations

$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha + \beta\int_0^1 \cos(\alpha+s\beta)\mathrm{ds} \\ &= \sin\alpha + \beta\eta_c(\alpha,\beta)\end{aligned} \qquad \begin{aligned}\cos(\alpha+\beta) &= \cos\alpha - \beta\int_0^1 \sin(\alpha+s\beta)\mathrm{ds} \\ &= \cos\alpha + \beta\eta_s(\alpha,\beta)\end{aligned}$$

where $\eta_c(\alpha,\beta) = \int_0^1 \cos(\alpha+s\beta)\mathrm{ds}, \;\; \eta_s(\alpha,\beta) = -\int_0^1 \sin(\alpha+s\beta)\mathrm{ds}.$

$|\eta_c(\alpha,\beta)| < 1, \; |\eta_s(\alpha,\beta)| < 1.$

Then, we have $\cos(\beta) = 1 + \beta\eta_s(0,\beta), \;\; \sin(\beta) = \beta\eta_c(0,\beta).$

3.1 3D Straight-line tracking

Control Design – vertical plane

Cascade form of vertical plane tracking model

$$\begin{cases} \Sigma_{V1}: \begin{bmatrix} \dot{z} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -u_c \sin\theta_d + w\cos\theta_d \\ -\dfrac{d_{33}}{m_{33}} w + \dfrac{m_{11}u_c}{m_{33}} \dfrac{k_\theta}{1+k_\theta^2 z^2} \left(-u_c \sin\theta_d + w\cos\theta_d\right) \end{bmatrix} + \begin{bmatrix} g_{z,\theta_e} & 0 \\ g_{w,\theta_e} & g_{w,q_e} \end{bmatrix} \begin{bmatrix} \theta_e \\ q_e \end{bmatrix} \\ \Sigma_{V2}: \begin{bmatrix} \dot{\theta}_e \\ \dot{q}_e \end{bmatrix} = \begin{bmatrix} q_e \\ -\dfrac{m_{11}-m_{33}}{m_{55}} u_c w - \dfrac{d_{55}}{m_{55}} q + \dfrac{1}{m_{55}} M - \ddot{\theta}_d \end{bmatrix} \end{cases}$$

nominal system

where $g_{z,\theta_e} = -u_c \eta_c(\theta_d,\theta_e) + w\eta_s(\theta_d,\theta_e)$, $g_{w,\theta_e} = \dfrac{m_{11}u_c}{m_{33}} \dfrac{k_\theta}{1+k_\theta^2 z^2} g_{z,\theta_e}$, $g_{w,q_e} = \dfrac{m_{11}}{m_{33}} u_c$.

Theorem: If the coefficient k_θ satisfies $k_\theta < \dfrac{d_{33}}{3m_{11}u_c}$, then the nominal system is GKES.

3.1 3D Straight-line tracking

Control Design – vertical plane

Proof:

Lyapunov function candidate $V_z = \frac{\lambda_z}{2} z^2 + \frac{1}{2} w^2, \quad \lambda_z > \frac{m_{11}}{m_{33}} u_c^2 k_\theta^2$

$$\dot{V}_z \le -\frac{u_c k_\theta \lambda_z}{2} \frac{z^2}{\sqrt{1+(k_\theta z)^2}} - \frac{d_{33}}{2m_{33}} w^2$$

$$-\frac{u_c k_\theta \lambda_z}{2} \left(\frac{z}{\sqrt{1+(k_\theta z)^2}} \right)^2 - \left(\frac{d_{33}}{2m_{33}} - \frac{m_{11}}{m_{33}} u_c k_\theta \right) w^2 + \lambda_z |w| \frac{|z|}{\sqrt{1+(k_\theta z)^2}}$$

If $k_\theta < \frac{d_{33}}{2m_{11} u_c}, \; u_c k_\theta \left(\frac{d_{33}}{m_{33}} - \frac{2m_{11}}{m_{33}} u_c k_\theta \right) > \lambda_z,$ $\quad \dot{V}_z \le -\frac{u_c k_\theta \lambda_z}{2\sqrt{1+(k_\theta z)}} z^2 - \frac{d_{33}}{2m_{33}} w^2$

Then $\dot{V}_z \le -cV_z, \quad c = \min\left\{ \frac{u_c k_\theta \lambda_z}{\sqrt{1+(k_\theta R)^2}}, \frac{d_{33}}{m_{33}} \right\}.$

3.1 3D Straight-line tracking

Control Design – vertical plane

Control design for System Σ_{V2}

Choose $M = m_{55}\ddot{\theta}_d - k_{M1}m_{55}\theta_e - k_{M2}m_{55}q_e + (m_{11} - m_{33})u_c w + d_{55}q, \quad k_{M1}, k_{M2} > 0$

Then we have $\begin{bmatrix} \dot{\theta}_e \\ \dot{q}_e \end{bmatrix} = \begin{bmatrix} q_e \\ -k_{M1}\theta_e - k_{M2}q_e \end{bmatrix}$

GES

Theorem: **with the control designed, the vertical subsystem is $\mathcal{K}$-exponential stable.**

Proof:

(1) The nominal system is GKES, and the Assumption 1 holds.

(2) The item

$$\left\| \begin{bmatrix} g_{z,\theta_e} & 0 \\ g_{w,\theta_e} & g_{w,q_e} \end{bmatrix} \right\| \le u_c \sqrt{1 + \left(\frac{m_{11}}{m_{33}} u_c k_\theta \right)^2 + \left(\frac{m_{11}}{m_{33}} \right)^2} + \left\| \mathbf{x}_{V1} \right\| \sqrt{1 + \left(\frac{m_{11}}{m_{33}} u_c k_\theta \right)^2}$$

Assumption 2 holds.

3.1 3D Straight-line tracking

Control Design – horizontal plane

$$\left\{\begin{array}{l}
\Sigma_{H1}:\begin{bmatrix}\dot{y}\\ \dot{v}\end{bmatrix}=\underbrace{\begin{bmatrix}u_c\sin\psi_d+v\cos\psi_d\\ \dfrac{m_{11}}{m_{22}}\dfrac{u_c k_\psi}{1+(k_\psi y)^2}(u_c\sin\psi_d+v\cos\psi_d)-\dfrac{d_{22}}{m_{22}}v\end{bmatrix}}_{\Sigma_{H1,n}}+\begin{bmatrix}g_{y,\psi_e} & 0\\ g_{v,\psi_e} & g_{v,r_e}\end{bmatrix}\begin{bmatrix}\psi_e\\ r_e\end{bmatrix}\\
\qquad+\begin{bmatrix}0 & 0 & g_{y,\theta} & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}z\\ w\\ \theta\\ q\end{bmatrix}\\
\Sigma_{H2}:\begin{bmatrix}\dot{\psi}_e\\ \dot{r}_e\end{bmatrix}=\underbrace{\begin{bmatrix}r_e\\ \left(\dfrac{m_{11}-m_{22}}{m_{66}}u_c v-\dfrac{d_{66}}{m_{66}}r+\dfrac{1}{m_{66}}N-\dot{r}_d\right)\Big/\cos\theta\end{bmatrix}}_{\Sigma_{H2,n}}+\begin{bmatrix}0 & 0 & g_{\psi_e,\theta} & 0\\ 0 & 0 & 0 & g_{r_e,q}\end{bmatrix}\begin{bmatrix}z\\ w\\ \theta\\ q\end{bmatrix}
\end{array}\right.$$

3.1 3D Straight-line tracking

Control Design – horizontal plane

Step 1: Design control *N* for $\sum_{H2,n}$

Lyapunov function candidate $V_\psi = \frac{1}{2}\psi_e^2 + \frac{m_{66}}{2}\left(r_e - \alpha_{r_e}\right)^2, \ \alpha_{r_e} = -k_{N1}\psi_e$

$$N = \cos\theta\left(-k_{N2}\left(r_e - \alpha_{r_e}\right) + \left(-\left(m_{11} - m_{22}\right)u_c v + d_{66} r + m_{66}\dot{r}_d\right)\right) - m_{66} k_{N1} r_e - \psi_e$$

$$\dot{V}_\psi = -k_{N1}\psi_e^2 - k_{N2}\left(r_e - \alpha_{r_e}\right)^2$$

⇒ $\left\{\sum_{H2}, \sum_V\right\}$ **global $\mathcal{K}$-exponential stable**

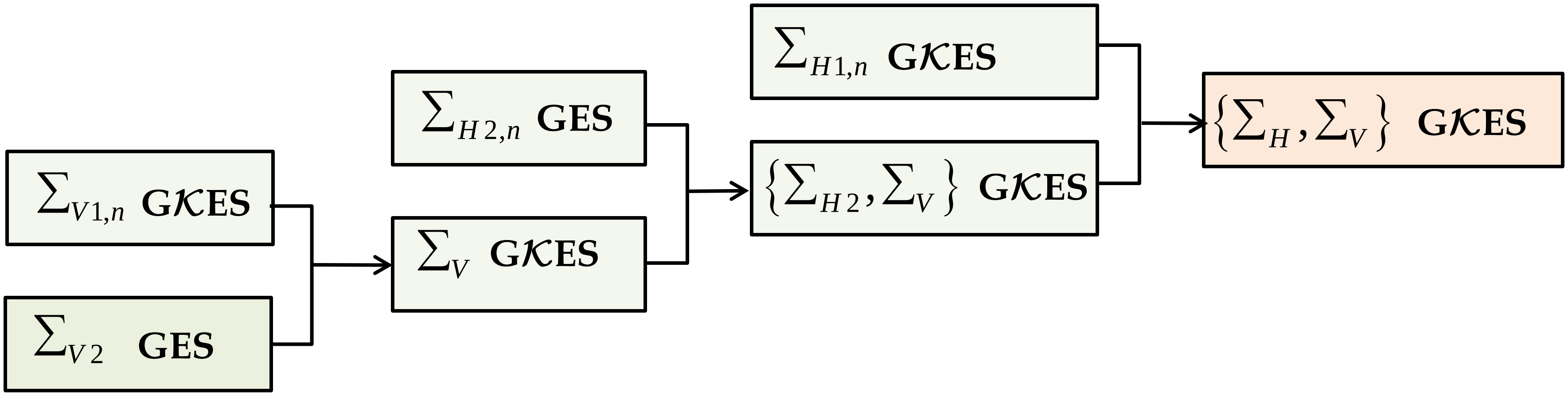
As the same techniques used as the vertical control design, we ignore the proof here.

Step 2: If $k_\psi < \frac{d_{22}}{3m_{11}u_c}$, $\sum_{H1,n}$ **is global $\mathcal{K}$-exponential stable.**

Theorem: **with the control *M* and *N* designed, system** $\left\{\sum_H, \sum_V\right\}$ **is $\mathcal{K}$-exponential stable.**

3.1 3D Straight-line tracking

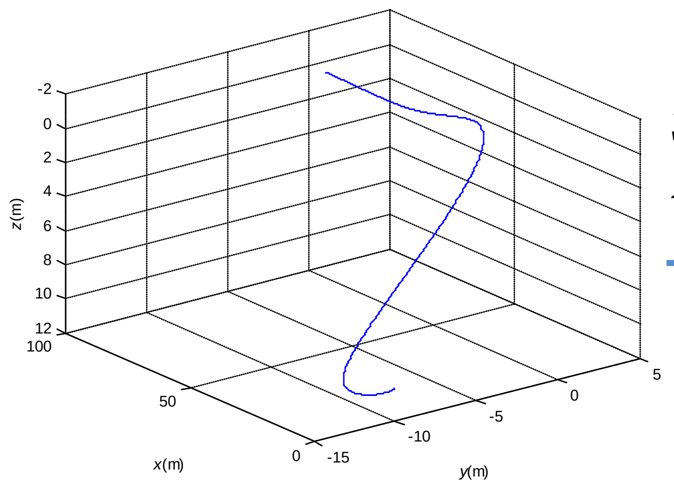
Whole system stability



G$\mathcal{K}$ES: globally $\mathcal{K}$-exponential stable
GES: globally exponential stable

3.1 3D Straight-line tracking

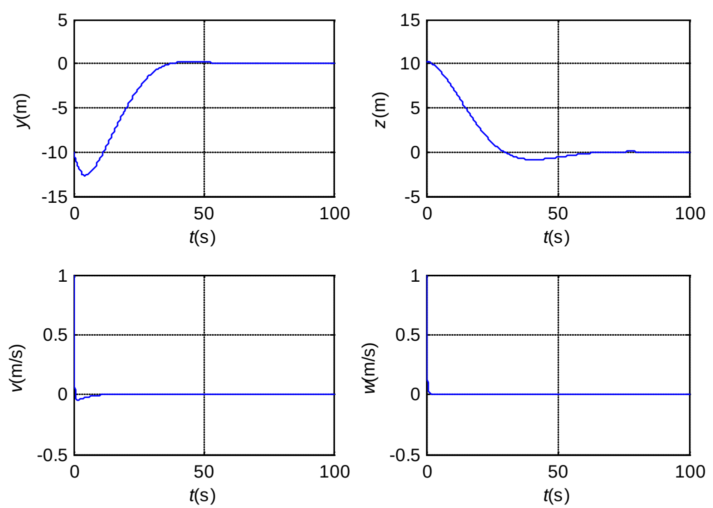
Simulation results

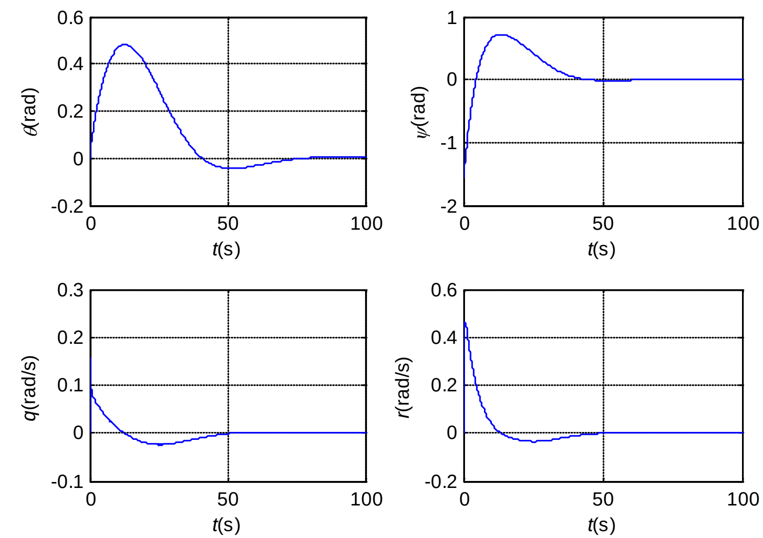


3D straight-line trajectory

Position tracking

Attitude tracking





3.2 2D Path following

Problem formulation

We consider the case that the AUV moves in horizontal plane.

Kinematics

$$\dot{x} = u\cos\psi - v\sin\psi$$
$$\dot{y} = u\sin\psi + v\cos\psi$$
$$\dot{\psi} = r$$

Dynamics

$$\dot{u} = \frac{m_{22}}{m_{11}}vr - \frac{d_{11}}{m_{11}}u + \frac{1}{m_{11}}X$$
$$\dot{v} = -\frac{m_{11}}{m_{22}}ur - \frac{d_{22}}{m_{22}}v$$
$$\dot{r} = \frac{m_{11} - m_{22}}{m_{33}}uv - \frac{d_{33}}{m_{33}}r + \frac{1}{m_{33}}N$$

No control input in y direction, i.e., Y=0.

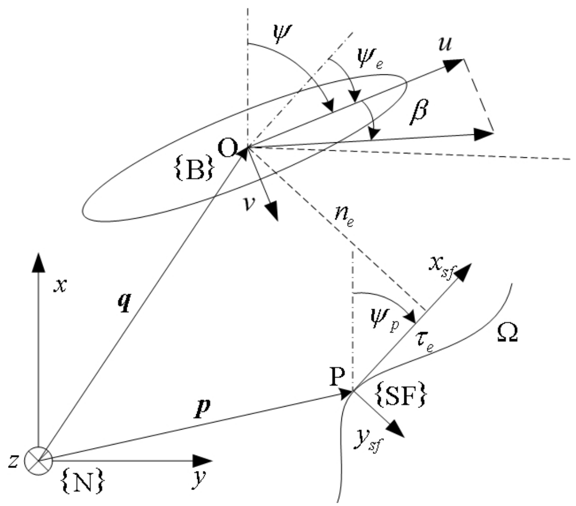
3.2 2D Path following

Problem formulation

Serret-Frenet Coordinate Frame: {SF}

Error dynamics

$$\dot{\tau}_e = -U_p + r_p n_e + u\cos\psi_e - v\sin\psi_e$$
$$\dot{n}_e = -r_p \tau_e + u\sin\psi_e + v\cos\psi_e$$
$$\dot{\psi}_e = r - r_p$$



$$P: \quad \left(x_p(\varpi), y_p(\varpi)\right)$$

$$U_p = \dot{\varpi}\sqrt{x_p'^2(\varpi) + y_p'^2(\varpi)}$$

$$\psi_p(\varpi) = \arctan\left(\frac{y_p'(\varpi)}{x_p'(\varpi)}\right)$$

Path following Problem:

Design the control inputs X, N and the parameter update law $\dot{\varpi}$ to ensure (τ_e, n_e) globally asymptotically stable, the forward velocity u converges to the desired velocity u_d.

3.2 2D Path following

Control design

Define the velocity error $u_e = u - u_d$

Error dynamics

$$\dot{\tau}_e = -U_p + r_p n_e + u_d \cos\psi_e - v\sin\psi_e + u_e \cos\psi_e$$

$$\dot{n}_e = -r_p \tau_e + u_d \sin\psi_e + v\cos\psi_e + u_e \sin\psi_e$$

$$\beta_d = \arctan\left(v/u_d\right)$$

$$\Psi = \psi - \psi_p + \beta_d$$

$$U_d = \sqrt{u_d^2 + v^2}$$

$$\dot{\tau}_e = -U_p + r_p n_e + U_d \cos\Psi + u_e \cos\psi_e$$

$$\dot{n}_e = -r_p \tau_e + U_d \sin\Psi + u_e \sin\psi_e$$

Backstepping design $\Psi_d = -\arctan\left(k_n n_e\right), \quad \Psi_d \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

heading error $\Psi_e = \Psi - \Psi_d, \ \Omega = \dot{\Psi} = r - r_p + \dot{\beta}_d$

$$\Omega_e = \dot{\Psi}_e = \Omega - \dot{\Psi}_d = r - r_p + \dot{\beta}_d - \dot{\Psi}_d$$

3.2 2D Path following

Control design

Error dynamics in cascaded form

$$\begin{cases} \Sigma_1 : \begin{bmatrix} \dot{\tau}_e \\ \dot{n}_e \end{bmatrix} = \underbrace{\begin{bmatrix} -U_p + r_p n_e + U_d \cos\Psi_d \\ -r_p \tau_e + U_d \sin\Psi_d \end{bmatrix}}_{\Sigma_{1,n}} + \begin{bmatrix} U_d \eta_s(\Psi_d, \Psi_e) & \cos\psi_e \\ U_d \eta_c(\Psi_d, \Psi_e) & \sin\psi_e \end{bmatrix} \begin{bmatrix} \Psi_e \\ u_e \end{bmatrix} \\ \Sigma_2 : \begin{bmatrix} \dot{\Psi}_e \\ \dot{\Omega}_e \\ \dot{u}_e \end{bmatrix} = \begin{bmatrix} \Omega_e \\ \frac{m_{11}-m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} N - \dot{r}_p + \ddot{\beta}_d - \ddot{\Psi}_d \\ \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} X - \dot{u}_d \end{bmatrix} \end{cases}$$

$$\eta_s = \begin{cases} \frac{\cos\Psi - \cos\Psi_d}{\Psi_e}, & \Psi_e \neq 0 \\ 1, & \Psi_e = 0 \end{cases}, \quad \eta_c = \begin{cases} \frac{\sin\Psi - \sin\Psi_d}{\Psi_e}, & \Psi_e \neq 0 \\ 1, & \Psi_e = 0 \end{cases}$$

3.2 2D Path following

Control design overview

Tracking errors in {SF}

Cascaded system design techniques

Backstepping design and Lyapunov analysis

Position errors

Velocity and attitude errors

Control inputs

3.2 2D Path following

Control design

Step 1: Stability of $\Sigma_{1,n}$

Lyapunov function candidate $V = \frac{1}{2}\left(\tau_e^2 + n_e^2\right)$

$$\dot{V} = -\tau_e\left(U_p - U_d\right) + \tau_e U_d\left(\cos\Psi_d - 1\right) + n_e U_d \sin\Psi_d$$

Choose $U_p = U_d + k_\tau \tau_e$

$$\dot{V} \le -\frac{k_\tau}{2}\tau_e^2 - \frac{U_d k_n}{2\sqrt{1+\left(k_n n_e\right)^2}} n_e^2 - \frac{k_\tau}{2}\tau_e^2 - \frac{U_d k_n}{2}\left(\frac{n_e}{\sqrt{1+\left(k_n n_e\right)^2}}\right)^2 + U_d k_n \frac{\left|\tau_e\right|\left|n_e\right|}{\sqrt{1+\left(k_n n_e\right)^2}}$$

$k_\tau > U_d k_n$

$$\dot{V} \le -\frac{k_\tau}{2}\tau_e^2 - \frac{U_d k_n}{2\sqrt{1+\left(k_n n_e\right)^2}} n_e^2$$

We can prove that $\Sigma_{1,n}$ **is GKES. The proof is omitted here.**

3.2 2D Path following

Control design

Step 2: Stability of Σ_2

Choose control

$$X = m_{11}\left(-k_u u_e + \dot{u}_d\right) - m_{22} v r + d_{11} u$$

$$N = m_{33}\left(-k_R \Omega_e - k_\Psi \Psi_e + \dot{r}_p - \ddot{\beta}_d + \ddot{\Psi}_d\right) - \left(m_{11} - m_{22}\right) u v + d_{33} r$$

Σ_2 is globally asymptotically stable

Cross item

$$\boldsymbol{g} = \begin{bmatrix} U_d \eta_s\left(\Psi_d, \Psi_e\right) & \cos\psi_e \\ U_d \eta_c\left(\Psi_d, \Psi_e\right) & \sin\psi_e \end{bmatrix} \longrightarrow \left\|\boldsymbol{g}\right\| \le \sqrt{2}\left(1 + U_d\right)$$

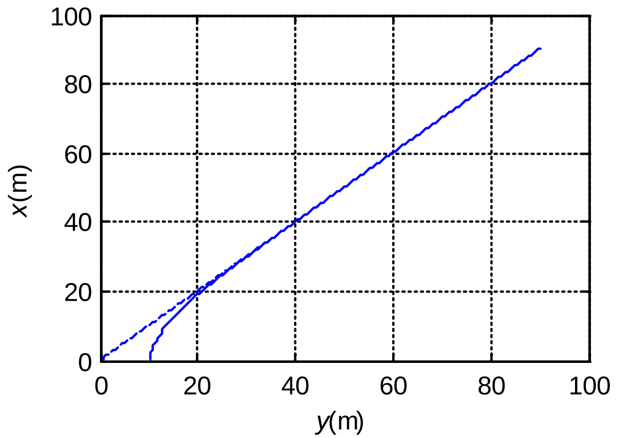
Assumptions 1-3 for the cascaded system are all fulfilled.

$\left\{\Sigma_1, \Sigma_2\right\}$ is GKES

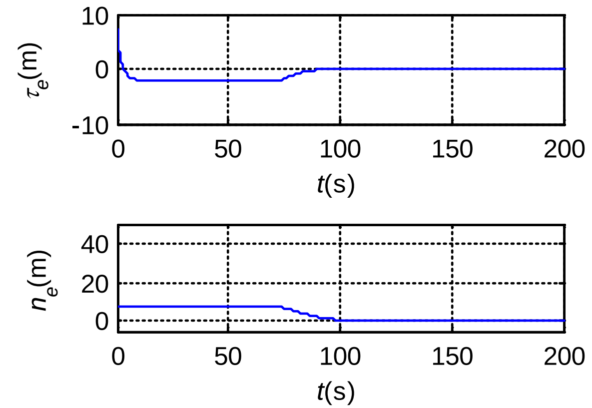
Loría A, Cascaded nonlinear time-varying systems: analysis and design, lecture notes, Minicourse at ECTS, France, 2004.

3.2 2D Path following

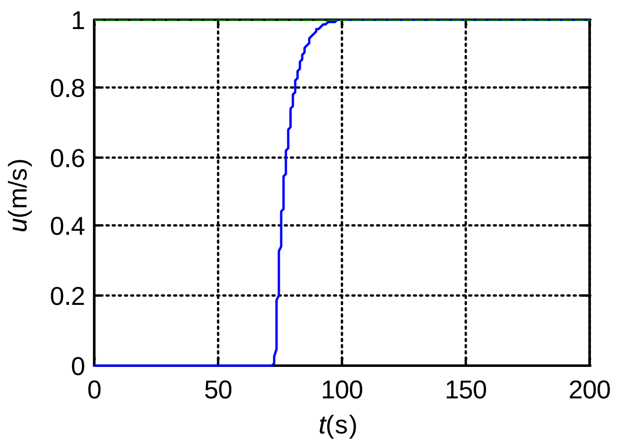
Simulation results - Straight-line path following



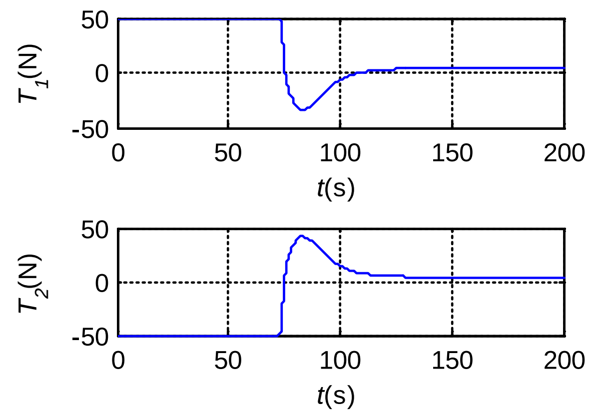
(a) path



(b) tracking errors



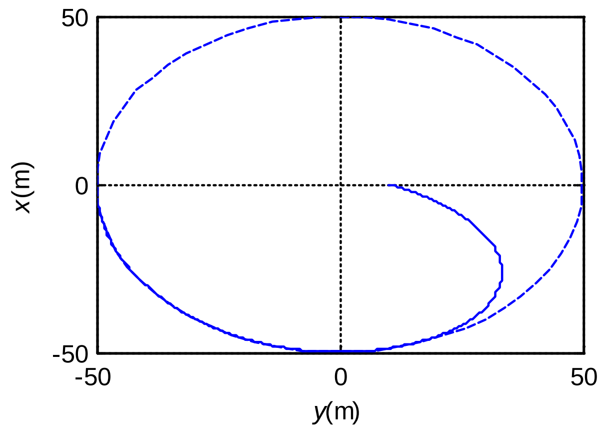
(c) Surge velocity



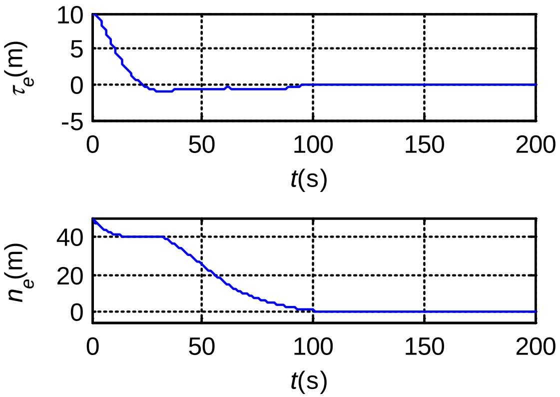
(d) Thrust forces

3.2 2D Path following

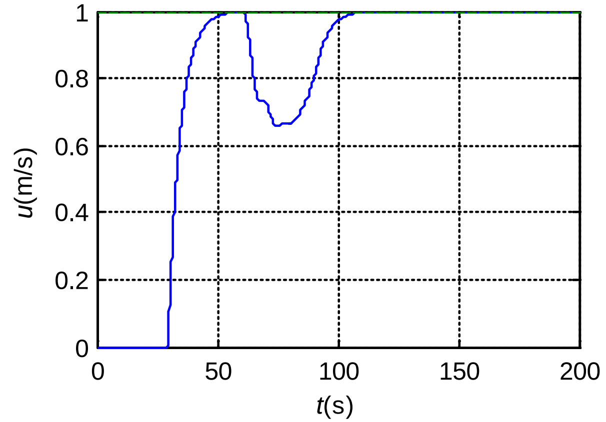
Simulation results - circle path following



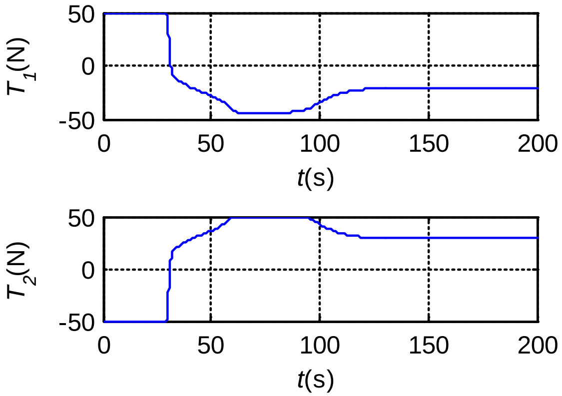
(a) path



(b) tracking errors



(c) Surge velocity



(d) Thrust forces

3.3 3D path following

Problem formulation

Two more coordinate frames

Velocity Coordinate Frame $(oxyz)$ $\{W\}$

{SF} in 3D $(cx_{sf}y_{sf}z_{sf})$ $\{SF\}$

Some definitions

The velocity in {W} $v_B^W = [V_t \quad 0 \quad 0]^T$ **The velocity in {SF}** $v_{SF}^{SF} = [\dot{\varpi} \quad 0 \quad 0]^T$

The velocity vector: {W} relative to the {SF}

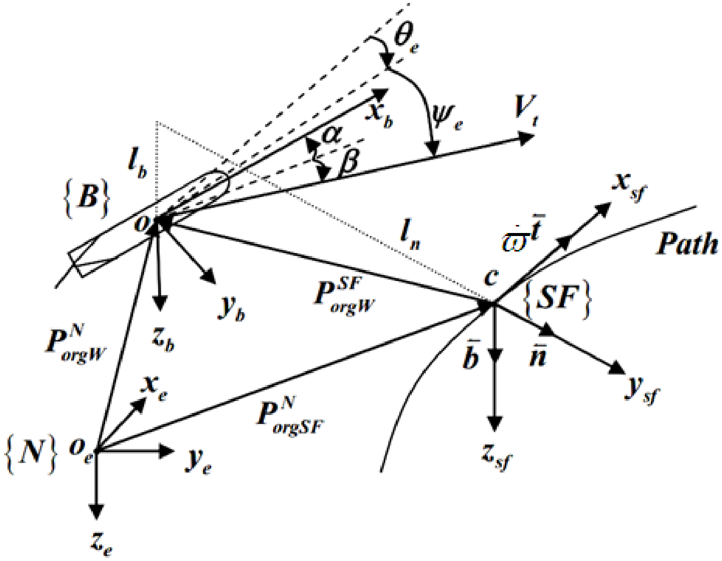
$$\frac{dP_{orgW}^{SF}}{dt} = [0 \quad i_n \quad i_b]^T$$

The angle velocity vector: {SF} relative to {N}

$$\omega_{SF}^{SF} = [\tau\dot{\varpi} \quad 0 \quad k\dot{\varpi}]^T$$

Angle of attack: α

Angle of sideslip: β



3.3 3D path following

Problem formulation

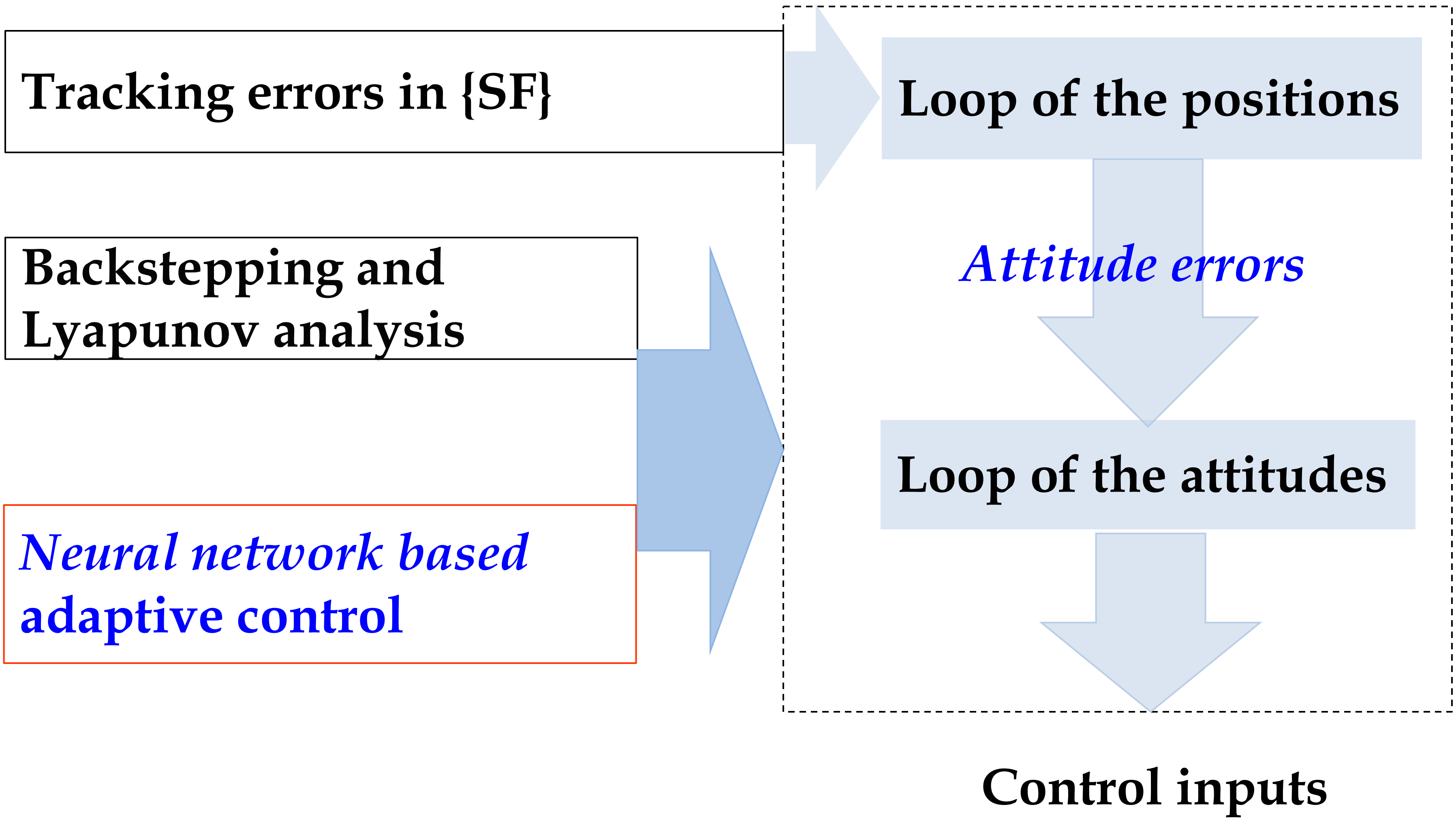
Coordinate transformation matrix : from {W} to {SF}

$$R_W^{SF}(\phi_e,\theta_e,\psi_e)=\begin{bmatrix} \cos\psi_e\cos\theta_e & \sin\psi_e\cos\theta_e & -\sin\theta_e \\ \begin{pmatrix}\cos\psi_e\sin\theta s\varphi_e \\ -\sin\psi_e\cos\varphi_e\end{pmatrix} & \begin{pmatrix}\cos\psi_e\cos\varphi_e \\ \sin\psi_e\sin\theta_e\sin\varphi_e\end{pmatrix} & \cos\theta_e\sin\varphi_e \\ \begin{pmatrix}\sin\psi_e\sin\varphi_e \\ \cos\psi_e\sin\theta_e\cos\varphi_e\end{pmatrix} & \begin{pmatrix}-\cos\psi_e\sin\varphi_e \\ \sin\psi_e\sin\theta_e\cos\varphi_e\end{pmatrix} & \cos\theta_e\cos\varphi_e \end{bmatrix}$$

Design the control to let the path following errors converge to zero neighborhood as time goes infinity.

3.3 3D path following

Control design overview



3.3 3D path following

The kinematics model

$$\begin{cases} \dot{\varpi} = \dfrac{V_t \cos\theta_e \cos\psi_e}{1 - kl_n} \\ \dot{l}_n = V_t \cos\theta_e \sin\psi_e + \tau l_b \dot{\varpi} \\ \dot{l}_b = -V_t \sin\theta_e - \tau l_n \dot{\varpi} \end{cases} \Rightarrow \begin{cases} \dot{l}_n = V_t \cos\theta_e \sin\psi_e + \dfrac{\tau l_b}{1 - l_n k} V_t \cos\theta_e \cos\psi_e \\ \dot{l}_b = -V_t \sin\theta_e - \dfrac{\tau l_n}{1 - l_n k} V_t \cos\theta_e \cos\psi_e \end{cases}$$

The Non-affine Nonlinear model

$$\dot{x}_1 = f_1(x_1, u_1) \qquad x_1 = [l_n, l_b]^T \qquad u_1 = [\theta_e, \psi_e]^T$$

where

$$f_1(x) = \begin{bmatrix} V_t \cos\theta_e \sin\psi_e + \dfrac{\tau l_b}{1 - l_n k} V_t \cos\theta_e \cos\psi_e \\ -V_t \sin\theta_e - \dfrac{\tau l_n}{1 - l_n k} V_t \cos\theta_e \cos\psi_e \end{bmatrix}$$

3.3 3D path following

Dynamics of the attitude errors

$$\omega_{W,SF}^{W} = \begin{bmatrix} \dot{\varphi}_e & \dot{\theta}_e & \dot{\psi}_e \end{bmatrix}^T = \omega_W^W - \omega_{SF}^W$$

$$\omega_W^W = \omega_{W,B}^W + \omega_B^W$$

$$\omega_{W,B}^W = \begin{bmatrix} 0 \\ 0 \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\dot{\alpha} \\ 0 \end{bmatrix}$$

$$\omega_B^W = R_B^W(\alpha,\beta)\omega_B^B = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\begin{bmatrix} \dot{\varphi}_e \\ \dot{\theta}_e \\ \dot{\psi}_e \end{bmatrix} = \begin{bmatrix} k_1^{\varphi_e} \\ k_2^{\theta_e} \\ k_3^{\psi_e} \end{bmatrix} + \begin{bmatrix} a_{11}^{\varphi_e} & a_{12}^{\varphi_e} & a_{13}^{\varphi_e} \\ 0 & a_{22}^{\theta_e} & a_{23}^{\theta_e} \\ 0 & a_{32}^{\psi_e} & a_{33}^{\psi_e} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

3.3 3D path following

Dynamics of the attitude errors

$$\dot{x}_2 = f_2(x_2) + g_2(x_2)u_2$$

where

$$x_2 = \begin{bmatrix} \varphi_e \\ \theta_e \\ \psi_e \end{bmatrix} \qquad f_2(x_2) = \begin{bmatrix} k_1^{\varphi_e} \\ k_2^{\theta_e} \\ k_3^{\psi_e} \end{bmatrix} \qquad g_2(x_2) = \begin{bmatrix} a_{11}^{\varphi_e} & a_{12}^{\varphi_e} & a_{13}^{\varphi_e} \\ 0 & a_{22}^{\theta} & a_{23}^{\theta} \\ 0 & a_{32}^{\psi_e} & a_{33}^{\psi_e} \end{bmatrix} \qquad u_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Dynamics of the rotary motion

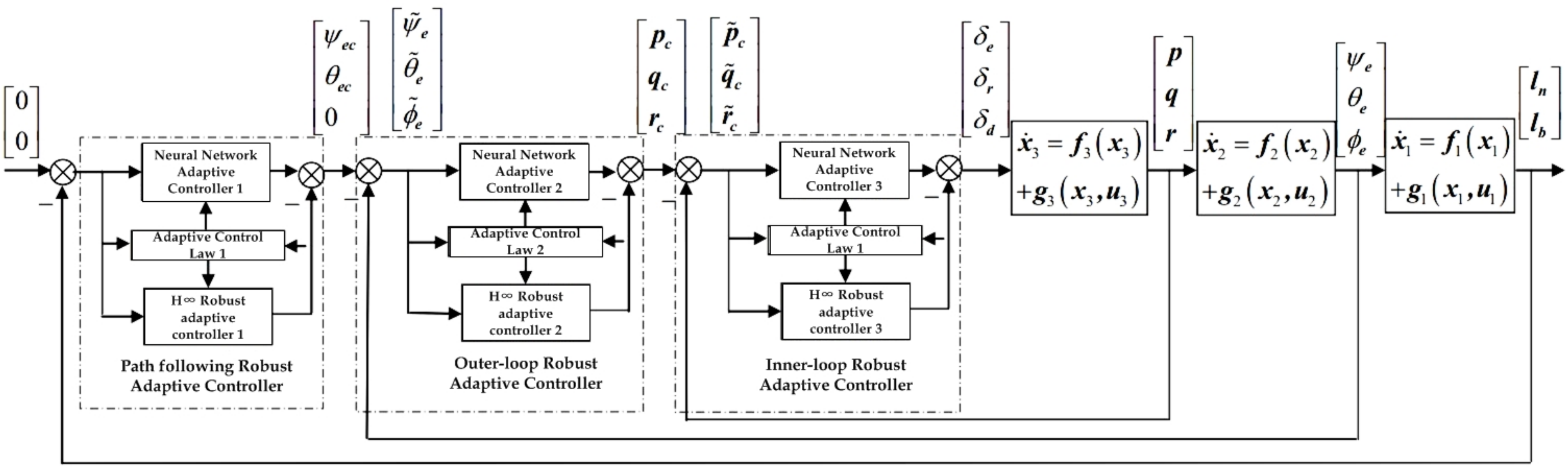
$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} k_1^p \\ k_2^q \\ k_3^r \end{bmatrix} + \begin{bmatrix} a_{11}^p & 0 & 0 \\ 0 & a_{22}^q & 0 \\ 0 & 0 & a_{33}^r \end{bmatrix} \begin{bmatrix} \delta_d \\ \delta_e \\ \delta_r \end{bmatrix} \Rightarrow \dot{x}_3 = f_3(x_3) + g_3(x_3)u_3$$

$$x_3 = u_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad u_3 = \begin{bmatrix} \delta_d \\ \delta_e \\ \delta_r \end{bmatrix} \qquad f_3(x_3) = \begin{bmatrix} k_1^p \\ k_2^q \\ k_3^r \end{bmatrix} \qquad g_3(x_3) = \begin{bmatrix} a_{11}^p & 0 & 0 \\ 0 & a_{22}^q & 0 \\ 0 & 0 & a_{33}^r \end{bmatrix}$$

3.3 3D path following

Neural network adaptive control

- Out-loop -- position control
- Inner-loop – attitude control



3.3 3D path following

RBF neural network adaptive control design for the inner-loop

Consider the following MIMO uncertain nonlinear systems

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = f(x) + g(x)\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} + d(x)$$

If the functions $f(x), g(x)$ **are known, then we have a nominal controller**

$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = g^{-1}(x)\left[-\begin{bmatrix} f_1(x) \\ \vdots \\ f_m(x) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} \right]$$

3.3 3D path following

If $f(x), g(x)$ **are unknown, then RBF neural networks can be applied to approximate** $f_i(x), g_{ij}(x)$ **as** $\hat{f}_i(x|w), \hat{g}_{ij}(x|w)$.

equivalent nonlinear robust adaptive controller

$$u_i = \frac{1}{\hat{g}_{ii}(x|w_{g_{ii}})}\left[-\hat{f}_i(x|w_{fi}) + y_{r_i}^{(n_i)} + K_c^T e_i - u_{ri}\right]$$

$$u_{ri} = -B_i^T P_i e_i / \eta_i$$

RBF NN approximation

$$\begin{cases} \hat{f}_i(x|w_{f_i}) = w_{f_i}^T \varphi_i(x) \\ \hat{g}_{ii}(x|w_{g_{ii}}) = w_{g_{ii}}^T \varphi_{ii}(x) \end{cases}$$

NN weight update law

$$\dot{w}_{fi} = -\mu_{1i}\varphi_{fi}(x)B_i^T P e_i$$

$$\dot{w}_{gii} = -\mu_{2i}\varphi_{gii}(x)B_i^T P e_i$$

3.3 3D path following

RBF neural network adaptive control design for the out-loop

MIMO Non-affine Nonlinear System

$$\begin{cases} y_1^{(r_1)} = F_1(x,u) \\ y_2^{(r_2)} = F_2(x,u) \\ \quad\vdots \\ y_m^{(r_m)} = F_m(x,u) \end{cases}$$

Corresponding affine system

$$F(x,u) = F(x,u^*) + \frac{\partial F(x,u)}{\partial u}\Big|_{u=u^*}(u-u^*) + O((u-u^*)^2)$$

$$f(x) = F(x,u^*) - \frac{\partial F(x,u)}{\partial u}\Big|_{u=u^*} u^* \qquad g(x) = \frac{\partial F(x,u)}{\partial u}\Big|_{u=u^*} u \qquad d = O((u-u^*)^2)$$

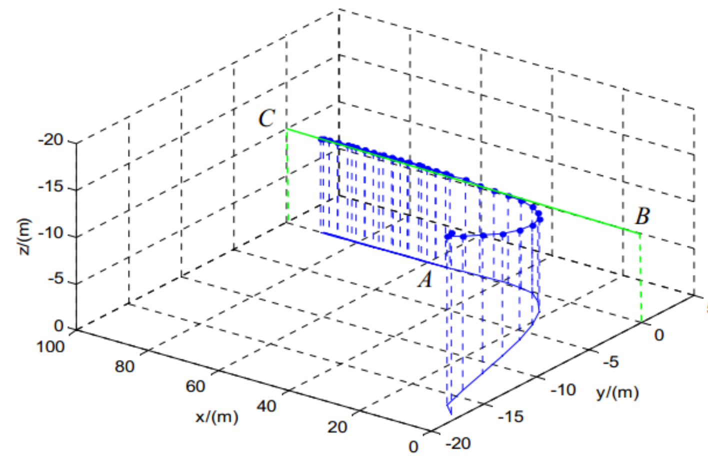
Then we can use the nominal affine system to design the control.

Through the Lyapunov analysis, we can prove the stability of the path following system.

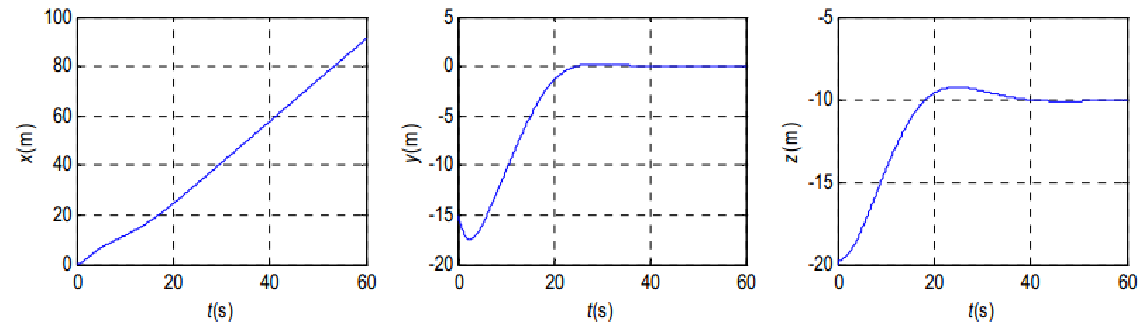
3.3 3D path following

Simulation results

Trajectory in 3D

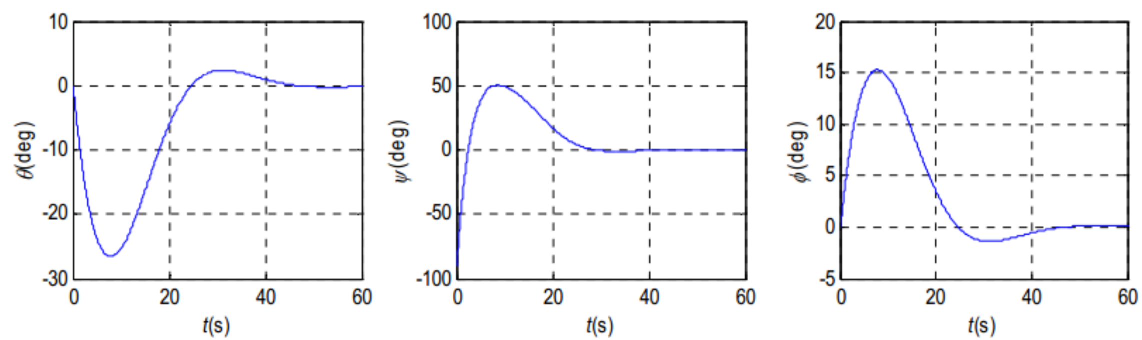


AUV displacement in *X,Y,Z* direction



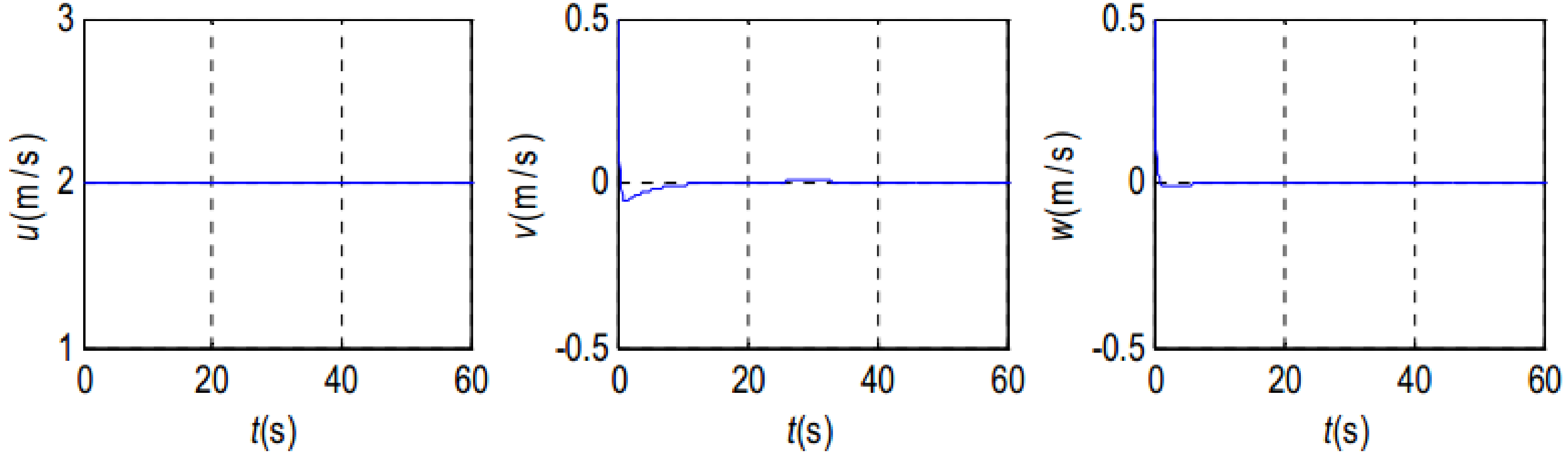
AUV attitude angles

θ, ψ, ϕ

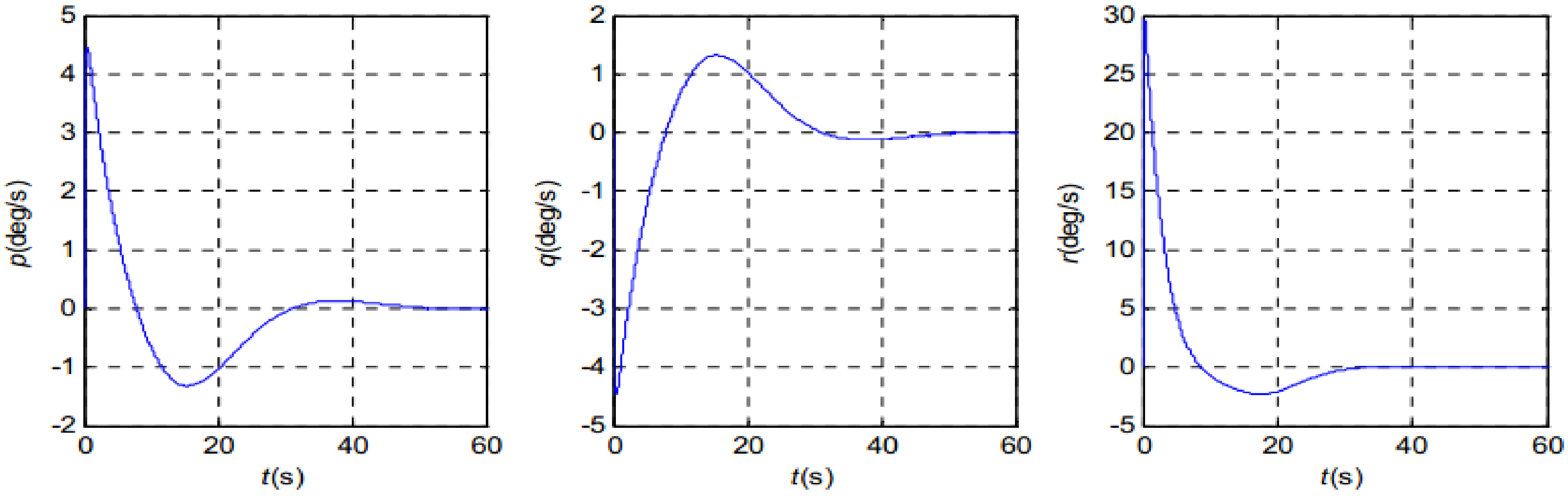


3.3 3D path following

Linear velocity, *u, v, w*



Angular velocity, *p, q, r*



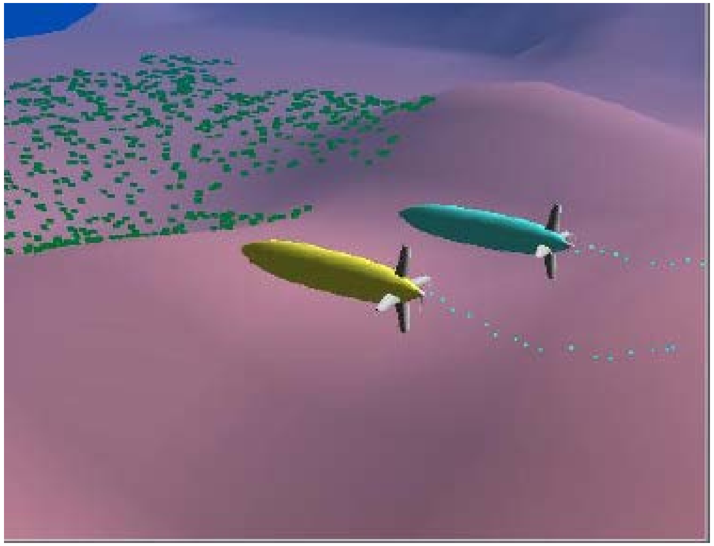
4 Formation control of multiple AUVs

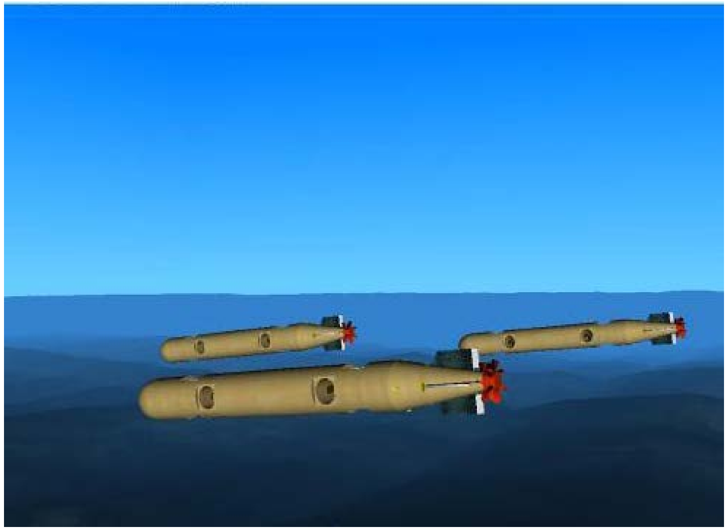
A Single AUV

- cannot perform complex tasks
- low efficiency, such as oceanography, counter mine
- task defeat if broken

Multiple AUVs Cooperation

- information sharing, enlarge the sensing range
- more efficiency, robust
- more low cost AUVs instead of a complex and costly AUV





4 Formation control of multiple AUVs

Outline

Formation of underactuated AUVs

4.1 Leader follower formation control

4.2 Consensus based decentralized formation control

4.1 Leader-follower formation

Problem formulation

The AUVs formation in horizontal plane at a constant depth.

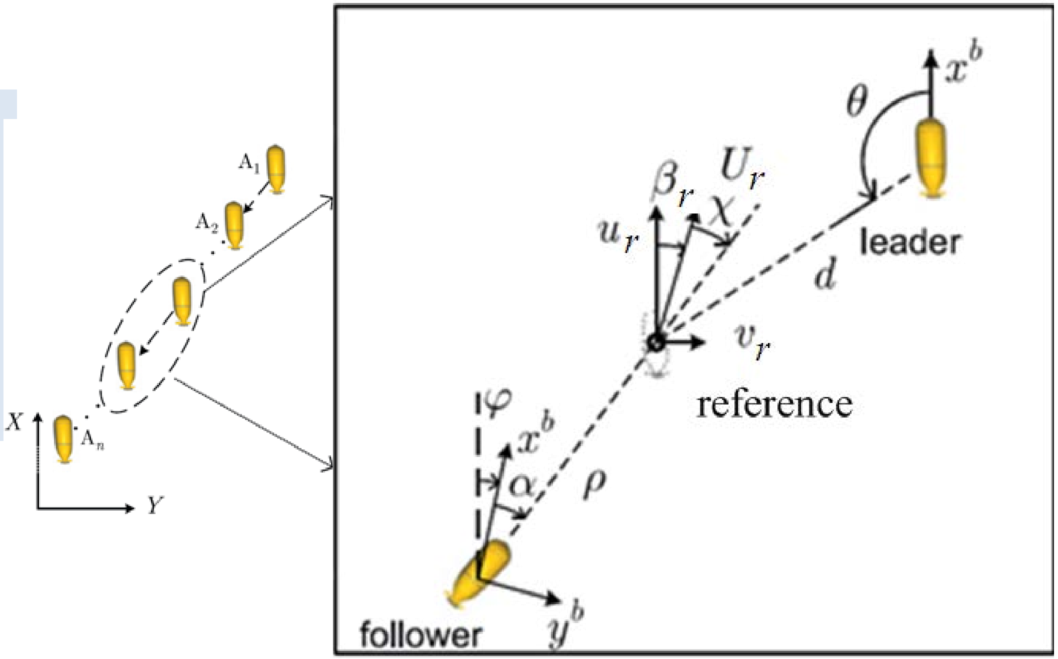
$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau + w$$

$$\nu = [u, v, r]^T, \eta = [x, y, \psi]^T, g(\eta) = 0, \quad \tau = [\tau_1, 0, \tau_2]^T$$

disturbances $w = [w_1, w_2, w_3]^T$

Technical challenges

- **the AUVs are underactuated**
- **model parameter uncertainties**
- **disturbances are unknown but bounded**



4.1 Leader-follower formation

Control design overview

Reference trajectory design

NN forward approximation and adaptive control

Cascaded system design and backstepping

Unknown model parameters

Unknown disturbances

Underactuated AUV

4.1 Leader-follower formation

Preliminaries

Definition 1 : Uniform semi global practical asymptotic stable (USPAS): Consider the following parameterized nonlinear time-varying systems $\dot{\xi} = f(t,\xi,\theta)$**, where** ξ **is a constant parameter and** $f(t,\xi,\theta)$ **is locally Lipschitz in and piecewise continuous in *t* for all** $\theta \in \mathbb{R}^m$ **.** ***[Chaillet and Lorı'a, 2008)]***

Lemma 1: For bounded initial conditions, if there exists a C^1 **continuous and positive definite Lyapunov function** $V(\xi)$ **satisfying** $\kappa_1(\|\xi\|) \le V(\xi) \le \kappa_2(\|\xi\|)$ **, such that** $\dot{V}(\xi) \le -\mu V(\xi) + c$ **, where** κ_1 **and** κ_2 **are** $\mathcal{K}$ **class functions and *c* is a positive constant, then the solution** $\xi = 0$ **is uniformly bounded .** ***[Ge and Wang 2004]***

Property 1: The function $f(\xi) = \log(\cosh(\xi))$ **has the following property:** $\dot{f}(\xi) = \dot{\xi}\tanh(\xi)$

4.1 Leader-follower formation

Reference path design

$$\eta_r = \eta_m + R(\psi_m)l$$

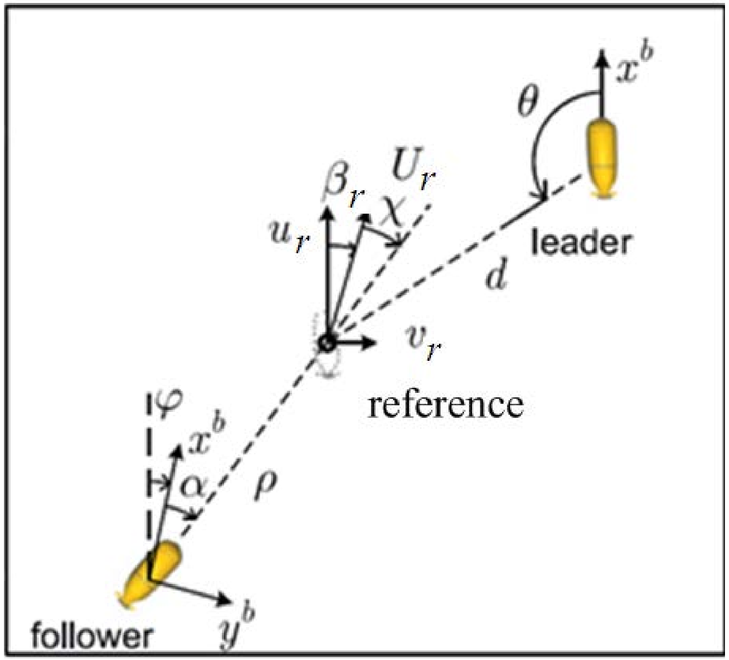
$$\dot{\eta}_r = R(\psi_m)v_r$$

$$v_r = [u_m - d_y r_m, v_m + d_x r_m, r_m]^T$$

$$l = \left[d_x, d_y, 0\right]^T$$

$$d_x = d\cos\theta$$

$$d_y = d\sin\theta$$



4.1 Leader-follower formation

Path following of the follower

The kinematic equations in the velocity frame

$$\dot{x}_r = U_r \cos\psi_w, \dot{y}_r = U_r \sin\psi_w, \dot{\psi}_w = r_r + \dot{\beta}_r, \quad U_r = \sqrt{u_r^2 + v_r^2}$$

We have following equations

$$\begin{cases} \rho = \sqrt{(x_f - x_r)^2 + (y_f - y_r)^2} \\ x_f - x_r = -\rho\cos(\varphi + \alpha) \\ y_f - y_r = -\rho\sin(\varphi + \alpha) \\ \varphi + \alpha = \arctan(\dfrac{y_f - y_r}{x_f - x_r}) \end{cases}$$

$$\dot{\rho} = -u_f \cos\alpha - v_f \sin\alpha + U_r \cos\chi$$

$$\dot{\alpha} = \frac{\sin\alpha}{\rho} u_f - \frac{\cos\alpha}{\rho} v_f - r_f - \frac{U_r \sin\chi}{\rho}$$

Control objective: α converges to zero and ρ converges to a small positive constant δ as t approaches infinity.

4.1 Leader-follower formation

Path following of the follower - Kinematic control

Kinematic control

$$r_f = \frac{\sin\alpha}{\rho}u_f - \frac{\cos\alpha}{\rho}v_f - \frac{U_v \sin\chi}{\rho} + k_4\alpha,\ k_4 > 0$$

$$u_f = -k_5(\rho - \delta) + U_v \cos\chi,\ 0 < k_5 < \frac{d_{22}}{m_{11}} - \frac{U_{v,\max}}{\delta}, \delta \in \mathbb{R}^+$$

Define $\bar{\rho} = \rho - \delta$

Cascaded form

$$\dot{\bar{\rho}} = -k_5\bar{\rho} + [\eta_s(0,\alpha)U_v \cos\chi - \eta_c(0,\alpha)v_f]\alpha$$

$$\dot{\alpha} = -k_4\alpha$$

Clearly, the assumptions for the cascaded system are fulfilled, then the system is UGAS.

4.1 Leader-follower formation

Trajectory tracking of the follower - Kinematic control

Dynamics of the uncontrolled sway velocity

$\dot{v}_f = -h_1(\cdot)v_f + h_2(\cdot)\rho + h_3(\cdot) + w_2'$ **Expressions of** $h_1(\cdot), h_2(\cdot), h_3(\cdot)$ **are omitted here.**

Lemma: With the designed kinematic control, for any bounded initial conditions, ρ and v_f are uniformly bounded.

Proof: Choose Lyapunov function candidate $V_1(\rho, v_f) = \frac{1}{2}\rho^2 + \frac{1}{2}v_f^2$

After some calculations, we have

$$\dot{V}_1 \le -2cV_1 + \frac{\bar{\zeta}_\infty}{2\mu_1} + \frac{h_\infty}{2\mu_1}$$

where c and $\frac{\bar{\zeta}_\infty}{2\mu_1} + \frac{h_\infty}{2\mu_1}$ are positive constants.

4.1 Leader-follower formation

Path following of the follower - dynamics control

Define the errors $z_1 = u_f - \alpha_u,\ z_2 = r_f - \alpha_r$

Select Lyapunov function candidate

$$V_2(t) = \frac{1}{2}\bar{\rho}^2 + \frac{1}{2}\alpha^2 + \frac{1}{2}m_{11}z_1^2 + \frac{1}{2}m_{33}z_2^2$$

We have

$$\begin{aligned}\dot{V}_2(t) = &-k_5\bar{\rho}^2 + g(\cdot)\bar{\rho}\alpha - k_4\alpha^2 \\ &+z_1\left(m_{22}v_f r_f - d_{11}u_f + \tau_1 + w_1 - m_{11}\dot{\alpha}_u + \frac{\sin\alpha}{\rho}\alpha\right) \\ &+z_2\left[(m_{11} - m_{22})u_f v_f - d_{33}r_f + \tau_2 + w_3 - \alpha - m_{33}\dot{\alpha}_r\right]\end{aligned}$$

where $g(\cdot) = [\eta_s(0,\alpha)U_v\cos\chi - \eta_c(0,\alpha)v_f]$

4.1 Leader-follower formation

Trajectory tracking of the follower - dynamics control

Model based control

$$\tau_1 = -m_{22} v_f r_f + d_{11} u_f + m_{11} \dot{\alpha}_u - \frac{\sin\alpha}{\rho}\alpha - k_6 z_1 - \mathrm{sgn}(z_1)\bar{w}_1$$

$$\tau_2 = -(m_{11} - m_{22}) u_f v_f + d_{33} r_f + \alpha + m_{33}\dot{\alpha}_r - k_7 z_2 - \mathrm{sgn}(z_2)\bar{w}_3$$

Noted that the model parameters m_{11}, m_{22}, etc and the disturbances w_i are all unknown, *NN approximation* is used to compensate for these uncertainties.

Adaptive control

$$\tau = -diag[k_6, k_7]z + \left[-\frac{\sin\alpha}{\rho}, \alpha\right]^T + \hat{W}^T S(Z)$$

$$\dot{\hat{W}} = -\Gamma_i\left(S_i(Z) z_i + \sigma_i \hat{W}_i\right)$$

$$\tau = [\tau_1, \tau_2]^T, z = [z_1, z_2]^T$$

$$W^{*T} S(Z) = \begin{bmatrix} -m_{22} v_f r_f + d_{11} u_f + m_{11}\dot{\alpha}_u - \mathrm{sgn}(z_1)\bar{w}_1 \\ -(m_{11} - m_{22}) u_f v_f + d_{33} r_f + m_{33}\dot{\alpha}_r - \mathrm{sgn}(z_2)\bar{w}_3 \end{bmatrix} - \varepsilon(Z)$$

4.1 Leader-follower formation

Path following of the follower - dynamics control

Lyapunov function candidate

$$V_3(t) = \frac{1}{2}\bar{\rho}^2 + \frac{1}{2}\alpha^2 + \frac{1}{2}m_{11}z_1^2 + \frac{1}{2}m_{33}z_2^2 + \frac{1}{2}\sum_{i=1}^{2}\tilde{W}_i^T\Gamma_i^{-1}\tilde{W}_i$$

where $\tilde{W}_i = \hat{W}_i - W_i^*$.

We have

$$\dot{V}_3 \le -\mu V_3 + C$$

$$\mu = \min\left\{2k_5 - \vartheta, 2k_4 - \vartheta, \frac{2k_6 - 1}{m_{11}}, \frac{2k_7 - 1}{m_{33}}, \min\left(\frac{\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}\right)\right\}$$

$$C = \sum_{i=1,2}\frac{\sigma_i}{2}\| W_i^* \|^2 + \frac{1}{2}\| \bar{\varepsilon} \|^2$$

4.1 Leader-follower formation

Path following of the follower - dynamics control

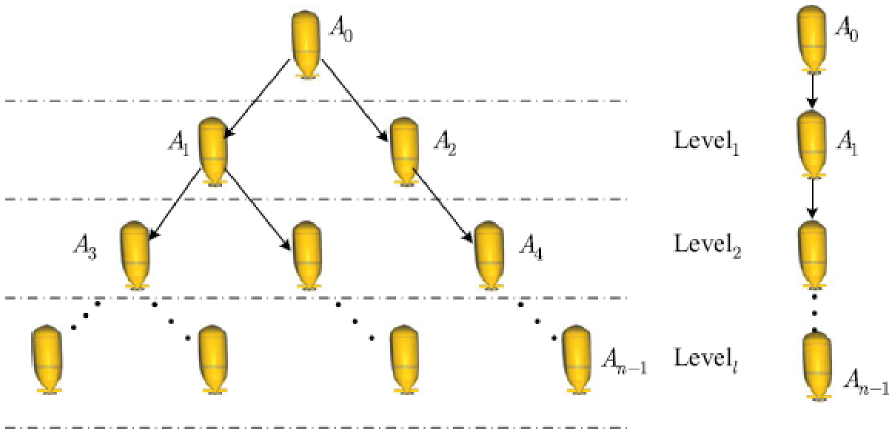
Theorem: With the desired adaptive control, for each compact set Ω_0, where $(n_f(0), v_f(0), \hat{W}_1(0), \hat{W}_2(0)) \in \Omega_0$, the trajectories of the position tracking closed-loop system are semiglobally uniformly bounded. The tracking error $[\bar{\rho}, \alpha]$ converges to a compact set $\Omega_{ZS} := \{\|[\bar{\rho}, \alpha]\| \le \sqrt{2C / \mu}\}$.

Proof: Following the methods found in *[Ge and Wang 2004]*, we can completes the proof.

4.1 Leader-follower formation

Extended to multiple leader-follower pairs.

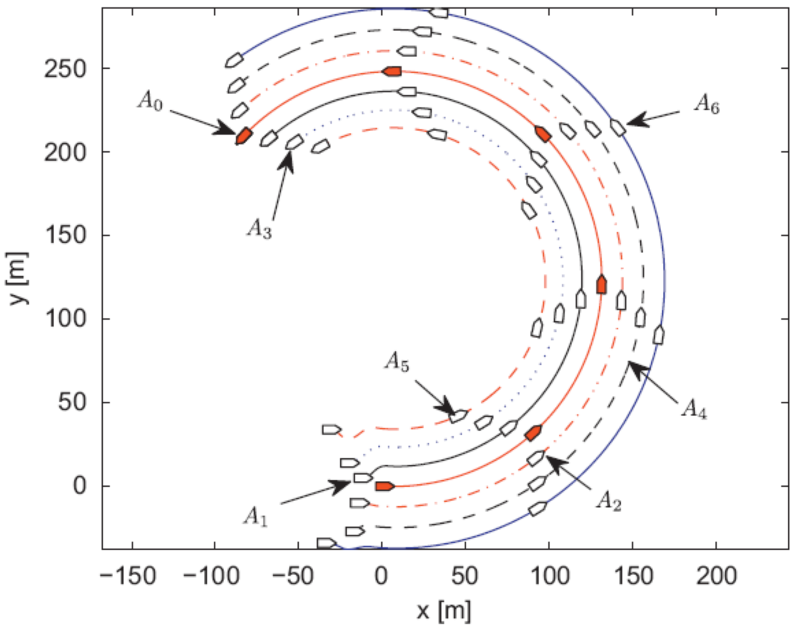
- **Series connection**
- **Parallel connection**
- **series-parallel connection**



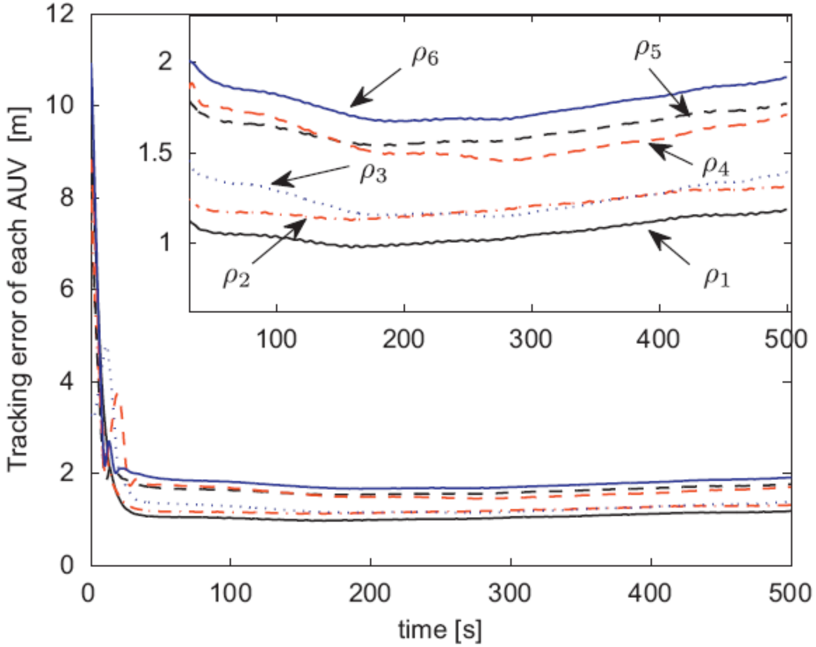
Consider leader–follower formation of *n* AUVs, each AUV using the designed control to regulate its position relative to its leader, for each Ω_0 compact set, where $\left(\eta_j(0), \nu_j(0), \hat{W}_{1,j}(0), \hat{W}_{2,j}(0)\right) \in \Omega_{0,j}$, the trajectories of the closed-loop system are semiglobally uniformly bounded. The tracking error of the formation is bounded and converges to a compact set $\bar{\Omega}_{zs} = \left\{\left\|\breve{p}, \breve{\alpha}\right\| \le \sqrt{2\bar{C}/\bar{\mu}}\right\}$, where $\bar{\mu}$ and $\bar{C}$ are positive constants.

4.1 Leader-follower formation

Simulation results



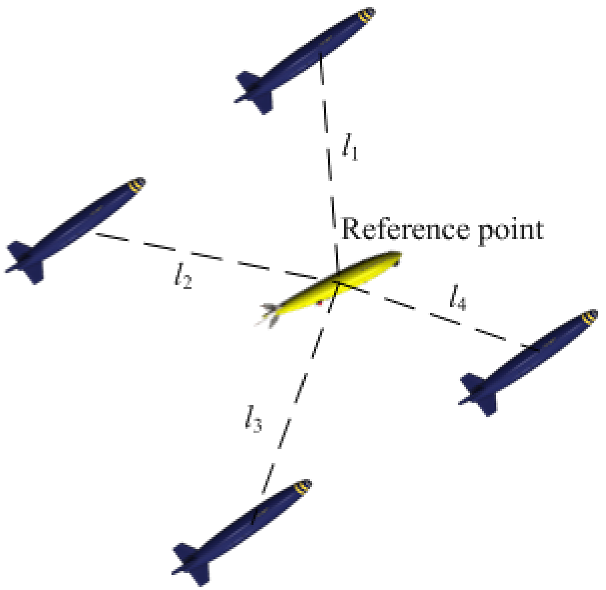
Trajectories



Errors

4.2 Consensus Based Decentralized Formation

Problem formulation – decentralized formation



$$\eta_{id}(\varpi_i) = \eta_d(\varpi_i) + R(\psi_d(\varpi_i))l_i$$

Velocity convergence

$$\lim_{t\to\infty} \dot{\varpi}_i(t) = v_l(t), \forall i \in \{1,\ldots,n\}$$

Path parameter consensus

$$\lim_{t\to\infty} \left|\varpi_i - \varpi_j\right| = 0, \forall i, j \in \{1,\ldots,n\}$$

Path following

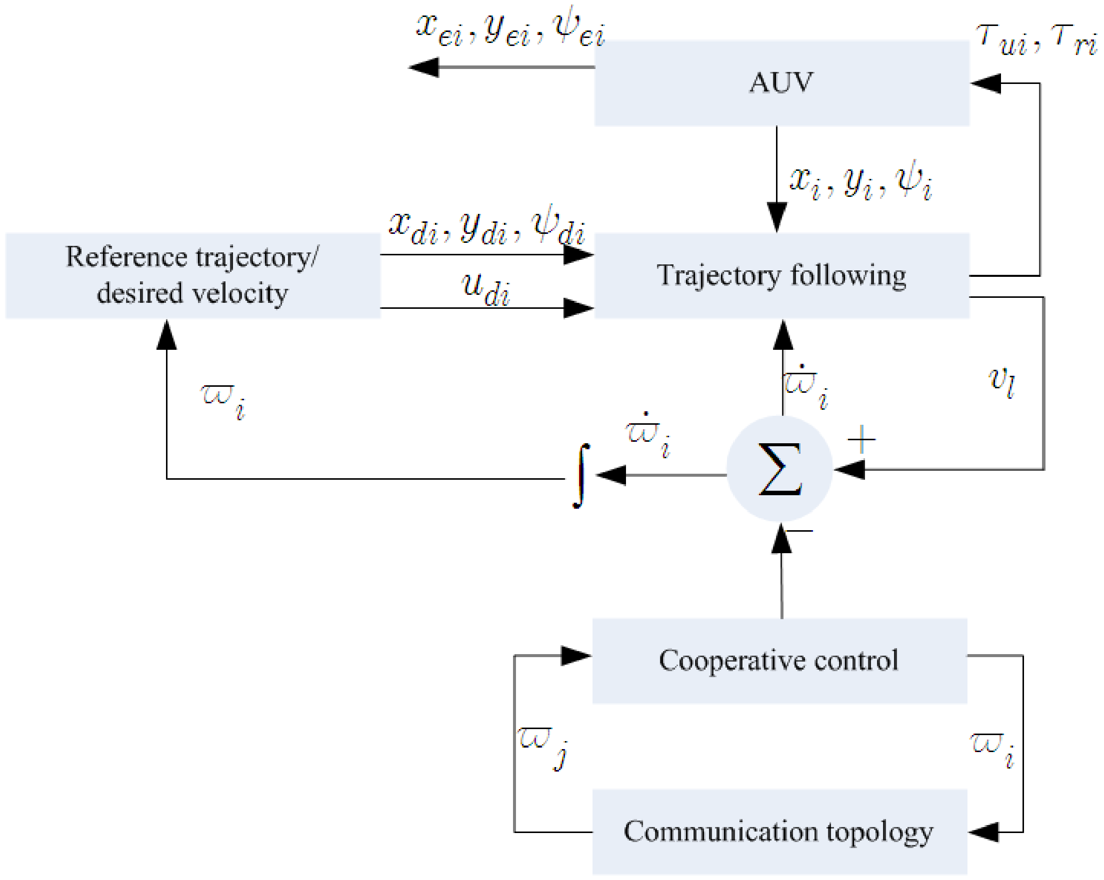
$$\lim_{t\to\infty} \left|\eta_i(t) - \eta_{id}(\varpi_i(t))\right| = 0, \forall i \in \{1,\ldots,n\}$$

4.2 Consensus Based Decentralized Formation

Control design overview

Control methods for a single AUV

Synchronization of path paramater



4.2 Consensus Based Decentralized Formation

Path following control

Model
$$M\dot{\nu}_i + SM\nu_i r_i + D\nu_i = u_{i1} + R^\top(\psi_i)b_i$$
$$m_{33}\dot{r}_i + (m_{22} - m_{11})u_i v_i + d_{33}r_i = \tau_{ir} + w_{i3}$$
$$M = diag\{m_{11}, m_{22}\} \quad D = diag\{d_{11}, d_{22}\} \quad \eta_i = [x_i, y_i]^\top \quad \nu_i = [u_i, v_i]^\top$$

$$e_i = R^\top(\psi_i)(\eta_i - \eta_{id}) \quad \Rightarrow \quad \dot{e}_i = -S(r_i)e_i + \nu_i - v_r R_i^\top \eta_{id}^{\varpi_i} - \tilde{\xi} R_i^\top \eta_{id}^{\varpi_i}$$

the first Lyapunov function candidate

$$V_{1i} = \frac{1}{2} e_i^\top e_i \quad \Rightarrow \quad \begin{aligned} \dot{V}_{1i} &= e_i^\top \dot{e}_i \\ &= e_i^\top(-S(r_i)e_i + \nu_i - v_r R_i^\top \eta_{id}^{\varpi_i} - \tilde{\xi} R_i^\top \eta_{id}^{\varpi_i}) \\ &= e_i^\top(\nu_i - v_r R_i^\top \eta_{id}^{\varpi_i}) - \tilde{\xi}_i^\top R_i^\top \eta_{id}^{\varpi_i}) \end{aligned}$$

Define the virtual tracking error $$z_{1i} = \nu_i - v_r R_i^\top \eta_{id}^{\varpi_i} + k_{ei} M^{-1} e_i$$

$$\dot{V}_{1i} = -k_{ei} e_i^\top M^{-1} e_i + e_i^\top z_{1i} + \alpha_{1i}\tilde{\xi}_i$$

4.2 Consensus Based Decentralized Formation

the second **Lyapunov function candidate** $V_{2i} = V_{1i} + \frac{1}{2}\mu_i^\top M^2 \mu_i + \frac{1}{2}\tilde{b}_i^\top \Lambda \tilde{b}_i$

$$\begin{aligned}
\dot{V}_{2i} &= \dot{V}_{1i} + \mu_i^\top M \cdot M\dot{z}_{1i} + \tilde{b}_i^\top \Lambda_i \tilde{b}_i \\
&= -k_{ei} e_i^\top M^{-1} e_i + e_i^\top \delta_i + \alpha_{2i}\tilde{\xi}_i + \tilde{b}_i^\top \Lambda_i (\dot{\hat{b}}_i - \Lambda_i^{-1} M R_i \mu_i) \\
&\quad + \mu_i^\top M (M^{-1} e_i + h_{2i} + \underbrace{v_r \Gamma_i r_i + S(M\delta_i) r_i + u_{1i}}_{\Delta_i} + R_i^\top \hat{b}_i) \\
&= -k_{ei} e_i^\top M^{-1} e_i + e_i^\top \delta_i + \alpha_{2i}\tilde{\xi}_i + \tilde{b}_i^\top \Lambda_i (\dot{\hat{b}}_i - \Lambda_i^{-1} M R_i \mu_i) \\
&\quad + \mu_i^\top M (\underbrace{\begin{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} & , v_r \Gamma_i + S(M\delta_i) \end{bmatrix} \cdot \begin{bmatrix} \tau_{iu} \\ 0 \\ \tau_{ir} \end{bmatrix}}_{\Delta_i} + M^{-1} e_i + h_{2i} + R_i^\top \hat{b}_i)
\end{aligned}$$

$$\dot{\hat{b}}_i = \Lambda_i^{-1} M R_i \mu_i \quad \Rightarrow \quad \dot{V}_{2i} = -k_{ei} e_i^\top M^{-1} e_i + e_i^\top \delta_i + \alpha_{2i}\tilde{\xi}_i - k_{\mu i}\mu_i^\top M \mu_i$$

4.2 Consensus Based Decentralized Formation

Angular velocity tracking error $z_{2i} = r_i - r_{id}$

the third Lyapunov function candidate $V_{3i} = V_{2i} + \frac{1}{2} m_{33} z_{2i}^2 + \frac{1}{2} \sigma_i \tilde{w}_{3i}^2$

$$
\begin{aligned}
\dot{V}_{3i} &= \dot{V}_{2i} + m_{33} z_{2i} \dot{z}_{2i} + \sigma_i \tilde{w}_{3i} \dot{\tilde{w}}_{3i} \\
&= -k_{ei} e_i^\top M^{-1} e_i + e_i^\top \delta_i + \alpha_{2i} \tilde{\xi}_i - k_{\mu i} \mu_i^\top M \mu_i \\
&\quad + z_{2i}(m_{33} \dot{r}_i - [0 \quad 0 \quad 1] \dot{U}_i) + \sigma_i \tilde{w}_{3i} \dot{\hat{w}}_{3i} \\
&= -k_{ei} e_i^\top M^{-1} e_i + e_i^\top \delta_i + \alpha_{2i} \tilde{\xi}_i - k_{\mu i} \mu_i^\top M \mu_i + \sigma_i \tilde{w}_{3i} (\dot{\hat{w}}_{3i} - \tfrac{1}{\sigma_i} z_{2i}) \\
&\quad + z_{2i}(-(m_{22} - m_{11}) u_i v_i - d_{33} r_i + \tau_{ir} + \hat{w}_{3i} - (0 \quad 0 \quad 1) \dot{U}_i)
\end{aligned}
$$

Update for the current estimation $\dot{\hat{w}}_{3i} = \frac{1}{\sigma_i} z_{2i}$

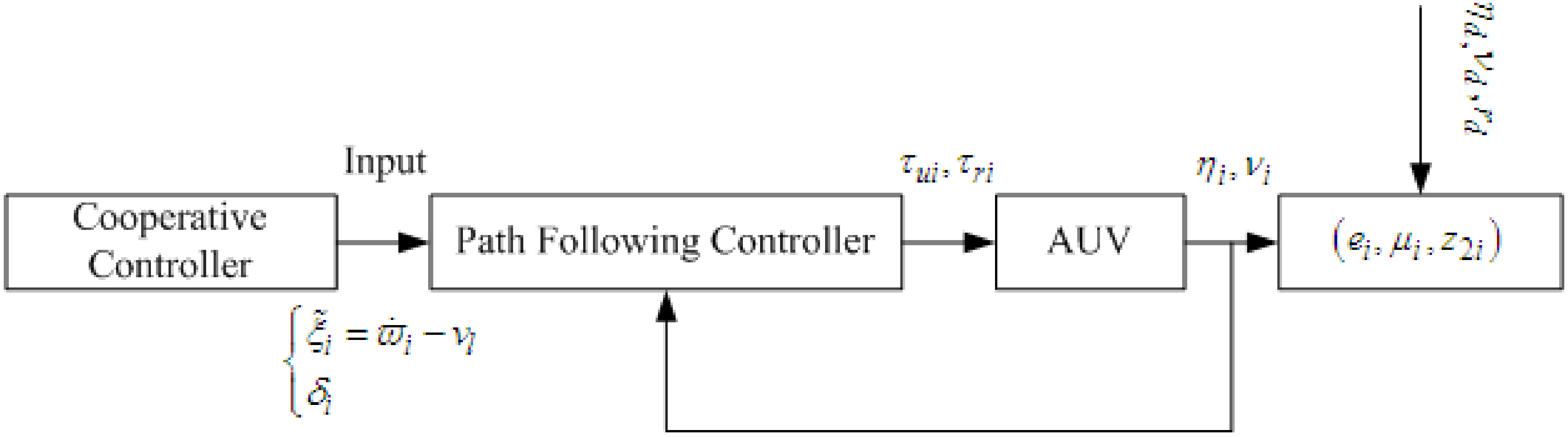
Control moment $\tau_{ir} = (m_{22} - m_{11}) u_i v_i + d_{33} r_i - \hat{w}_{3i} + (0 \quad 0 \quad 1) \dot{U}_i - k_{z2i} z_{2i}$

$$\dot{V}_{3i} = -k_{ei} e_i^\top M^{-1} e_i + e_i^\top \delta_i + \alpha_{2i} \tilde{\xi}_i - k_{\mu i} \mu_i^\top M \mu_i - k_{z2i} z_{2i}^2$$

4.2 Consensus Based Decentralized Formation

The path following error (e_i, μ_i, z_{2i}) **is Input-State-Stable (ISS) respect to** δ_i **and** $\tilde{\xi}_i$ **.**

The cascade connection of cooperative controller and path following controller



4.2 Consensus Based Decentralized Formation

Path parameter consensus design

Fixed communication topology

$$\dot{\varpi}_i = -\alpha \sum_{j=0}^{n} a_{ij}(\varpi_i - \varpi_j) - \beta \operatorname{sgn}[\sum_{j=0}^{n} a_{ij}(\varpi_i - \varpi_j)]$$

Lemma: **Assume the communication topology $\mathcal{G}$ is connected and fixed, and at least one a_{i0} is non-zero, if there is $\beta > \varpi_l$, then we have $\varpi_i(t) = \varpi_0(t)$, $t \geq \bar{t}$.**

$$\bar{t} = \frac{\sqrt{\tilde{\varpi}^\top(0)\lambda_{\max}(M)\tilde{\varpi}(0)}\sqrt{\lambda_{\max}(M)}}{(\beta - \gamma_l)\lambda_{\min}(M)}$$

Proof:

$$\dot{V} \leq -(\beta - \varpi_l)\|M\tilde{\varpi}\|_2$$

After some calculations

$$= -(\beta - \varpi_l)\sqrt{\tilde{\varpi}^\top \lambda_{\min}^2(M)\tilde{\varpi}} \leq -(\beta - \varpi_l)\sqrt{\lambda_{\min}^2(M)\|\tilde{\varpi}\|_2^2}$$

$$= -(\beta - \varpi_l)\lambda_{\min}(M)\|\tilde{\varpi}\|_2 \leq -(\beta - \varpi_l)\frac{\sqrt{2}\lambda_{\min}(M)}{\sqrt{\lambda_{\max}(M)}}\sqrt{V}$$

4.2 Consensus Based Decentralized Formation

Path parameter consensus design

Time varying communication topology

$$\dot{\varpi}_i = -\alpha \sum_{j \in \bar{N}_i(t)} b_{ij}(\varpi_i - \varpi_j) - \beta \operatorname{sgn}[\sum_{j \in \bar{N}_i(t)} b_{ij}(\varpi_i - \varpi_j)]$$

Lemma: **Assume the communication topology is time varying, and at least one follower can connect to the virtual leader all the time, if** $\beta > \varpi_l$ **then we have** $\varpi_i(t) \to \varpi_0(t)$ $t \to \infty$ **.**

Proof:

$$V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} V_{ij} + \sum_{i=1}^{n} V_{i0}$$

$$V^o = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\frac{\partial V_{ij}}{\partial \varpi_i}\dot{\varpi}_i + \frac{\partial V_{ij}}{\partial \varpi_j}\dot{\varpi}_j\right] + \sum_{i=1}^{n}\left[\frac{\partial V_{i0}}{\partial \varpi_i}\dot{\varpi}_i + \frac{\partial V_{i0}}{\partial \varpi_0}\dot{\varpi}_0\right]$$

$$\le -\alpha \tilde{\varpi}^\top \left[\hat{M}(t)\right]^2 \tilde{\varpi} - \beta \left\|\hat{M}(t)\tilde{\varpi}\right\|_1 + \dot{\varpi}_0 \left\|\hat{M}(t)\tilde{\gamma}\right\|_1$$

$$\le -\alpha \tilde{\varpi}^\top \left[\hat{M}(t)\right]^2 \tilde{\varpi} - (\beta - \varpi_\ell)\left\|\hat{M}(t)\tilde{\varpi}\right\|_1$$

4.2 Consensus Based Decentralized Formation

Path parameter consensus design

Communication with time delay

$$\dot{\varpi}_i = -\frac{1}{\sum_{j=0}^{n} a_{ij}} \sum_{j=0}^{n} a_{ij} \left\{ \dot{\varpi}_j(t-\tau) - \left[\varpi_i - \varpi_j(t-\tau) \right] \right\}$$

Lemma: **If the communication topology contains a spanning tree, then** $\tilde{\varpi}_i = \varpi_i - \varpi_0, i = 1, \ldots, n$ **converges to zero as time goes infinity.**

Proof: $V(\tilde{\varpi}) = \tilde{\varpi}^\top \tilde{\varpi}$

$$\dot{V}(\mathcal{D}\tilde{\varpi}_t) = (\mathcal{D}\tilde{\varpi}_t)^\top [-\tilde{\varpi}(t) + \mathcal{A}\tilde{\varpi}(t-\tau) + R_{fft}]$$

neutral functional differential equation-NFDE

$$= (\mathcal{D}\tilde{\varpi}_t)^\top (\mathcal{D}\tilde{\varpi}_t) + (\mathcal{D}\tilde{\varpi}_t) R_{fft}$$

$$\dot{V}(\mathcal{D}\tilde{\varpi}_t) \leq -\left\| \mathcal{D}\tilde{\varpi}_t \right\| \left(\left\| \mathcal{D}\tilde{\varpi}_t \right\| - \left\| R_{fft} \right\| \right)$$

If $\left\| \mathcal{D}\tilde{\varpi}_t \right\| > \left\| R_{fft} \right\|$***, we have*** $\dot{V}(\mathcal{D}\tilde{\varpi}_t) < 0$ **.**

4.2 Consensus Based Decentralized Formation

Stability proof of the formation system

Lyapunov function candidate

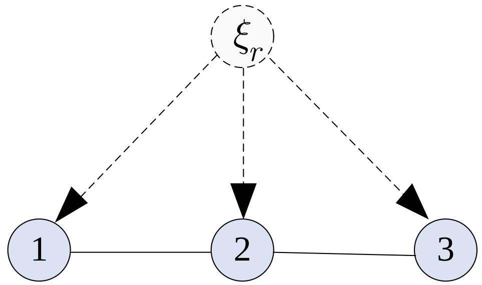
$$V_c = \underbrace{\sum_{i=1}^{n} V_{3i}}_{path\ following} + \underbrace{\frac{1}{2}\tilde{\xi}^\top M \tilde{\xi}}_{coordination}$$

$$\begin{aligned}
\dot{V}_c &\leq -\sum_{i=1}^{n}\left(\frac{1}{2}k_{ei}m\left\|e_i\right\|^2 + \frac{1}{2}k_{\mu i}\left\|\mu_i\right\|^2 + k_{z2i}\left\|z_{2i}\right\|^2 + \lambda_1\left\|\delta_i\right\|^2\right) \\
&\quad + \lambda_4\sum_{i=1}^{n}\left|\tilde{\xi}_i\right|^2 - (\beta - \varpi_l)\lambda_{\min}(M)\left\|\tilde{\xi}\right\|_2^2 \\
&= -\sum_{i=1}^{n}\left(\frac{1}{2}k_{ei}m\left\|e_i\right\|^2 + \frac{1}{2}k_{\mu i}\left\|\mu_i\right\|^2 + k_{z2i}\left\|z_{2i}\right\|^2 + \lambda_1\left\|\delta_i\right\|^2\right) \\
&\quad + \left(\lambda_4 - (\beta - \varpi_l)\lambda_{\min}(M)\right)\sum_{i=1}^{n}\left|\tilde{\xi}_i\right|^2
\end{aligned}$$

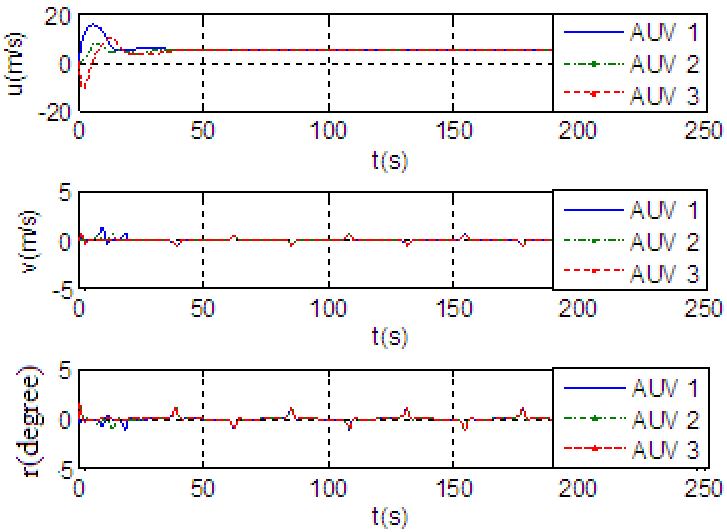
$$\dot{V}_c \leq -\sum_{i=1}^{n}\left(\frac{1}{2}k_{ei}m\left\|e_i\right\|^2 + \frac{1}{2}k_{\mu i}\left\|\mu_i\right\|^2 + k_{z2i}\left\|z_{2i}\right\|^2 + \lambda_1\left\|\delta_i\right\|^2\right) \leq 0$$

4.2 Consensus Based Decentralized Formation

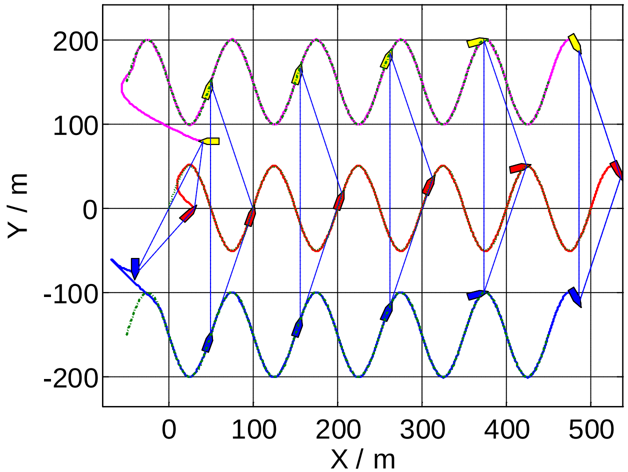
Simulation results



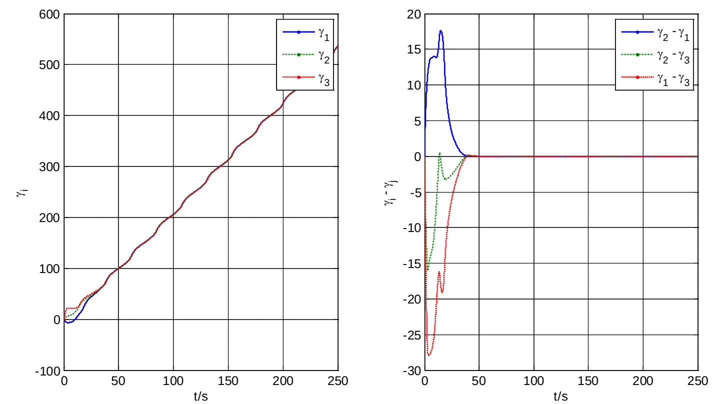
Communication topology



Velocities

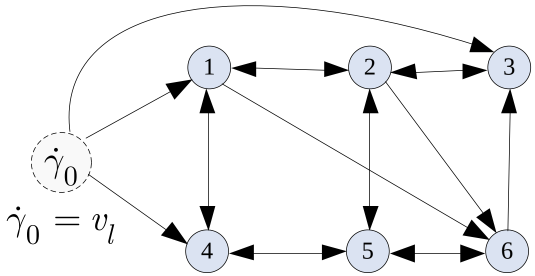


Formation trajectories

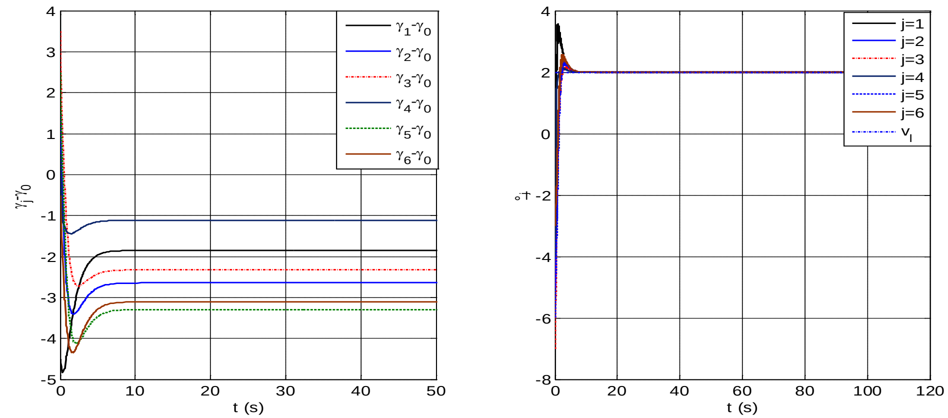


Path parameter consensus

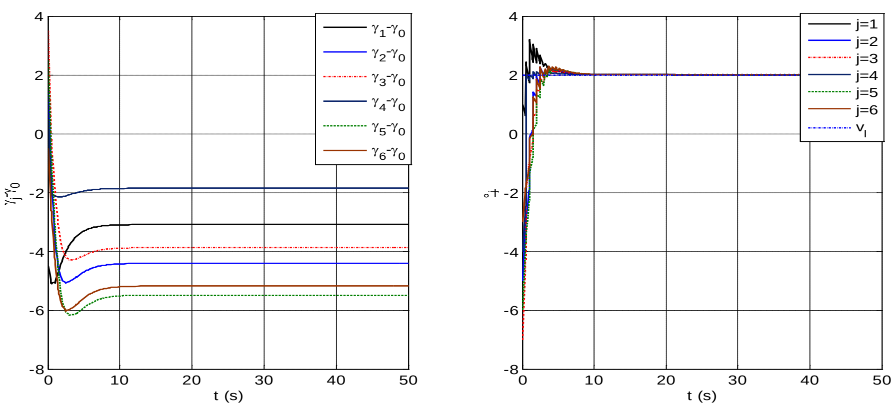
4.2 Consensus Based Decentralized Formation



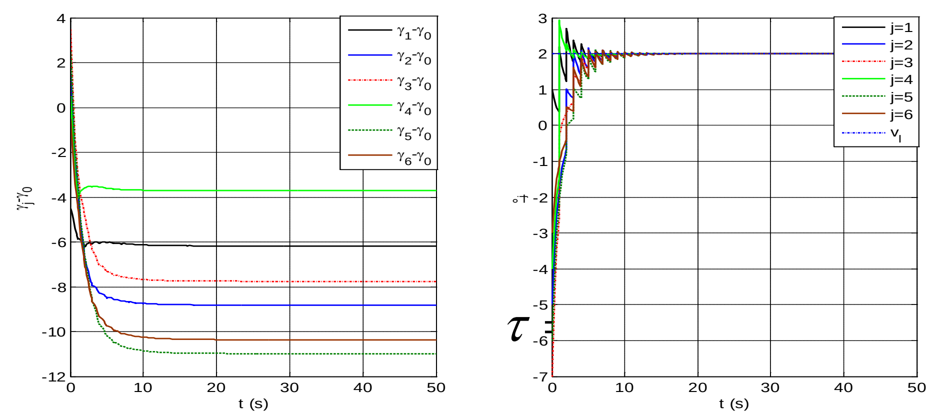
Communication topology



$\tau = 0.3s$



$\tau = 0.5s$

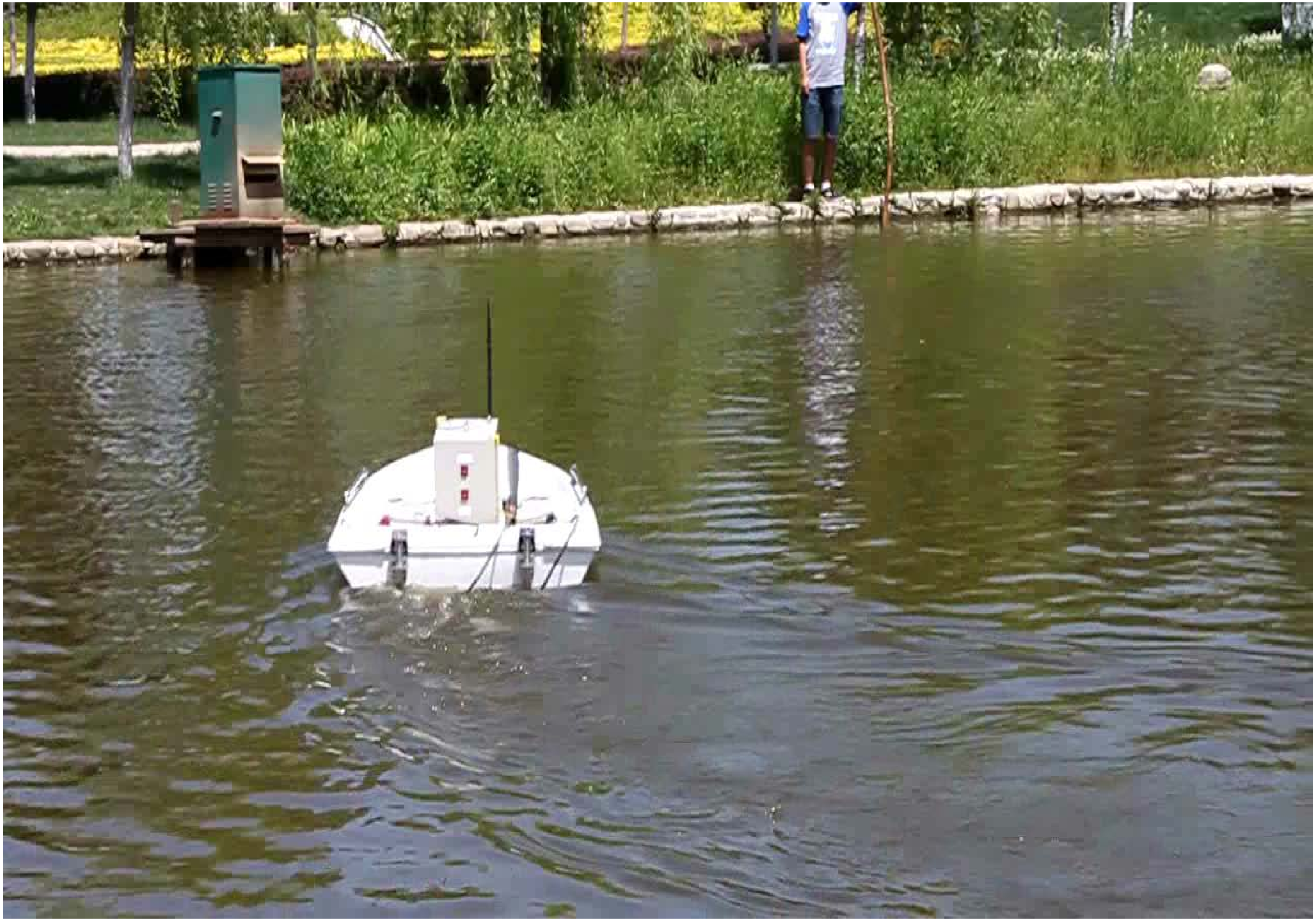


$\tau = 1s$

5 Conclusions

1. **Modeling of AUVs**
2. **Straight line trajectory tracking in 3D space**
3. **Path following in 2D and 3D space**
4. **Leader follower formation**
5. **Consensus based decentralized formation**

- **Cascaded system**
- **Serret-Frenet Coordinate Frame**
- **Lyapunov analysis and backstepping design**
- **Neural network approximation**



Thank you!