Model-Based Optimization and Control of Subsurface Flow in Oil Reservoirs

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Upstream oil industry

- Seismic imaging
- Production
- Drilling
- Reservoir modeling
- Geological modeling
Upstream oil industry characteristics

• **Capital intensive:** well: $1-100 \cdot 10^6$ US$, field: $0.1-10 \cdot 10^9$ US$

• **Uncertainty:** geology, oil price, limited data

• **Stretched in time scales:**
  • production operations: day – weeks
  • field development – years
  • reservoir management: 10s of years

• **Slow in response**

• **Many disciplines involved:** geology, geophysics, reservoir engineering, production, drilling

• **Remote**

• **New technology:** horizontal drilling, multi-laterals, time lapse seismics, smart fields ….
Oil Production

- Production from Oil Reservoirs

  - Porous rock with oil in pores
  - Geological structure heterogeneous
    - Very different rock properties within reservoir
  - $10^1 - 10^4$ km$^2$ in size
  - $10^3$m – $10^4$m underground
  - Difficult locations
Oil production mechanisms

• Primary recovery – natural flow
  (depletion drive, 5-15% recovery)

• Secondary recovery – injection of water or gas to maintain reservoir pressure and displace oil actively
  (water flooding, gas flooding, 20-70% recovery)

• Tertiary recovery – injection of steam or chemicals (polymers, surfactants) to change the in-situ physical properties (e.g. viscosity, surface tension)
  (steam flooding, polymer flooding, 20-90% recovery)
Waterflooding

- Involves the injection of water through the use of injection wells
- Goal is to increase reservoir pressure and displace oil by water
- Production is terminated when ratio between produced oil and water is no longer economically viable
Waterflooding (WF)

Essentially a batch process
  • Life time in the order of decades
  • Potential to recover 20-70% of the oil

Limited in practice by lack of operational strategy
Reservoir characteristics

- 3D
- heterogeneity of reservoir
- flow dynamics determined by geological structure (permeability)

(Gijs van Essen et al., CAA 2006)
• Smart wells allow control over (distributed) local valve settings in injectors and producers, and local measurements.
The Objective

**Inputs:** control valve settings of the wells (injectors and producers)
- Smart wells: multiple (subsurface) valves

**Outputs:** (fractional) flow rates and/or bottomhole pressures
- Smart wells: multiple (subsurface) measurement devices

**Objective:** Economic operational strategy that optimizes performance (life cycle)
Contents

• Introduction
• Closed-loop reservoir management
• Current limitations and challenges

• Robust optimization
• Balancing long-term and short-term objectives (hierarchical)
• Parameter estimation
• Time-scale separation – 2-level approach
• Control-relevant models

• Discussion
The models

Mass balance:

\[ \nabla \cdot (\rho_i u_i) + \frac{\partial}{\partial t}(\phi \rho_i S_i) = 0 \quad i = \{o, w\} \]

Momentum (Darcy’s law):

\[ u_i = -k \frac{k_{ri}}{\mu_i} \nabla p_i \quad i = \{o, w\} \]

Variables: \( p_o, p_w, S_o, S_w \)

Saturations satisfy: \( S_o + S_w = 1 \)

Simplifying assumptions, a.o.: \( p_o = p_w \)
Discretization in space and time

After discretization in space (and time):

\[
g(x_{k+1}, x_k, u_k, \theta) = 0 \quad \text{dim}(x) \approx 10^4 - 10^6
\]
\[
y_k = h(x_k)
\]
\[
y_k^T = [p_{\text{well}}^T q_{\text{well}, o}^T q_{\text{well}, w}^T]
\]
\[
x_k^T = [p_o^T S_w^T]
\]

and \( \theta \) typically the permeabilities in each grid block

- Models are large-scale
- Nonlinear
- Long simulation time
- Typically used off-line
- Actually a batch-type process
Model-based Life-Cycle Optimization

Net present value (NPV):

\[
J_K = \sum_{k=1}^{K} \left[ \frac{r_o \cdot q_{o,k}(y_k) - r_w \cdot q_{w,k}(y_k) - r_{inj} \cdot q_{inj,k}(u_k)}{(1 + b)^{t_k/t_t}} \right] \Delta t_k
\]

Optimization problem:

\[
\max_{u \in Q} J_K(u, x_0),
\]

subject to \( g(u, x) = 0, \quad x_0 = \bar{x}_0, \)

and \( c(x, u) \leq 0, \quad d(x, u) = 0 \)

Non-convex optimization, solved by gradient-based method:
Adjoint-variables calculation through backward integration of
the related (Hamiltonian based adjoint) equation.
(feasible for systems of this size)

[Ramirez, 1987; Brouwer & Jansen, SPE J, 2004]
12-well example

- 3D reservoir
- 8 injection / 4 production wells
- Period of 10 years
- High-permeability channels
- 18,553 grid blocks
- Minimum rate of 0.1 $/stb/d
- Maximum rate of 400 $/stb/d
- No discount factor
- $r_o = 20/$/stb, $r_w = 3/$/stb and $r_i = 1/$/stb
- Optimization of economic revenues (NPV)
- Model-based optimal control with a known model

(Gijs van Essen et al., CAA 2006)
Reactive versus optimal control

**Reactive Control**

**Optimal Control**

---

**Cumulative Data**

- **Oil Production:** $0.00 \times 10^6 \text{ bbl}$
- **Water Production:** $0.00 \times 10^6 \text{ bbl}$
- **Water Injection:** $0.00 \times 10^6 \text{ bbl}$

- **Revenue:** $0.0 \text{ M\$}$

---

**Cumulative Data**

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- **Water Injection:** $0.00 \times 10^6 \text{ bbl}$

- **Revenue:** $0.0 \text{ M\$}$

---

`time = 0.00 \text{ year}`

---

TU/e

Technische Universiteit Eindhoven

University of Technology
Reactive versus optimal control

Net Present Value

Reactive Control

Optimal Control

time = 0.00 year
Closed-loop Reservoir Management

• Moving from (batch-wise) open-loop optimization to on-line closed-loop control
  • State estimator
  • Optimized plant input through NMPC in receding/shrinking horizon
  • No trajectory following but dynamic RTO

But how about the model?
Obtaining a model

- **First-principle models** (geology) are highly uncertain

- Opportunities for **identification** are limited
  (nonlinear behaviour dependent on front-location, single batch process, experimental limitations)

- Option: **estimate** physical parameters (permeabilities) in first principles model; starting with initial guess
Closed-loop Reservoir Management

Several options for nonlinear state and parameter estimation:

Available from oceanographic domain:

Ensemble Kalman filter (EnKF) (Evensen, 2006)

- Kalman type estimator, with analytical error propagation replaced by Monte Carlo approach (error cov. matrix determined by processing ensemble of model realizations)
- Ability to handle model uncertainty (in some sense)
- In reservoir engineering used for estimation of states and parameters (history matching)

Note: # parameters = # states
Ensemble Kalman Filter

• As prior information an ensemble of initial states \( \{\hat{x}_k|k\} \) is generated from a given distribution.

• By simulating every ensemble member, corresponding ensembles \( \{\hat{x}_{k+1}|k\} \) and \( \{\hat{y}_{k+1}|k\} \) are generated, and stored as columns of matrices \( \hat{X} \) and \( \hat{Y} \) respectively.

• The measurement update of a EKF is applied to every element of the ensemble, where the covariance matrices are replaced by sampled estimates on the basis of \( \hat{X} \) and \( \hat{Y} \).

• The update becomes: 
  \[
  \hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}[y_{k+1} - \hat{y}_{k+1|k}],
  \]
  where \( K_{k+1} \) is given by:
  \[
  K_{k+1} = \hat{X}\hat{Y}^T \cdot [\hat{Y}\hat{Y}^T + R]^{-1}
  \]
  (BLUE)

• The result is a new ensemble \( \{\hat{x}_{k+1|k+1}\} \).
Closed-loop Reservoir Management
Closed-loop simulation example

- 3 study cases: reactive control, optimal open-loop control based on perfect (‘reality’) model, optimal closed-loop control
**Closed-loop simulation example**

Parameter updates at different times

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<th>Layer</th>
<th>Real</th>
<th>t = 0</th>
<th>t = 240</th>
<th>t = 390</th>
<th>t = 690</th>
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Closed-loop reservoir management

Relatively poor looking models may work quite well!
Limitations and challenges

• Model uncertainty: uncertain geology
• Model complexity: geological vs control models
• Measurement data: limited knowledge
• Nonlinearity: dynamics change over lifetime
• Process configuration: (dynamic) well placement
• Introduction
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• Control-relevant models

• Discussion
Robust optimization

- Reservoir models / permeability structure are highly uncertain

- Use 100 realizations in a max-mean approach:

\[ \max_u \left( \frac{1}{M} \sum_{i=1}^{M} J_K(u, \theta_i) \right) \]

[Gijs van Essen et al., Proc. CCA, 2006]
Robust optimization example

3 control strategies applied to set of 100 realizations:
- reactive control, nominal optimization (100 strategies), robust optimization

(each strategy is applied to 100 “systems”)

[Van Essen et al, SPE J, 2009]
Hierarchical optimization

- Focussing on life-time (long-term) NPV has limitations:
  - Compromise in short-term production
  - Erratic operational strategy

[Van Essen et al, SPE J, 2009]
Hierarchical optimization

• Distinguish two different objective functions:

\[ u^* = \arg \max_u J^{(1)}_K(u, x_0) \]

with \( J^{(1)}_K \) the long-term NPV

• Additionally optimize for short-term production:

\[ \tilde{u}^* = \arg \max_u J^{(2)}_K(u, x_0) \]

such that \( J^{(1)}_K(u, x_0) \geq J^{(1)}_K(u^*, x_0) - \varepsilon \)

with \( J^{(2)}_K \) the short-term NPV

Utilize degrees of freedom in the input to optimize short-term production

[Van Essen et al, SPE J, 2011]
Objective function with ridges
Hierarchical optimization example

Optimization of secondary objective function - constrained to null-space of primary objective

Optimization of secondary objective function - unconstrained

![Graphs showing iterative changes in Net Present Value for primary and secondary objectives with and without discounting.]
Hierarchical optimization example

Short-term NPV

long term NPV

short-term NPV under constraints on long-term NPV

[Van Essen et al, SPE J, 2011]
Hierarchical optimization example

Long-term optimized

Short-term constrained optimized
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Parameter Estimation

Typical approach in geological models:

- Use EnKF to estimate states
- Consider parameters (grid-block permeabilities) as extended states
- Estimate parameters and states, based on an initial ensemble
  - Data not sufficiently informative to estimate all parameters
  - Parameters are updated only in directions where data contains information

Result and reliability is crucially dependent on initial state/model
Bayesian approach:

\[ V_p(\theta) := V(\theta) + \frac{1}{2}(\theta - \theta_p)P_{\theta_p}^{-1}(\theta - \theta_p) \]

Lack of identifiability: Hessian of \( V \) is poorly conditioned

- The Bayesian estimate becomes heavily determined by priors

Parametrization can be reduced by projecting unto the (locally) identifiable subspace  
[Van Doren et al, IFAC 2008,2011]

Several alternatives for reduction of parameter space  

Capturing long-term behaviour (nonlinear) is the challenge
Testing local identifiability in model estimation

- Consider quadratic identification criterion based on prediction errors

\[ V(\theta) := \frac{1}{2} \epsilon(\theta)^T P_v^{-1} \epsilon(\theta), \quad \epsilon(\theta) = y - \hat{y} = y - h(\theta, u; x_0), \]

- Hessian given by

\[ \frac{\partial^2 V(\theta)}{\partial \theta^2} = \frac{\partial \hat{y}^T}{\partial \theta} P_v^{-1} \left( \frac{\partial \hat{y}^T}{\partial \theta} \right)^T + S \]

- Local identifiability test in \( \hat{\theta} = \arg \min V(\theta) : \) Hessian > 0

- With quadratic approximation of cost function around \( \hat{\theta} \):

  Hessian given by

\[ \frac{\partial \hat{y}^T}{\partial \theta} P_v^{-1} \left( \frac{\partial \hat{y}^T}{\partial \theta} \right)^T \]
Testing local identifiability in identification

- Rank test on Hessian through SVD

\[
\frac{\partial \hat{y}^T}{\partial \theta} P_v^{-\frac{1}{2}} \bigg|_{\theta = \hat{\theta}} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}
\]

- If \( \Sigma_2 = 0 \) then lack of local identifiability

- SVD can be used to reparameterize the model structure through

\[
\theta = U_1 \rho, \quad \dim(\rho) << \dim(\theta)
\]

in order to achieve local identifiability in \( \rho \)

- Columns of \( U_1 \) are basis functions of the identifiable parameter space
Testing local identifiability in identification

\[
\frac{\partial \hat{y}^T}{\partial \theta} P_v^{-\frac{1}{2}} \bigg|_{\theta = \hat{\theta}} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}
\]

• What if \( \Sigma_2 \neq 0 \) but contains (many) small singular values?

No lack of identifiability, but possibly very poor variance properties

• Identifiability mostly considered in a yes/no setting: qualitative rather than quantitative [Bellman and Åström (1970), Grewal and Glover (1976)]

• Approach: **quantitative** analysis of appropriate parameter space, maintaining physical parameter interpretation
Approximating the identifiable parameter space

Asymptotic variance analysis: \[ \text{cov}(\hat{\theta}) = J^{-1} = \left( \mathbb{E} \left[ \frac{\partial^2 V(\theta)}{\partial \theta^2} \right] \right)^{-1} \]

with \( J = \text{Fisher Information Matrix} \).

- Sample estimate of parameter variance, on the basis of \( V(\theta) \):

\[
\text{cov}(\hat{\theta}) = \begin{cases} 
\left[ \begin{array}{cc} \Sigma_1^{-2} & 0 \\ 0 & \Sigma_2^{-2} \end{array} \right] \left[ \begin{array}{c} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{array} \right] & \text{for } \Sigma_2 > 0 \\
\infty & \text{for } \Sigma_2 = 0
\end{cases}
\]

\[
\text{cov}(\mathbf{U}_1 \hat{\rho}) = \mathbf{U}_1 \Sigma_1^{-2} \mathbf{U}_1^T
\]

\[ \text{cov}(\hat{\theta}) > \text{cov}(\mathbf{U}_1 \hat{\rho}) \quad \text{if } \Sigma_2 > 0 \]
Discarding singular values that are small, reduces the variance of the resulting parameter estimate.

Particularly important in situations of (very) large numbers of small s.v.’s.

Model structure approximation (local).

Quantified notion of identifiability – related to parameter variance.

\[ \text{cov}(\hat{\theta}) > \text{cov}(U_1\hat{\rho}) \quad \text{if } \Sigma_2 > 0 \]
• Interpretation:
  Remove the parameter directions that are poorly identifiable (have large variance)

• This is different from removing the (separate) parameters for which the value 0 lies within the confidence bound [Hjalmarsson, 2005]
Simple reservoir example

2D two-phase example
(top view)

21 x 21 grid block permeabilities
5 wells; 3 permeability strokes

1 injector (centre)
4 producers (corners)

5 inputs: 1 injector flow-rate, and 4 bottom hole pressures
8 outputs: producer flow rates (water and oil)
Simple reservoir example

Using the reduced parameter space –iteratively- in estimation:

<table>
<thead>
<tr>
<th>exact field</th>
<th>initial estimate</th>
<th>final estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(local point)</td>
<td></td>
</tr>
</tbody>
</table>

Observation:

Only grid block permeabilities around well are identifiable.
Time-scale separation – Two-level approach

Reasoning

• Optimization on the basis of nonlinear reservoir models suffers from model uncertainties

• Optimization on the basis of identified (linear) models suffers from a lack of predictive capabilities beyond the –local– measurement interval

Combine the two
Two-level approach
Example: 3D reservoir

local grid refinement

‘truth’ model
- time step size: 0.25 days
- 8 injection wells, 4 production wells

modeling error in main flow direction

reservoir model
- time step size: 30 days
- Modeling error due to geological uncertainty & undermodeling of fast, local dynamics
Example: 3D reservoir

3 production strategies

1. Reactive control
   • Maximal injection rates/minimal bottom-hole pressures
   • Shut-in wells when watercut >0.90

2. Open-loop life-cycle optimization
   • Optimize inputs based on reservoir model
   • Apply to ‘truth’ model

3. Combined dynamic optimization & MPC control
   • Life-cycle optimization on reservoir model to obtain references
   • Excitation on ‘truth’ model to identify low-order model
   • MPC on ‘truth’ model to track references
Low-order linear modeling (system id)

Persistently exciting inputs = Injection rates & Producer BHP's

System Identification → identified model

Virtual asset

Liquid production flow rates
Example: Identification Experiment

Input excitation for identification

sub-space identification

Simulation fit of \(8^{th}\) order identified model
**Example: Results**

<table>
<thead>
<tr>
<th>Case</th>
<th>NPV</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: Reactive Control</td>
<td>550 M$</td>
<td>-</td>
</tr>
<tr>
<td>Case 2: Open-loop Optimization</td>
<td>558 M$</td>
<td>+1.5%</td>
</tr>
<tr>
<td>Case 3: Two-level Control</td>
<td>594 M$</td>
<td>+8.0%</td>
</tr>
<tr>
<td>Maximum based on reservoir model</td>
<td>596 M$</td>
<td>+8.4%</td>
</tr>
</tbody>
</table>

[Van Essen et al., CDC 2010]
Example: Results

Production rates at the 4 producers

- Optimized model output traj
- Designed input applied to plant
- MPC tracking controlled plant
Control relevant models

• What are the important physical phenomena in the reservoir that are essential for the optimized operational strategy?

• There is a serious gap between the reservoir models with geological relevance, and goal-oriented models that are fit for control/optimization.
Summary

• Challenging problems in model-based operation on the basis of highly uncertain information
• Systems and control tools play an important role
• Size of the prize: .... field tests

• Key elements:
  • Model-based optimization under physical constraints and geological uncertainties
  • Appropriate merging of physical and measured data in low-order reliable and goal-oriented models
  • Capturing the essential non-linear behaviour of reservoirs
  • Challenging parametrization issues, in relation to control-relevance and identifiability (control-relevant geological models?)
  • Learning the optimal strategy in one shot (batch)
The Team:

- Jan Dirk Jansen
- Arnold Heemink
- Gijs van Essen
- Paul Van den Hof
- Sippe Douma
- Okko Bosgra
- Malgorzata Kaleta
- Jan
- Jorn Van Doren
- Remus Hanea
- Ali Vakili
- Mariya Krymskaya
Further reading


• Several control-related papers and theseses available at: www.dcsc.tudelft.nl/~pvandenhoof/publications.htm or through: p.m.j.vandenhof@tue.nl
Thank you for your attention

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