SUPERNETWORK AND ITS APPLICATION

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We are living in an era of networks. Networks in different forms build the backbones of our societies and economies.

Different physical networks; i.e., transportation and logistical networks, communication networks, energy and power networks has been transformed our societies and economies to more abstract networks comprising: financial networks, environmental networks, social, and knowledge networks, as well as therecombinations. Today people benefit from the networks in traveling, transportation of goods, communication, finance, etc. At same time suffer from infection of diseases, spread of viruses, and break down of the whole power systems by local fault.
Networks in different areas now interweave into complex networks of networks, called **supernetworks**.
The origins of supernetworks can be traced to the study of transportation networks, telecommunication networks, and, interestingly, to biology, as well as the interwoven networks.

In order to investigate the network of networks, Y. Sheffi used the concept and term “hypernetwork”, which he later redefined as a “supernetwork.”

According to A. Nagurney, “the super networks may be thought of as networks that are above and beyond existing networks, which consist of nodes, links, and flows, with nodes corresponding to locations in space, links to connections in the form of roads, cables, etc., and flows to vehicles, data, etc.” This is the first clear describing of supernetwork.
In the present stage, the supernetwork is only a concept with fuzzy boundary, and has not yet exact definition.

The supernetworks have at least one of the following features:

- Network of networks
- Multi-tiered
- Multi-level
- Multi-mode network flows
- Congestion
- Alternative behavior of users of the network
- Multi-criteria
Supernetwork concept has a wide range of applications and only a small part of those applications has been explored thus far. Some specific applications of supernetworks are: supernetworks consisting of social networks interacting with supply chain networks, supernetworks consisting of social networks interacting with financial networks, and knowledge supernetworks.

This framework captures the different interacting networks in one model. It allows one to compute optimal solutions under different scenarios and to test how the equilibrium will change when certain cost and benefit functions are changed.
Nowadays there appear a lot of so-called “system of systems”.

The “system of systems” has the following main features:

- Operational independence
- Managerial independence
- Geographic distribution
- Heterogeneity
- Evolutionary and emergent behaviors

If the systems can be modeled as networks, the system of systems can be modeled as supernetwork.
Typical examples of “system of systems”.
- Sustainable environment management
- Transportation system
- Communication and navigation in space
- Renewable energy system
- Global service system
- Healthcare system
Application areas of supernetwork models

- Supply Chain Networks with Electronic Commerce
Application areas of supernetwork models

➢ We have developed a close-loop supply chain network model and optimization method.
Application areas of supernetwork models

- Teleshopping/Shopping Decision-Making
Application areas of supernetwork models

- Telecommuting/Commuting Decision-Making
In a world influenced by ever growing networks, new paradigms are necessary. The supernetwork framework provides us with tools to study interrelated networks. It allows for the application of efficient algorithms for computation, and it provides visual aids to see the dynamic changes.

Tools That Have Been Using:

- Network theory;
- Optimization theory;
- Game theory;
- Variational inequality theory;
- Projected dynamical systems theory;
- Network visualization tools
Supernetworks Model

- Formal definition of Supernetworks Model

\[ S = (N, R, N', R', F, E, Re) \]

- \( N \) is the set of nodes with the characteristic of networks and directly belonging to the whole supernetwork model

- \( N' \) is the set of nodes in each sub-network. All the nodes are confined within the corresponding boundary of sub-network system
\[ S = (N, R, N', R', F, E, Re) \]

- \( R \) is the set of hierarchy relationships among the nodes in different sub-networks. The fundamental concern of these hierarchy relationships is the balance between coordination and autonomy.

- \( R' \) represents the set of all the relationships among the nodes in the same sub-network.
**S** = \((N, R, N', R', F, E, Re)\)

- The implications of \(F, E\) and \(Re\) is the same as that of traditional system theory (Churchman, 1968; Gad&Michael, 1985), that is function, environment and resources

- The **function**, or **role** of a system represents its intended impact on its environment

- The **environment** is the set of entities and conditions outside of the system boundary that affects the system or is affected by it

- The system **resources** are the elements that used or consumed in building and operating the system
Building of Supernetworks Model

- Identification of the theme of supernetwork model
- Identification of $N$ related to the theme
- Identification of $N^*$ for all the sub-networks (N)
- Identification of $R^*$ for all the sub-networks (N)
- Modeling of all the sub-networks
- Building of supernetwork model
Step 1: Identify the theme of supernetwork. This is a critical step and should not be ignored. The theme determines the range of system and is also helpful to the following tasks;

Step 2: Identify the nodes (N) directly belonging to the theme acquired from Step 1. It’s necessary to get the related experts involved in this step;

Step 3: Identify the hierarchy relationships (R) among the sub-networks;
Step 4: Identify the nodes types and specific representations (N’) for each sub-network;

Step 5: For each sub-network, identify all the relationships (R’) among the nodes;

Step 6: According to the nodes types and the relationships acquired from step 4 and 5, build the network model for each sub-network;

Step 7: Build the supernetwork model with the hierarchy relationships R and all the sub-networks model having been built.
Knowledge intensive organizations are typical supernetworks. Today, such as news organizations, intelligence agencies, and/or global financial institutions, are facing numerous challenges. Examples of some major challenges include: how to respond in a timely manner to new events, how to ensure the efficiency of their various production allocation processes, and how to best manage the scale and scope of their coverage and ultimate reach which is global in dimension.
The knowledge supernetwork consists of three networks

\[ G_p = (P, E_{p-p}) \]

\[ G_k = (K, E_{k-k}) \]

\[ G_m = (M, E_{m-m}) \]

\[ KSN = f(G_k, G_p, G_m) \]

\[ = (K, P, M, E_{k-k}, E_{p-p}, E_{m-m}, E_{p-k}, E_{p-m}, E_{m-k}) \]
Knowledge Organization Model Based on Supernetworks

- Types of nodes and relationships in the sub-networks

<table>
<thead>
<tr>
<th>Sub-Networks</th>
<th>Nodes Types</th>
<th>Relationships Types</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collaboration</td>
<td>Knowledge</td>
<td>Collaboration</td>
</tr>
<tr>
<td>Network</td>
<td>Personnel</td>
<td></td>
</tr>
<tr>
<td>Concept Network</td>
<td>Concepts</td>
<td>Semantic</td>
</tr>
<tr>
<td>Topic Network</td>
<td>Topics</td>
<td>Affiliation</td>
</tr>
<tr>
<td>Resource Network</td>
<td>Knowledge</td>
<td>Association</td>
</tr>
<tr>
<td></td>
<td>Resources</td>
<td></td>
</tr>
</tbody>
</table>
APPLICATIONS IN KNOWLEDGE SYSTEMS

in the collaboration network. The optimization problem can, hence, be expressed as:

\[
\text{Maximize } \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{O} u_{ijk}(x_{ijk}, x_{jik})
\]

subject to:

\[
\sum_{j=1}^{N} \sum_{k=1}^{O} t_{ijk}x_{ijk} \leq T_i, \quad i = 1, \ldots, N,
\]

\(t\) – actual time spent

\(T\) – time budget

\(u\) – utility of collaborating researcher

\(x \in \mathbb{R}^{N \times N \times O}_+\).
In some cases the use of simple or directed graphs to represent complex networks does not provide a complete description of the real-world systems under investigation. For instance Figure (a) shows three binary telephone conversation relations between 3 persons.

The conference call structure of Figure (b) is that of a 3-ary relation which enables everyone to hear what the others say, Pairwise interaction is an inefficient way for three people to communicate.
Another example is in a collaboration network represented as a simple graph. We only know whether scientists have collaborated or not, but we can not know whether three or more authors linked together in the network were coauthors of the same paper or not.

A possible solution to this problem is to represent the collaboration network as a bipartite graph in which a disjoint set of nodes represents papers and another disjoint set represents authors. However, in this case the “homogeneity” in the definition of nodes is lost. This distinction between two classes of nodes with completely different interpretations may lead to artifacts in the data.
A natural way of representing these systems is to use a generalization of graphs known as hypergraphs.

In a graph a link relates only a pair of nodes, but the edges of the hypergraph — known as hyper-edges — can relate groups of more than two nodes. Thus, we can represent the collaboration network as a hypergraph in which nodes represent authors and hyper-edges represent the groups of authors that published papers together.

A hypergraph is represented by a pair \( H=(V,E) \), where \( V \) is the set of \( n \) nodes; \( E \) is the set of \( m \) hyper-edges. Each hyper-edge is a subset of \( V \).

A complex system represented by hypergraph can be called “Hybernetwork”.

"Hypernetwork"
4 arches assembled from 8 blocks by different combinations

Incidence matrix for the assembly

<table>
<thead>
<tr>
<th></th>
<th>b₁</th>
<th>b₂</th>
<th>b₃</th>
<th>b₄</th>
<th>b₅</th>
<th>b₆</th>
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The bipartite graph of the assembly relation

The hypergraph defined by the elements of A being related to subsets of B
We can also define the dual hypergraph by the elements of B being related to subsets of A like

A complex system represented by hypergraph can be called “Hypernetwork” (Estrada and Rodríguez-Velázquez).

“Hypernetwork” and “Supernetwork” all belong to network of networks and conceptually have the same or similar meaning. We can consider them as synonyms. But the supernetwork has more broad covering than hypernetwork.
Using Hypergraph Representations for Documents and Terms:

In the knowledge organizing works, people often need to represent the relations between documents and related terms. The relations often have the many to many features and ordinary graphs are insufficient to represent them. The hypernetwork excels in this aspect.

If we disregard the structure of text documents, we can view any collection of documents as hypergraph $H=(T,D)$, where each node $t$ of $T$ corresponds to a term, each hyperedge $d$ of $D$ corresponds to a document.
In the hypergraph $H=(T,D)$, a hyperedge $d$ is a multiset with elements in $T$, representing the abstraction of a document as a bag of terms. We call this a document-centered hypergraph. For example,

$H = (T,D) = (\{1,2,3,4\}, \{A,B,C\})$ and $A = \{1,2\}, B = \{2,2\}, C = \{2,3,4\}$, the hypergraph is like:

- Circles represent hyperedges
- Triangles represent nodes.
- The value associated with the connection stands for the number of occurrences of the node in the hyperedge
Corresponding to $H=(T,D)$, there is a dual hypergraph $H^*=(D,T)$, whose nodes correspond to documents and hyperedges correspond to terms. It is a term-centered hypergraph like:

- Circles represent hyperedges
- Triangles represent nodes

The hypergraph or hypernetwork representation is convenient for the knowledge organizing at the lower level.
The weighted hypergraph can be created by the consideration of the importance of terms (documents).
The hypergraphs can be combined and integrated by similarity and cooccurrence relations.
Networks in different forms, as the backbones of our societies and economies, are now weaving more and more together. New supernetworks are growing up everyday.

Up to now, the supernetwork is only a concept with ambiguous boundary, and without a common exact definition.

We can find and clarify the unique features of supernetworks as a starting point to define it. Researches on complex network offer some successful experiences.

A very important feature of supernetwork is the integration of physical network (railway, highway, communication network, etc) with the virtual network (information network, knowledge network, social network, etc). This approach can be widely applied to Electronic Commerce, Electronic Government, Management of Innovation, etc.
In the past 10 years, research works around the supernetwork were focused on the normal operations and optimizations.

Recently natural disasters, terrorist attacks and power blackouts impressively and dramatically demonstrated the vulnerability of our societies and economies when networks stop functioning.

This is due to our dependence on different network systems and their interdependence. Furthermore, these events have also shown how immense many networks are and that national borders seldom restrict them in their impacts.

It is crucial task to investigate the robustness and vulnerability of the supernetwork for the emergency control.
At the theoretical level, the modeling of supernetwork may bridge the gap between the lump-parameter model and distributed-parameter model. There are some complex objects with loose-coupled components. They can not be described by lump-parameter model, and also without continuum characteristics that can be modeled by distributed-parameter model. Network approach, especially the supernetwork concept, may be the ideal way to tackle such problem.

At the methodological level, research work on supernetwork may help the people to understand “why the complex system is complex”. 
Traditional approaches of system modeling includes discrete system (with lumped parameter) and continuum system (with distributed parameter) in essence.

Two frequently adopted means:

- discretization of continuum system
- continuation of discrete system
Both the discretization of continuum system and continuation of discrete system are approximate treatments and will inevitably lose some information.

The appearance of complex network, especially the supernetworks, provides a possible route to access the modeling of the complex system with information loss as low as possible.
References:

- Supernetworks: An Introduction to the Concept and its Applications with a Specific Focus on Knowledge Supernetworks, Anna Nagurney and Tina Wakolbinger, 2004; see http://supernet.som.umass.edu

We have published a book on supernetwork in Chinese:
THANK YOU