Grinding mill modelling and Control: past, present and future

Ian Craig

Department of Electrical, Electronic
and Computer Engineering

University of Pretoria
South Africa

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Denkleiers • Leading Minds • Dikgopolo tša Dihlalefi

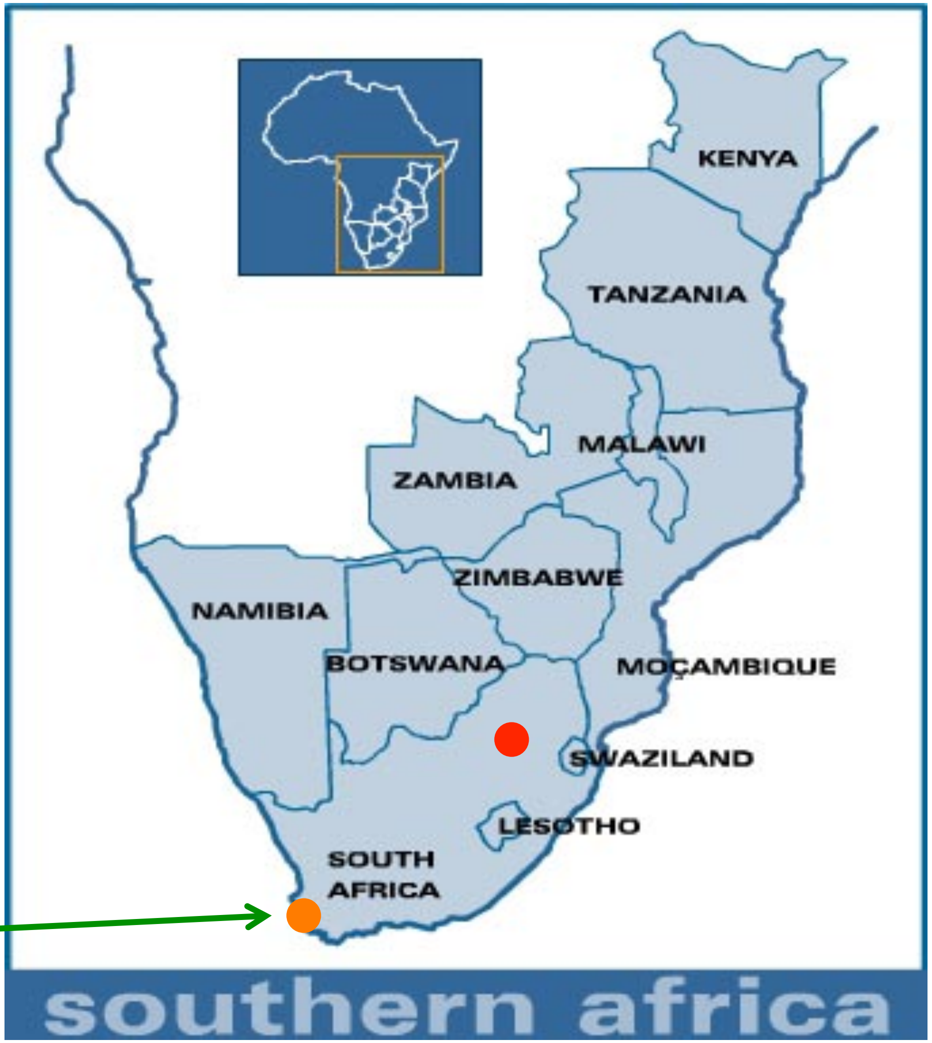
Pretoria

Latitude:
25° 43′ South

Longitude:
28° 11′ East

Elevation:
1,370 m

Time:
GMT +2 hours

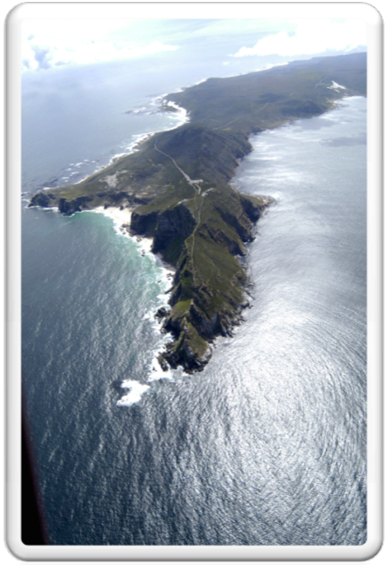


Cape Town

1,500 km

19th IFAC World Congress
Cape Town, 24-29 Aug. 2014

www.ifac2014.org

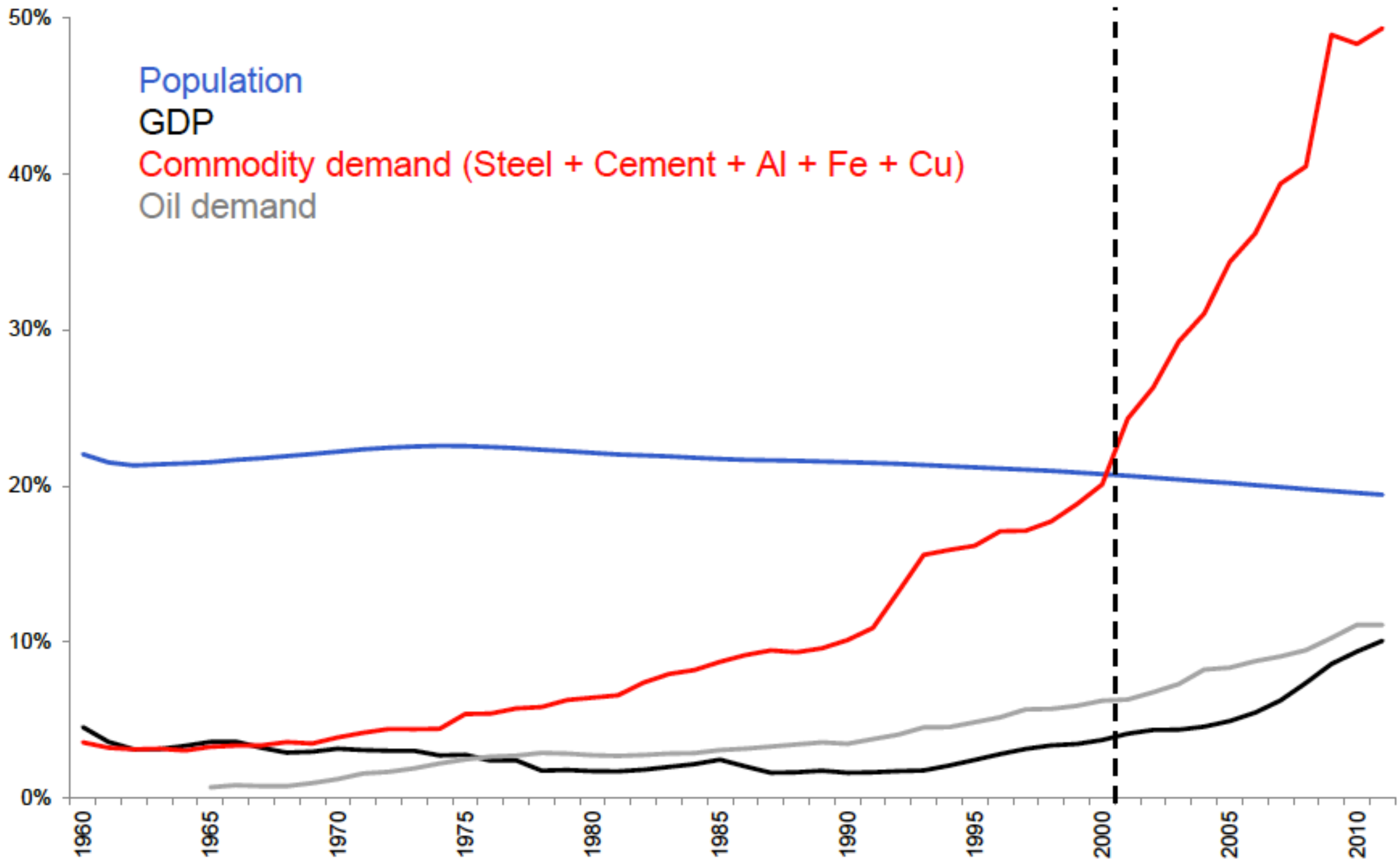


CAPE TOWN
& Western Cape

Contents

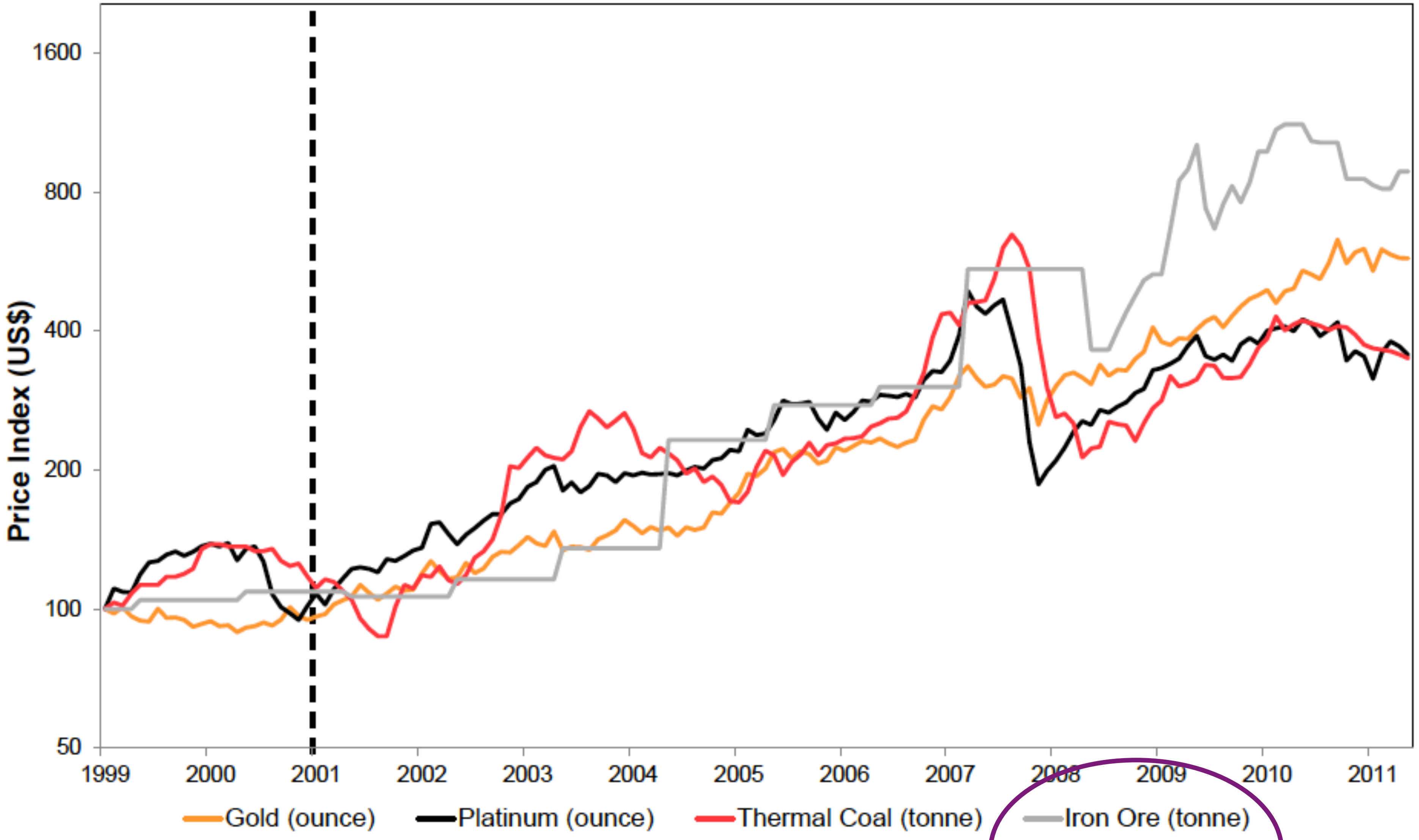
- The big picture for commodity prices
- Why study grinding mill control?
- Mineral processing and grinding process overview
- Grinding circuit modelling
  - Simplified grinding circuit model for control studies
- History and current status of grinding mill control
- Grinding circuit control objectives
- Control examples:
  - μ-synthesis
  - Robust nonlinear MPC
- Current research projects
- Future research challenges

China's share of the world (1960-2011)*



* Worldbank, BP, UBS, Allan Gray Proprietary Limited

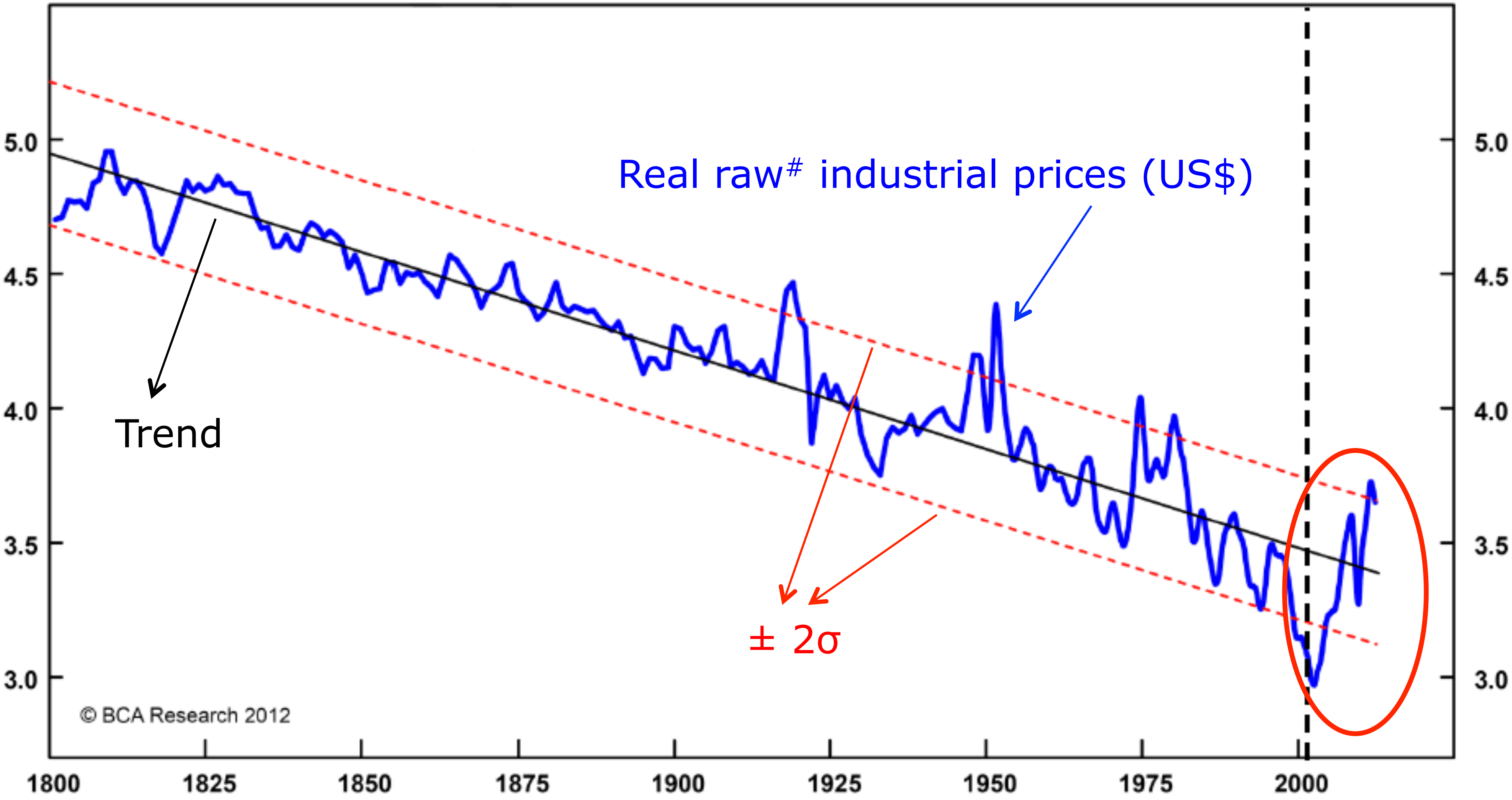
Key commodity prices (1999-2011)*



10-yr CAGR: 18.8% 11.3% 12.7% 22.5%

* I-Net Bridge, UBS, Allan Gray Proprietary Limited

Long-term commodity prices (1800-2011)*



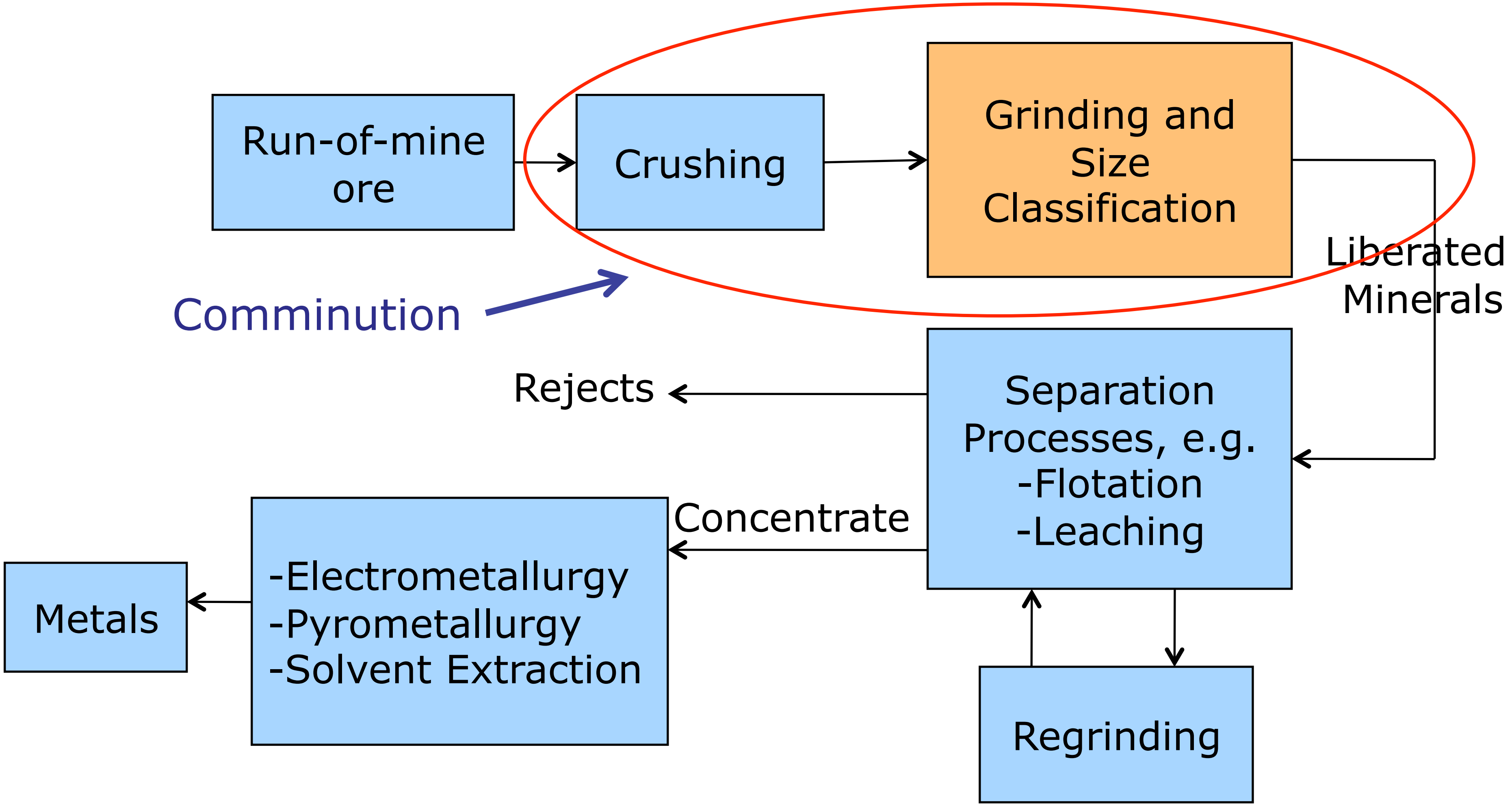
Adjusted by U.S. GDP deflator; shown as natural logarithm

* BCA Research, Allan Gray Proprietary Limited

Why grinding mill control?

- Mineral processing processes make for challenging control problems:
  - Poor process models
  - Large (unmeasured) disturbances
  - Lack of on-line measurements
  - Difficult to establish quantitative economic control objectives
- Costly and energy intensive process
- Part of a team that introduced advanced process control to the grinding community
- Close links with industry

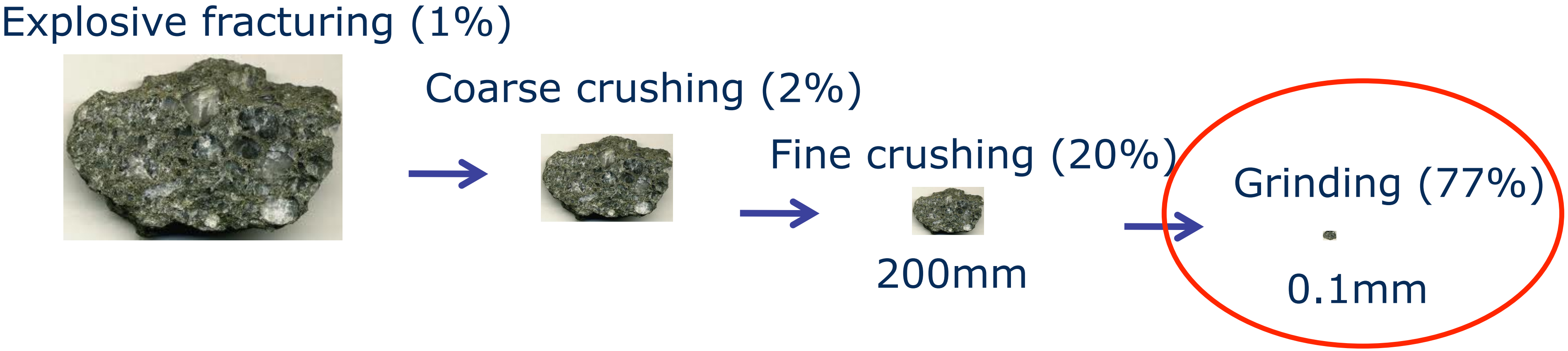
Chain of processes in mineral processing and metallurgical plants*



* Adapted from Hodouin, D., *Journal of Process Control*, 21 (2011), 211-225.

Cost of comminution

- Fifty percent of mineral processing operating cost associated with comminution (crushing and grinding)
- World wide cost of comminution*
  - Energy consumed =26 billion US$/year
  - Wear parts consumed = 5 billion US$/year
- Average energy consumption (kWh/t): 2.2 for crushing, 11.6 for grinding, and 2.6 for separation
- Typical breakdown of comminution costs*:


Explosive fracturing (1%)

Coarse crushing (2%)

Fine crushing (20%)

200mm

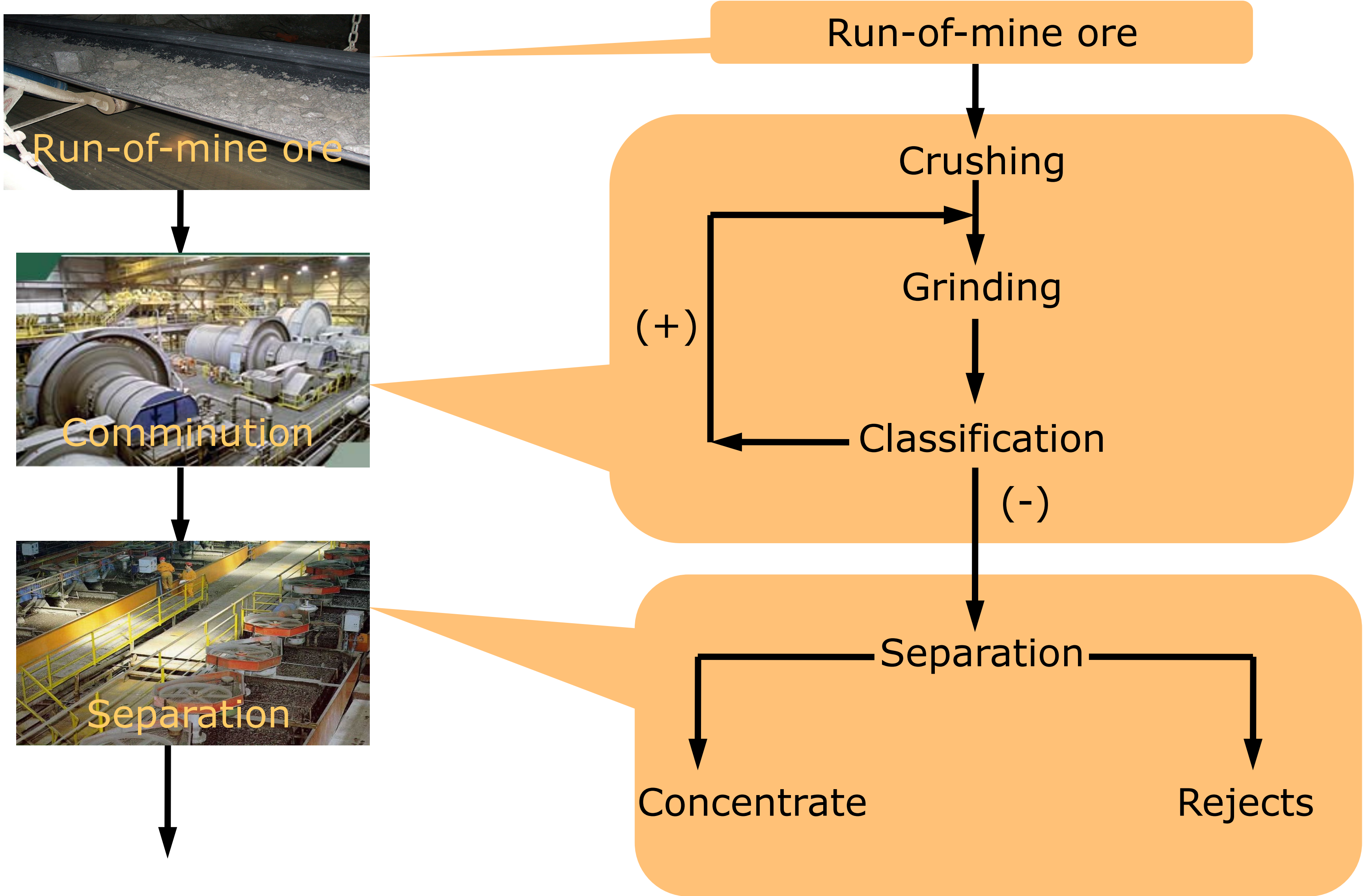
Grinding (77%)

0.1mm

---

* JA Herbst and Associates, 2002.

Mineral processing steps*



Run-of-mine ore

Crushing

Grinding

(+)

Classification

(-)

Separation

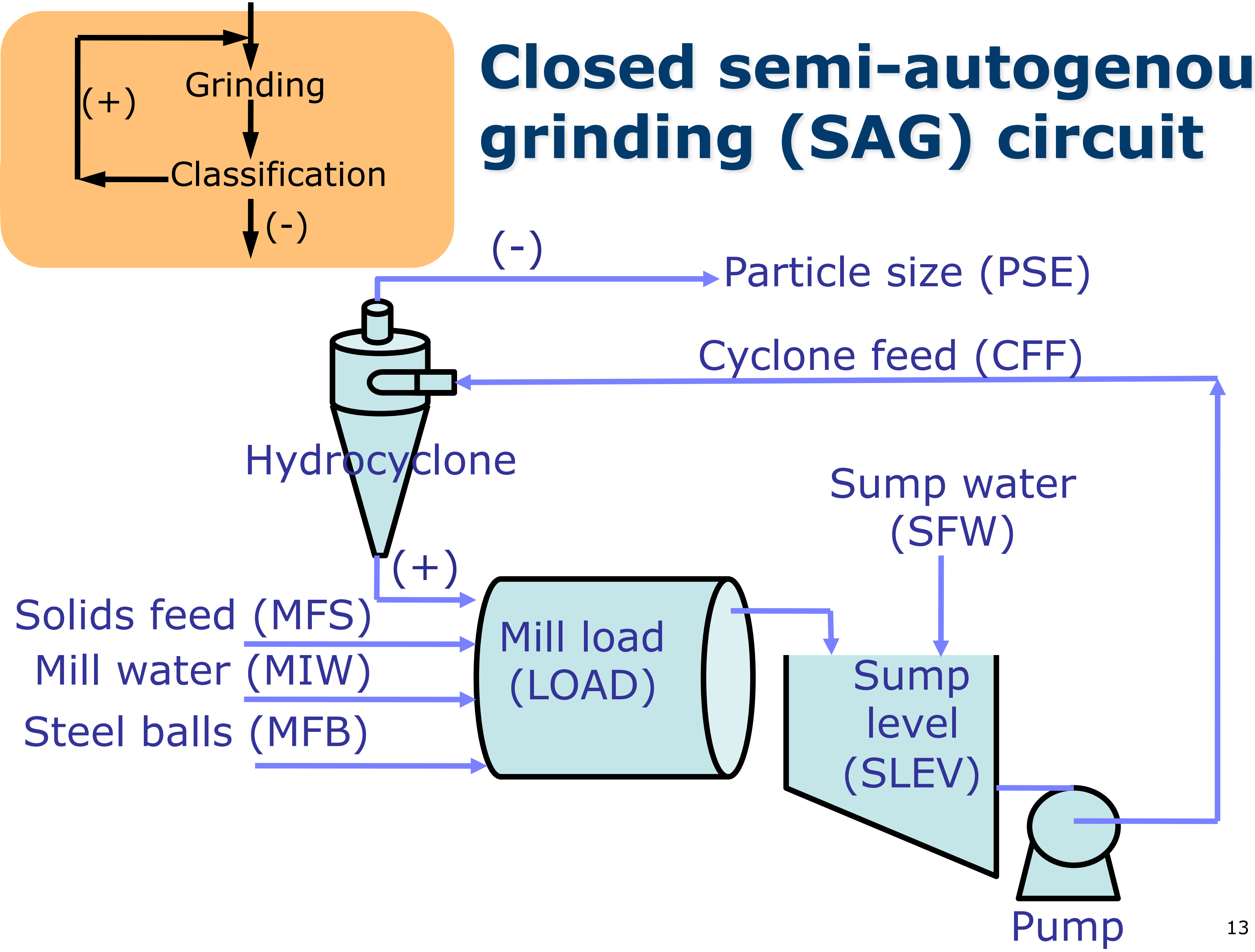
Concentrate

Rejects

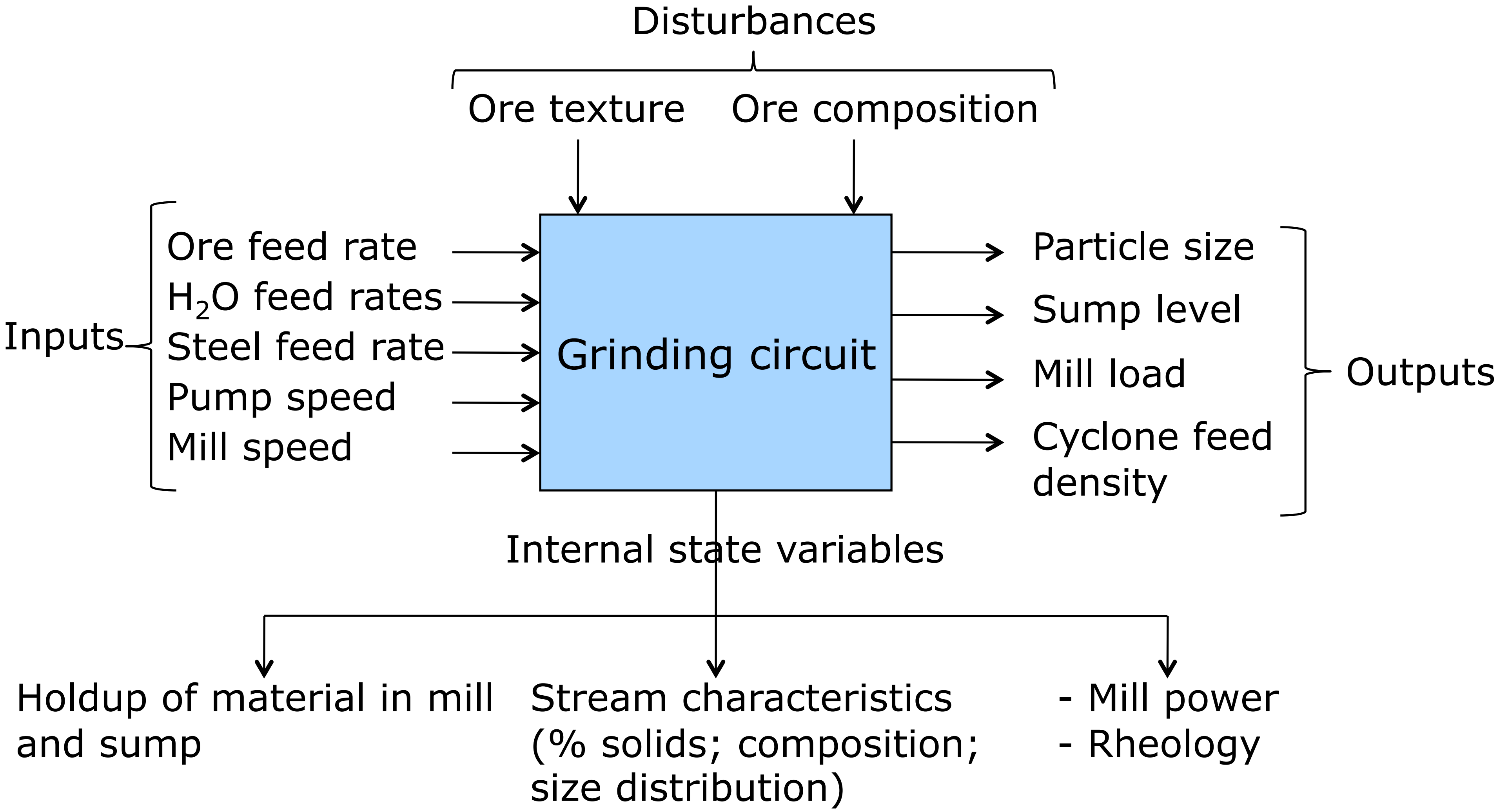
---

* Adapted from Wills and Napier-Munn, *Wills' Mineral Processing Technology (7e), 2005,* Butterworth-Heinemann, *p13.*

Closed semi-autogenous grinding (SAG) circuit



Grinding circuit process variables*



Disturbances

Ore texture

Ore composition

Inputs

- Ore feed rate
- H_2O feed rates
- Steel feed rate
- Pump speed
- Mill speed

Grinding circuit

Outputs

- Particle size
- Sump level
- Mill load
- Cyclone feed density

Internal state variables

Holdup of material in mill and sump

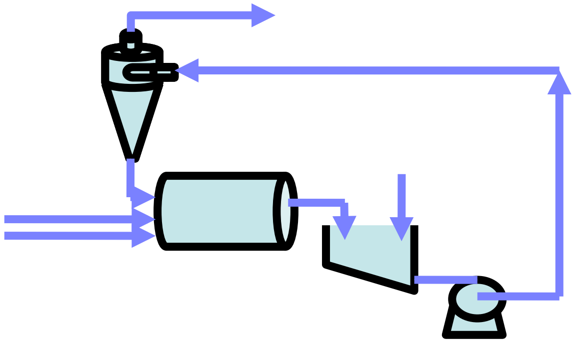
Stream characteristics (% solids; composition; size distribution)

- Mill power
- Rheology

* Adapted from Hodouin, D., *Journal of Process Control,* 21 (2011), 211-225.

Contents

- The big picture for commodity prices
- Why study grinding mill control?
- Mineral processing and grinding process overview
- Grinding circuit modelling
  - Simplified grinding circuit model for control studies
- History and current status of grinding mill control
- Grinding circuit control objectives
- Control examples:
  - μ-synthesis
  - Robust nonlinear MPC
- Current research projects
- Future research challenges



Grinding circuit models

- Mineral liberation is difficult to model due to complexity of:
  - naturally occurring mineralogical textures
  - the fracture processes that occur when the ore is crushed and ground
- Type of model depends on purpose
- Phenomenological model-based grinding circuit simulators are used for process design and optimization, and also training
- The population balance method of modelling provides a unifying framework
- Empirical LTI models used for model-based controller design
- “New” simplified phenomenological model used for controller design and analysis case studies

“Standard” phenomenological grinding circuit model I

- SAG mill process modelled by 35 state equations
- Solids*: $Accumulation = In - Out + Generation - Consumption$

$$\frac{\partial s_i}{\partial t} = f_i - p_i + \sum_{j=1}^{i-1} r_j s_j a_{ij} - (1 - a_{ii}) r_i s_i \ , i = 1, \ldots, 27 \ ; \qquad p_i = d_0 c_i s_i$$

Parameter	Description	Unit
s_i	mill rock charge particles in size i	tons
f_i	feedrate of particles in size i	tons/hour
p_i	mill discharge of particles in size i	tons/hour
d_0	maximum mill discharge rate constant	hour^{-1}
c_i	grate classification function for size i	fraction
r_i	breakage rate of particles in size i	hour^{-1}
a_{ij}	appearance function describing the amount of material selected for breakage and the distribution of material after breakage occurred	fraction

* Napier-Munn et al., JKMRC, 1996.

“Standard” phenomenological grinding circuit model II

Generation term: $\sum_{j=1}^{i-1} r_j s_j a_{ij} \quad i = 1, \ldots, 27$

Summation of the product of the rock charge mass in the size fractions above size i, s_j , and their respective breakage rates, r_j , and the fraction appearing into size i from the breakage occurring above, a_{ij} , results in the generation term for size i.

Consumption term: $(1 - a_{ii}) r_i s_i \quad i = 1, \ldots, 27$

The appearance function, a_{ij} is in a mass fraction retained format, therefore the diagonal of the appearance function, a_{ii}, indicates (by difference) how much of the material in a given size is broken and distributed into the size fractions below.

$$r_i = f(\text{equipment parameters, operating conditions})$$

$$a_{ij} = f(\text{ore characteristics, operating conditions})$$

* Apelt et al, *Minerals Engineering*, 15, (2002), 1043-1053.

"Standard" phenomenological grinding circuit model* III

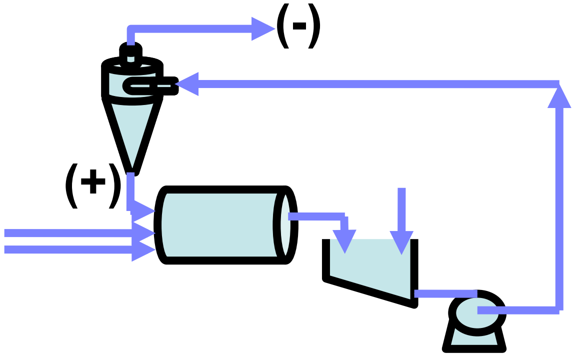
Water $$\frac{\partial s_w}{\partial t} = f_w - p_w; \quad p_w = d_o s_w$$

Balls $$\frac{\partial b_{ci}}{\partial t} = b_i - b_{ei} + b_{wi-1} - b_{wi}; i = 1, \ldots, 7$$

Parameter	Description	Unit
s_w	Mill water charge	tons
f_w	Mill water feedrate	tons/hour
p_w	mill water discharge rate	tons/hour
d_o	maximum mill discharge rate constant	hour^{-1}
b_{ci}	Mill ball charge for balls in size i	tons
b_i	Mill ball feedrate for balls in size i	tons/hour
b_{ei}	Mill ball charge ejection rate for balls in size i	tons/hour
b_{wi}	Mill ball charge wear rate balls out of size i into size i+1	tons/hour

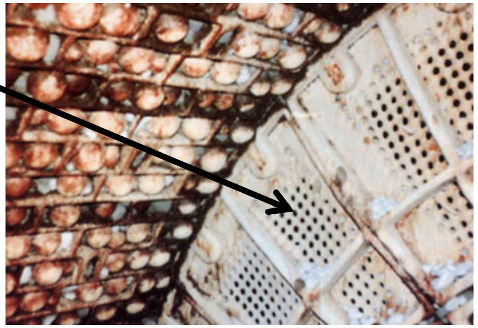
---

* Apelt et al, *Minerals Engineering*, 15, (2002), 1043-1053.

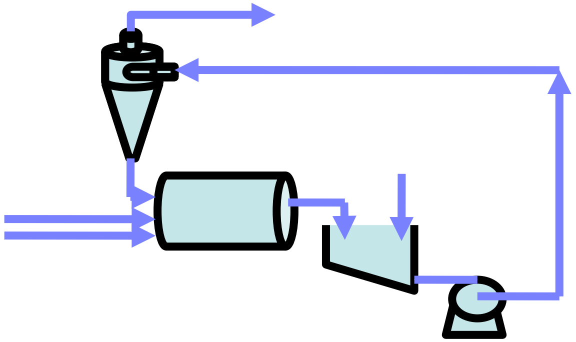


Simplified grinding circuit model* I

- Model has 4 modules
  - Feeder, mill, sump, hydrocyclone
  - Modularized structure allows for arbitrary circuit configurations
- Nonlinear SAG mill model with 5 states
  - Water, Solids, Fines, Rocks, Balls
  - Solids: Consists of Coarse ((+) out-of-specification) and Fines ((-) in-specification) material.
  - Fines < 75μm (milling circuit product)
  - Solids < Discharge grate size < Rocks
- Generation & consumption terms include effects of:
  - Slurry rheology and Mill power
- Hydrocyclone module
  - Empirical and algebraic model



*Coetzee, Craig and Kerrigan, *IEEE T. Control Systems Technology*, 18, (2010), 222-229.



Simplified grinding circuit model* II

5 mill state equations

$$\frac{\partial X_{mw}}{\partial t} = V_{mwi} - V_{mwo}$$

$$\frac{\partial X_{ms}}{\partial t} = V_{msi} - V_{mso} + RC$$

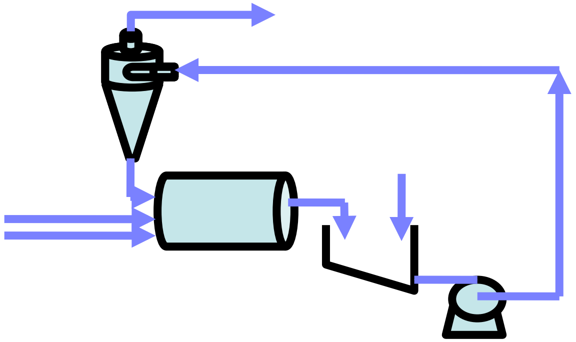
$$\frac{\partial X_{mf}}{\partial t} = V_{mfi} - V_{mfo} + FP$$

$$\frac{\partial X_{mr}}{\partial t} = V_{mri} - RC$$

$$\frac{\partial X_{mb}}{\partial t} = V_{mbi} - BC$$

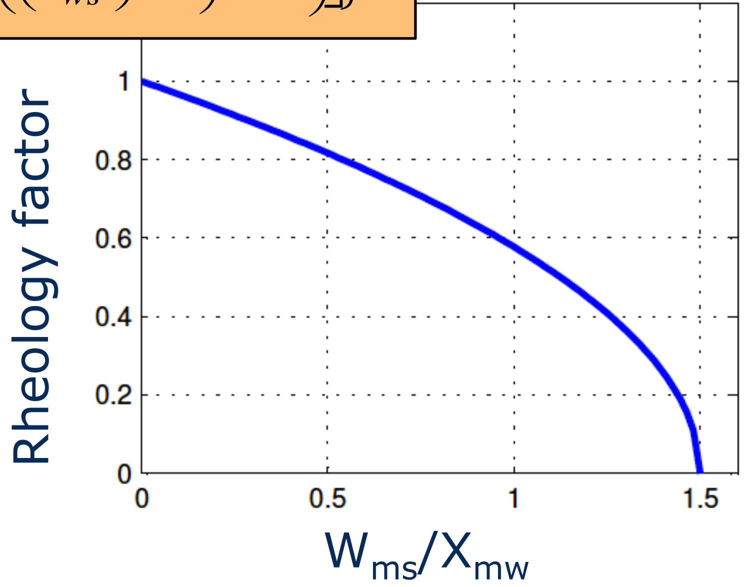
	Description	**Unit**
X_{mw}	Holdup of water in the mill	m^3
X_{ms}	Holdup of solids in the mill	m^3
X_{mf}	Holdup of fines in the mill	m^3
X_{mr}	Holdup of rocks in the mill	m^3
X_{mb}	Holdup of balls in the mill	m^3
$V_{mxi\ or\ o}$	Flow-rate of water/solids/ balls in to/out of the mill	m^3/ hour
RC	Rock consumption	m^3/ hour
FP	Fines produced	m^3/ hour

*Coetzee, Craig and Kerrigan, *IEEE T. Control Systems Technology*, 18, (2010), 222-229.



Simplified grinding circuit model* III

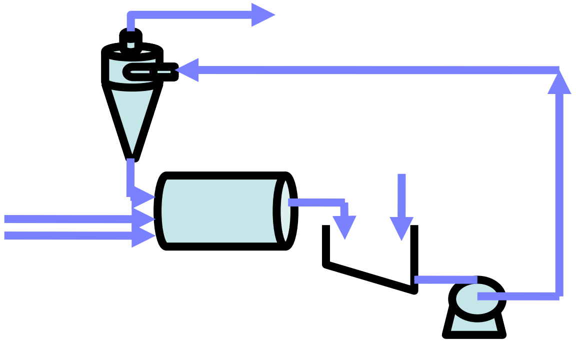
Rheology factor: $$\varphi = \left\{ \frac{1}{X_{mw}} \max\left[0, \left(X_{mw} - \left(\left(\frac{1}{\varepsilon_{ws}}\right) - 1\right) X_{ms}\right)\right]\right\}^{0.5}$$



Mill Load

Mill power: $$P_{mill} = P_{\max}\left\{1 - \left(\frac{\boxed{X_{mb}+X_{mr}+X_{ms}+X_{mw}}}{v_{P_{\max}}v_{mill}} - 1\right)^2 - \left(\frac{\varphi}{\varphi_{P_{\max}}} - 1\right)^2\right\}\left(\alpha_{speed}\right)^{\alpha_p}$$

*Coetzee, Craig and Kerrigan, *IEEE T. Control Systems Technology*, 18, (2010), 222-229.



Simplified grinding circuit model* III

Rock Consumption: $RC = \frac{P_{mill}\varphi}{D_s\phi_r}\left(\frac{X_{mr}}{X_{mr}+X_{ms}}\right)$

Fines Production: $$FP = \frac{P_{mill}}{D_s\phi_f\left\{1+\alpha_{\phi_f}\left(\frac{X_{mw}+X_{mr}+X_{ms}+X_{mb}}{v_{mill}}-v_{P_{\max}}\right)\right\}}$$

Ball Consumption: $BC = \frac{P_{mill}\varphi}{D_b\phi_b}\left(\frac{X_{mb}}{X_{mr}+X_{ms}+X_{mb}}\right)$

---

*Coetzee, Craig and Kerrigan, *IEEE T. Control Systems Technology*, 18, (2010), 222-229.

Empirical grinding circuit models I

- Properties available in simulators such as ore grindability, slurry rheology, grinding media size distribution are not measured on-line and are impossible/very difficult to infer from other measurements
- Plant models for model-based controller design are therefore typically LTI models obtained from plant identification tests, e.g. (time constants in seconds)*

$$\begin{bmatrix}\Delta\text{Product size}\\ \Delta\text{Mill load}\\ \Delta\text{Sump level}\end{bmatrix} = \begin{bmatrix} \frac{0.105}{83s+1}e^{-65s} & \frac{-0.082}{1766s+1}e^{-80s} & \frac{-0.0575}{167s+1}e^{-460s}\\ \frac{-0.0468}{1864s+1}e^{-140s} & \frac{0.000122}{s} & \frac{0.115}{1981s+1}e^{-120s}\\ \frac{0.00253}{s} & 0 & \frac{-0.00299}{s}\end{bmatrix}\begin{bmatrix}\Delta\text{Sump water feed}\\ \Delta\text{Mill feed solids}\\ \Delta\text{Cyclone feed}\end{bmatrix}$$

---

* Craig and MacLeod, *Control Engineering Practice*, 3, (1995), 621-630.

Empirical grinding circuit models II

- Empirical LTI models can be very tedious to obtain:
  - Repeated tests to cover a suitable range of operating conditions
  - Tests can take a long time because of slow plant dynamics
  - Frequent plant stoppages and equipment failure
  - Large uncertainties associated with some transfer function parameters (parameter standard deviation in %)*:

$$g_{ij} = \frac{k_{ij}}{\tau_{ij}s+1} e^{-\theta_{ij}s} \quad \text{or} \quad g_{ij} = \frac{k_{ij}}{s} e^{-\theta_{ij}s}$$

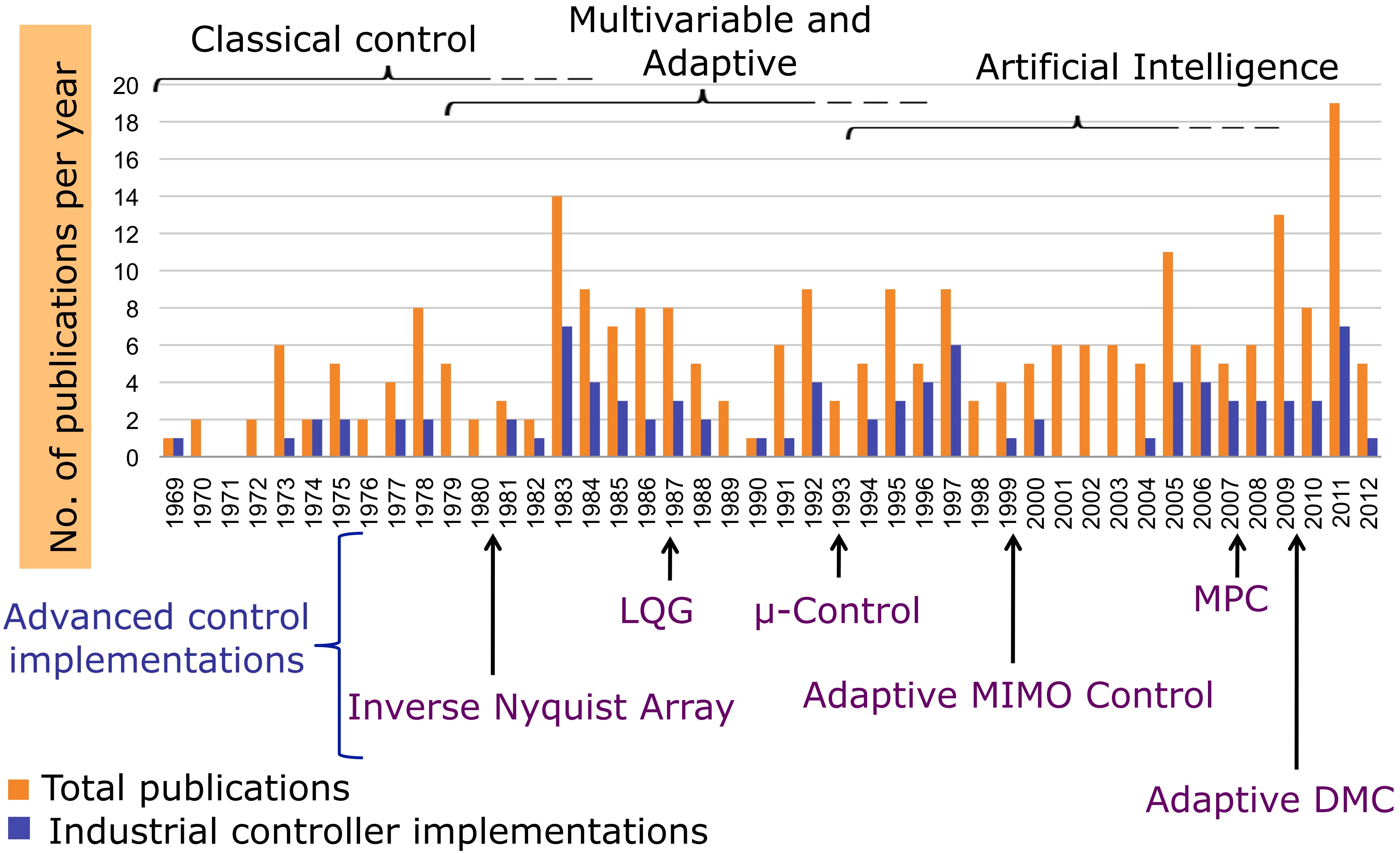
$$k_{ij}: \begin{bmatrix} 35\% & 14\% & 31\% \\ 16\% & 11\% & 65\% \\ 0\% & 0\% & 0\% \end{bmatrix} \quad \tau_{ij}: \begin{bmatrix} 19\% & 0\% & 18\% \\ 60\% & 0\% & 40\% \\ 0\% & 0\% & 0\% \end{bmatrix} \quad \theta_{ij}: \begin{bmatrix} 0\% & 0\% & 27\% \\ 43\% & 0\% & 0\% \\ 0\% & 0\% & 0\% \end{bmatrix}$$

* Craig and MacLeod, *Control Engineering Practice*, 3, (1995), 621-630.

Contents

- The big picture for commodity prices
- Why study grinding mill control?
- Mineral processing and grinding process overview
- Grinding circuit modelling
  - Simplified grinding circuit model for control studies
- History and current status of grinding mill control
- Grinding circuit control objectives
- Control examples:
  - μ-synthesis
  - Robust nonlinear MPC
- Current research projects
- Future research challenges

Milling control literature: Historical development



Classical control

Multivariable and Adaptive

Artificial Intelligence

Advanced control implementations

Inverse Nyquist Array

LQG

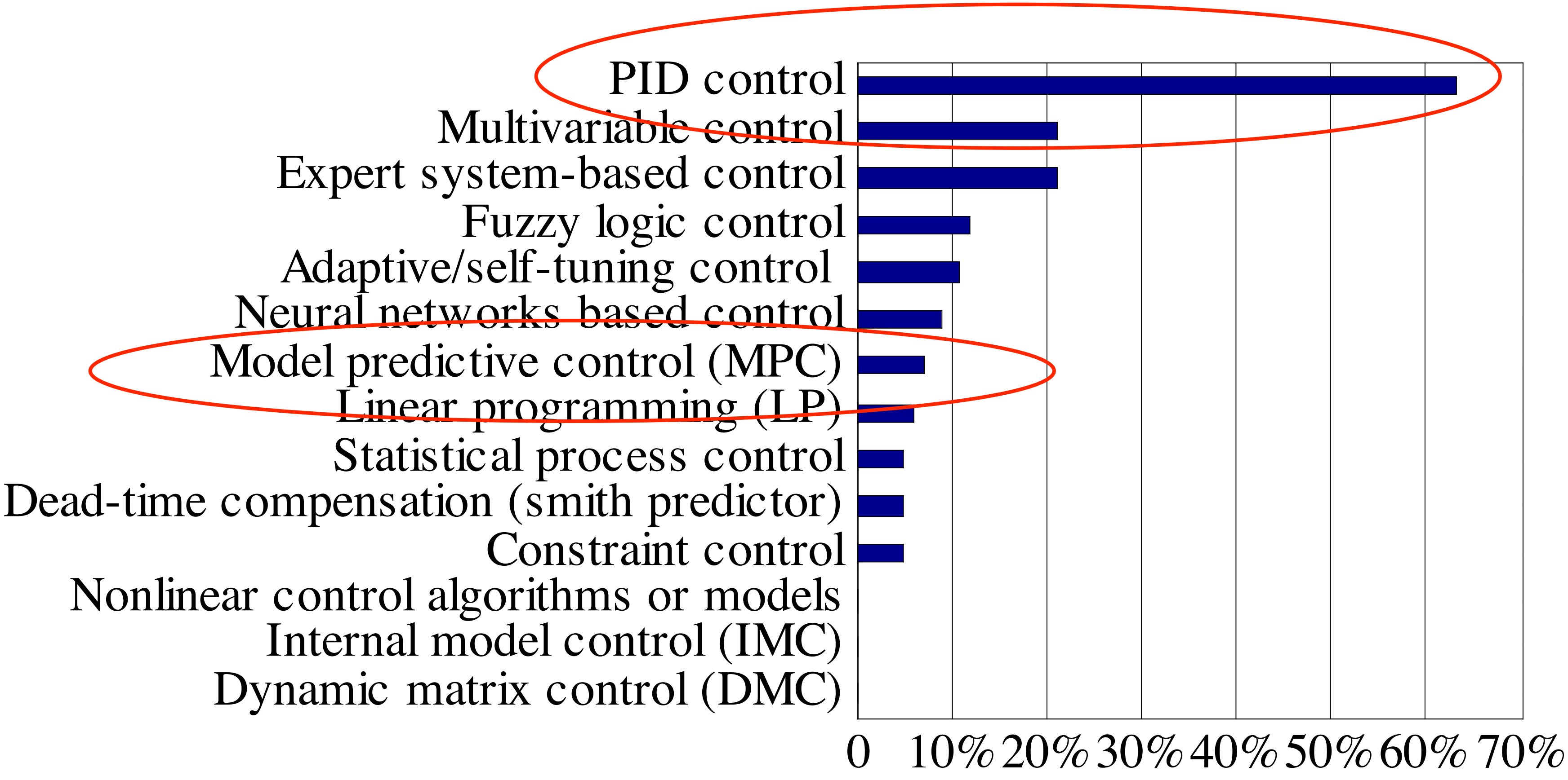
µ-Control

Adaptive MIMO Control

MPC

Adaptive DMC

Grinding mill control: Adopted control technologies*



* Wei and Craig, *International Journal of Minerals Processing*, 90, (2009), 56-66.

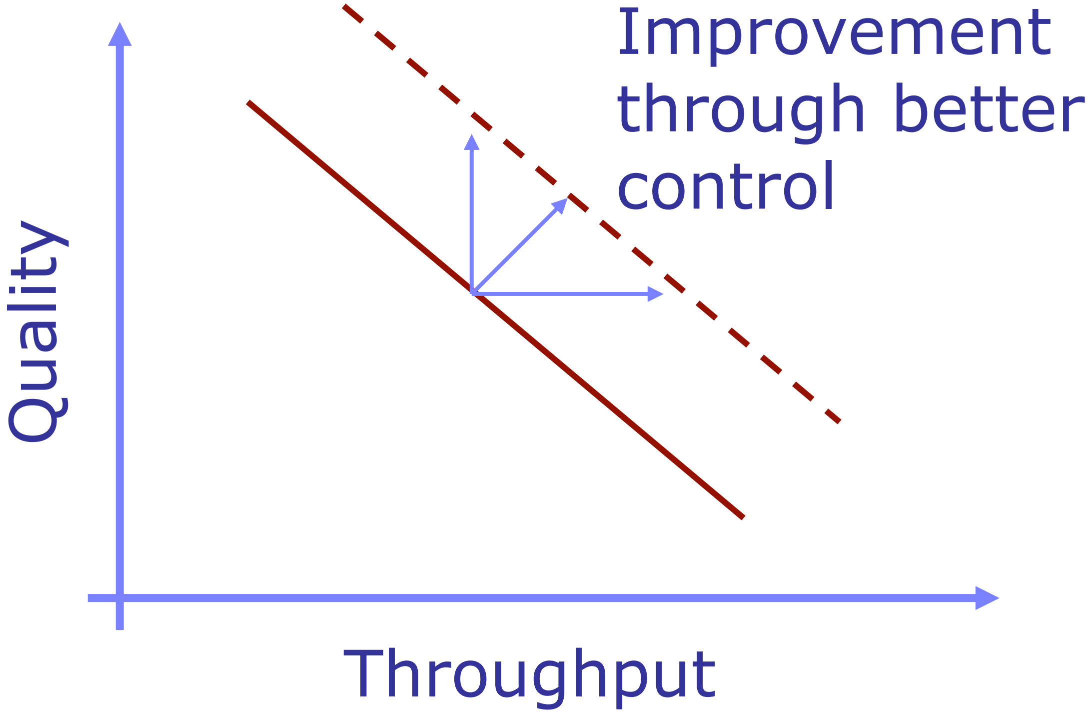
Contents

- The big picture for commodity prices
- Why study grinding mill control?
- Mineral processing and grinding process overview
- Grinding circuit modelling
  - Simplified grinding circuit model for control studies
- History and current status of grinding mill control
- Grinding circuit control objectives
- Control examples:
  - µ-synthesis
  - Robust nonlinear MPC
- Current research projects
- Future research challenges

Grinding circuit control objectives

- Stabilize the circuit
  - Mill load and Sump level are open-loop unstable as they act as flow integrators
- Improve product quality (particle size)
  - Maintain particle size setpoint at value determined by the subsequent separation process
  - Decrease particle size variance
- Maximise throughput given the desired particle size setpoint
- Circuit control objective can also be formulated as an objective function to be optimised:
  - Throughput or recovery maximization at a constant grade
  - Net revenue maximization using net smelter return (NSR)
  - Constraints on input, output and other process variables should also be considered

Quality/throughput trade-off

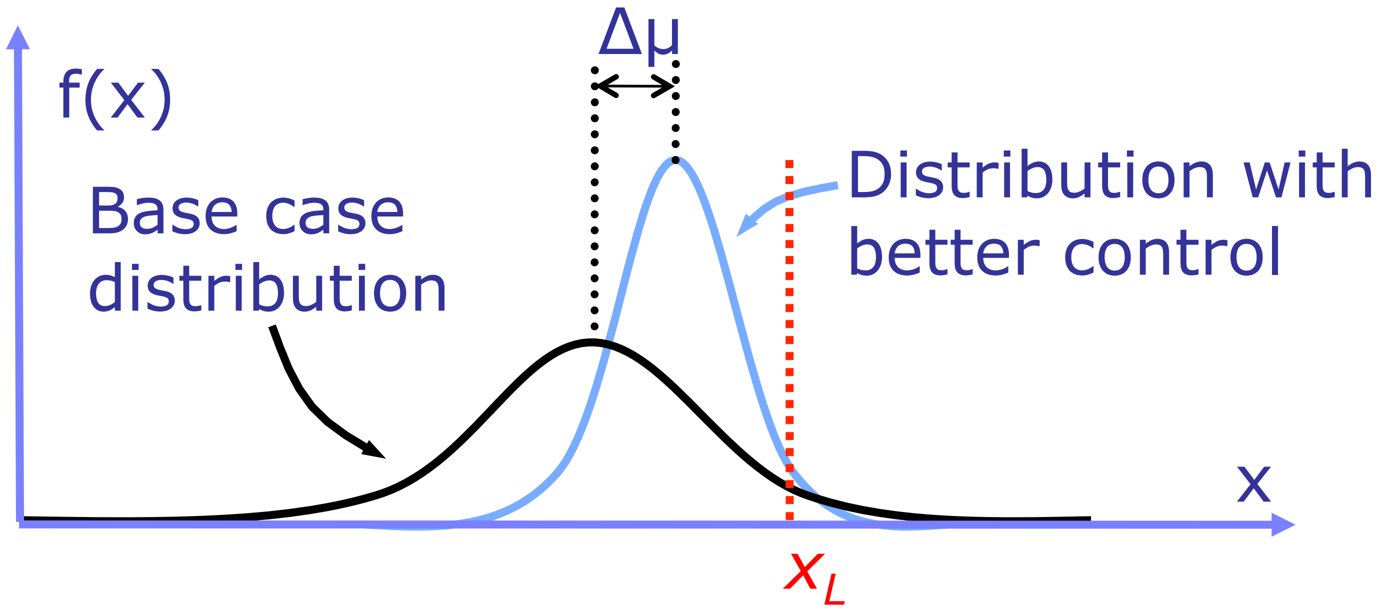


Improvement through better control

Improvement through better product quality control

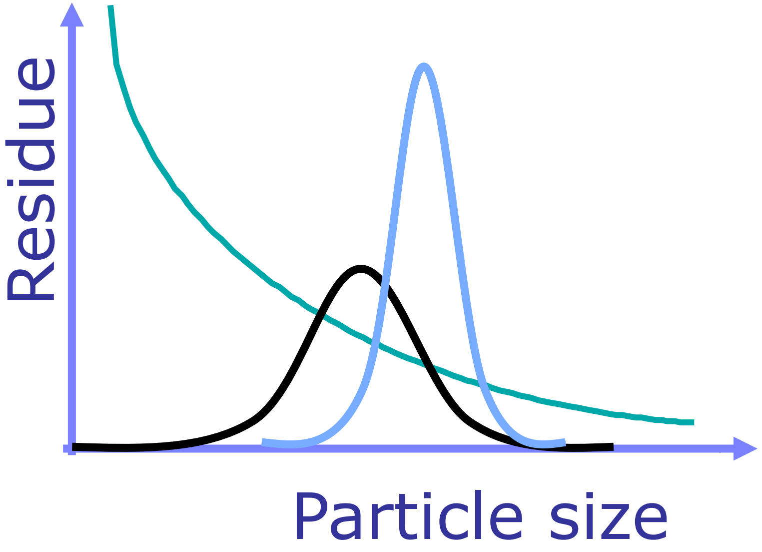
Improve product quality (particle size) control via variance reduction and subsequent optimization:

- Average μ can be shifted closer to constraint x_L by Δμ
- f(x): pdf of process variable x (e.g. particle size)



Separation process and particle size setpoint selection I

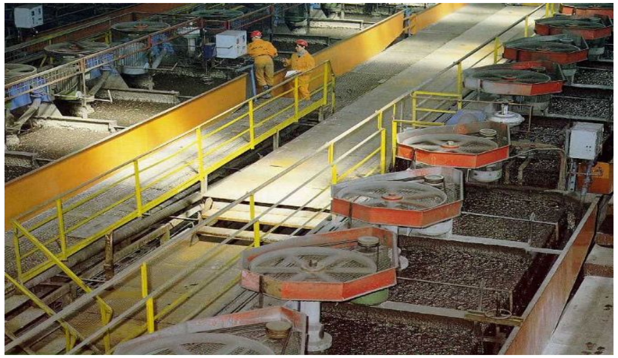
- Leaching as downstream separation process
- Economic benefit obtained from reducing grind variations
- Residue-Particle size relationship*
  - Residue: (g/t) of metal not recovered
- The finer the grind the better
  - Size constrained by throughput and operating costs

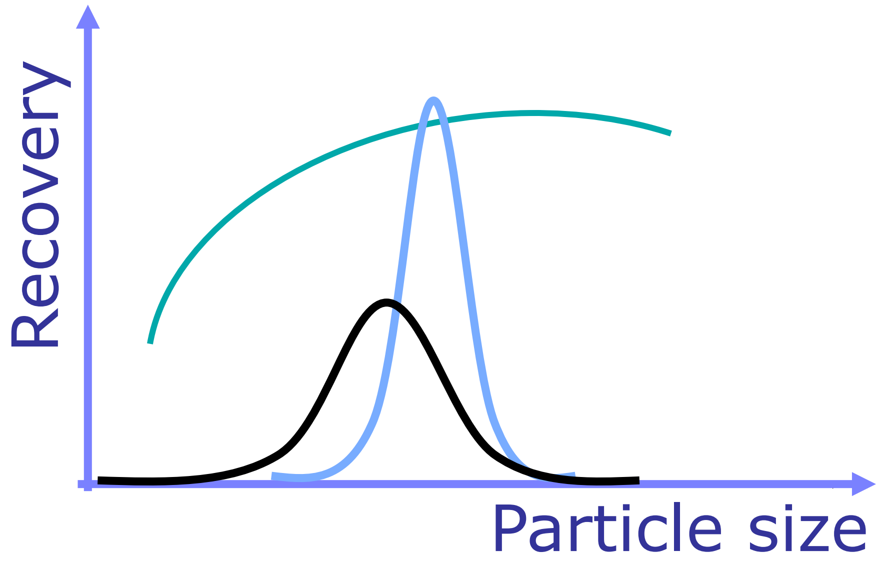


* Craig et al, *J.S. Afr. Inst. Min. Metall.*, 92, (1992), 69-176.

Separation process and particle size setpoint selection II

- Flotation as downstream separation process
- The optimum grind size of the ore is the particle size at which the most economic recovery can be obtained
  - Depends not only on the grindability of the ore but also on its floatability.
  - $ improvement from both variance reduction and better setpoint*
- Recovery - Particle size relationship

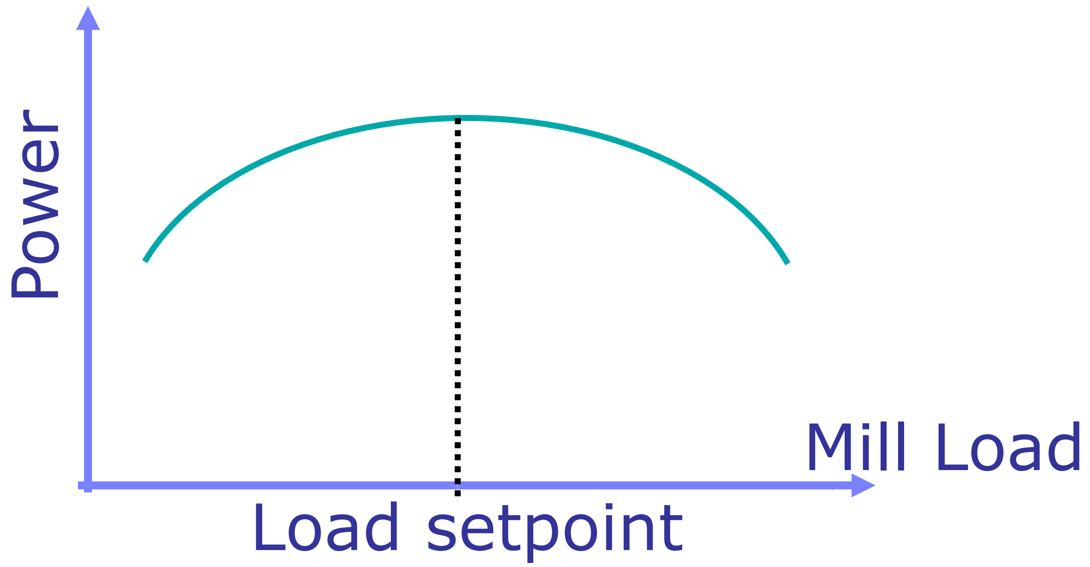




* Craig and Koch, *Control Engineering Practice*, 11, (2003), 57-66.

Maximising throughput

- Maximise throughput given desired product size
- Assumption: Throughput is maximised when maximum power is drawn from mill motor
- Power is quadratic in total load volume so optimise Load setpoint for maximum power draw



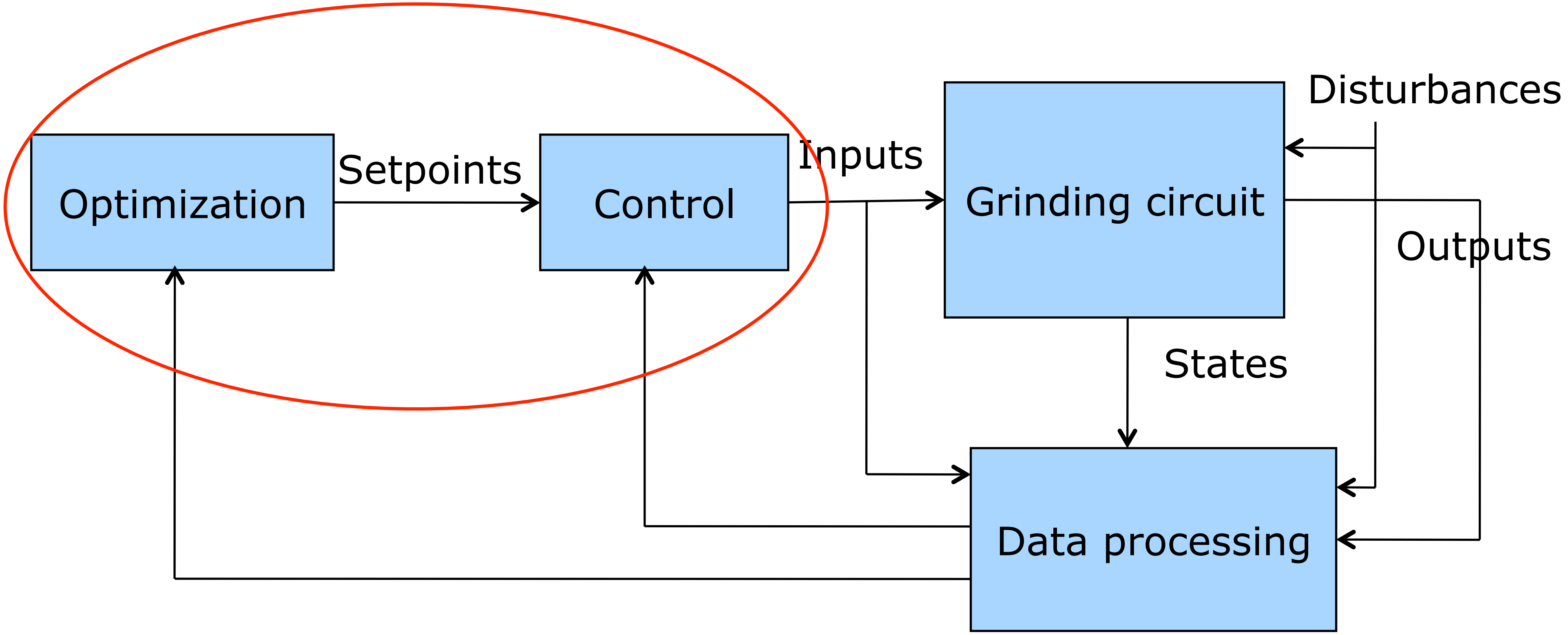
* Craig et al, *J.S. Afr. Inst. Min. Metall.*, 92, (1992), 169-176.

Contents

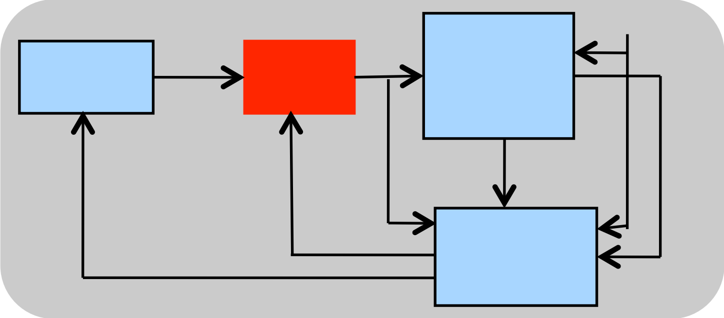
- The big picture for commodity prices
- Why study grinding mill control?
- Mineral processing and grinding process overview
- Grinding circuit modelling
  - Simplified grinding circuit model for control studies
- History and current status of grinding mill control
- Grinding circuit control objectives
- Control examples:
  - µ-synthesis
  - Robust nonlinear MPC
- Current research projects
- Future research challenges

Grinding circuit control loop*

Real-time optimization (RTO)

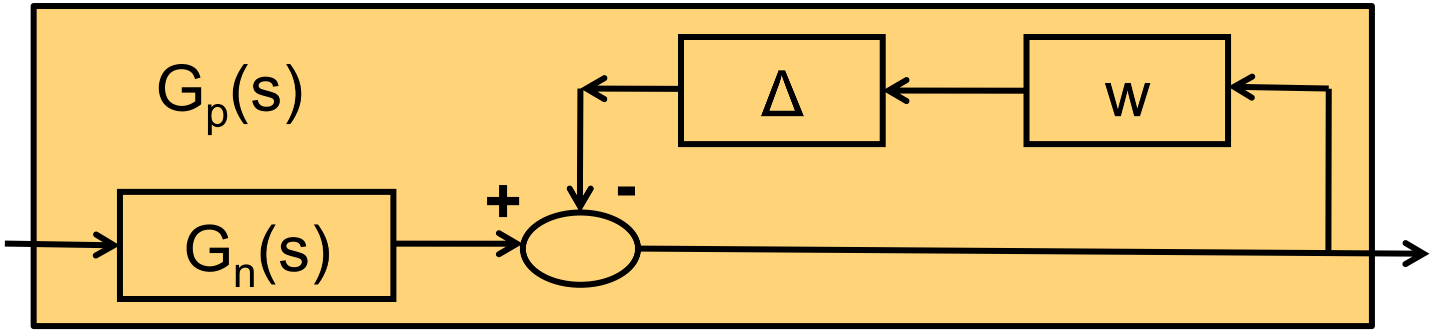


* Adapted from Hodouin, D., *Journal of Process Control,* 21 (2011), 211-225.

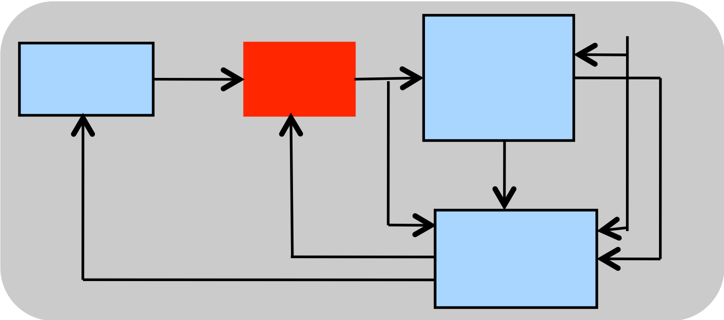


Control example: μ-synthesis I

- Model uncertainties are represented by multiplicative and inverse multiplicative norm-bounded perturbations and frequency dependent weighting functions
- Example of an inverse multiplicative uncertainty description:
  - Time constant that are dependent directly on the rheology of the slurry inside the mill are correlated and can be grouped together
  - Load/Sum water feed rate and Load/Cyclone feed rate time constants are similar to the hold-up time of the mill and can be combined



* Craig and MacLeod, *Control Engineering Practice*, 3, (1995), 621-630.



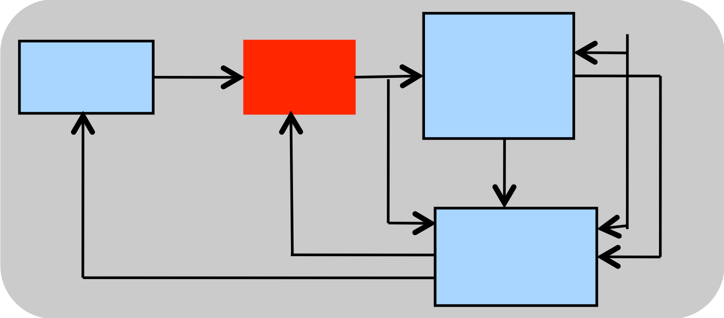
Control example: μ-synthesis II

- Gain (k_{ij}) and gain and time delay (k_{ij} and θ_{ij}) uncertainties represented as multiplicative
- Time constant (τ_{ij}) uncertainties represented as inverse-multiplicative

$$g_{ij} = \frac{k_{ij}}{\tau_{ij} s + 1} e^{-\theta_{ij} s} \quad \text{or} \quad g_{ij} = \frac{k_{ij}}{s} e^{-\theta_{ij} s}$$

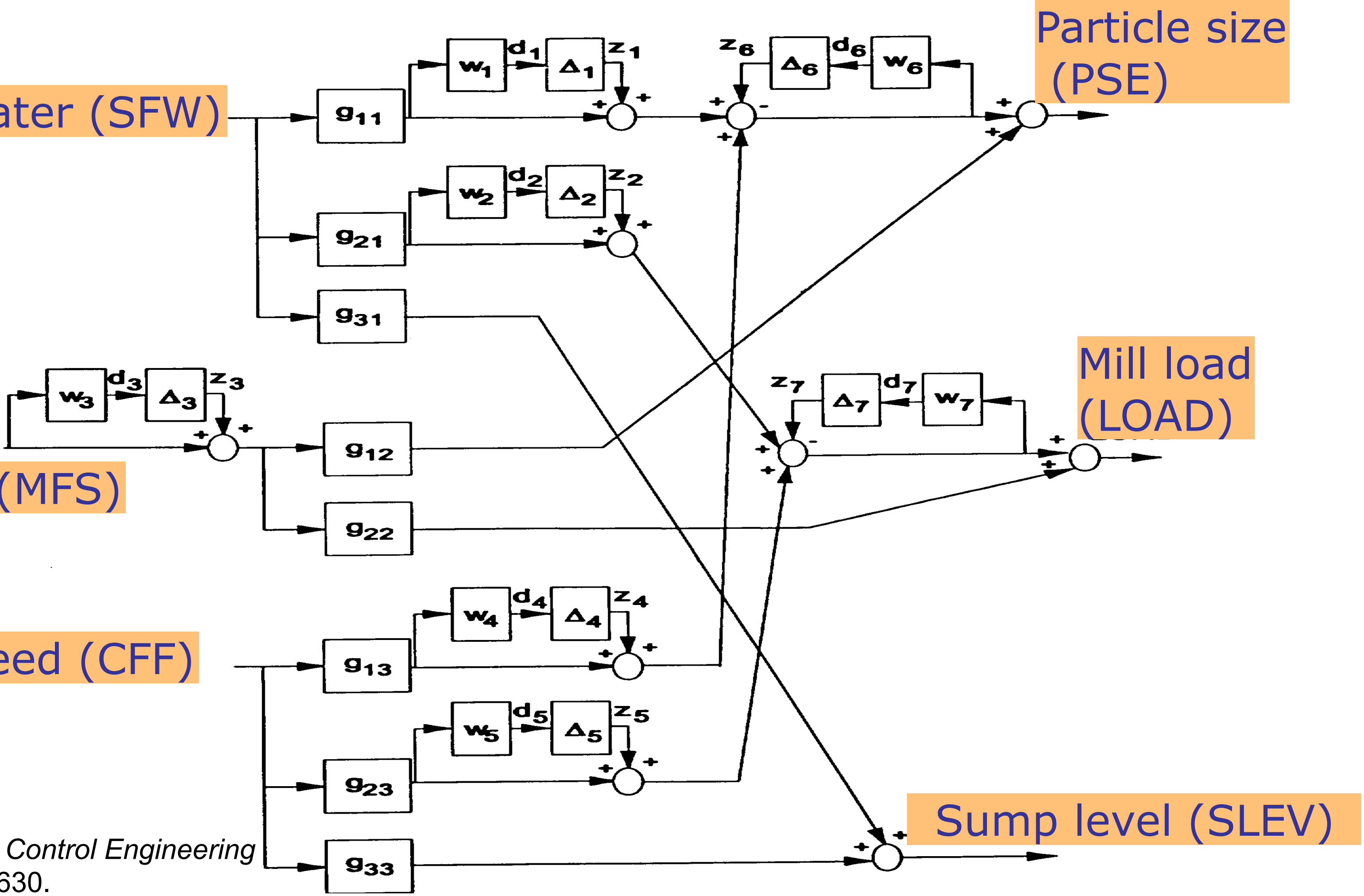
$$k_{ij} : \begin{bmatrix} 35\% & 14\% & 31\% \\ 16\% & 11\% & 65\% \\ 0\% & 0\% & 0\% \end{bmatrix} \qquad \tau_{ij} : \begin{bmatrix} 19\% & 0\% & 18\% \\ 60\% & 0\% & 40\% \\ 0\% & 0\% & 0\% \end{bmatrix} \qquad \theta_{ij} : \begin{bmatrix} 0\% & 0\% & 27\% \\ 43\% & 0\% & 0\% \\ 0\% & 0\% & 0\% \end{bmatrix}$$

* Craig and MacLeod, *Control Engineering Practice*, 3, (1995), 621-630.



Control example: μ-synthesis III

Nominal plant with uncertainty description



Sump feed water (SFW)

Solids feed (MFS)

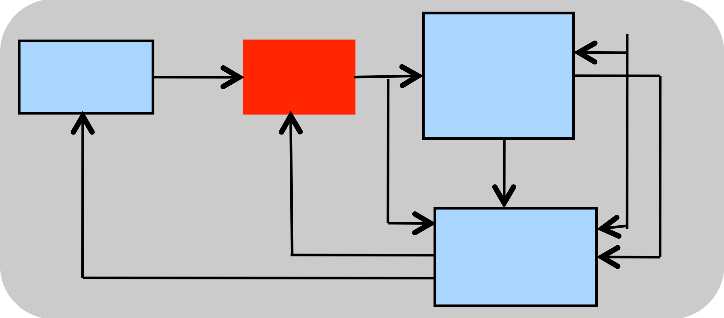
Cyclone feed (CFF)

Particle size (PSE)

Mill load (LOAD)

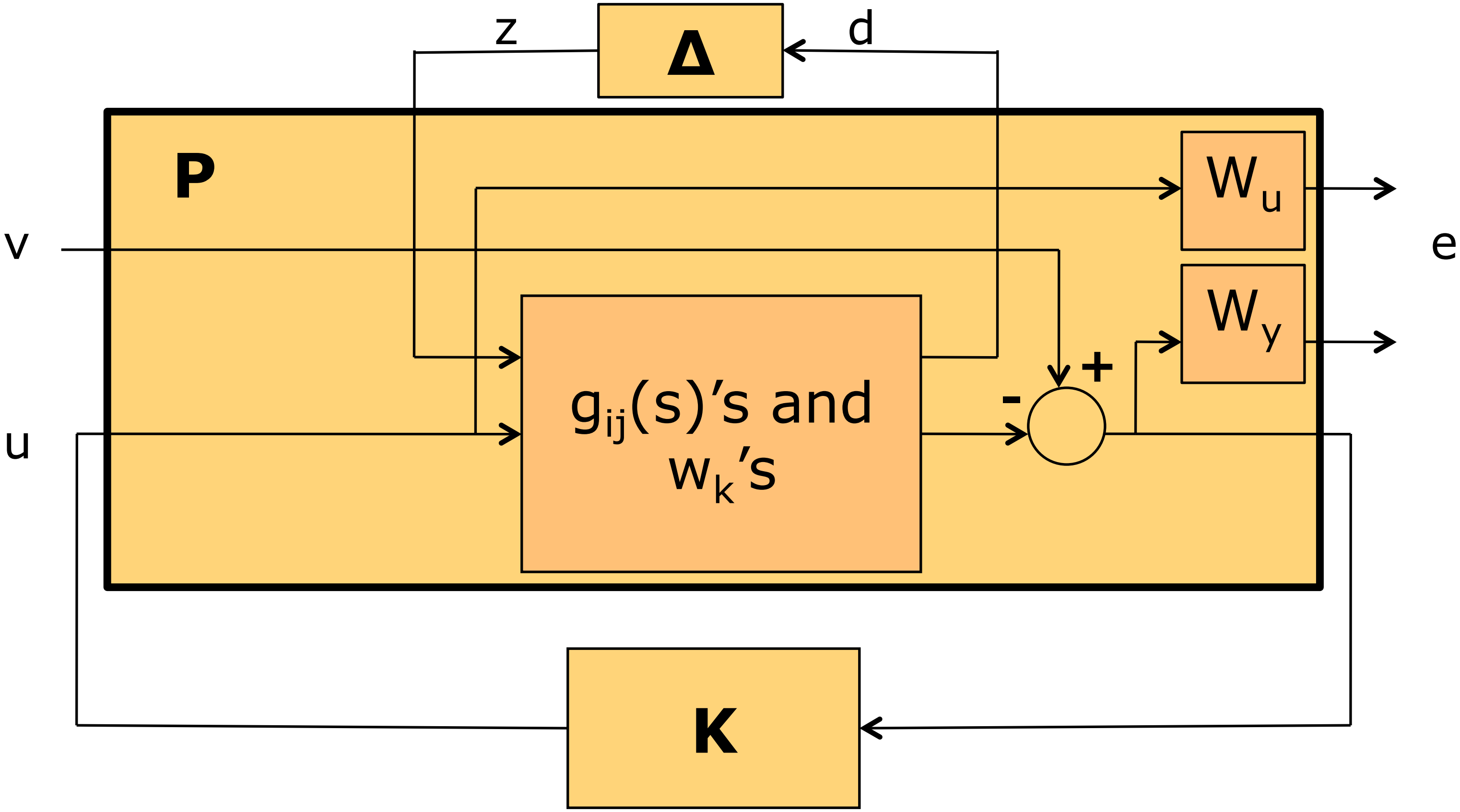
Sump level (SLEV)

* Craig and MacLeod, *Control Engineering Practice*, 3, (1995), 621-630.

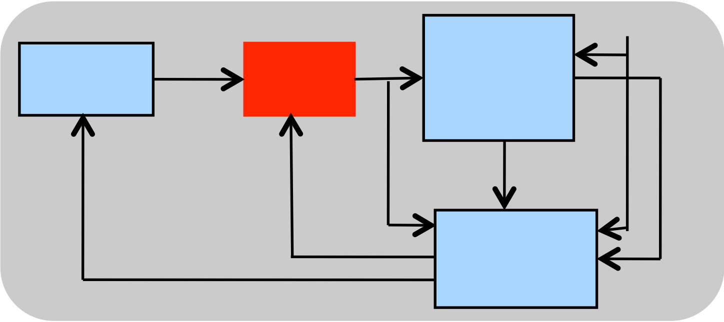


Control example: μ-synthesis IV

- Convert specifications to performance weights W_u and W_y
- General configuration for controller synthesis

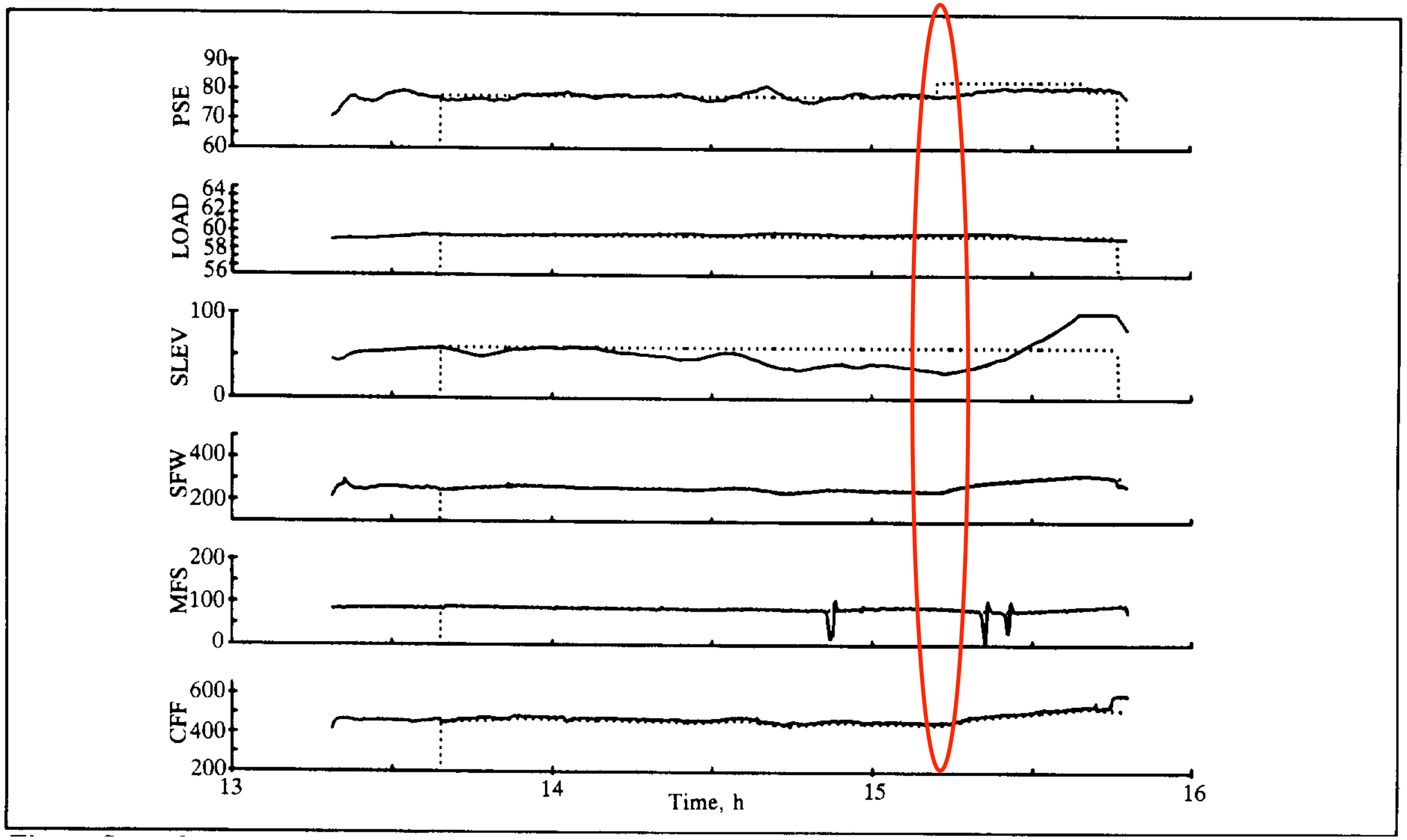


* Craig and MacLeod, *Control Engineering Practice*, 3, (1995), 621-630.

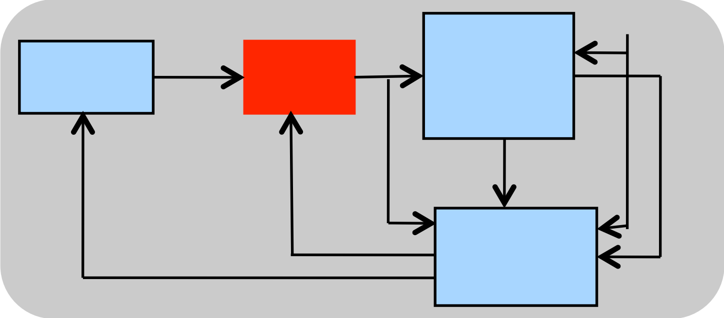


Control example: μ-synthesis V

Plant trial



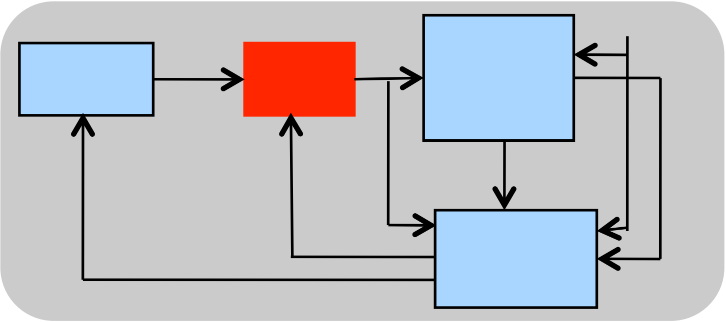
* Craig and MacLeod, *Control Engineering Practice*, 4, (1996), 1-12.



Control example: Robust non-linear MPC I

- Design a robust nonlinear model predictive controller
  - Explicitly incorporate uncertainty in design
  - Deal with simplified nonlinear grinding circuit model without approximation
- Nonlinear MPC is robustified to parameter uncertainty
  - Calculate worst-case objective and the constraint functions by maximizing these functions with regard to the model parameter sequence and state values
  - The worst case objective function is then minimized by choosing the control moves subject to the worst-case constraints
  - Min-max optimization problem converted to an easier to solve minimization problem using an approximate robust counterpart formulation

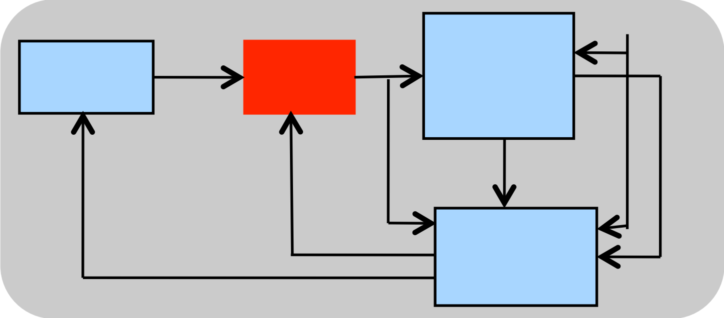
*Coetzee, Craig and Kerrigan, *IEEE T. Control Systems Technology*, 18, (2010), 222-229.



Control example: Robust non-linear MPC II

- Verify RNMPC through simulation study
- Plant disturbances
  - Feed ore hardness change: increase energy needed to produce a ton of fines by 50% at time 10 minutes
  - Feed ore composition change: increasing the fraction of the feed consisting of rock by 50% at time 100 minutes
  - These disturbances are very large but not uncommon in practice
- Strong points: Good disturbance rejection and constraint satisfaction in the face of large disturbances
- Drawbacks
  - Computational time longer than required sampling time
  - Full-state feedback assumed

*Coetzee, Craig and Kerrigan, *IEEE T. Control Systems Technology*, 18, (2010), 222-229.



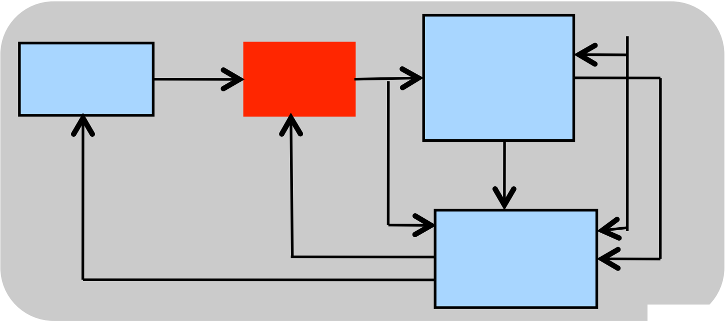
Control example: Robust non-linear MPC III

Parameter uncertainty description

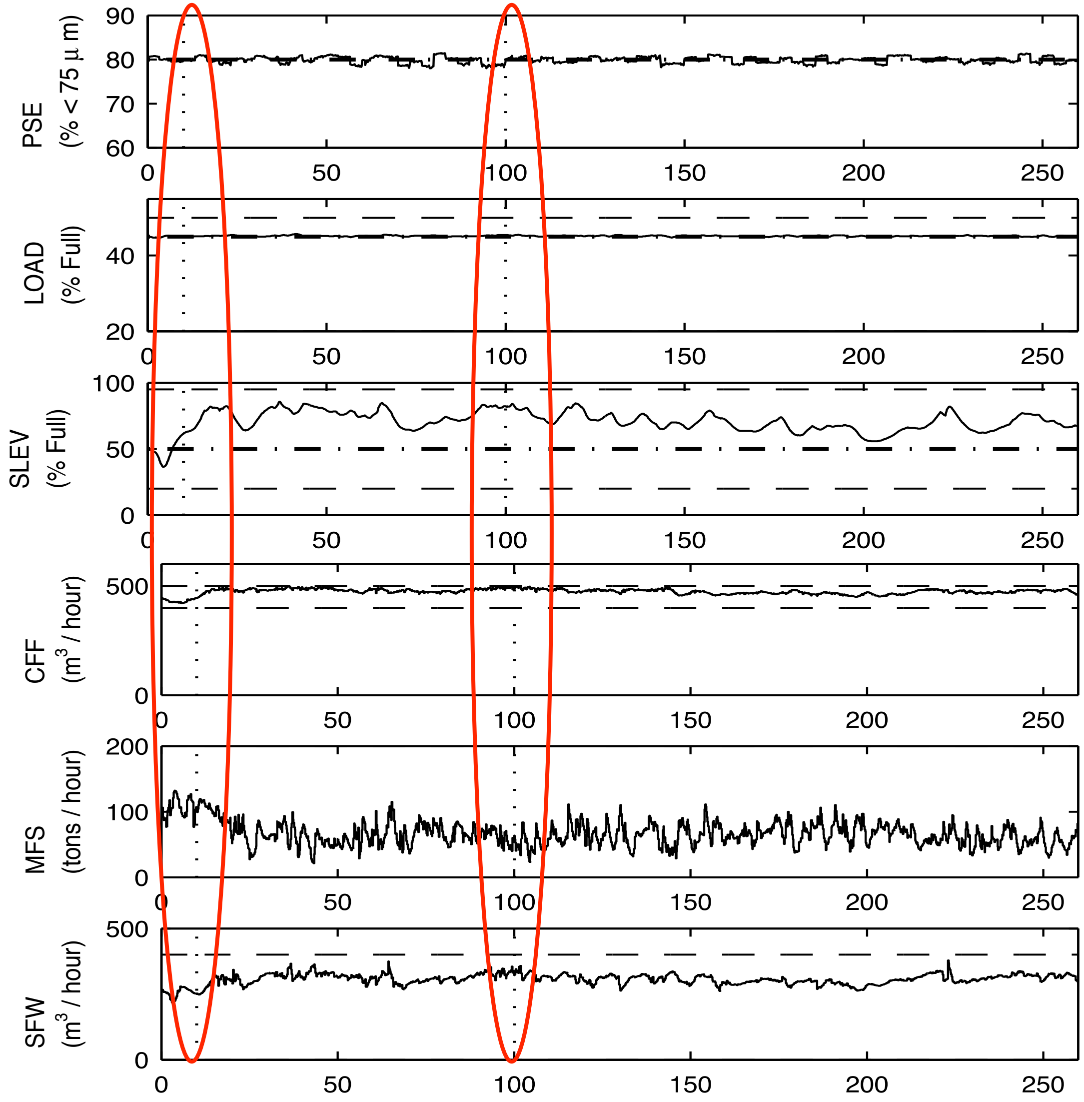
$$P_{box} = \left\{ p \in R^{n_p} \middle| p_l \leq p \leq p_u \right\} = \left\{ p \in R^{n_p} \middle| \left\| \operatorname{diag}\left(\frac{p_u - p_l}{2} \right)^{-1} \left(p - \frac{p_l - p_u}{2} \right) \right\|_\infty \leq 1 \right\}$$

Parameter (p)	**Min=p_l**	**Max=p_u**	**Description**
α_f	0.05	0.15	Fraction of fines in the ore [dimensionless]
α_r	0.05	0.15	Fraction of rock in the ore [dimensionless]
ϕ_f	14	42	Energy needed for a ton of fines produced [kWh/t]
ϕ_r	55	83	Rock abrasion factor [kWh/t]
ϕ_b	89	99	Steel abrasion factor [kWh/t]

*Coetzee, Craig and Kerrigan, *IEEE T. Control Systems Technology*, 18, (2010), 222-229.



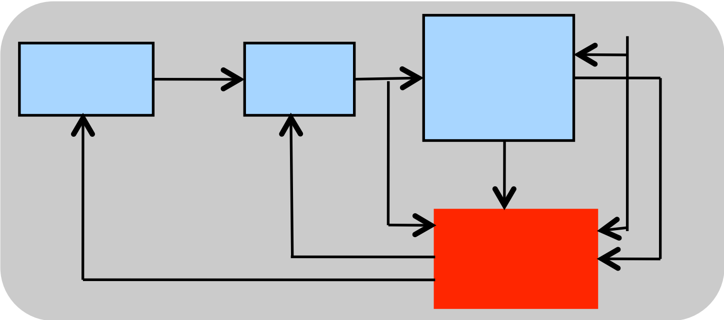
Robust non-linear MPC IV



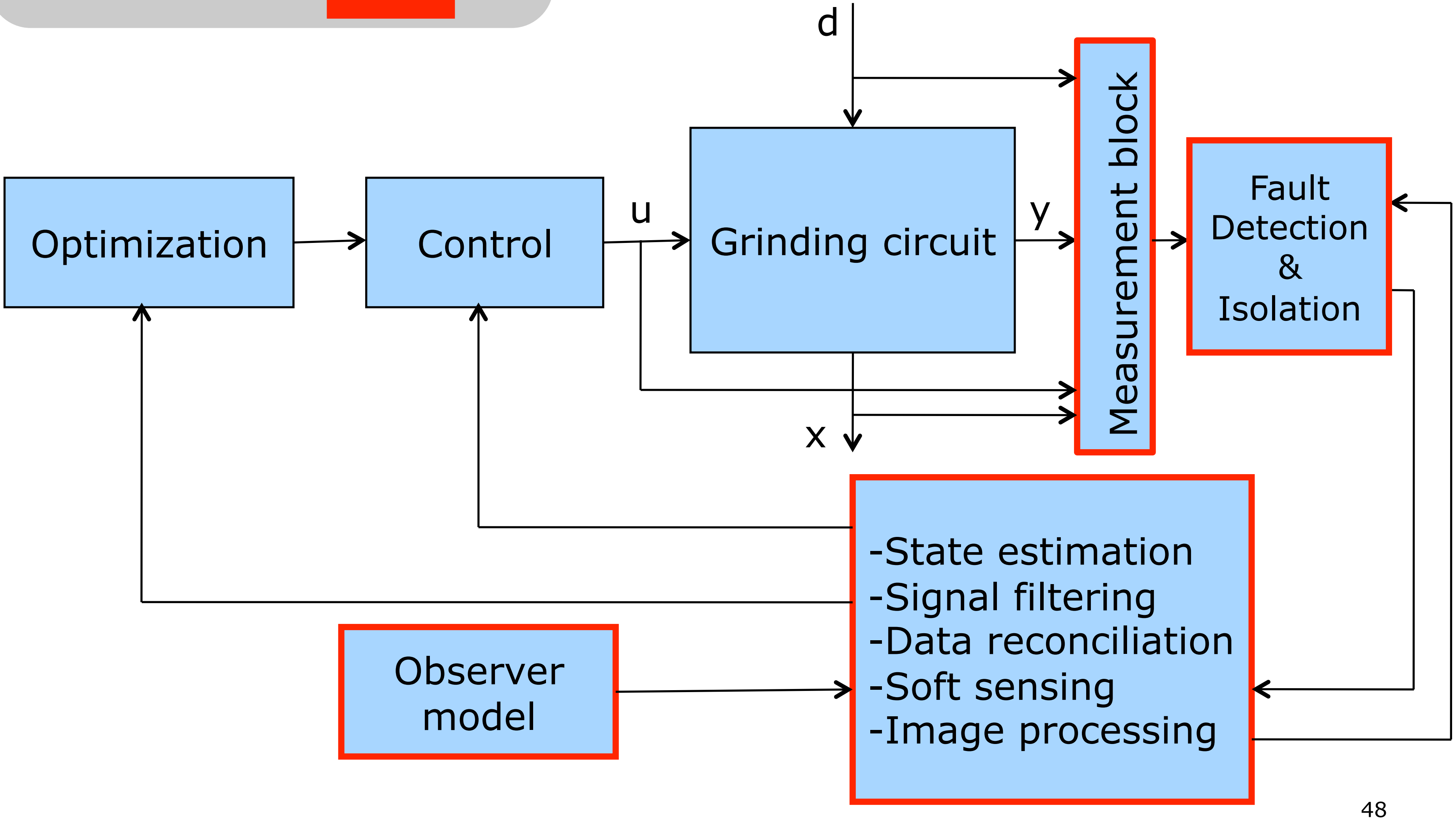
*Coetzee, Craig and Kerrigan,
IEEE T. Control Systems Technology, 18,
(2010), 222-229.

Contents

- The big picture for commodity prices
- Why study grinding mill control?
- Mineral processing and grinding process overview
- Grinding circuit modelling
  - Simplified grinding circuit model for control studies
- History and current status of grinding mill control
- Grinding circuit control objectives
- Control examples:
  - µ-synthesis
  - Robust nonlinear MPC
- Current research projects
- Future research challenges



Data processing block*



d

Optimization

Control

u

Grinding circuit

y

Measurement block

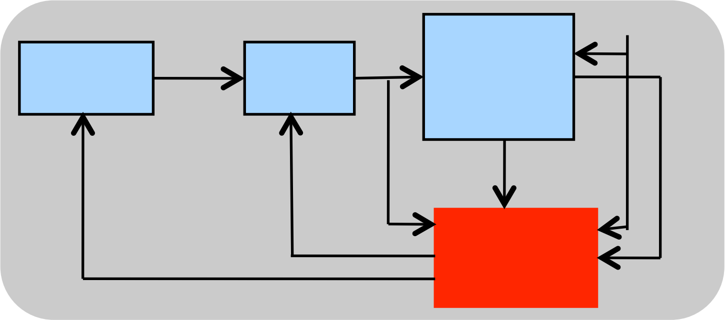
Fault Detection & Isolation

x

Observer model

- -State estimation
- -Signal filtering
- -Data reconciliation
- -Soft sensing
- -Image processing

* Adapted from Hodouin, D., *Journal of Process Control,* 21 (2011), 211-225.



Current projects

- Parameter estimation and disturbance observer design
  - Olivier, L.E., Huang, B., and Craig, I.K., Dual particle filters for state and parameter estimation with application to a run-of-mine ore mill, *Journal of Process Control*, Vol. 22, No. 4, 2012, pp. 710-717.
  - Olivier, L.E., Craig, I.K., and Y.Q. Chen, Fractional Order and BICO Disturbance Observers for a Run-of-Mine Ore Milling Circuit, *Journal of Process Control*, Vol. 22, No. 1, 2012, pp. 3-10.
- Model analysis and verification
  - Le Roux, D.J., and Craig, I.K., Identifiability of run-of-mine ore grinding mill circuit parameters, *10th IEEE Region 8 AFRICON*, Zambia, 13-15 Sep., 2011.
  - Le Roux, D.J., Craig, I.K., Hulbert, D.G., and A.L. Hinde, Analysis and validation of a run-of-mine ore grinding mill circuit model for process control, submitted to *Minerals Engineering.*

Contents

- The big picture for commodity prices
- Why study grinding mill control?
- Mineral processing and grinding process overview
- Grinding circuit modelling
  - Simplified grinding circuit model for control studies
- History and current status of grinding mill control
- Grinding circuit control objectives
- Control examples:
  - µ-synthesis
  - Robust nonlinear MPC
- Current research projects
- Future research challenges

Future research challenges*

- Energy efficiency
  - More holistic perspectives on energy use and emissions reduction in industrial processes, including minerals processing, is required
- Very-Large-Scale Integrated Process Control (VLSIPC)
  - Use of economic-performance-optimizing MPCs in the form of dynamic real-time optimization (D-RTO).
  - Integration of mining, and mineral and metal extraction processes
- Generating good process models at low cost by e.g. easing the modelling effort
- Development of a practical high-fidelity milling circuit observer

* Craig et al., Control in the Process Industries, in "*The Impact of Control Technology*", IEEE CSS, 2011.

Conclusions

- The big picture for commodity prices
- Why study grinding mill control?
- Mineral processing and grinding process overview
- Grinding circuit modelling
  - Simplified grinding circuit model for control studies
- History and current status of grinding mill control
- Grinding circuit control objectives
- Control examples:
  - μ-synthesis
  - Robust nonlinear MPC
- Current research projects
- Future research challenges

Acknowledgements

- YangQuang Chen (Utah State, Logan, UT)
- Loutjie Coetzee (Mintek, South Africa)
- Adrain Hinde (AH Consulting)
- Biao Huang (University of Alberta)
- Dave Hulbert (Mintek, South Africa)
- Derik Le Roux (University of Pretoria)
- Bjorn Matthews (University of Pretoria)
- Myrin Naidoo (University of Pretoria)
- Laurentz Olivier (University of Pretoria)