# Cyber-Physical Systems Design Using Dissipativity

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## Cyber-Physical Systems (CPS)

-As computers become ever-faster and communication bandwidth ever-cheaper, computing and communication capabilities will be embedded in all types of objects and structures in the physical environment.

-***Cyber-physical systems (CPS)*** are physical, biological and engineered systems whose ***operations are monitored, coordinated,*** **controlled** ***and integrated by a computing and communication core***.

-This intimate coupling between the cyber and physical will be manifested from the nano-world to large-scale wide-area systems of systems. And at multiple time-scales.

-Applications with enormous societal impact and economic benefit will be created. Cyber-physical systems will transform how we interact with the physical world just like the Internet transformed how we interact with one another.

-**We should care about CPS because our lives depend on them**

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# Thanks to:

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- Vijay Gupta, Bill Goodwin*e*

- *NSF CPS Large: Science of Integration for CPS, Vanderbilt, Maryland, Notre Dame, GM R&D*



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# Some Applications


5

## Medical care and health

- Pacemakers, infusion pumps, medical delivery devices, connected to the patient for life-critical functions
- Life-supporting micro-devices, embedded in the human body; wireless connectivity enabling body area sensor nets; mass customization of heterogeneous, configurable personalized medical devices, and natural, wearable sensors (clothing, jewelry) and benignly implantable devices

6

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## Energy

- Centralized generation, Supervisory Control and Data Acquisition (SCADA) Systems for transmission and distribution
- Systems for more efficient, effective, safe and secure generation, transmission, and distribution of electric power, integrated through the smart grid; smart ("net-zero energy") buildings for energy savings; systems to keep nuclear reactors safe

7

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## Transportation and Mobility

- Vehicle-based safety systems, ABS, traction and stability control, powertrain management; precision GPS-enabled agriculture
- Vehicle-to-vehicle communications for enhanced safety and convenience ("zero fatality" highways), drive-by-wire, autonomous vehicles; next generation air transportation system (NextGen); autonomous vehicles for off-road and military mobility applications

8

## Manufacturing

- Computer controlled machine tools and equipment; robots performing repetitive tasks, fenced off from people
- Smarter, more connected processes for agile and efficient production; manufacturing robotics that work safely with people in shared spaces; computer-guided printing or casting of composites, design for manufacturabillity, programmable foundries

9

## Materials and other sectors

- Relatively few, highly specialized applications of smart materials– predominantly passive materials and structures
- Sustainable mass production of “smart” fabrics and other “wearables” with applications in many areas; Actively controlled buildings and structures to improve safety by avoiding or mitigating accidents; electronics provide versatility without recourse to a silicon foundry; emerging materials such as carbon fiber and polymers offer the potential to combine capability for electrical and/or optical (hence NIT) functionality with important physical properties (strength, durability, disposability

10

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# What Drives CPS

11

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## Technological and Economic Drivers

•The decreasing cost of computation, networking, and sensing.

• A variety of social and economic forces will require us to use national infrastructures more efficiently.

• Environmental pressures will mandate the rapid introduction of technologies to improve energy efficiency and reduce pollution.

•As the national population ages, we will need to make more efficient use of our health care systems, ranging from facilities to medical data and information.

12

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# Drivers of Cyber-Physical Systems (CPS)

- Networking and Information Technology (NIT) have been increasingly used as universal system integrator in human–scale and societal–scale systems
- Functionality and salient system characteristics emerge through the interaction of networked physical and computational objects
- Engineered products turn into Cyber-Physical Systems (CPS): networked interaction of physical and computational processes

13

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# PCAST Report

Leadership Under Challenge:
Information Technology R&D in a Competitive World
An Assessment of the Federal Networking and Information Technology R&D Program
President's Council of Advisors on Science and Technology
August 2007

New Directions in Networking and Information Technology (NIT)

Recommendation: No 1 Funding Priority:
**NIT Systems Connected with the Physical World**

14

## CPS Workshops

· Workshop on "High Confidence Medical Device Software and Systems (HCMDSS)", June 2 - 3, 2005, Philadelphia, PA.
· Workshop on "Aviation Software Systems: Design for Certifiably Dependable Systems", October 5-6, 2006, Alexandria, TX.

· Workshop on "Cyber-Physical Systems", October 16-17, 2006, Austin,TX.
· Meeting on "Beyond SCADA: Networked Embedded Control for Cyber Physical Systems", November 8-9, 2006, Pittsburgh, PA.
· Workshop on "High-Confidence Automotive Cyber-Physical Systems," April 3-4, 2008, Troy, MI
· Workshop on "High-Confidence Transportation Cyber-Physical Systems: Automotive, Aviation & Rail," Nov 18-20, 2008, Tyson's Corner, VA
· Workshop on "Developing Dependable and Secure Automotive Cyber-Physical Systems from Components," March 17-18, 2011, Troy, MI

· CPS Week, 2008 (St. Louis), 2009 (San Francisco), 2010 (Stockholm), 2011(Chicago), 2012 (Beijing)

Series of topical workshops under NSF sponsorship started in 2006.



# CPS Challenges

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# CPS Characteristics

What cyber physical systems have as defining characteristics:

- Cyber capability (i.e. networking and computational capability) in every physical component
- They are networked at multiple and extreme scales
- They are complex at multiple temporal and spatial scales.
- They are dynamically reorganizing and reconfiguring
- Control loops are closed at each spatial and temporal scale. Maybe human in the loop.
- Operation needs to be dependable and certifiable in certain cases
- Computation/information processing and physical processes are so tightly integrated that it is not possible to identify whether behavioral attributes are the result of computations (computer programs), physical laws, or both working together.

17

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# CPS Issues

There is a set of pervasive underlying problems for CPS not solved by current technologies:

• How to build predictable real time, networked CPS at all scales?

• How to build and manage high-confidence, secure, dynamically-configured systems?

• How to organize and assure interoperability?

• How to avoid cascading failure?

• How to formulate an evidential (synthetic and analytic) basis for trusted systems?  Certified.

18

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## Research Challenges

- We need systems that are compositional, scalable, and evolvable
  - Big and small components
  - One component to billions of components
  - New and old technology co-exist
- We need ways to measure and certify the "performance" of cyber-physical systems
  - Time and space, but multiple degrees of resolution
  - New metrics, e.g., energy use
  - New properties, e.g., security, privacy-preserving
- We need new engineering processes for developing, maintaining, and monitoring CPS
  - Traditional methods will not work or are too costly

Source: NSF

19

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## Connections & Personal Motivation

**Linear Feedback Systems**

– Polynomial Matrix Descriptions

**Autonomous Intelligent Control**

- 20 years ago – CSS Report on Defining Intelligent Control
- Hierarchies – Degrees of Autonomy
- To DES (PN), Hybrid Systems, Networked Control Systems (MB)
- To Cyber Physical Systems

**Quest for Autonomy–from ancient water clocks to UAVs**

**The Power of Feedback**

**– in natural and engineered systems**

**– Feedback Transcends Models**

20

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# Approaches to Meet the Challenges

21

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## Passivity and Symmetry in CPS

-In real world CPS-automotive, medical-overall system dynamics emerge from the interaction of physical dynamics, computational dynamics and communication network

-**Heterogeneity causes major challenges**. In addition network uncertainties-time-varying delays, data rate limitations, packet losses.

-**We impose passivity constraints on the components and use wave variables, and the design becomes insensitive to network effects. Human-interaction. Symmetry.**

- **NSF CPS Large Project:** "Science of Integration of CPS" (with Vanderbilt).

22

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# Topics

- Passivity of CT/DT systems
- Stability of passive systems
- Interconnections of passive systems
- Dissipativity of CT/DT systems
- Conic systems
- Passivity indices
- Dissipativity for CT switched systems
- Passivity for CT switched systems
- Networked passive systems
- Networked conic systems
- Dissipativity for DT switched systems (QSR)
- LMI methods for showing pass/diss for LTI systems
- SOS methods for showing pass/diss for nonlinear systems
- Compensation for quantization of passive systems
- Passivity indices for symmetric systems
- Passivity of systems in series
- Event triggered control of passive systems
- Output synchronization of passive systems using event-driven communication



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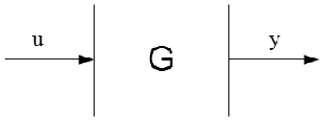
# Background on Passivity

# Definition of Passivity in Continuous-time

- Consider a continuous-time nonlinear dynamical system

$$\dot{x} = f(x,u)$$
$$y = h(x,u).$$




- This system is *passive* if there exists a continuous storage function *V(x) ≥ 0 (for all x)* such that

$$\int_{t_1}^{t_2} u^T(t)y(t)dt + V(x(t_1)) \geq V(x(t_2))$$

for all $t_2 \geq t_1$ and input *u(t) ϵ U*.

- When V(x) is continuously differentiable, it can be written as:

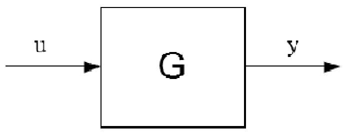
$$u^T(t)y(t) \geq \dot{V}(x(t))$$

25

# Passivity in Discrete-time

- Passivity can also be defined for discrete-time systems. Consider a nonlinear discrete time system

$$x(k+1) = f(x(k),u(k))$$
$$y(k) = h(x(k),u(k)).$$




- This system is *passive* if there exists a continuous storage function *V(x) ≥ 0* such that

$$\sum_{k=k_1}^{k_2} u^T(k)y(k) + V(x(k_1)) \geq V(x(k_2))$$

for all $k_1$, $k_2$ and all inputs *u(k) ϵ U*.

26

# Extended Definitions of Passivity

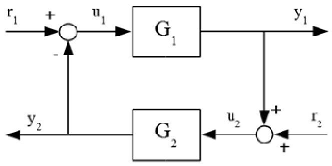
| | |
|---|---|
| Passive | $u^T y \geq \dot{V}(x)$ |
| Lossless | $u^T y = \dot{V}(x)$ |
| Strictly Passive | $u^T y \geq \dot{V}(x) + \psi(x)$ |
| Strictly Output Passive | $u^T y \geq \dot{V}(x) + \varepsilon y^T y$ |
| Strictly Input Passive | $u^T y \geq \dot{V}(x) + \delta u^T u$ |

- Note that *V(x)* and *Ψ(x)* are positive definite and continuously differentiable. The constants ε and δ are positive. These equations hold for all times, inputs, and states.

27

# Interconnections of Passive Systems

- One of the strengths of passivity is when systems are interconnected. Passive systems are stable and passivity is preserved in many practical interconnections.
- For example, the negative feedback interconnection of two passive systems is passive.

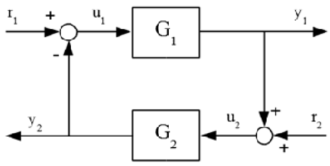



- If $u_1 \rightarrow y_1$ and $u_2 \rightarrow y_2$ are passive then the mapping $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \rightarrow y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ is passive
- Note: the other internal mappings ($u_1 \rightarrow y_2$ and $u_2 \rightarrow y_1$) will be stable but may not be passive

28

## Interconnections of Stable Systems

- Compared with passive systems, the feedback interconnection of two stable systems is not always stable




- One notable special case is the small gain theorem where if $G_1$ and $G_2$ are finite-gain $L_2$ stable with gains $\gamma_1$ and $\gamma_2$ then the interconnection is stable if $\gamma_1\gamma_2 < 1$.
- Both Passivity theory and the small gain theorem are special cases of larger frameworks including the conic systems theory and the passivity index theory.



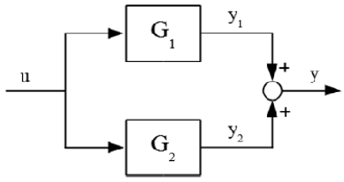
## Stability of Passive Systems

- Strictly passive systems ($\psi(x)>0$) are asymptotically stable
- Output strictly passive systems ($\delta>0$) are $L_2$ stable
- The following results hold in feedback
  - Two passive systems → passive and stable loop
  - Passive system and a strictly passive system → asymptotically stable loop
  - Two output strictly passive systems → $L_2$ stable loop
  - Two input strictly passive systems ($\varepsilon>0$) → $L_2$ stable loop

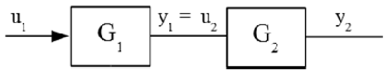
$$u^T y \geq \dot{V}(x) + \varepsilon u^T u$$

## Other Interconnections

- The parallel interconnection of two passive systems is still passive




- However, this isn't true for the series connection of two systems




- For example, the series connection of any two systems that have 90° of phase shift have a combined phase shift of 180°



# Dissipativity, conic systems, and passivity indices

## Definition of Dissipativity (CT)

- This concept generalizes passivity to allow for an arbitrary energy supply rate *ω(u,y).*
- A system is *dissipative* with respect to supply rate *ω(u,y)* if there exists a continuous storage function *V(x) ≥ 0* such that

$$\int_{t_1}^{t_2} \omega(u,y)dt \geq V(x(t_2)) - V(x(t_1))$$

for all $t_1$, $t_2$ and the input *u(t) ϵ U*.

- A special case of dissipativity is the QSR definition where the energy supply rate takes the following form:

$$\omega(u,y) = y^T Q y + 2 y^T S u + u^T R u.$$

- *QSR dissipative systems are* $L_2$ *stable when Q<0*

33

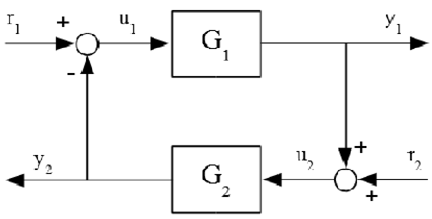
## QSR Dissipativity (CT)

- Consider the feedback interconnection of $G_1$ and $G_2$
  - $G_1$ is QSR dissipative with $Q_1$, $S_1$, $R_1$
  - $G_2$ is QSR dissipative with $Q_2$, $S_2$, $R_2$
- The feedback interconnection

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \rightarrow y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

is stable if

$$\tilde{Q} = \begin{bmatrix} Q_1 + R_2 & S_1 - S_2^{\ T} \\ S_1^{\ T} - S_2 & Q_2 + R_1 \end{bmatrix} < 0$$




- Other mappings ($r_1 \rightarrow y_2$ and $r_2 \rightarrow y_1$) are stable but may not be passive
- Large scale sytems (with multiple feedback connections) can be analyzed using QSR dissipativity to show stability of the entire system

34

## Definition of Dissipativity (DT)

- The concept of dissipativity applies to discrete time systems for an arbitrary supply rate *ω(u,y).*
- A system is *dissipative* with respect to supply rate *ω(u,y)* if there exists a continuous storage function *V(x) ≥ 0* such that

$$\sum_{k=k_1}^{k_2} \omega(u,y) \geq V(x(k_2)) - V(x(k_1))$$

  for all $k_1$, $k_2$ and the input *u(k) ϵ U*.
- A special case of dissipativity is the QSR definition where the energy supply rate takes the following form:

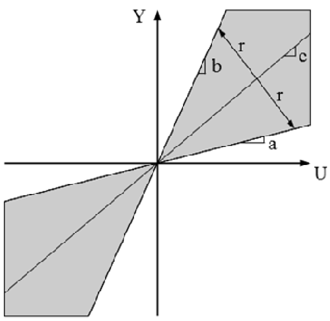
$$\omega(u,y) = y^T Q y + 2 y^T S u + u^T R u.$$

- Dissipative DT systems are stable when *Q<0*

35

## Conic Systems

- A conic system is one whose input-output behavior is constrained to lie in a cone of the *U x Y* inner product space




- A system is conic if the following dissipative inequality holds for all $t_2 \geq t_1$

$$\int_{t_1}^{t_2} \left[ (1+\frac{a}{b}) u^T y - a u^T u - \frac{1}{b} y^T y \right] dt$$
$$\geq V(x(t_2)) - V(x(t_1))$$

36

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# Passivity Indices

- Conic systems and passivity indices capture similar information about a system

- A passivity index measures the level of passivity in a system
- Two indices are required to characterize the level of passivity in a system
  1. The first measures the level of stability of a system
  2. The second measures the extent of the minimum phase property in a system
- They are independent in the sense that knowing one provides no information about the other
- Each has a simple physical interpretation

37

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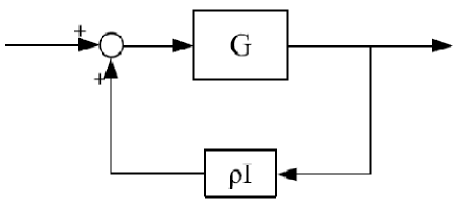
# Output Feedback Passivity Index

The output feedback passivity index (OFP) is the largest gain that can be put in positive feedback with a system such that the interconnected system is passive.



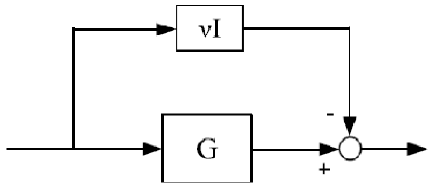

Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y\,dt \geq V(x(t_2)) - V(x(t_1)) + \rho \int_{t_1}^{t_2} y^T y\,dt$$

38

# Input Feed-Forward Passivity Index

The input feed-forward passivity index (IFP) is the largest gain that can be put in a negative parallel interconnection with a system such that the interconnected system is passive.



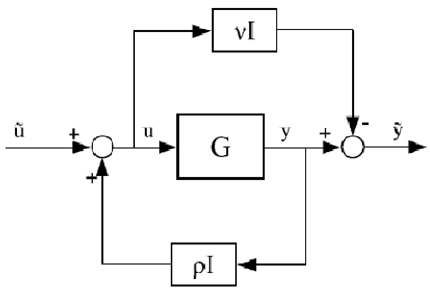

Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y\,dt \geq V(x(t_2)) - V(x(t_1)) + \nu \int_{t_1}^{t_2} u^T u\,dt$$



# Simultaneous Indices

When applying both indices the physical interpretation as in the block diagram




Equivalent to the following dissipative inequality holding for G

$$(1+\rho\nu) \int_{t_1}^{t_2} u^T y\,dt \geq V(x(t_2)) - V(x(t_1)) + \rho \int_{t_1}^{t_2} y^T y\,dt + \nu \int_{t_1}^{t_2} u^T u\,dt$$

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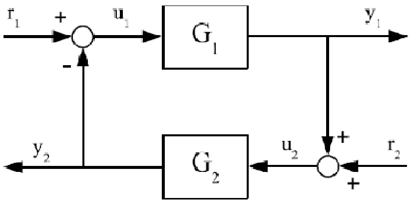
# Stability

We can assess the stability of an interconnection using the indices for $G_1$ and $G_2$




$G_1$ has indices $\rho_1$ and $\nu_1$

$G_2$ has indices $\rho_2$ and $\nu_2$

The interconnection is $\mathcal{L}_2$ stable if the following matrix is positive definite

$$\begin{bmatrix} (\rho_1 + \nu_2)I & \frac{1}{2}(\rho_2\nu_2 - \rho_1\nu_1)I \\ \frac{1}{2}(\rho_2\nu_2 - \rho_1\nu_1)I & (\rho_2 + \nu_1)I \end{bmatrix} > 0$$

41

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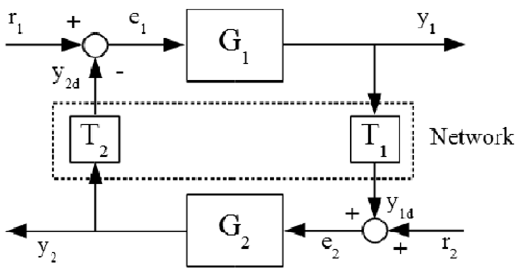
# Networked passive systems

42

# Networked Systems

- Motivating Problem: The feedback interconnection of two passive systems is passive and stable. However, when the two are interconnected over a delayed network the result is not passive so stability is no longer guaranteed. How do we recover stability?

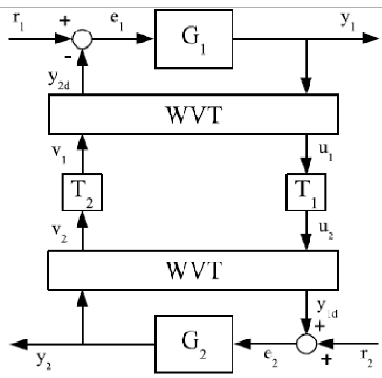



The systems $G_1$ and $G_2$ are interconnected over a network with time delays $T_1$ and $T_2$



# Stability of Networked Passive Systems

- One solution to interconnecting passive systems over a delayed network is to add an interface between the systems and the network
- The wave variable transformation forces the interconnection to meet the small gain theorem. Stability is guaranteed for arbitrarily large time delays
- The WVT is defined below




$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_1 \\ y_{2d} \end{bmatrix} \qquad \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_{1d} \\ y_2 \end{bmatrix}$$

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# Computational methods for showing passivity and dissipativity

45

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## Showing Passivity and Dissipativity

- Passivity and dissipativity are powerful properties for analysis and synthesis of dynamical systems.
- Requires finding a positive storage function *V* and an appropriate $\omega$ in the case of dissipativity.

$$\int_{t_1}^{t_2} \omega(u(t), y(t))dt + V(x(t_1)) \geq V(x(t_2))$$

- In a switched system with *m* subsystems, dissipativity requires finding *m* storage functions and ~$m^2$ dissipative rates
- In the worst case, this is a search similar to finding a Lyapunov function
- In many practical cases, this can be automated so that a program can generate an energy storage function (for LTI systems this is done using LMIs)

46

## LMI Methods – Passivity

- There are computational methods to find storage functions. For LTI passive systems, can always assume there exists a quadratic storage function

$$V(x) = \frac{1}{2} x^T P x \quad \text{where} \quad P = P^T > 0.$$

- For continuous-time system this leads to the following LMI

$$\begin{bmatrix} A^T P + PA & PB - C^T \\ B^T P - C & -D - D^T \end{bmatrix} \leq 0$$

- In discrete-time the LMI becomes the following

$$\begin{bmatrix} A^T PA - P & A^T PB - C^T \\ B^T PA - C & B^T PB - D - D^T \end{bmatrix} \leq 0$$



## LMI Methods – QSR Dissipativity

- The same can be done to demonstrate that an LTI system is QSR dissipative. Once again, a quadratic storage function is used

$$V(x) = \frac{1}{2} x^T P x \quad \text{where} \quad P = P^T > 0.$$

- For continuous-time system this leads to the following LMI

$$\begin{bmatrix} A^T P + PA - C^T QC & PB - C^T QD - C^T S \\ B^T P - D^T QC - S^T C & -D^T QD - S^T D - D^T S - R \end{bmatrix} \leq 0$$

- In discrete-time the LMI becomes the following

$$\begin{bmatrix} A^T PA - P - C^T QC & A^T PB - C^T QD - C^T S \\ B^T PA - D^T QC - S^T C & B^T PB - D^T QD - S^T D - D^T S - R \end{bmatrix} \leq 0$$

## Passivity and CPS

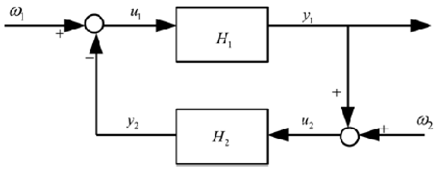
1. A Passivity Measure Of Systems In Cascade Based On Passivity Indices
2. Passivity-Based Output Synchronization With Application To Output Synchronization of Networked Euler-Lagrange Systems Subject to Nonholonomic Constraints
3. Event-Triggered Output Feedback Control for Networked Control Systems using Passivity
4. Output Synchronization of Passive Systems with Event-Driven Communication
5. Quantized Output Synchronization of Networked Passive Systems with Event-driven Communication



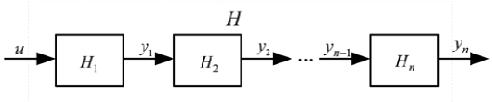
## Passivity of systems in series

## A Passivity Measure Of Systems In Cascade Based On Passivity Indices (Yu & Antsaklis, CDC10)

➢Problem Statement




• The negative feedback interconnection of two passive systems is still passive.




• Will the cascade interconnection of several passive systems still be passive?

H.Yu and Panos J. Antsaklis, "A Passivity Measure Of Systems In Cascade Based On The Analysis Of Passivity Indices", Proceedings of IEEE Conference on Decision and Control, Atlanta, Georgia, USA, December 15-17, 2010.





## A Passivity Measure Of Systems In Cascade Based On Passivity Indices (Yu & Antsaklis, CDC10) : main results

Each $H_i$ is IF-OFP($\nu_i$,$\rho_i$) such that $\dot{V}_i \leq u_i^T y_i - \rho_i y_i^T y_i - \nu_i u_i^T u_i$ with $\nu_i, \rho_i \in \mathbb{R}$. If for some $\hat{\rho}, \hat{\nu} \in \mathbb{R}$ and

$$A = \begin{bmatrix} -\nu_1 + \hat{\nu} & \frac{1}{2} & 0 & \cdots & -\frac{1}{2} \\ \frac{1}{2} & -\nu_2 - \rho_1 & \frac{1}{2} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{2} & -\nu_n - \rho_{n-1} & \frac{1}{2} \\ -\frac{1}{2} & 0 & \cdots & \frac{1}{2} & -\rho_n + \hat{\rho} \end{bmatrix},$$

such that $-A$ is quasi-dominant,
then cascade system admits a storage function of the form

$$V = \sum_{i=1}^{n} d_i V_i, \; d_i > 0, \; n \geq 2$$

and the cascade interconnection is IF-OFP($\hat{\nu}$,$\hat{\rho}$).

# Event-triggered control of passive systems



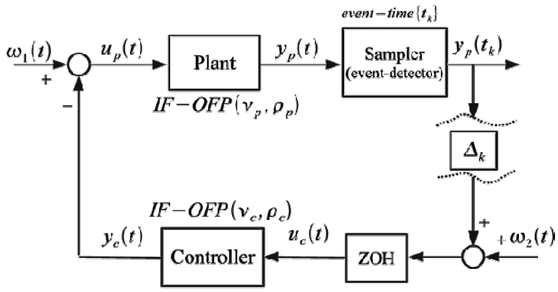
# Event-Triggered Control for Networked Systems using Passivity

- Event triggered control is used to reduce communication in networked control systems
- This approach studies event-triggered control from an input-output perspective instead of a state based model
- The implementation of our event-triggered control strategy does not impose constraints on the maximal admissible network induced delays

[Yu and Antsaklis 2011 CDC]

# Event-Triggered Network

- When an event occurs the sampled output $y_p(t_k)$ is sent to the networked controller
- Events occur when a triggering condition is met




- One such triggering condition is to maintain closed loop stability
- This can be done by using passivity indices of the plant *($\rho_p$* and *$v_p$)* and controller *($\rho_c$* and *$v_c$)*



# Triggering Condition

- Closed loop stability can be maintained using the passivity indices of the plant *($\rho_p$* and *$v_p$)* and controller *($\rho_c$* and *$v_c$)*
- For now, assume the network delay is zero
- An appropriate triggering condition can be found according to

$$\|\tilde{e}_p(t)\|_2 = \frac{\delta}{\zeta}\left[\sqrt{\beta(\rho_p+\nu_c)+\frac{\nu_c^2}{\zeta^2}}-\frac{|\nu_c|}{\zeta}\right]\|y_p(t)\|_2, \;\forall t \geq 0,$$

$$\text{where } \; \tilde{e}_p(t) = y_p(t) - y_p(t_k), \quad \text{for } \; t \in [t_k, t_{k+1}],$$

$$\zeta = \left[\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c\right]^{\frac{1}{2}},$$

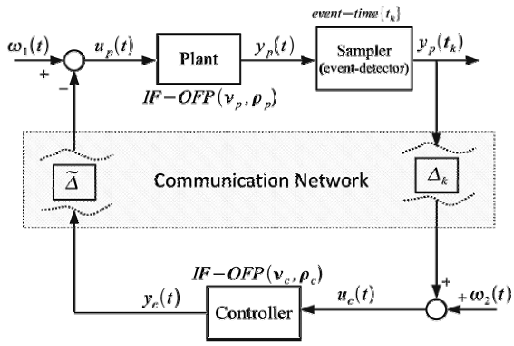
where α, β, and δ are constants between 0 and 1.

- Then the closed loop system with the network is *$L_2$* stable

[Yu & Antsaklis 2011 CDC]

# Delay Over the Network

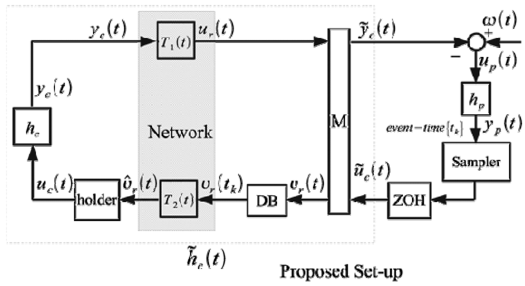
Many control architectures have communication delays on both directions of the network



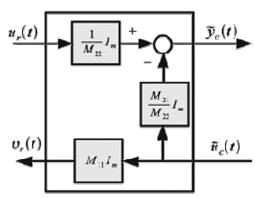





# Compensating for Delays




Proposed Set-up



Local controller M

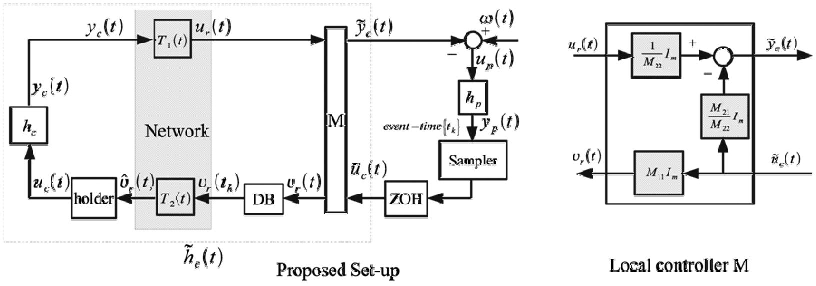
The proposed design method decouples the design of a controller from the design of the network with delays

The network controller $h_c$ has been designed for stability

$$0 < \rho_c + \nu_p < \infty, \quad 0 < \rho_p + \nu_c < \infty$$

The transformation M can be designed to compensate for delays

## Defining an Appropriate Transformation




Proposed Set-up

Local controller M

$$M_{11}^2 = \frac{\frac{1}{4\rho_c} - \nu_c}{\frac{1}{2\rho_c} + |\nu_c|}, \quad M_{21}^2 = \frac{1}{2(1-D_1)\rho_c^2}$$

$$M_{22}^2 = \frac{2}{1-D_1}, \quad M_{21}M_{22} < 0,$$

$$\begin{bmatrix} v_r(t) \\ u_r(t) \end{bmatrix} = M = \begin{bmatrix} M_{11}I_m & 0 \\ M_{21}I_m & M_{22}I_m \end{bmatrix} \begin{bmatrix} \tilde{u}_c(t) \\ \tilde{y}_c(t) \end{bmatrix}$$

[Yu & Antsaklis 2011 CDC]



# Output synchronization of passive systems using event-driven communication

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# Traditional Output Synchronization

- Output synchronization is the agreement of a group agents on a particular output value
- Communication is modeled using graph theory – the graph can change over time but it is assumed that the communication graph is at least weakly connected
- Each agent updates its own output value by using the output values of its neighboring agents

$$u_i(t) = \sum_{j \in \mathcal{N}_i} K\left[y_j(t) - y_i(t)\right], \quad i = 1, 2, \ldots, N$$

- Previous work has shown that the group of agents will synchronize using this update law when communication is periodic



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# Output Synchronization Using Event-Driven Communication

- Using event-driven communication, the update law holds previous values between update times

$$u_i(t) = \sum_{j \in \mathcal{N}_i} a\left[\widehat{y}_j(t) - \widehat{y}_i(t)\right]$$

$$\widehat{y}_j(t) = y_j(t^j_{k'}),\ \text{for } t \in [t^j_{k'}, t^j_{k'+1}] \text{ with } j \in \mathcal{N}_i;\ \widehat{y}_i = y_i(t^i_k),\ \text{for } t \in [t^i_k, t^i_{k+1}].$$

- The error between the actual value and held value can be written

$$e_i(t) = y_i(t) - \widehat{y}_i$$

- Output synchronization can be achieved if each agent transmits its current output $y_i$ when the error grows large enough to meet the following triggering condition

$$\left\|e_i(t)\right\|_2 > \frac{\delta_1 \sum_{j \in \mathcal{N}_i} \left\|\widehat{y}_j - \widehat{y}_i\right\|_2^2}{\left\|\sum_{j \in \mathcal{N}_i} (\widehat{y}_j - \widehat{y}_i)\right\|_2},\ \forall t \geq 0$$

- Weak connectivity is still assumed

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# Event-driven Communication with Quantization

- When there is quantization in the network, the agents cannot synchronize exactly but the error can be bounded
- The same update law can be used

$$u_i(t) = \sum_{j \in \mathcal{N}_i} a\left[q(\widehat{y}_{k_j}) - q(\widehat{y}_{k_i})\right],$$

- Each agent transmits its current output when the following triggering condition is satisfied

$$\|e_i(t)\|_2 > \delta_3\left(\frac{1-\kappa}{2} - \frac{1}{2\beta}\right)\frac{1}{|\mathcal{N}_i|}\sum_{j \in \mathcal{N}_i}\left\|q(\widehat{y}_{k_j}) - q(\widehat{y}_{k_i})\right\|_2,$$

$$\text{where } \delta_3 \in (0,1),\ 0 < \kappa < 1 \text{ and } 1 < \tfrac{1}{1-\kappa} < \beta,$$

- It can be shown that the error in the output synchronization algorithm is bounded by the quantization error

63

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# Passivity and Dissipativity in Networked Switched Systems

64

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## Passivity for Switched Systems

- The notion of passivity has been defined for switched systems

$$\dot{x} = f_\sigma(x,u)$$
$$y = h_\sigma(x,u)$$

A *switched system* is *passive* if it meets the following conditions

1. Each subsystem *i* is passive when active:

$$\int_{t_1}^{t_2} u^T y dt \geq V_i(x(t_2)) - V_i(x(t_1))$$

2. Each subsystem *i* is dissipative w.r.t. $\omega_j^i$ when inactive:

$$\int_{t_1}^{t_2} \omega_j^i(u,y,x,t)dt \geq V_j(x(t_2)) - V_j(x(t_1))$$

3. There exists an input *u* so that the cross supply rates ($\omega_j^i$) are integrable on the infinite time interval.

[McCourt & Antsaklis 2010 ACC, 2010 CDC]

65

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## Dissipativity for Switched Systems

**Definition of Dissipativity in Discrete-time**

A discrete-time switched system is dissipative if for each subsystem $i$ there exists a positive function $V_i$ such that the following conditions hold

1. Each subsystem $i$ is dissipative while it is active with respect to $\omega_i(u,y)$:

$$\omega_i(u(t),y(t)) \geq V_i(x(t+1)) - V_i(x(t))$$

2. Each subsystem is dissipative when inactive with respect to $\omega_{ij}(u,y,x,t)$ for each active subsystem $j$:

$$\omega_{ij}(u(t),y(t),x(t),t(t)) \geq V_i(x(t+1)) - V_i(x(t))$$

**Stability for Dissipative Discrete-time Systems**

***Theorem.*** *Dissipative switched systems are stable when $\omega_i < 0$ for all i and there exists an infinitely summable function $\phi(t)$ so that the inactive energy is bounded,*

$$\phi(t) \geq \omega_{ij}(u,y,x,t).$$

66

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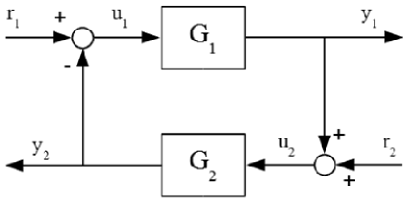
## QSR Dissipativity for Switched Systems

- QSR dissipativity uses a quadratic supply rate to capture energy

$$\omega_i(u,y) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} Q_i & S_i \\ S_i^T & R_i \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix}$$

- Stability of switched systems can be assessed using $Q_i$




- Dissipativity of the feedback interconnection of two switched systems can be assessed with $Q_i$, $S_i$, $R_i$ of both systems
- Large scale systems can be analyzed or designed using QSR dissipativity to ensure that the entire system is stable
- When dealing with passive switched systems ($Q_i = 0$, $S_i = {}^1\!/_2 I$, $R_i = 0$ ), any sequential combination of systems in feedback or parallel can be shown to be passive and stable

[McCourt &Antsaklis 2012 ACC]



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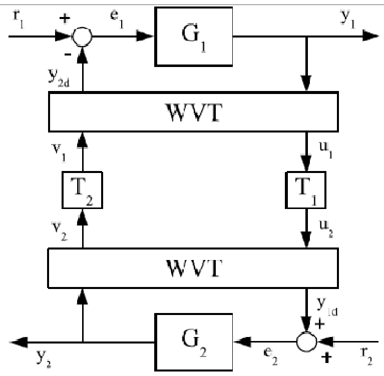
## Stability of Networked Passive Systems

- When interconnecting passive discrete-time switched systems over a network, delays must be considered
- The transformation approach can be generalized to apply to switched systems
- The approach can compensate for time-varying delays
- The wave variable transformation is defined below




$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_1 \\ y_{2d} \end{bmatrix} \qquad \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_{1d} \\ y_2 \end{bmatrix}$$

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# Compensating for quantization in passive systems



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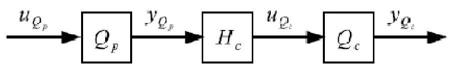
## Motivating Problem

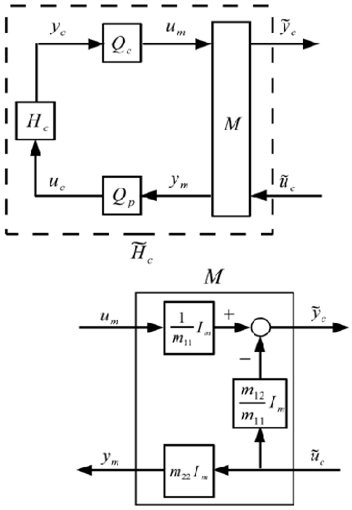
- When analyzing digital controllers or continuous systems that are sampled, discretization and quantization must be considered
- There are existing results on how to preserve passivity when a continuous system is discretized
- A passive system $H_C$ may not be passive after input-output quantization




- Can recover passivity when making some simple assumptions on the quantization

## One Solution to Quantization

- The given scheme recovers passivity for an output strictly passive (OSP) system ($H_C$) with passive quantizers ($Q_C$ and $Q_P$) using an input/output transformation ($M$)
- Can be applied to passive switched systems
  - Switch input/output transformation according the current active system.
  - Stability conditions can be applied



[Zhu, Yu, McCourt,&Antsaklis 2012 HSCC]



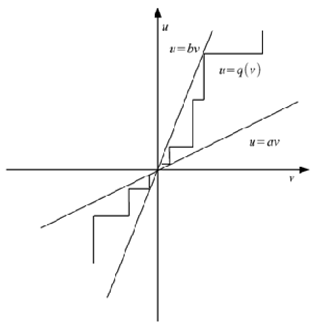
## Class of Quantization

- This work applies with a restricted set of quantization – specifically quantizers that are restricted to lie in a cone

**Definition** *A quantizer is called a passive quantizer if its input $v$ and output $u$ satisfy*

$$av^2 \leq uv \leq bv^2$$

*where $u = q(v)$ and $0 \leq a \leq b < \infty$.*




- This definition requires very small values to map to zero
- Includes many practical quantizers

# Preserving Passivity…

THEOREM . *Consider an OSP system $H_C$ in the proposed scheme shown in Fig. 5 with passive quantizers $Q_c$ and $Q_p$. If a transformation $M$ is chosen such that*

$$m_{21} = 0, \quad m_{11}^2 = 2b_c^2$$
$$m_{11}m_{12} = \frac{-b_c^2}{\rho_c}\ , \quad m_{12}^2 = \frac{b_c^2 b_p^2}{\rho_c^2} m_{22}^2,$$

*then the subsystem $\widetilde{H}_c : \tilde{u}_c \to \tilde{y}_c$ is output strictly passive such that*

$$\Delta V_c(k) = V_c(k+1) - V_c(k) \leq \tilde{u}_c^T(k)\tilde{y}_c(k) - \rho_c \tilde{y}_c^T(k)\tilde{y}_c(k).$$

- This result is valid for continuous-time OSP systems.

- The negative feedback interconnection of two OSP systems is passive and thus stable.

-The same idea can be extended to switched systems

73

# Extension to Switched Systems

THEOREM *Consider an output strictly passive discrete-time switched system $H_C$. This system is placed in the structure with passive quantizers defined by the constants $a_c$, $b_c$, $a_p$, and $b_p$. This control structure preserves the output strict passivity property of system $H_C$ if the transformation $M(k)$ is chosen according to the following time-varying equations*

$$m_{21}(k) = 0, \quad m_{11}^2(k) = 2b_c^2$$
$$m_{11}m_{12}(k) = \frac{-b_c^2}{\rho(k)}\ , \quad m_{12}^2(k)(t) = \frac{b_c^2 b_p^2}{\rho^2(k)} m_{22}^2(k),$$

- This theorem guarantees that the switched system will be passive even with input and output quantization.

-Since the feedback interconnection of two passive switched systems is also passive and thus stable, this result can be used to further interconnect systems.

74

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# Symmetry and Dissipativity



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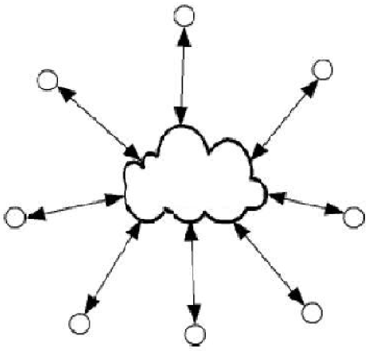
# Introduction: Symmetry in Systems

- Symmetry: A basic feature of shapes and graphs, indicating some degree of repetition or regularity
  - Symmetry in characterizations of information structure
  - Identical dynamics of subsystems
  - Invariance under group transformation e.g. rotational symmetry

- Why Symmetry?
  - Decompose into lower dimensional systems with better understanding of system properties such as stability and controllability
  - Construct symmetric large-scale systems without reducing performance if certain properties of low dimensional systems hold

## Simple Examples

**Star-shaped Symmetry**



$$u = u_e - \tilde{H}y$$

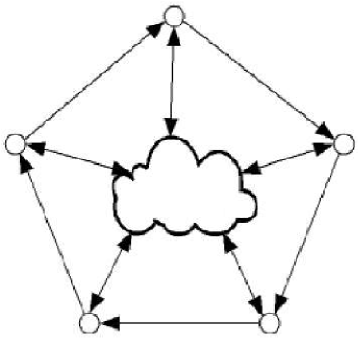
$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

$$u_0 = u_{e0} - Hy_0 - by_1 - \cdots - by_m$$
$$u_1 = u_{e1} - cy_0 - hy_1$$
$$\vdots$$
$$u_m = u_{em} - cy_0 - hy_m$$

77

## Simple Examples

**Cyclic Symmetry**



$$u = u_e - \tilde{H}y$$

$$\tilde{H} = \begin{bmatrix} H & b \quad \cdots \quad b \\ c & \\ \vdots & \tilde{h} \\ c & \end{bmatrix}$$

$$\tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

$$u_0 = u_{e0} - Hy_0 - by_1 - \cdots - by_m$$
$$u_1 = u_{e1} - cy_0 - v_1 y_1 - v_2 y_2 - \cdots - v_m y_m$$
$$\vdots$$
$$u_m = u_{em} - cy_0 - v_2 y_1 - v_3 y_2 - \cdots - v_1 y_m$$

78

## Problem

-A system is (Q, S, R) – dissipative if there exists a positive semi-definite storage function V(x) and a specific supply rate $\omega(u, y)$ such that the following inequality holds

$$V(x(0)) + \int_0^T \omega(u(t), y(t))dt \geq V(x(T))$$

where

$$\omega(u, y) = y^T Q y + 2 y^T S u + u^T R u$$

-Nonlinear symmetric distributed systems with dissipativity

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i \qquad y_i = h_i(x_i)$$

Feedback interconnections

$$u_i = u_{ei} - \sum_{j=1}^{N} H_{ij} y_j \qquad H = [H_{ij}] \quad \Rightarrow \quad u = u_e - Hy$$

79

## Main Result (1)

Theorem (Star-shaped Symmetry)
Consider a (Q, S, R) – dissipative system extended by *m* star-shaped symmetric (q, s, r) – dissipative subsystems. The whole system is finite gain input-out stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(c^T rc + \beta(\hat{q} - b^T Rb)^{-1}\beta^T)}, \frac{\hat{q}}{b^T Rb}\right)$$

where

$$\hat{Q} = -H^T RH + SH + H^T S^T - Q > 0$$
$$\hat{q} = -h^T rh + sh + h^T s^T - q > 0$$
$$\beta = Sb + c^T s^T - H^T Rb - c^T rh$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

80

## Main Result (2)

Theorem (Cyclic Symmetry)
Consider a (Q, S, R) – dissipative system extended by *m* cyclic symmetric (q, s, r) – dissipative subsystems. The whole system is finite gain input-out stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(c^T rc + \beta_m \Lambda^{-1} \beta_m{}^T)}, \frac{-r\sigma(\tilde{h})\overline{\sigma(\tilde{h})} + s(\sigma(\tilde{h}) + \overline{\sigma(\tilde{h})}) - q}{b^T Rb}\right)$$

where

$$\tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

$$\sigma(\tilde{h}) = \sum_{j=1}^{m} v_j \lambda_i^j = \sum_{j=1}^{m} v_j e^{\frac{2\pi ij}{m}}$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$



## Main Result (3)

(cont.)

$$\Lambda = -r\tilde{h}^T \tilde{h} + s(\tilde{h}^T + \tilde{h}) - q \otimes I_m - b^T Rb \otimes circ([1,1,\ldots,1])$$

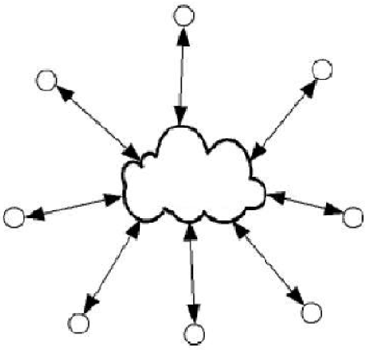
$$\beta = Sb + c^T s^T - H^T Rb - c^T r\tilde{h}$$

$$\beta_m = [\underbrace{\beta\beta\ldots\beta}_{m}]$$

the spectral characterization of $\tilde{h}$ should satisfy

$$\| \sigma(\tilde{h}) - \frac{s}{r} \| < \sqrt{\frac{s^2}{r^2} - \frac{q + mb^T Rb}{r}}$$

# Simple Examples



$$u = u_e - \tilde{H}y$$

$$\tilde{H} = \underbrace{\begin{bmatrix} 0.9 & -0.8 & -0.8 & \cdots & -0.8 \\ -0.8 & 0.1 & 0 & \cdots & 0 \\ -0.8 & 0 & 0.1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -0.8 & 0 & 0 & \cdots & 0.1 \end{bmatrix}}_{m+1}$$

$$Q = q = -I, S = s = 0, R = r = \tfrac{1}{4} I$$

$$\Rightarrow \quad m < \min(3.11, 6.25) = 3.11$$

Remark: $(-I, 0, \alpha^2 I) - \text{dissipative}$ systems corresponding to systems with gain less or equal to $\alpha$ (here $\alpha = \frac{1}{2}$)



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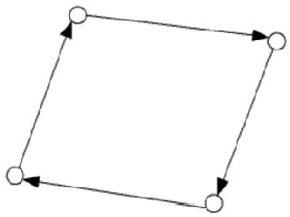
# Simple Examples



$$u = u_e - \tilde{H}y \qquad q = 0, s = \tfrac{1}{2}, r = 1$$

$$\tilde{H} = \tilde{h} = \underbrace{\begin{bmatrix} 0.1 & 0.2 & 0 & \cdots & 0 \\ 0 & 0.1 & 0.2 & \cdots & 0 \\ 0 & 0 & 0.1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.2 & 0 & 0 & \cdots & 0.1 \end{bmatrix}}_{m}$$

The cyclic symmetric system is stable if

$$|\,\sigma(\tilde{h}) - \frac{s}{r}\,\| = \| \sum_{j=0}^{m-1} v_j e^{\frac{2\pi i j}{m}} \| \le 0.3 < 0.5 = \sqrt{\frac{s^2}{r^2} - \frac{q}{r}}$$

The above stability condition is always satisfied. Also

$$m < \min(+\infty, +\infty)$$

Thus the system can be extended with infinite numbers of subsystems without losing stability.

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# Main Result (4)

Theorem (Star-shaped Symmetry for Passive Systems)
Consider a passive system extended by *m* star-shaped symmetric passive subsystems. The whole system is finite gain input-output stable if

$$m < \frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(\beta \hat{Q}^{-1} \beta^T)}$$

where

$$\hat{Q} = \frac{H + H^T}{2} > 0$$

$$\hat{q} = \frac{h + h^T}{2} > 0$$

$$\beta = \frac{b + c^T}{2}$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

85

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# Main Result (5)

Theorem (Cyclic Symmetry for Passive Systems)
Consider a passive system extended by *m* cyclic symmetric passive subsystems. The whole system is finite gain input-output stable if

where

$$m < \frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(\beta_m \Lambda^{-1} \beta_m{}^T)}$$

$$\hat{Q} = \frac{H + H^T}{2} > 0 \qquad \Lambda = \frac{\tilde{h} + \tilde{h}^T}{2}$$

$$\beta = \frac{b + c^T}{2} \qquad \beta_m = [\underbrace{\beta\beta\ldots\beta}_{m}]$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

$$\sigma(\tilde{h}) = \sum_{j=1}^{m} v_j \lambda_i^j = \sum_{j=1}^{m} v_j e^{\frac{2\pi ij}{m}} \qquad \tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

86

# Concluding Remarks

- Main points
  - **New ways of thinking needed to deal effectively with the CPS problems. New ways to determine research directions.**
  - Passivity/Dissipativity and Symmetry are promising
  - Need deeper understanding of fundamentals that cut across disciplines.

    **CPS**, Distributed, Embedded, Networked Systems. Analog-digital, large scale, life cycles, safety critical, end to end high-confidence.
  - **Need to expand our horizons. Control Systems at the center.**
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# Cyber-Physical Systems Design Using Dissipativity

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**Abstract:** In Cyber-Physical Systems large numbers of heterogeneous cyber and physical subsystems are networked, are interacting tightly, may change dynamically and may expand or contract. Designing and preserving properties of a CPS over its lifespan is very challenging. Passivity and dissipativity are energy like concepts that offer great promise in guaranteeing properties, such as stability, in complex heterogeneous interconnected systems that are changing dynamically. Passivity indices that provide a measure of the degree of passivity are used to generalize classical results in interconnected systems, and results for continuous, discrete and switched systems in networks with delays, event triggered architectures, conic systems and systems with symmetries are shown.

## 1 Introduction

Recent technological developments in sensing, communications, control and computation have created an emerging class of complex systems, called Cyber-Physical Systems (CPS). Cyber-Physical Systems are characterized by large numbers of tightly integrated heterogeneous components in a network, which may expand and contract dynamically. Cyber-Physical Systems are very common and are becoming increasingly ubiquitous. Examples of CPS may be found in smart transportation systems, smart medical devices, smart buildings, smart energy systems, the smart grid. The control of such systems presents huge challenges and requires designs drawn from approaches such as those in traditional control, hybrid control systems, discrete event systems, and networked control. In addition, robustness, reliability and security issues for reconfiguring dynamical systems must also be addressed. This integration of different technologies and scientific domains presents new and challenging fundamental problems underlying the theoretical foundations for this class of systems.

There has been a series of research activities in CPS over the past 7 years and information may be found at the CPS Virtual Organization website (http://cps-vo.org/). It should be noted that the importance of CPS was recognized in the 2007 report of the USA President's Council of Advisors on Science and Technology (PCAST) "Leadership Under Challenge: Information Technology R&D in a Competitive World," PCAST report, August 2007 (http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-07-nitrd-review.pdf) and reaffirmed in the 2010 PCAST report "Designing a Digital Future: Federally Funded Research and Development in Networking and Information Technology," PCAST Report, December 2010 (http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-nitrd-report-2010.pdf).

In the design of CPS one has to guarantee certain properties of the whole system even though the system consists of networked heterogeneous subsystems, the number of which may expand or contract. The energy like concepts of passivity and dissipativity appear to offer promise towards that goal and this presentation describes our recent research efforts towards establishing design methodologies for CPS. Our research work on passivity and CPS is in collaboration with Vanderbilt University, University of Maryland and GM R&D [1] and it is being supported by the National Science Foundation (Grant No. CNS-1035655); this support is gratefully acknowledged.

In the following an outline of our research on the design of CPS using passivity and dissipativity is presented. In particular, Section 2 covers the problem of networking passive switched systems. This uses an accepted definition of passivity for switched systems and assumes these systems are connected over a network with possible delays, lost data, and quantization. Section 3 focuses on the application of event-triggered control to passive systems. Event-triggered control has been used to reduce communication in networks to guarantee a specified level of performance. In Section 4, passivity is applied to multi-agent systems that exhibit a form of symmetry in their interconnections. Finally, concluding remarks are made in Section 5.

## 2 Networking Passive Switched Systems

In CPS, physical processes are modeled using differential or difference equations with a strong dependence on time. The cyber processes evolve based on the occurrence of events, both physically and in software, and are modeled using discrete-event models such as finite automata or Petri nets. The combination of these different components results in system models that are hybrid or switched.

One challenging aspect of CPS is that these complex systems are often made up of varying components. The property of compositionality is crucial to analyzing stability of these systems. One approach to compositionality is using passivity or dissipativity theory [2]. Passivity is an energy inspired property of dynamical systems that is preserved when two systems are interconnected in parallel or in negative feedback. Under mild assumptions, passivity implies stability [3]. Using these two properties together, large-scale stable systems can be built up by sequentially connecting passive components together. Dissipativity properties can also be studied in the switched system framework. Although dissipative systems may not be stable, dissipativity is a property that is preserved in feedback and stability may be assessed from the dissipative rate of the interconnection.

The compositionality that passivity provides may be exploited in CPS by a generalized passivity property for switched systems (see [4] and the references therein). A general model of nonlinear switched systems may be used. Switching between subsystems is assumed to be bounded on any finite time interval so to avoid the Zeno phenomenon. In general terms, a switched system is passive if the following two conditions hold.

1) Each subsystem is passive when it is active.
2) Each subsystem is dissipative (of a special form) when it is inactive.

For the second condition, the form of dissipativity is general, but it is restricted since there must exist inputs to ensure that switching only adds a finite amount of energy over the infinite time horizon. This definition generalizes the expected properties of passive systems. First, passivity is preserved when passive switched systems are interconnected in negative feedback. Second, when the definitions are made slightly more restrictive, expected stability results are shown. This includes strictly passive implying asymptotic stability and output strictly passive implying $\mathcal{L}_2$ stability (bounded input, bounded output stability).

Although many practical systems are passive, some applications include switched systems that aren't necessarily passive. One approach is the area of passivity indices where the feedback stability result can be extended to these non-passive systems. This framework generalizes the property by quantifying the level of passivity in a given system. In order to completely characterize the level in a system, two indices are required. The first is a measure of the level of stability of the system. The second is a measure of the extent of the minimum phase property in a system. This framework has close ties to conic systems theory [5].

The main difference in applying the indices to switched systems is that the indices become *time-varying*. Each subsystem has values for the two indices and the overall switched system takes on the values of the indices over the time intervals where that subsystem is active. With this definition, the passivity indices for switched systems are piecewise constant. With the earlier assumption that there is finite switching on any finite time interval, the switching signals are well behaved and the time varying indices are well defined. The results based on the indices generalize to the case of switched systems. Conceptually, when considering the feedback interconnection of two systems, a shortage of passivity of one system can be compensated by an excess of passivity in the other system. Specifically, a shortage of stability in one system can be compensated by an excess of the minimum phase property in the other system and the other way around [6]. Once the indices have been assessed for a given interconnection of two switched systems, the verification that the interconnection is stable is as simple as checking whether a matrix is positive definite. This means that stable feedback loops can be designed even when the systems in the loop aren't passive or even stable.

Another challenging area in CPS is that many components are interconnected over communication channels that include delayed data and lost packets. Although passivity is preserved when two switched systems are interconnected in feedback, this no longer holds when delays are introduced. Since passivity is an energy based property with a strong dependence on time, it requires instant transmission of the energy on one side of the network to the other.

One solution to this issue is a network interface that decouples the notion of energy defined by passivity. The interface is an invertible, input-output coordinate transformation to wave variables. Instead of an inner product, energy is decoupled into a wave going out to the network and a wave coming in from the network. The wave variable transformation is well established for non-switched systems, but many areas of CPS require models that are switched. This approach can be used to handle delays that are time-varying with an upper bound and lost data due to packet drops [7].

Networking CPS has other issues such as discretization and quantization. These are common issues when continuous-time physical processes are sampled to be controlled by digital controllers or when signals are quantized to use digital networks. While discretization of passive systems has been well-studied, *quantization* has been largely ignored. The main problem is that the stability results that are provided by passivity theory no longer hold when quantization is present.

This problem is addressed in [8]. That paper introduced a control framework under which passivity for switched and non-switched systems can be maintained despite input and output quantization. The quantizers may be general with non-uniform levels as long as the gains of the quantizers are finite. This framework centers on the use of an input-output coordinate transformation to recover passivity. The transformation is not unique, but under mild assumptions, a transformation can always be found to preserve passivity despite quantization.

## 3 Event Triggered Control of Passive Systems

Recently, several researchers have suggested the idea of event-based control as a promising technique to reduce communication and computation load for the purpose of control in many control applications. In a typical event-based implementation, the control signals are kept constant until the violation of an "event triggering condition" on certain signals which triggers the re-computation of the control actions. Compared with time-driven control, where constant sampling period is applied to guarantee stability in the worst case scenario, the possibility of reducing the number of computations, and thus of transmissions, while guaranteeing desired levels of performance makes event-based control very appealing in networked control systems (NCSs). A comparison of time-driven and event-driven control for stochastic systems favoring the latter can be found in [9]; a deterministic event-triggered control strategy is introduced in [10]; similar results on deterministic self-triggered feedback control have been reported in [11], [12]; output-based event-triggering control with guaranteed $L_\infty$-gain for linear time-invariant systems has been studied in [15]; event-triggering stabilization for distributed networked control systems has been studied in [13]; in [14], a self-triggered coordination strategy for optimal deployment of mobile robotics is proposed.

Most of the results on event-triggered control are obtained under the assumption that the feedback control law provides input-to-state stability (ISS) with respect to the state measurement errors. However, in many control applications the full state information is not available for measurement, so it is important to study stability and performance of event-triggered control systems with dynamic and static output feedback controllers. In [16], a static output feedback based event-triggered control scheme is introduced for stabilization of passive and output feedback passive (OFP) NCSs. A static output feedback gain and a triggering condition are derived based on the output feedback passivity indices of the plant. In [17], a dynamic output feedback based event-triggered control scheme is introduced for stabilization of Input Feed-forward Output Feedback Passive (IF-OFP) NCSs, which expands our previous work in [16] for stabilization of more general dissipative systems. The triggering condition is derived based on the passivity theorem which allows us to characterize a large class of output feedback stabilization controllers. We show that under the triggering condition derived in [17], the control system is finite gain $L_2$ stable in the presence of bounded external disturbances. The interactions between the triggering condition, the achievable $L_2$ gain of the control system and the inter-sampling time have been studied in terms of the passivity indices of the plant and the controller. Based on the results in [17], we further propose a dynamic output feedback based event-triggered control set-up for NCSs which allows us to consider network induced delays both from the sampler to the controller and from the controller to the plant [18]. We show that based on the proposed set-up, finite-gain $L_2$ stability can be achieved in the presence of arbitrary constant network induced delays or delays with bounded jitters.

Event-based distributed control in cooperative control of multi-agent systems is of interest because of the potential to reduce communication load and implementation cost. In [19], we propose a distributed event-driven communication strategy for stabilization of large scale networked control systems with finite-gain $L_2$ stability. Each subsystem broadcasts its output information to its neighbors only when the subsystem's local output error exceeds a specified threshold. The triggering condition is related to the topology of the underlying communication graph. We also provide analysis of the time intervals between two consecutive communication broadcasts (the inter-event time). Our analysis shows that the topology of the underlying communication graph plays an important role on the performance of the NCSs with event-driven communication. In [16], we study the quantized output synchronization problem of networked passive systems with event-driven communication, in which the data transmissions among networked agents are event-based and quantized measurements are exchanged among neighboring agents. We show that with the event-driven communication strategy proposed in [16], output synchronization errors of the networked passive systems are bounded by the quantization errors of the signals transmitted in the communication network.

## 4 Passivity and Symmetry

Symmetry, as a basic feature of shapes and graphs, appears in many real-world networks, such as the Internet and power grid, resulting from the process of tree-like or cyclic growing. Since symmetry is related to the concept of a high degree of repetitions or regularities, the study of symmetry has been appealing in many scientific areas, such as Lie groups in quantum mechanics and crystallography in chemistry. In the classical theory of dynamical systems, symmetry has also been extensively studied. For example, to simplify the analysis and synthesis of large-scale dynamic systems, it is always of interest to reduce the dynamics of a system into smaller symmetric subsystems, which potentially simplifies control, planning or estimation tasks. When dealing with multi-agent systems with various information constraints and protocols, under certain conditions such systems can be expressed as or decomposed into interconnections of lower dimensional systems, which may lead to better understanding of system properties such as stability and controllability. Then the existence of symmetry here means that the system dynamics are invariant under transformations of coordinates.

In our work, stability conditions for large-scale systems are derived by categorizing agents into symmetry groups and applying local control laws under limited interconnections with neighbors [21]. Particularly, stability for dissipative systems is considered. Dissipativity is a generalization of passivity where the energy supplied to the system can take different forms. Several properties of dynamical systems can be captured by varying the energy supply rate. When subsystems of a symmetric system are dissipative, overall stability properties can be studied. Conditions are derived for the maximum number of subsystems that may be added while preserving stability and these results may be used in the synthesis of large-scale systems with symmetric interconnections.

Let the dynamics of interconnected nonlinear distributed systems $\Sigma_0,\ \Sigma_1, \ldots, \Sigma_m$ be given by

$$\Sigma_i: \quad \begin{array}{l} \dot{x}_i = f_i(x_i) + g_i(x_i)u_i \\ y_i = h_i(x_i) \\ u_i = u_{ei} - \sum_{j=0}^{m} H_{ij}y_j \end{array}$$

where $i = 0, \cdots, m$, $u_i$ is the input to subsystem $i$, $y_i$ is its output, $u_{ei}$ is an external input, and the $H_{ij}$ are constant matrices. If we define $y = [y_1^T, \ldots, y_n^T]^T$, $\widetilde{H} = [H_{ij}]$, and define $u$, $u_e$ similarly, then the interconnected system can be represented by

$$u = u_e - \widetilde{H}y$$

Symmetries may be introduced into interconnected systems via identical dynamics of subsystems, as well as same characterizations of information structure. For instance, let the systems be interconnected with star-shaped symmetry. That is starting with the base system $\Sigma_0$, a group of systems $\Sigma_i$ are connected to it without interconnections among each other. Therefore let

$$\widetilde{H} = \begin{bmatrix} H & b & \ldots & b \\ c & h & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \ldots & h \end{bmatrix}$$

.

Theorem: Consider a $(Q, S, R) - dissipative$ system $\Sigma_0$ extended by $m$ star-shaped symmetric $(q, s, r) - dissipative$

subsystems $\Sigma_i$. The whole system is asymptotically stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(c^T rc + \beta(\hat{q} - b^T Rb)^{-1}\beta^T)}, \frac{\hat{q}}{b^T Rb}\right)$$

where

$$\begin{aligned}\hat{Q} &= -H^T RH + SH + H^T S^T - Q > 0\\ \hat{q} &= -h^T rh + sh + h^T s^T - q > 0\\ \beta &= Sb + c^T s^T - H^T Rb - c^T rh\end{aligned}$$

The above theorem shows that there exists an upper bound on the number of subsystems that can be added so to preserve stability of dissipative systems. Besides star-shaped symmetries, there are similar results for interconnections with cyclic symmetries.

When we consider passivity as a special case of dissipativity, passivity indices can be used for interconnections of agents to assess the level of passivity. Motivated by the interest of sufficient stability conditions in [21], passivity indices for both linear and nonlinear multi-agent systems with feedforward and feedback interconnections are derived with the distributed setup in [22]. For linear systems, the passivity indices are explicitly characterized, while the passivity indices in the nonlinear case are characterized by a set of matrix inequalities. We also focus on symmetric interconnections and specialize stability results to this case.

## 5 Conclusions

This paper summarizes a large body of research on the control of CPS related to the concepts of passivity, dissipativity, and symmetry. The work here focuses on key areas of CPS including networking and interconnecting systems that may change dynamically. Specific areas that are addressed include stability of interconnected passive switched systems that contain cyber and physical dynamics over delayed networks, stability of interconnected passive systems using an event-triggered scheme, and networking multi-agent agent passive systems over a structure that contains symmetries. Throughout this research, the energy concepts of passivity and dissipativity have been invaluable. These concepts will continue to be used in CPS as these areas develop.

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