Quantum Feedback Control

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Outline

Historical Perspective

'Technology seems to advance in waves. Small advances in science and technology accumulate slowly ... until a critical level...

'Woven into the rich fabric of technological history is an invisible thread that has a profound effect on each of these waves...

'This thread is the idea of feedback control.

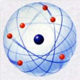
Dennis Bernstein, *History of Control*, 2002

Control Engineering Timeline

classical control | **modern control**







1788 | 1930 | 1960 | 2000

PID, Bode diagrams, gain and phase margins, Nyquist stability criteria, root locus, etc

optimal control, LQG, Kalman filtering, estimation, multivariable control, adaptive control, robust control, nonlinear control, stochastic control, systems biology, etc

quantum control

classical mechanics

quantum mechanics

Quantum Technology

Quantum technology is the application of quantum science to develop new technologies.

This was foreshadowed in a famous lecture:

1959: Richard Feynman, *Plenty of Room at the Bottom*

> *"What I want to talk about is the problem of manipulating and controlling things on a small scale."*

Key drivers for quantum technology:

- Miniaturization - quantum effects can dominate
  - Microelectronics - feature sizes of 10s nm (Moore's Law)
  - Nanotechnology - nano electromechanical devices have been made sizing 10s nm
- Exploitation of quantum resources
  - Quantum Information - (ideally) perfectly secure communications
  - Quantum Computing - algorithms with exponential speed-ups
  - Metrology - ultra-high precision measurements

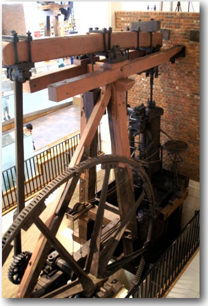
[Dowling-Milburn, 2003]

Quantum technology revolutions

[Dowling-Milburn, 2003]

- First: [QM used to understand what exists]
  - wave-particle duality
  - semiconductors
  - information age
- Second: [QM used to engineer new things]
  - artificial atoms
  - man-made quantum states
  - quantum engineering

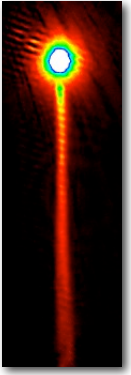
Quantum Control



[Boulton and Watt, 1788, London Science Museum]

Watt used a governor to control steam engines
- very macroscopic.

Now we want to control things at the quantum level
- e.g. atoms



[ANU atom laser, 2007, Canberra]

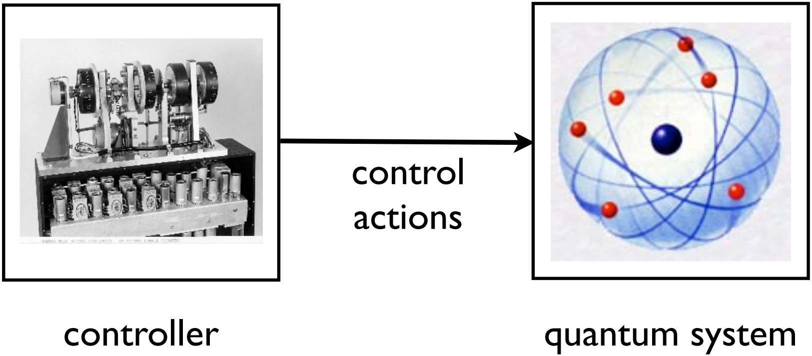
Quantum control concerns the control of physical systems whose behavior is dominated by the laws of *quantum mechanics*.

2003: Dowling and Milburn:

> *"The development of the general principles of quantum control theory is an essential task for a future quantum technology."*

Types of Quantum Control:

Open loop - control actions are predetermined, no feedback is involved.



On the controllability of quantum-mechanical systems

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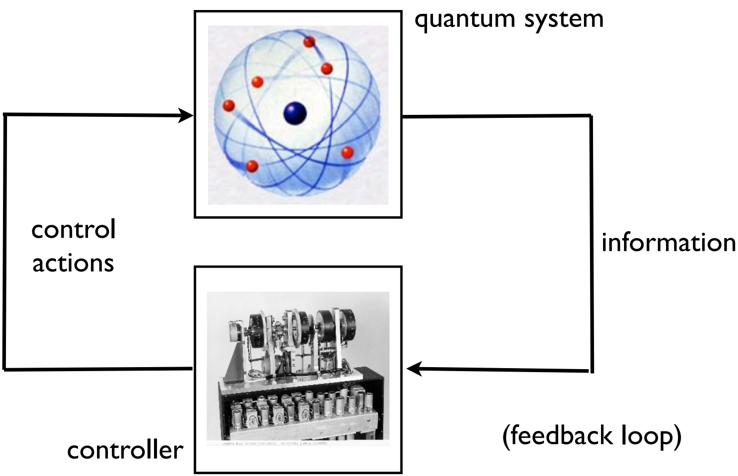
(Received 25 August 1981; accepted for publication 10 June 1983)

The systems-theoretic concept of controllability is elaborated for quantum-mechanical systems, sufficient conditions being sought under which the state vector ψ can be guided in time to a chosen point in the Hilbert space $\mathscr{H}$ of the system. The Schrödinger equation for a quantum object influenced by adjustable external fields provides a state-evolution equation which is linear in ψ and linear in the external controls (thus a bilinear control system). For such systems the existence of a dense analytic domain $\mathscr{D}_\omega$ in the sense of Nelson, together with the assumption that the Lie algebra associated with the system dynamics gives rise to a tangent space of constant finite dimension, permits the adaptation of the geometric approach developed for finite-dimensional bilinear and nonlinear control systems. Conditions are derived for global controllability on the intersection of $\mathscr{D}_\omega$ with a suitably defined finite-dimensional submanifold of the unit sphere $S_{\mathscr{H}}$ in $\mathscr{H}$. Several soluble examples are presented to illuminate the general theoretical results.

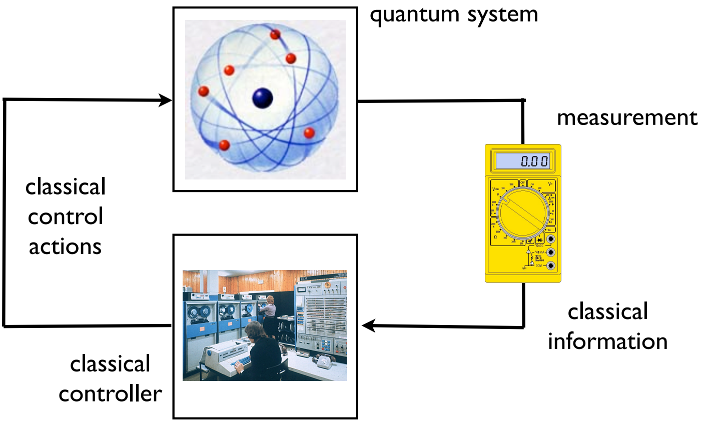
PACS numbers: 03.65.Bz, 02.20.Sv

Closed loop - control actions depend on information gained as the system is operating.



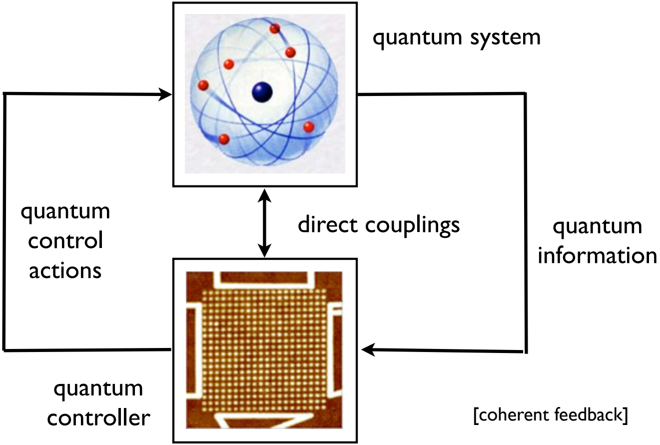
Types of Quantum Feedback: Using measurement

The classical measurement results are used by the controller (e.g. classical electronics) to provide a classical control signal.



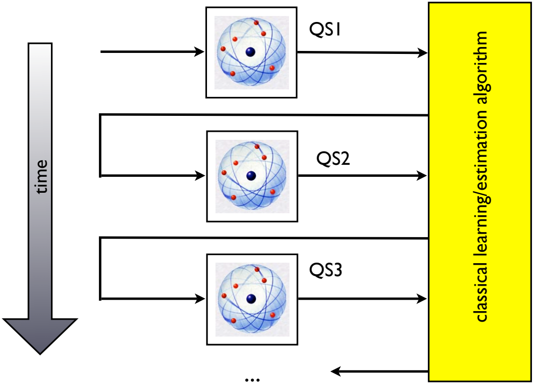
Not using measurement

The controller is also a quantum system, and feedback may involve a flow of quantum information, as well as direct couplings.



Iterative learning control

Same scheme for estimation from repeated identical experiments.
Fresh quantum system in each iteration.



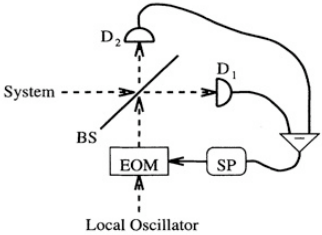
Examples of quantum feedback control

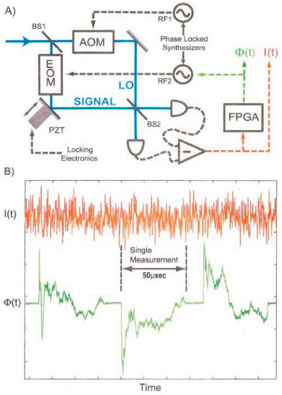
Adaptive phase measurement

[Wiseman 1995]

- the first quantum measurement feedback control experiment (a very important experimental test)

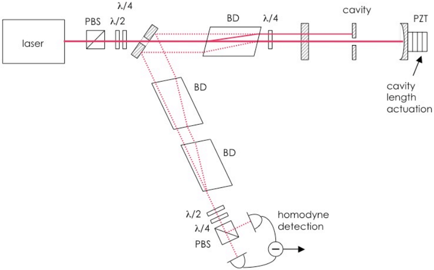
[Armen, Au, Stockton, Doherty, Mabuchi 2002]



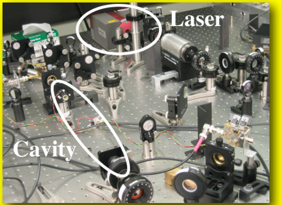


Laser-cavity locking

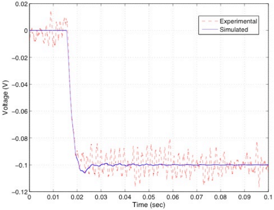
- quantum LQG measurement feedback control experiment



[Huntington, James, Petersen, Sayed Hassen, Heurs, 2009]



[Huntington Lab, ADFA@UNSW]

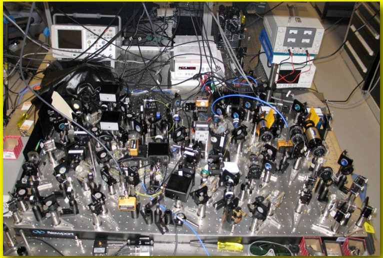


Coherent quantum feedback control

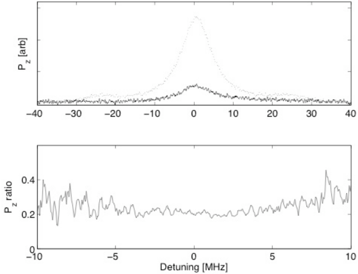
- quantum coherent feedback control experiment

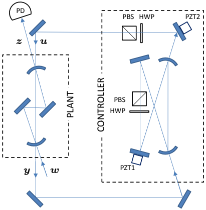
[Mabuchi, 2008]

[James, Nurdin, Petersen, 2008]



[Mabuchi Lab, Stanford]



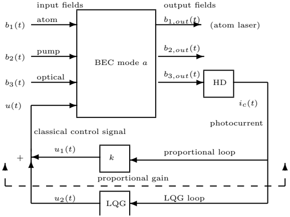


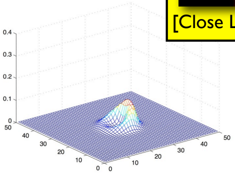
BECs and Atom Lasers

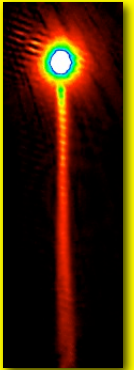
- measurement feedback control of atom laser coherence (theory) [Thomsen, Wiseman, 2002]

- stabilization via measurement feedback (theory) [Wilson, Carvalho, Hope, James 2007]

- multiloop measurement feedback (theory) [Yanagisawa, James 2008]






[Close Lab, ANU]

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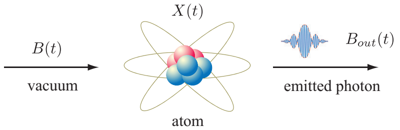
Some Quantum Mechanics

A little history

- Black body radiation (Plank)
- Photoelectric effect (Einstein)
- Atomic quantization (Bohr)
- Quantum probability (Born)
- Spontaneous and stimulated emission of light (Einstein)
- Matter waves (De Broglie)
- Matrix mechanics, uncertainty relation (Heisenberg)
- Wave functions (Schrodinger)
- Entanglement (EPR)
- Axiomatization, quantum probability (von Neumann)



Non-commuting observables

$$[Q, P] = QP - PQ = i\hbar\, I$$

Expectation

$$\langle Q \rangle = \int q|\psi(q,t)|^2 dq$$

Heisenberg uncertainty

$$\Delta Q \Delta P \geq \frac{1}{2}|\langle i[Q, P]\rangle| = \frac{\hbar}{2}$$

Schrodinger equation

$$i\hbar \frac{\partial \psi(q,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(q,t)}{\partial q^2} + V(q)\psi(q,t)$$

The Postulates of Quantum Mechanics

- Observables - self-adjoint operators on a Hilbert space $\mathfrak{H}$

$$X \equiv \begin{bmatrix} x_1 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & x_n \end{bmatrix}$$

Represent physical quantities

- position Q
- momentum P
- spin σ_z
- energy H

- States - allow for the calculation of probabilities and expectations of observables

$$\mathbb{E}[X] = \langle \psi | X | \psi \rangle, \text{ or } \mathbb{E}[X] = \mathrm{Tr}[\rho X].$$

  - Pure states $|\psi\rangle \in \mathfrak{H}$
    E.g.

$$\psi(x) = C \exp(-\frac{1}{4}x^2) \text{ or } \psi = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

  - Density operators ρ (self-adjoint, non-negative, trace one)
    E.g.

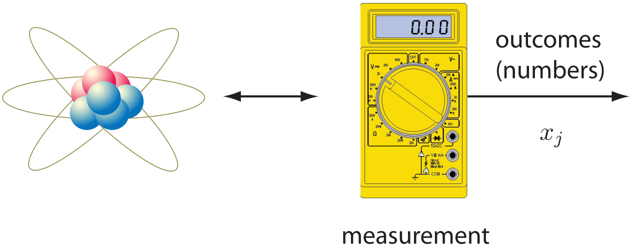
$$\rho = \frac{1}{2} \begin{bmatrix} 1 & c \\ c^* & 1 \end{bmatrix}$$

- Measurement - in a measurement, the numerical outcomes are the eigenvalues of observables.

$$X \equiv \begin{bmatrix} x_1 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & x_n \end{bmatrix}$$

Probability of outcomes:

$$\mathsf{Prob}[x_j] = \mathrm{Tr}[\rho P_{x_j}]$$



- Conditioning - if a measurement result x_j occurs, the state changes to

$$\begin{array}{rcl} \rho & \mapsto & \dfrac{P_{x_j}\rho P_{x_j}}{\mathrm{Tr}[\rho P_{x_j}]} \\ \text{before} & & \text{after} \\ \text{measurement} & & \text{measurement} \end{array}$$

This is known as the "projection postulate".

- Evolution - $U(t)$ unitary satisfies *Schrodinger equation*

$$i\hbar \frac{d}{dt} U(t) = H(t)U(t)$$

  - states [Schrodinger picture]

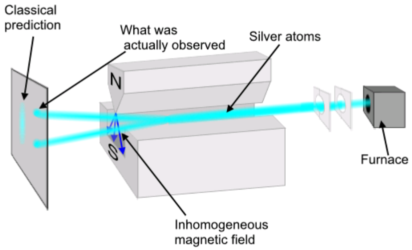
$$|\psi(t)\rangle = U(t)|\psi\rangle$$

$$\rho(t) = U(t)\rho U^*(t)$$

  - observables [Heisenberg picture]

$$X(t) = U(t)^* X U(t),$$

Example - Stern-Gerlach experiment



The observable representing spin in the z-direction is a 2×2 complex matrix

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Measurement values are

$$\{1, -1\}$$

which correspond to spin up and spin down, respectively.

Compatible and incompatible observables

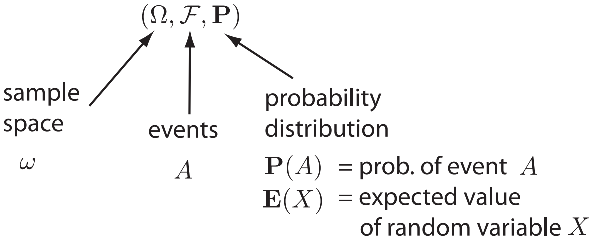
One of the key differences between classical and quantum mechanics concerns the ability or otherwise to simultaneously measure several physical quantities. In general it is not possible to exactly measure two or more physical quantities with perfect precision if the corresponding observables do not commute, and hence they are *incompatible*.

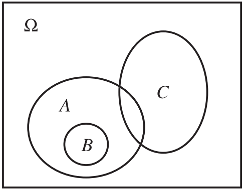
A consequence of this is lack of commutativity is the famous *Heisenberg uncertainty principle*.

We may think of quantum mechanics as the description of physical systems using a *non-commutative probability theory*.

Classical probability

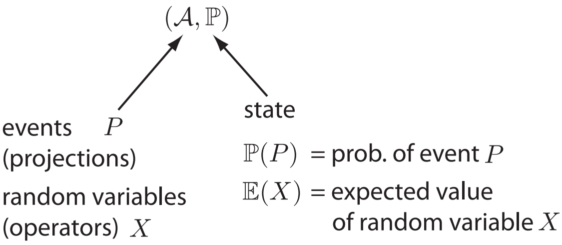
Classical physics is built on foundations of classical logic, which is closely related to classical probability.





Quantum probability

We may think of quantum mechanics as the description of physical systems using a *non-commutative probability theory*.



States may be defined using pure states $|\psi\rangle$ or density operators ρ:

$$\mathbb{E}[X] = \langle\psi|X|\psi\rangle, \text{ or } \mathbb{E}[X] = \mathrm{Tr}[\rho X].$$

Algebras $\mathcal{A}$ of events describe information in both classical and quantum probability.

The *spectral theorem* tells us that a commutative quantum probability space is equivalent to a classical probability space.

$$\underset{\text{commutative}}{(\mathscr{C}, \mathbb{P})} \longleftrightarrow (\Omega, \mathscr{F}, \mathbf{P})$$

This is the mathematics corresponding to the measurement postulate.

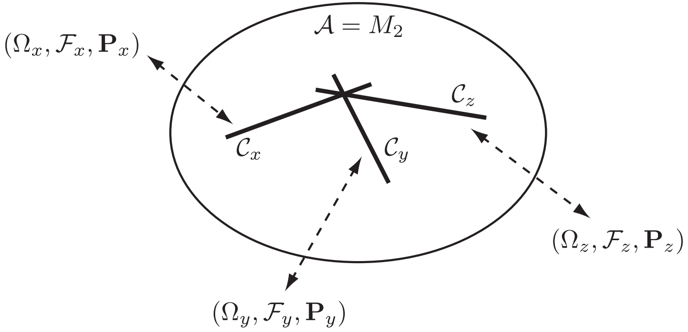
Example (spin)

Measurement in the x, y and z directions correspond to non-commuting observables

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and so are incompatible (cannot be simultaneously diagonalized).

These correspond to distinct commutative subspaces:

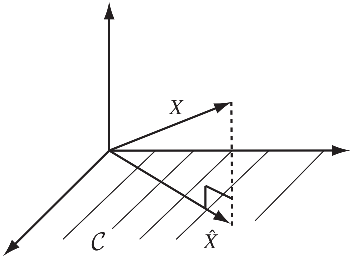


Conditional expectation

Let X commute with a commutative subspace $\mathcal{C}$. The conditional expectation

$$\hat{X} = \pi(X) = \mathbb{E}[X|\mathcal{C}]$$

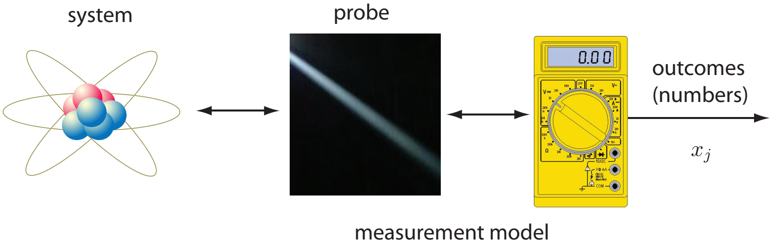
is the orthogonal projection of $X \in \mathcal{A}$ onto $\mathcal{C}$.



$\hat{X}$ is the minimum mean square estimate of X given $\mathcal{C}$.

By the spectral theorem, $\hat{X}$ is equivalent to a classical random variable.

Probe model for quantum measurement



Information about the system is transferred to the probe.

Quantum conditional expectation is well defined.

The von Neumann "projection postulate" is a special case.

In continuous time, this leads to *quantum filtering*.

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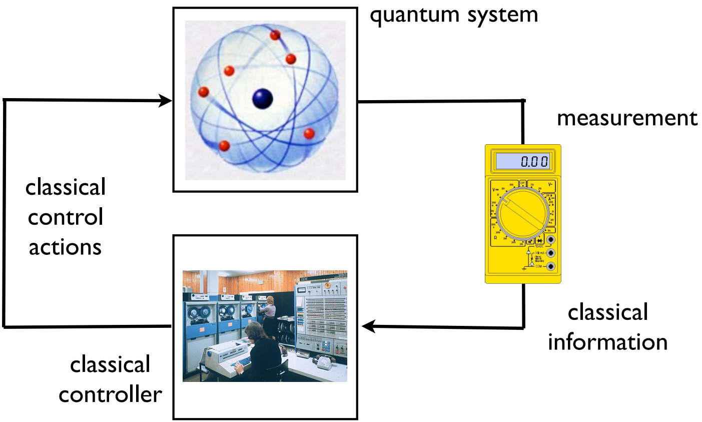
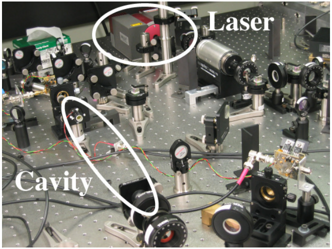
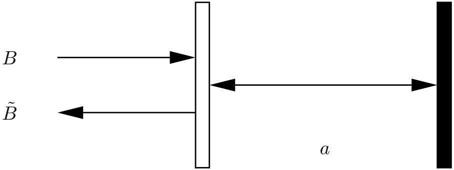
Measurement Feedback Quantum Control



Quantum systems with inputs and outputs





external free field with
input and output components

cavity mode

Schrodinger equation

$$dU(t) = \{LdB^*(t) - L^*dB(t) - (\frac{1}{2}L^*L + iH(u))dt\}U(t)$$

where $B(t)$ is a *quantum Wiener process*.

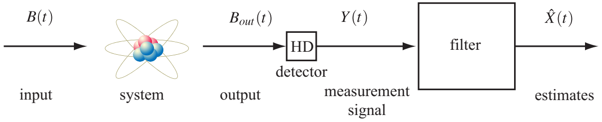
[Hudson-Parthasarathy (1984), Gardiner-Collett (1985)]

System operators X and output field $\tilde{B}(t)$ evolve in the Heisenberg picture:

$$X(t) = j_t(X) = U^*(t)(X \otimes I)U(t)$$
$$\tilde{B}(t) = U^*(t)(I \otimes B(t))U(t)$$

Measurement of the output field (e.g. amplitude quadrature observables)

$$Y(t) = \tilde{B}(t) + \tilde{B}^*(t)$$



Dynamics for $X(t) = j_t(X)$—a *quantum Markov process* (given u)—and output $Y(t)$:

$$\begin{aligned} dj_t(X) &= j_t(\mathcal{L}^{u(t)}(X))dt + dB^*(t)j_t([X, L]) + j_t([L^*, X])dB(t) \\ dY(t) &= j_t(L + L^*)dt + dB(t) + dB^*(t) \end{aligned}$$

where

$$\mathcal{L}^u(X) = -i[X, H] + \frac{1}{2}L^*[X, L] + \frac{1}{2}[L^*, X]L$$

Quantum conditional expectation

$$\pi_t(X) = \mathbb{E}[j_t(X) | Y(s), 0 \leq s \leq t]$$

Quantum filter [stochastic Schrodinger equation]

$$\begin{aligned} d\pi_t(X) &= \pi_t(\mathcal{L}^{u(t)}(X))dt \\ &\quad + (\pi_t(XL + L^* X) - \pi_t(X)\pi_t(L + L^*))(dY(t) - \pi_t(L + L^*)dt) \end{aligned}$$

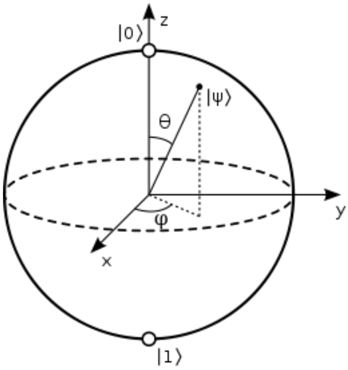
[Belavkin (1993), Carmichael (1993)]

Conditional density operator $\hat{\rho}(t)$ is defined by

$$\pi_t(X) = \mathrm{tr}[\hat{\rho}(t)X]$$

For a two-level spin system, we use Bloch sphere coordinates:

$$\hat{\rho}(t) = \frac{1}{2}(I + \hat{x}(t)\sigma_x + \hat{y}(t)\sigma_y + \hat{z}(t)\sigma_z),$$



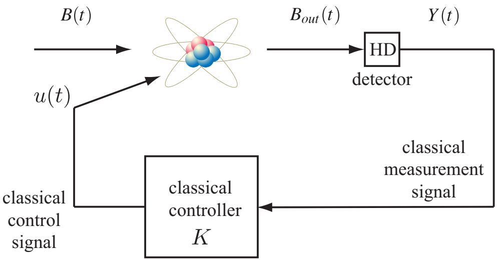
The quantum filter is then given by

$$\begin{aligned}
d\hat{x}(t) &= (-\omega\hat{y}(t) - \frac{\kappa}{2}\hat{x}(t))dt \\
&\qquad +\sqrt{\kappa}\,(1+\hat{z}(t)-\hat{x}^2(t))dW(t) \\
d\hat{y}(t) &= (\omega\hat{x}(t) - \frac{\kappa}{2}\hat{y}(t))dt \\
&\qquad +\sqrt{\kappa}\,\hat{x}(t)\hat{y}(t)dW(t), \\
d\hat{z}(t) &= (-\kappa\hat{z}(t) - \kappa)dt \\
&\qquad -\sqrt{\kappa}\;\hat{x}(t)(1+\hat{x}(t))dW(t).
\end{aligned}$$

The innovations process is given by $dW(t) = dY(t) - \hat{x}(t)dt$.

The quantum filter is driven by the measurement signal $Y(t)$ and can be used for measurement feedback control.

Quantum optimal control (measurement feedback)



Problem: minimize

$$J(K) = \mathbb{E}[\int_0^T C_1(s)ds + C_2(T)]$$

with respect to the controller K, where

$$C_1(u) = \begin{pmatrix} \frac{c_1}{2}|u|^2 & 0 \\ 0 & 1 + \frac{c_1}{2}|u|^2 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0 & 0 \\ 0 & c_2 \end{pmatrix}$$

Using properties of conditional expectation, the cost function can be expressed in terms of the quantum conditional expectation

$$\begin{aligned} J(K) &= \mathbb{E}[\int_0^T \pi_s(C_1(u(s)))ds + \pi_T(C_2)] \\ &= \mathbf{E}[\frac{1}{2}\int_0^T (1 - \hat{z}(t) + c_1|u(t)|^2)dt + \frac{c_2}{2}(1 - \hat{z}(T))]. \end{aligned}$$

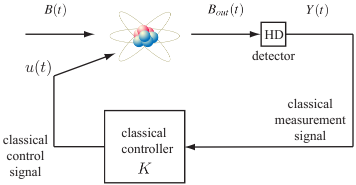
This converts a quantum measurement feedback problem to a classical full information control problem that can be solved using standard classical optimal control methods.

Optimal measurement feedback controller:

$$
\begin{aligned}
d\pi_t(X) &= \pi_t(\mathcal{L}^{u(t)}(X))dt \\
&\quad +(\pi_t(XL + L^*X) - \pi_t(X)\pi_t(L + L^*))(dY(t) - \pi_t(L + L^*)dt) \\
u(t) &= \mathbf{u}^\star(\pi_t, t)
\end{aligned}
$$

Note the *separation structure*:

- estimation part (filter, the equation for π_t)
- control part ($\mathbf{u}^\star$)



[Belavkin (1983), Doherty-Jacobs (1999), James (2005)]

Quantum risk-sensitive optimal control (measurement feedback)

Let $R(t)$ be defined by

$$\frac{dR(t)}{dt} = \frac{\mu}{2} C_1(t) R(t), \quad R(0) = I.$$

Risk-sensitive cost (average of exponential cost)

$$J^\mu(K) = \mathbb{E}[R^*(T) e^{\mu C_2(T)} R(T)].$$

[James (2004, 2005)]

Can define an *information state* $\sigma_i^{\mu}(X)$ so that

$$J^{\mu}(K) = \mathbb{E}^0[\sigma_T^{\mu}(e^{\mu C_2})]$$

Optimal risk-sensitive measurement feedback controller:

$$\begin{aligned} d\sigma_t^{\mu}(X) &= \sigma_t^{\mu}((\mathcal{L}^{u(t)} + \mu C_1(u(t)))X))dt + \sigma_t^{\mu}(L + L^*)dY(t) \\ u(t) &= \mathbf{u}^{\mu\star}(\sigma_t^{\mu}, t) \end{aligned}$$

Modified stochastic Schrodinger equation: [James (2004, 2005)]

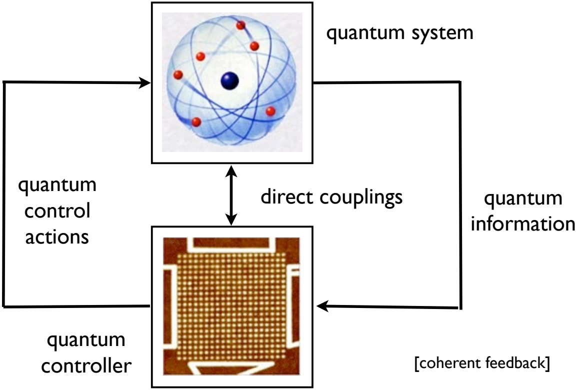
- knowledge
- purpose

Schrodinger's cat

The study of quantum feedback control has practical and fundamental value.

Quantum Feedback Networks (QFN)

- Quantum information is lost when measurements are made.
- Coherent feedback loops need not involve measurements, and so allow for the flow of quantum information. The controller is another quantum system.

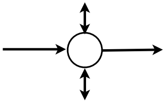


Modern electrical circuit theory...

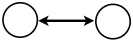
- builds on laws of physics (Maxwell, Faraday, Ohm,...)
- evolved to meet the needs of electrical system designers (Thevenin, Kirchhoff,...)
- includes
  - device models
  - rules for interconnection
  - methods for analysis, simplification, and synthesis

In a quantum feedback network (QFN)

- The nodes are open quantum systems

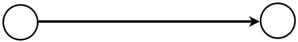


- The branches are
  - direct physical couplings



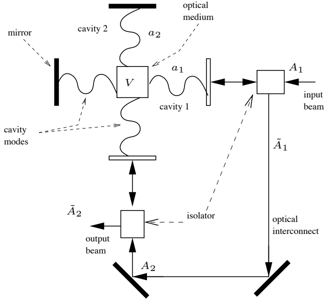
or

  - indirect couplings using freely travelling *quantum fields* serving as *'quantum wires'*.

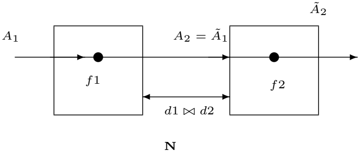


[Yurke-Denker (1984), Carmichael (1993), Gardiner (1993), Wiseman-Milburn (1994), Yanagisawa-Kimura (2001), Gough-James (2008,2010)]

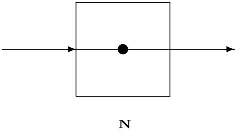
A quantum feedback network theory



[Wiseman-Milburn, 1994]



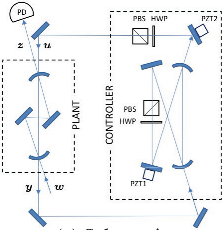
$$\begin{aligned}
\mathbf{N} &= \mathbf{G}_1 \wedge \mathbf{G}_2 = (\mathbf{G}_{f2} \triangleleft \mathbf{G}_{f1}) \boxplus (\mathbf{G}_{d1} \bowtie \mathbf{G}_{d2}) \\
&= (1, \sqrt{\gamma_2}\, a_2 + \sqrt{\gamma_1}\, a_1, _, \\
&\quad \Delta_1 a_1^* a_1 + \Delta_2 a_2^* a_2 - ig(a_2 a_1^* - a_2^* a_1))
\end{aligned}$$



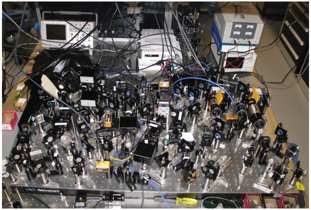
[Gough and James (2008, 2010)]

- According to Mabuchi 2008:

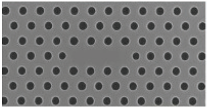
  "... a <u>genuinely</u> new category of control-theoretic problems as it encompasses non-commutative signals and quantum-dynamical transformations thereof" and "... relatively little is yet known about the systematic control theory of coherent feedback".



(a) Schematic



(b) Quantum optics experiment



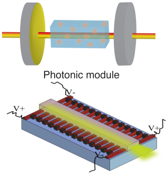
(c) Photonic crystal cavity

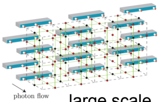
[James, Nurdin & Petersen (2008)]

[Mabuchi (2008)]

Future Directions

Quantum Technology

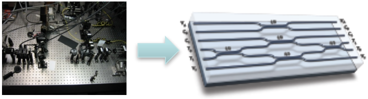




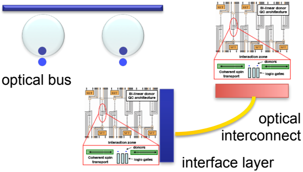
large scale entanglement

waveguided memory

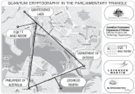
- quantum computers
- quantum communications
- quantum metrology
- other quantum technologies

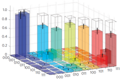


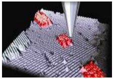
On-chip circuits and active elements

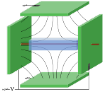


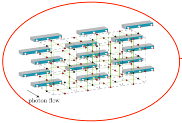
Quantum Communication	Optical Quantum Computation	Silicon Quantum Computation	Quantum Resources & Integration
Quantum crypto in the Parliamentary Triangle Quantum repeater	Multi-qubit gates Quantum circuits Si quantum photonics	Multi-qubit gates Spin qubit transport Optical-silicon interface	Quantum memory Photon sources Photon detectors



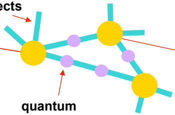




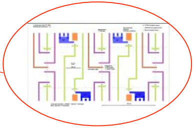




photonic interconnects



quantum repeaters



photonic quantum processor

silicon quantum processor

Ultra-Secure Global Quantum Network

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Role of Systems and Control Researchers

- To develop holistic, systems-oriented concepts, theories, methods and tools based on quantum mechanics (in place of classical mechanics)
- Participate in the development of quantum technologies

...quantum mechanics as a science has matured
...quantum engineering as a technology is now emerging...

[Dowling-Milburn, 2003]

Quantum feedback control

- quantum feedback network *modelling* and *analysis*
  - large scale quantum networks
  - dynamical behavior
  - quantum coherence and entanglement
  - quantum - classical systems
- fully quantum coherent feedback *design*
  - non-commutative *variables* and *signals*
  - design by interconnection
  - optimization

'...the most fruitful areas for growth of sciences were those ... between various established fields.'

'It is these boundary regions of science which offer the richest opportunities to the qualified investigator.'

Norbert Wiener, *Cybernetics*, 1948

谢谢