Control over Communication Networks
- Trends and Challenges

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Why Networked Control?

- Situation reactive multi-junction traffic lights control
- Real-time vehicle routing and road pricing for congestion control
- Sensor networks for indoor/outdoor environments
- Multi-level control for energy efficiency
- Diagnostics, control and safety networks
- Wireless manufacturing

- Smart metering / real-time pricing
- Renewable energy
- Smart power networks

- D2H2
- Elderly care

- Cooperative detection/tracking
- Cooperative actions
Basic Features of Networked Control Systems

NCS is a control system wherein the control loops are closed through a real-time network.

- Devices are spatially distributed and may operate in an asynchronous but coordinated manner.
- Multi control loops are closed over a network of hierarchical multi-layer control structure connected via digital networks.
- Different types of networks (CAN based such as DeviceNet, SCADA, Ethernet based networks) for control, diagnostics and safety.

Image Source: SmartFactory, Germany
Today’s industrial communication architecture

Centralized control system with low-level loops closed over wired network
Towards Wireless Sensor/Actuator Networks

Reduced wiring and cost.

Point-to-Point Connections → Wired Fieldbuses → Wireless Networks

Enhanced flexibility, efficiency, etc.

Main challenges for wireless control networks:

- Existing wireless networks are designed for applications such as data comm (wireless LAN), voice comm (mobile phone), sensor networks (Zigbee)
- Determinism of data transmission, interferences, fading and time-varying throughput, reliability and security, etc
- Conflict between contention based communications and time based control
- Lack of standards for industrial control and rigorous control design methods
Example of Existing Wireless Control Networks

Wireless HART

- To support wide range of applications from monitoring to closed loop control
- Mesh topology
- Mixed TDMA and CSMA protocol

- Time delay: average 20 msec per hop
- Accuracy of time synchronization: 1 msec
- Testing results show that the Wireless HART allows for secure, highly reliable and low latency control
- Provide built-in 99.9% end-to-end reliability in all industrial environments.

Others include WIA-PA (中国科学院沈阳自动化所) and ISA100.
Two paradigm shifts:

- Network must now be explicitly considered in feedback system
- Renewed emphasis on distributed control due to wide availability of low cost processing and comm devices.
Research Issues in NCS

- Sensor sampling strategies (periodic, event-driven sampling, or random sampling)
- Sensor data processing
- Communication strategies such as packet generation and scheduling

TDMA = Time division multiple access, CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance
Research Issues in NCS

- Network design for control, e.g., network topology, MAC protocol design (e.g., TDMA, CSMA/CA, or hybrid), etc.

TDMA = Time division multiple access, CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance
Research Issues in NCS

- Communication constrained control design
- Communication and control co-design
- Robustness

TDMA = Time division multiple access, CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance
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- Why networked control
- Basic features of networked control systems
- Shannon channel capacity and channel capacity in control
- Stabilization over fading channels
  - State feedback
  - Output feedback w/o channel feedback
- Network topology for multi-agent consensus
- Conclusions

The focus is on minimal information and information flow for control
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Shannon Channel Capacity

- **Channel capacity**: Highest rate in bits per channel use at which information can be transmitted reliably over channel.

- **Shannon channel capacity**:

  \[
  C = \max_{p(x)} I(X, Y) = \max_{p(x)} (H(X) - H(X | Y))
  \]

  ✓ Shannon’s capacity is a universal upper bound for all comm. schemes.
  ✓ Shannon capacity may be achieved by long LDP code which results in significant delay.
  ✓ If code length is limited, a scaled capacity can be achieved. Compromise between delay and capacity is to be made.
  ✓ There is no constraint on the coding complexity.

- **Zero-error capacity** $C_0$: zero probability of estimation error
**Configuration of Communication System**

- **AWGN Channel** *(satellite comm, air-to-air, optical comm)*
  - $n(t)$ is white Gaussian with variance $\sigma_n^2$.
  - Power of channel input is bounded by $P$.
  - Signal-to-noise ratio (SNR): $\gamma = P / \sigma_n^2$. (constant)
  - $C = \frac{1}{2} \log(1 + \gamma)$

- **Fading Channel** *(urban, underwater comm)*
  - $n(t)$ is white Gaussian with variance $\sigma_n^2$.
  - Power of channel input is bounded by $P$.
  - Channel side information: $\xi(t)$.
  - Instantaneous SNR: $\gamma = \frac{P\xi^2(t)}{\sigma_n^2}$. (time-varying)
Channel Capacity of Fading Channel

If only the distribution of $\gamma$ is known at the transmitter and receiver, the achievable channel capacity remains open in general.

With CSI at receiver, Shannon channel capacity:

$$C = \frac{1}{2} \int_0^\infty \log_2 (1 + \gamma)p(\gamma)d\gamma$$

Based on Jensen’s inequality,

$$C = \frac{1}{2} E[\log_2 (1 + \gamma)] \leq \frac{1}{2} \log_2 (1 + E[\gamma])$$

i.e., capacity is upper bounded by the capacity of AWGN channel with the average SNR.
Channel capacity in control

- What is the required network resource for control?
- How various network conditions affect the capability of effective information transmission for control?
- How to allocate network resources?
- How to design coder/decoder/controller to achieve control objectives?
Mahler Measure

- **Mahler measure (Kurt Mahler, 1960)**
  \[ M(A) = \prod_{|\lambda_i(A)| \geq 1} |\lambda_i(A)| \]

- **Mahler measure is related to the minimal energy control:**
  \[ \inf_{K: A + BK \text{ stable}} \| T_{du} \|_2^2 = M(A)^2 \]

- **Mahler measure is also related to optimal H-infinity performance (Gu and Qiu, 2008):**
  \[ \inf_{K: A + BK \text{ stable}} \| T_{du} \|_\infty = M(A) \]

We focus on its relation to constraint of information flow and patterns of information flow for control.
Channel Capacity in Control

Stabilization over \textbf{noiseless} discrete channel (Nair and Evans, SICON, 2004)

✓ Perfect channel: no transmission error, no packet drop, no delay
✓ What determines the minimal data rate for stabilization?

For any $x_0$ and $w_k$ satisfying some mild conditions, the system is M.S. stabilizable

$$R > \log_2 M(A)$$

Related work: Wong and Brockett (1995, 1997); Baillieul (1999); Delchamps (1990); Brockett and Liberzon (2000); Elia and Mitter (2001); Fu and Xie (2005); etc
Stabilization over digital erasure channel

- **Channel with i.i.i. Packet drop**
  - Channel packet drop characterized by an i.i.d. process with dropout rate $p$
  - Question is that can we exactly quantify the additional bit rate required?

Maximum pack drop out rate for MS stabilizability under no constraint on data rate (Sinopoli et al., 2004)

Single input system is M.S. stabilizable if

\[ p < \frac{1}{M(A)^2} \]

\[ R > \log_2 M(A) + \frac{1}{2} \log_2 \frac{1-p}{1-pM(A)^2} \]
Channel with Markovian Packet drop (You and Xie, TAC11)

- Channel conditions are usually correlated
- Use Markov packet drop to characterize channel correlations
- Assume that there exists a feedback channel
- Can we exactly quantify the additional amount of information required?
- Packet drop correlations make the problem much more complicated

Scalar system is M.S. stabilizable

\[ q > 1 - \frac{1}{|\lambda|^2}; \]

\[ R > \log_2 |\lambda| + \frac{1}{2} \log_2 \frac{(1-p)+|\lambda|^2 (p+q-1)}{1-(1-q)|\lambda|^2}. \]
Stabilization over AWGN Channel

- $n$ is zero mean, Gaussian white and $\nu$ is zero mean, stationary.
- **Channel Capacity:**
  \[
  C_c = C = \frac{1}{2} \log_2 \left(1 + \text{SNR}\right); \quad \text{SNR} = \frac{P}{\sigma_n^2}
  \]
  (Braslavsky, Middleton and Freudenberg, TAC 2007)

  The system is M.S. stabilizable $\iff C_c > \log_2 M(A)$
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**Channel Model**

- **Stochastic multiplicative model (fading)**

\[ u = \xi v \]

- Real fading is often used as the phase component can be estimated and compensated for at the receiver.
- \( \xi \) is white with mean \( \mu_\xi \) and variance \( \sigma_\xi^2 \).
- (Elia, SCL, 2005) Mean Square Capacity:

\[
C_c = C_{MS} = \frac{1}{2} \log_2 \left( 1 + \frac{\mu_\xi^2}{\sigma_\xi^2} \right).
\]

- Different from capacity defined in information theory.
- Examples: Rayleigh and Nakagami fading, packet drop.
- Deterministic case: logarithmic quantization (Fu and Xie, 2005)
Plant $G(z)$: strictly proper with realization $(A,B,C)$, where $A$ is unstable, $(A,B,C)$ is stabilizable & detectable.

- Design parameter: $K(z)$
- Fading channel(s): $\bar{h}(t) = \xi(t)\bar{g}(t)$.
- Single (series transmission) or multiple (parallel transmission) channels
- Channel feedback, pre and post channel processing
Stabilization over Single Transmission Channel

- If all input signals are sent over the same channel one by one whose fading is constant within one time step,
  \[ \xi_i(t) = \xi_j(t), \forall i, j. \]

- Capacity:
  \[ C_{MS,1} = \frac{1}{2} \log_2 \left( 1 + \frac{\mu_{\xi_i}^2}{\sigma_{\xi_i}^2} \right). \]

Theorem: The minimal overall capacity for stabilizability is

\[ C_{MS,1} > \log_2 \nu \]

\[ \nu = \begin{cases} 
M(A) & m = 1 \\
\rho(A) & m = n \\
\inf_{S>0,Y} \sqrt{g_1} & m \neq n 
\end{cases} \]

\[
\begin{bmatrix}
-S & (AS + BY)' & (\sqrt{\frac{1}{g_1-1}}BY)' \\
AS + BY & -S & 0 \\
\sqrt{\frac{1}{g_1-1}}BY & 0 & -S
\end{bmatrix} < 0
\]
Stabilization over Multiple Fading Channels

- Multiple fading channels: \( \xi(t) = \text{diag}(\xi_1(t), \xi_2(t), \ldots, \xi_m(t)) \).

- \( \xi_i(t), i = 1, 2, \ldots, m \) are scalar-valued white noise processes with

\[
E\{\xi_i(t)\} = \mu_i \neq 0, \quad E\{(\xi_i(t) - \mu_i)(\xi_j(t) - \mu_j)\} = \sigma_{ij}
\]

- In general MIMO comm., the numbers of transmitters and receivers could be different.

- Fading between channels could be uncorrelated if an orthogonal access scheme is adopted (Goldsmith, 2005).

- Overall capacity:

\[
C_{MS} = \sum_{i=1}^{m} C_{MSi} = \sum_{i=1}^{m} \frac{1}{2} \log_2 \left( 1 + \frac{\mu_{\xi_i}^2}{\sigma_{\xi_i}^2} \right).
\]

We focus on independent fading channel
Stabilizability via State Feedback over Independent Fading Channel

M.S. stabilization with independent fading is equivalent to

\[ \inf_{\Theta, K} \left\| \Theta^{-1} T \Theta \right\|_{MS}^2 < 1. \]

where \( \left\| T \right\|_{MS}^2 \) is the maximal row sum of \( \left\| T_{ij} \right\|_2^2 \).

For MI cases, the search of the minimal overall capacity for stabilization is non-convex and has no explicit solution in general.
Communications & Control Co-design

Communication resource (e.g., overall capacity) is allocatable towards control objectives (e.g., stabilization)

Question: What is the minimal resource requirement and how to distribute it?

Assumption 1: The fading channels are independent, and the overall capacity $C_{MS} = \sum_{i=1}^{m} C_{MS,i}$ can be allocated among the fading channels.

Theorem: Under Assumption 1, the minimal overall capacity for stabilizability is

$$C_{MS} > \log_2 M(A)$$
Stabilizability via Dynamic Output Feedback (without channel feedback)
Theorem: Minimal capacity for stabilizability is

\[ C_{MS} > \frac{1}{2} \log_2 \left\{ M(G)^2 + \eta(G) + \varphi(G) \right\} \]

where \( \eta(G) \) and \( \varphi(G) \) are nonnegative and are functions of the anti-stable poles, NMP zeros and relative degree of \( G(z) \).

Note:

\[ \frac{1}{2} \log_2 \left\{ M(G)^2 + \eta(G) + \varphi(G) \right\} \geq \log_2 M(G) \]

- Equality holds only when \( G(z) \) is MP with relative degree one.
- The requirement on capacity in the output feedback case is generally more stringent than that in the state feedback.
There always exists a unimodular matrix $U_N$ such that

$$U_N G = \begin{bmatrix} L_G \\ 0 \end{bmatrix}$$

where $L_G$ is either lower or upper triangular.

Assume $\begin{bmatrix} L(G) \end{bmatrix}_{ii}$ has no or has distinct NMP zeros.

**Theorem:** Under Assumption 1, necessary overall capacity for stabilizability is

$$C_{MS} > \log_2 M(G),$$

and sufficient overall capacity for stabilizability is

$$C_{MS} > \frac{1}{2} \sum_{i=1}^{q} \log_2 \left\{ M(\begin{bmatrix} L_G \end{bmatrix}_{ii})^2 + \eta(\begin{bmatrix} L_G \end{bmatrix}_{ii}) + \phi(\begin{bmatrix} L_G \end{bmatrix}_{ii}) \right\}.$$
Dynamic Output Feedback with Coding & Channel Feedback

Theorem: Under Assumptions 1, the minimal overall capacity for stabilizability is

\[ C_{MS} > \log_2 M(G). \]

- The scaling factor and the decoder are redundant.
- The channel feedback plays a key role in eliminating the limitation induced by NMP zeros and high relative degree.
Example: Vehicle Platooning

\[ x_i \quad e_i \quad \delta_i \quad x_{i-1} \]

- Follower
- Follower
- Leader

Remote Controller
Vehicle model: \( X_i(z) = G_i(z)U_i(z) + \frac{zx_i(0)}{z-1} \).

Vehicle separation: \( e_i(t) \equiv x_i(t) - x_{i-1}(t) + \tau \), and \( \tau \) is the target separation.

Fading channel: \( v_i(t) = \xi_i(t)e_i(t) \).

Define \( E(z) = [E_1(z) \ E_2(z) \cdots E_p(z)]' \), \( U(z) = [U_1(z) \ U_2(z) \cdots U_p(z)]' \), then
\[
E(z) = G(z)U(z) - [1 \ 0 \ \cdots \ 0]'X_0(z) \quad \text{with}
\]
\[
G(z) = \begin{bmatrix}
G_1(z) & 0 & 0 & \cdots & 0 \\
-G_1(z) & G_2(z) & 0 & \cdots & 0 \\
0 & -G_2(z) & G_3(z) & \ddots & 0 \\
& \vdots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & -G_{p-1}(z) & G_p(z)
\end{bmatrix}.
\]

Note: 
\( G(z) \) has a lower triangular structure.
Simulation Result

- Number of vehicles: 5
- Dynamics of the followers:
  \[ G_i(z) = \frac{1}{(z-1)^2}, \quad i = 1, 2, 3, 4. \]
- Fading model:
  \[ \xi_i(t) = \Omega_i(t)\gamma_i(t), \quad i = 1, 2, 3, 4. \]
  - \( \Omega_i(t) \) is the packet-loss process with loss rate 0.2
  - \( \gamma_i(t) \) is Nakagami distributed with mean power gain 2 and severity of fading 2

The trajectories of vehicles in the platoon with sufficient capacity.

![Graph showing trajectories of vehicles over time](image-url)
Channel Fading and SNR Constraint Co-exist

**Channel Model One:**
\[ u = \xi v + n \]

- Suitable for general analog fading

**Channel Model Two:**
\[ u = \xi (v + n) \]

- Suitable for digital erasure

- \( n \) is zero mean, white with variance \( \sigma_n^2 \); the power of \( v \) is bounded by \( P \). Further let \( \gamma = P/\sigma_n^2 \).

- \( \xi \) is white with mean \( \mu_\xi \) and variance \( \sigma_\xi^2 \).

A necessary and sufficient condition for stabilizability is

\[
\begin{align*}
\frac{\mu_\xi^2 \gamma}{\sigma_\xi^2 \gamma + 1} + 1 > M(A)^2,
\end{align*}
\]

for channel model one;

\[
\begin{align*}
\frac{\mu_\xi^2 \gamma}{\sigma_\xi^2 \gamma + \mu_\xi^2 + \sigma_\xi^2} + 1 > M(A)^2,
\end{align*}
\]

for channel model two.

- The scaling factor is essential in satisfying the power constraint.
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Cooperative Control and Consensus

✓ Networked agents.
✓ Each agent can only talk to its neighboring agents.
✓ Information may propagate across the network.
✓ The goal is to carry out coordinated tasks.
✓ We are interested in network topology (information flow) for distributed consensus (simultaneous stabilization)
Graph Representation

- Both data rate and network topology are important for cooperative control performance.
- Weighted digraph $G = \{V, E, A\}$
- Neighborhood $N_i = \{j \in V \mid (j, i) \in E\}$
- Weighted adjacency matrix $A = [a_{ij}]_{N \times N}$, $a_{ij} \geq 0$, $a_{ij} > 0 \Leftrightarrow j \in N_i$
- Degree
  \[
  \deg_{in}(i) = \sum_{j=1}^{N} a_{ij}, \quad \deg_{out}(i) = \sum_{j=1}^{N} a_{ji}
  \]
- Degree matrix $D = \text{diag}(\deg_{in}(1), \ldots, \deg_{in}(N))$
- Laplacian matrix $L = D - A$
  \[
  \lambda_i(L), \ i = 1, 2, \ldots, N \quad \text{Crucial for MAS cooperation}
  \]
Distributed consensus

- Identical linear agent dynamic:
  \[ X_i(k + 1) = AX_i(k) + Bu_i(k), k = 0, 1, \ldots, N \]

- **Static** distributed protocol with perfect local states
  \[ u_i(k) = K \sum_{j=1}^{N} a_{ij} (X_j(k) - X_i(k)) \]

- Consensusable if for any finite \( X_i(0) \), there exists a control gain \( K \) such that:
  \[ \lim_{k \to \infty} \| X_i(k) - X_j(k) \| = 0, \forall i, j. \]

- Assuming no constraint on data rate, what are conditions on the network topology and the agent dynamics so that consensus can be achieved?
Consensusability via state feedback

- Consensus can be shown to be equivalent to simultaneous stabilization of $A + \lambda_i BK$, $i = 2, 3, \ldots, N$.
- Like channel capacity, network topology needs to meet certain condition related to Mahler measure of agent.

**Theorem:** Assume A has all poles or outside unit circle. The multi-agent system is consensusable if and only if

- $(A, B)$ is a stabilizable pair.
- $M(A) < \frac{1 + \lambda_2 / \lambda_n}{1 - \lambda_2 / \lambda_n}$, Network synchronizability

**Protocol gain:**

$$K = \frac{2}{\lambda_2 + \lambda_n} \frac{B^T PA}{B^T PB}$$

P is a positive definite solution to

$$P - A^T PA + \frac{A^T PBB^T PA}{B^T PB} > 0.$$
Key observations:

- Mahler measure imposes a lower bound on network synchronizability for distributed consensus, similar to its constraint on channel capacity in single loop control.

- For an unstable agent, consensusability implies $\lambda_2 > 0$, i.e. the graph must be connected.

- For the complete network, $\lambda_2 = \lambda_N = 1$, consensusability can be achieved for any unstable system.

- The protocol requires global knowledge on network topology, what if the information is unavailable?

- What if there is also a constraint on data rate?
Consensus with Limited Data Rate

(Li, Fu, Xie and Zhang, TAC, 2010; Li and Xie; Automatica, 2011)

\[ G = \{V, E, A\} \]

States are real-valued, but only finite bits of information are transmitted at each time-step

\[ x_i(t) \xrightarrow{\text{encoding}} x_{ij}(t) \xrightarrow{\text{transmission}} \hat{x}_{ij}(t) \xrightarrow{\text{decoding}} x_j(t) \]
Consensus vs Data Rate

- **Problem 1:** How many bits does each pair of neighbors need to exchange at each time step to achieve consensus of the whole network?

- **Problem 2:** What is the relationship between the consensus convergence rate and bit rate?
Key techniques

- Quantized innovation for bit rate efficiency

- Self-compensation to achieve exact consensus

\[ u_i(t) = \sum_{j \in N_i} a_{ij} (\hat{x}_{ji}(t) - \xi_i(t)), \quad t = 0, 1, \ldots \]
Scalar multi-agent systems

For a connected network, average-consensus can be achieved with exponential convergence rate based on a single-bit exchange between each pair of neighbors at each time step.

Asymptotic convergence rate:

$$\lim_{N \to \infty} \inf_{(h, \gamma) \in \Omega_K} \gamma \exp \left\{ - \frac{KQ_N^2}{2\sqrt{N}} \right\} = 1$$

$$Q_N = \frac{\lambda_2}{\lambda_N}$$

Synchronizability

Barahona & Pecora
PRL 2002

Donetti et al.
PRL 2005
Networked control simulation platform

- Aim to simulate and visualize control performance with incorporation of comm network for data transmissions
- OMNet++ can simulate data transmissions under specified environment
- UNREAL engine enables visualizing control system behavior
Integrated simulation platform

- Test Omnet
- Communicator
Integrated Simulation of UAV Formation
Cooperative Control of Multi-robot System
Cooperative Control of Multi-robot System
Target Tracking with Sensor Network
Concluding Remarks

- There has been much work on interplay between stability theory and information theory for single loop systems.
- Multi-loop control over networks are of practical importance.
- More research is needed for performance control over realistic communication channels and associated robustness.
- Network design for control and communication co-design are interesting research problems.
- Complex networked systems remain challenging.